

Spring Block 2

Fractions B

Small steps

Step 1

Multiply a unit fraction by an integer

Step 2

Multiply a non-unit fraction by an integer

Step 3

Multiply a mixed number by an integer

Step 4

Calculate a fraction of a quantity

Step 5

Fraction of an amount

Step 6

Find the whole

Step 7

Use fractions as operators



Multiply a unit fraction by an integer

Notes and guidance

In this small step, children encounter multiplication number sentences with fractions, multiplying unit fractions by an integer. Make links to multiplication as repeated addition: if children know that $\frac{1}{5} \times 4 = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$, this will link back to previous learning and avoid the common misconception of multiplying both the numerator and the denominator by the integer.

Bar models are a useful representation and can show the calculations in multiple or single bars. When answers are greater than 1, encourage children to write their answers as a mixed number. They may also find a number line useful.

This learning is built upon in the next few steps, when children multiply non-unit fractions and mixed numbers.

Things to look out for

- Children may think that when multiplying, the answer is always greater than both of the numbers. For example, they may think the result of $3 \times \frac{1}{10}$ must be greater than 3
- Children may multiply both the numerator and the denominator by the integer, and not recognise that this is the process for finding equivalent fractions, not for multiplying fractions by integers.

Key questions

- How can you write this multiplication as a repeated addition? How does this help you to work it out?
- How can you represent this question as a bar model?
- When you multiply a fraction by an integer, what happens to the numerator? What happens to the denominator?
- What is your answer as a mixed number? What is it as an improper fraction?
- What happens if the integer you are multiplying by is the same as the denominator? Does this always happen?

Possible sentence stems

- $\frac{1}{\square} \times \underline{\hspace{2cm}} = \frac{1}{\square} + \dots + \frac{1}{\square}$
- To multiply a fraction by an integer, I multiply the _____ by the integer and the _____ remains the same.

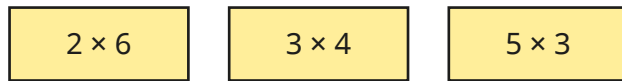
National Curriculum links

- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

Multiply a unit fraction by an integer

Key learning

- Write the multiplications as repeated additions.



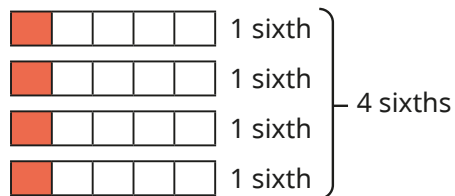
How would you write $\frac{1}{4} \times 3$ as a repeated addition?

- Write the multiplications as repeated additions.

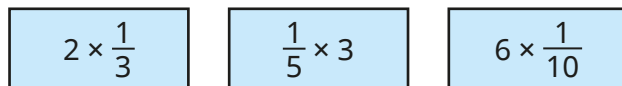


What do you notice?

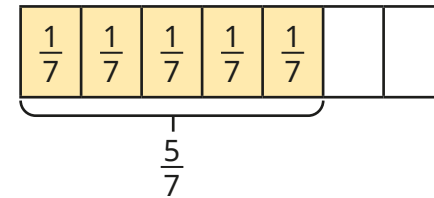
- Ron uses bar models to work out $\frac{1}{6} \times 4 = \frac{4}{6}$



Use Ron's method to work out the multiplications.



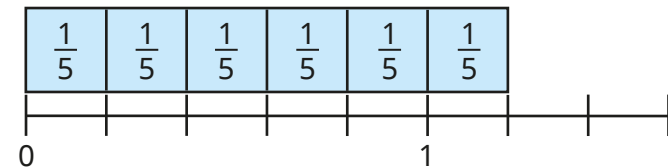
- Alex uses a bar model to work out $5 \times \frac{1}{7} = \frac{5}{7}$



Use Alex's method to work out the multiplications.

- $3 \times \frac{1}{7}$
- $4 \times \frac{1}{10}$
- $7 \times \frac{1}{8}$
- $\frac{1}{10} \times 3$

- Filip uses a number line to work out $\frac{1}{5} \times 6 = \frac{6}{5} = 1\frac{1}{5}$



Use Filip's method to work out the multiplications.

- $\frac{1}{6} \times 7$
- $\frac{1}{3} \times 5$
- $5 \times \frac{1}{5}$
- $9 \times \frac{1}{4}$

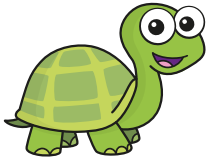
- Complete the multiplications.
Give your answers as mixed numbers.

- $6 \times \frac{1}{4}$
- $\frac{1}{3} \times 4$
- $10 \times \frac{1}{8}$
- $\frac{1}{10} \times 19$

Multiply a unit fraction by an integer

Reasoning and problem solving

Tiny is multiplying a unit fraction by an integer.



$$\frac{1}{5} \times 5 = \frac{5}{25}$$

Explain Tiny's mistake.

Tiny has multiplied the denominator as well as the numerator.

Is the statement always true, sometimes true or never true?

When you multiply a unit fraction by the same number as its denominator, the answer will be 1 whole.

Explain your answer.

always true



I am thinking of a unit fraction.

When I multiply it by 4, it is equivalent to $\frac{1}{2}$

When I multiply it by 2, it is equivalent to $\frac{1}{4}$

$$\frac{1}{8}$$

6

What is Mo's fraction?

What does Mo need to multiply his fraction by so that his answer is equivalent to $\frac{3}{4}$?

Write a similar question for a partner to solve.

Multiply a non-unit fraction by an integer

Notes and guidance

In this small step, children build on the previous step to multiply non-unit fractions by integers.

As in the previous step, children make the link between multiplication and repeated addition, and use bar models and number lines to support calculations. However, they should become more fluent and recognise the generalisation that they need to multiply the numerator by the integer and leave the denominator the same.

Children need to be able to convert improper fractions to mixed numbers and could use number lines or other representations to help.

In the next small step, children combine their learning from the first two steps to multiply mixed numbers by integers.

Things to look out for

- Children may think that when multiplying, the answer is always greater than both of the numbers. For example, they may think the result of $3 \times \frac{3}{10}$ must be greater than 3
- Children need to be confident in converting between improper fractions and mixed numbers.

Key questions

- How can you write this multiplication as a repeated addition?
- How can you represent this multiplication as a bar model?
- When you multiply a fraction by an integer, what happens to the numerator? What happens to the denominator?
- What is your answer as a mixed number? What is it as an improper fraction?
- How do you know that $\frac{3}{5} \times 2 = \frac{6}{10}$ can not be correct?

Possible sentence stems

- $\frac{\square}{\square} \times \text{_____} = \frac{\square}{\square} + \dots + \frac{\square}{\square}$
- To multiply a fraction by an integer, I multiply the _____ by the integer and the _____ remains the same.

National Curriculum links

- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

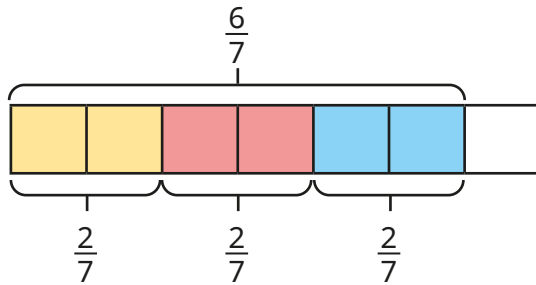
Multiply a non-unit fraction by an integer

Key learning

- Write the multiplications as repeated additions.

$2 \times \frac{3}{8}$	$\frac{3}{10} \times 3$	$\frac{2}{9} \times 4$	$5 \times \frac{3}{19}$
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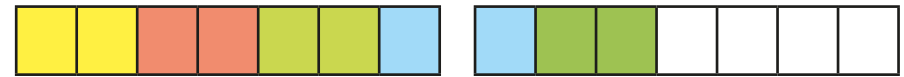
- Brett uses a bar model to work out $3 \times \frac{2}{7} = \frac{6}{7}$



Use Brett's method to work out the multiplications.

$2 \times \frac{3}{8}$	$4 \times \frac{2}{11}$	$\frac{3}{10} \times 3$	$\frac{3}{8} \times 2$
$3 \times \frac{5}{19}$	$\frac{3}{10} \times 2$	$\frac{2}{13} \times 5$	$4 \times \frac{2}{9}$

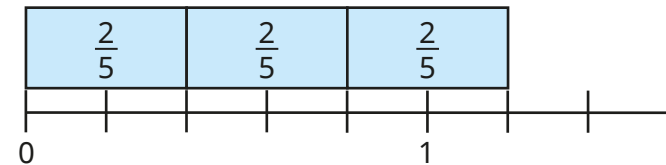
- Dani uses bar models to work out $\frac{2}{7} \times 5 = \frac{10}{7} = 1\frac{3}{7}$



Use Dani's method to work out the multiplications.

▶ $\frac{2}{7} \times 6$ ▶ $\frac{3}{7} \times 5$ ▶ $3 \times \frac{4}{7}$

- Huan uses a number line to help work out $\frac{2}{5} \times 3 = \frac{6}{5} = 1\frac{1}{5}$



Use Huan's method to work out the multiplications.

▶ $\frac{2}{5} \times 4$ ▶ $2 \times \frac{3}{5}$ ▶ $\frac{3}{10} \times 5$ ▶ $\frac{2}{9} \times 9$

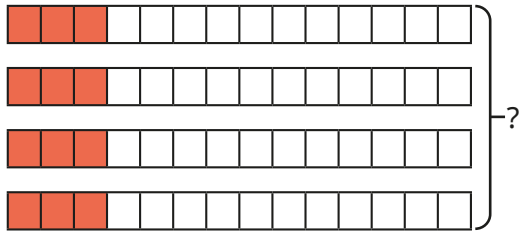
- Complete the multiplications.
Give your answers as mixed numbers.

▶ $\frac{3}{13} \times 5$ ▶ $6 \times \frac{5}{7}$ ▶ $\frac{6}{11} \times 9$ ▶ $8 \times \frac{7}{12}$

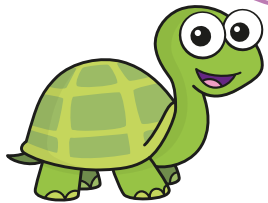
Multiply a non-unit fraction by an integer

Reasoning and problem solving

Tiny has worked out $4 \times \frac{3}{14}$



From the bar model, I can see that $4 \times \frac{3}{14} = \frac{12}{56}$



Do you agree with Tiny?
Explain your answer.

No

The bar model shows 4 lots of $\frac{3}{14}$

$$\frac{3}{14} + \frac{3}{14} + \frac{3}{14} + \frac{3}{14} = \frac{12}{14}$$

Work out the calculations.

$$\frac{3}{4} \times 3$$

$$4 \times \frac{4}{5}$$

$$\frac{5}{6} \times 5$$

What patterns do you notice?

What comes next in the sequence?

$$2\frac{1}{4}, 3\frac{1}{5}, 4\frac{1}{6}$$

$$6 \times \frac{6}{7} = 5\frac{1}{7}$$

Use the digit cards to complete the multiplication.

You can use a card once only in each multiplication.



$$\square \times \frac{\square}{\square} = \frac{\square}{\square}$$

Is there more than one possible answer?

Are there any of the cards you cannot use?

multiple possible answers, e.g.

$$2 \times \frac{1}{3} = \frac{4}{6}$$

Multiply a mixed number by an integer

Notes and guidance

In this small step, children build on their learning from the first two steps to multiply mixed numbers by integers. Children need to be secure in their understanding of multiplying proper fractions by integers before adding the extra challenge of multiplying mixed numbers.

Children explore a range of methods to complete the calculations and discuss the efficiency of each. To build understanding, initially calculations should not involve converting improper fractions to mixed numbers. Once children are secure in using the methods, they can explore questions where in the answer, the fractional part of the calculation is greater than 1 and needs converting to a mixed number before combining the totals.

Things to look out for

- Children may write their answer as a whole number and an improper fraction rather than a mixed number.
- Children may use an inefficient method to solve a calculation, for example using improper fractions to work out $4 \times 8\frac{3}{15}$
- Children may make errors converting between improper fractions and mixed numbers.

Key questions

- How could you partition this mixed number?
- When you multiply a fraction by an integer, what happens to the numerator? What happens to the denominator?
- What do you need to do if you have an improper fraction in your answer?
- Could you work it out another way? Which way is most efficient?
- Have you written your answer in its simplest form?

Possible sentence stems

- I can partition $\square \frac{\square}{\square}$ into \square and $\frac{\square}{\square}$
- When I multiply a fraction by an integer, I multiply the _____ by the integer and the _____ remains the same.
- To multiply a mixed number by an integer, I multiply the _____ by the integer and the _____ by the integer.

National Curriculum links

- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

Multiply a mixed number by an integer

Key learning

- Rosie is working out $1\frac{1}{5} \times 3$

$$\text{I know that } 1\frac{1}{5} \times 3 = 1\frac{1}{5} + 1\frac{1}{5} + 1\frac{1}{5} = 3\frac{3}{5}$$

Use Rosie's method to work out the multiplications.

$$1\frac{1}{7} \times 3$$

$$2\frac{2}{10} \times 3$$

$$3 \times 5\frac{3}{10}$$

$$2 \times 4\frac{3}{11}$$

- Amir is working out $3 \times 5\frac{1}{10}$



I will partition $5\frac{1}{10}$ into 5 and $\frac{1}{10}$ and multiply each part by 3

$$3 \times 5 = 15 \quad 3 \times \frac{1}{10} = \frac{3}{10} \quad 3 \times 5\frac{1}{10} = 15\frac{3}{10}$$

Use Amir's method to work out the multiplications.

$$3 \times 4\frac{1}{10}$$

$$5\frac{3}{10} \times 3$$

$$6 \times 2\frac{1}{7}$$

$$3\frac{4}{9} \times 2$$

- Whitney is working out $3 \times 2\frac{2}{5}$ by partitioning the mixed number into a whole number and a fraction.

$$3 \times 2 = 6$$

$$3 \times \frac{2}{5} = \frac{6}{5} = 1\frac{1}{5}$$

$$3 \times 2\frac{2}{5} = 6 + 1\frac{1}{5} = 7\frac{1}{5}$$

Use Whitney's method to work out the multiplications.

$$4 \times 2\frac{2}{5}$$

$$3\frac{3}{8} \times 4$$

$$5 \times 3\frac{7}{9}$$

$$3 \times 5\frac{3}{7}$$

- Scott uses improper fractions to work out $4 \times 1\frac{3}{8} = 5\frac{1}{2}$

$$4 \times 1\frac{3}{8} = 4 \times \frac{11}{8} = \frac{44}{8} = 5\frac{4}{8} = 5\frac{1}{2}$$

Use Scott's method to work out the multiplications.

$$5 \times 1\frac{5}{6}$$

$$1\frac{9}{10} \times 3$$

$$6 \times 2\frac{3}{5}$$

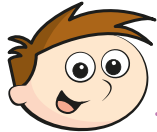
$$4 \times 12\frac{7}{8}$$

Is Scott's method always efficient?

Multiply a mixed number by an integer

Reasoning and problem solving

Teddy is working out $5 \times 2\frac{3}{5}$



$$5 \times 2 = 10$$

$$5 \times \frac{3}{5} = \frac{15}{5}$$

The answer is $10\frac{15}{5}$

Do you agree with Teddy?

Explain your answer.

Yes, but he has not simplified his answer.

$\frac{15}{5} = 3$, so the answer is 13

Annie is working out $2 \times 3\frac{2}{7}$



If I work out $3 \times 2\frac{2}{7}$, I will get the same answer.

Do you agree with Annie?

Explain your answer.

No

$$2 \times 3\frac{2}{7} = 6\frac{4}{7}$$

$$3 \times 2\frac{2}{7} = 6\frac{6}{7}$$

Jack runs $2\frac{2}{3}$ miles three times per week.

Mo runs $3\frac{3}{4}$ miles twice a week.

Who runs further during the week?

Explain your answer.

Jack

Find the missing numbers.

$$2\frac{\square}{8} \times \square = 7\frac{7}{8}$$

Explain how you worked out the missing numbers.

$$2\frac{5}{8} \times 3 = 7\frac{7}{8}$$

Calculate a fraction of a quantity

Notes and guidance

In this small step, children calculate a fraction of a quantity, building on understanding from previous years. The step focuses on using concrete and pictorial representations to support learning.

Children begin by using real-life objects or counters and sharing them into equal groups. This helps children to identify the relationship between dividing by the denominator and multiplying by the numerator. They start by finding unit fractions of amounts and, when they are secure in their understanding, move on to non-unit fractions.

Children will build on this understanding in the next step, in which they focus on more abstract methods.

Things to look out for

- Children may divide by the numerator rather than by the denominator.
- Children may find it more difficult to find non-unit fractions of amounts, as it involves more than one step and requires more cognitive load.
- If using place value counters, children may not exchange and may believe they cannot find, for example, $\frac{1}{4}$ of 52

Key questions

- How can you share the counters equally?
- How do you know the counters are in equal groups?
- If you know $\frac{1}{\square}$ of a number, how do you find $\frac{2}{\square}$ of the number?
- What do you need to do when you cannot share your tens counters equally?
- How do you find a fraction of an amount?

Possible sentence stems

- If I know $\frac{1}{\square}$ of a quantity, then to find $\frac{\square}{\square}$ I need to multiply by _____
- To find $\frac{3}{4}$ of _____, I need to divide by _____ and multiply by _____
- I need to divide by the _____ and multiply by the _____

National Curriculum links

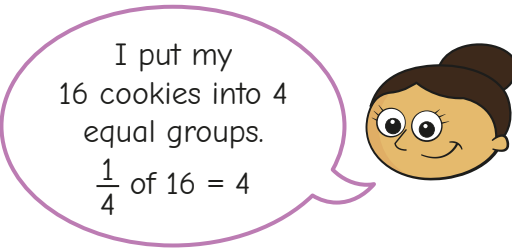
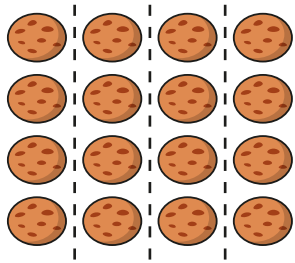
- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

Calculate a fraction of a quantity

Key learning

- Dora is sharing 16 cookies between 4 friends.

She needs to find $\frac{1}{4}$ of 16

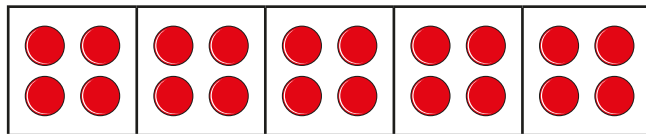


Use Dora's method to work out the fractions of amounts.



- The bar model shows 20 counters shared equally into 5 groups.

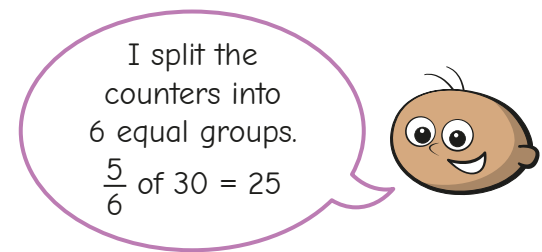
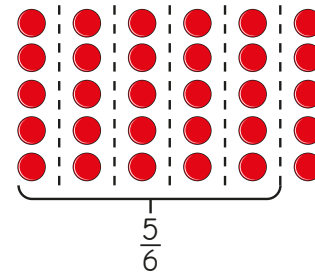
Use the bar model to find the fractions of amounts.



▶ $\frac{1}{5}$ of 20 ▶ $\frac{2}{5}$ of 20 ▶ $\frac{3}{5}$ of 20 ▶ $\frac{4}{5}$ of 20 ▶ $\frac{5}{5}$ of 20

What do you notice?

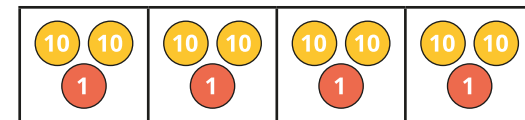
- Tommy uses an array of counters to find $\frac{5}{6}$ of 30



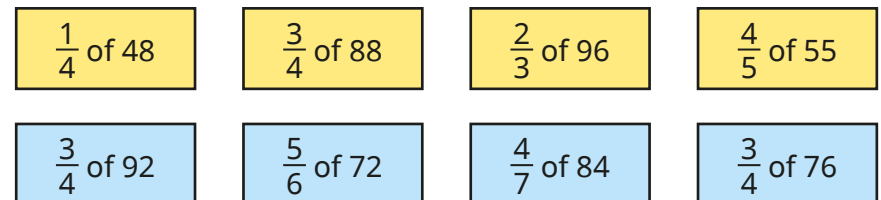
Use Tommy's method to work out the fractions of amounts.



- Nijah uses place value counters to find $\frac{1}{4}$ of 84 = 21



Use Nijah's method to work out the fractions of amounts.



Why were the last calculations more challenging?

Calculate a fraction of a quantity

Reasoning and problem solving

Alex is finding $\frac{3}{5}$ of 15



$$15 \div 3 = 5 \quad \frac{3}{5} \text{ of } 15 = 5$$

Do you agree with Alex?

Explain your answer.



No

Ron is trying to find $\frac{4}{8}$ of 20



It is impossible to find $\frac{4}{8}$ of 20, as I cannot split 20 into 8 equal groups.

Do you agree with Ron?

Explain your answer.



No

Sort the calculations into the table.



$$\frac{1}{2} \text{ of } 16$$

$$\frac{1}{5} \text{ of } 30$$

$$\frac{3}{7} \text{ of } 14$$

$$\frac{2}{9} \text{ of } 36$$

Equal to 6	Equal to 8

Write another calculation in each column.

How many different answers can you find?



$$= 6: \frac{1}{5} \text{ of } 30, \frac{3}{7} \text{ of } 14$$

$$= 8: \frac{1}{2} \text{ of } 16, \frac{2}{9} \text{ of } 36$$

multiple possible answers, e.g.

$$= 6: \frac{1}{2} \text{ of } 12, \frac{2}{10} \text{ of } 30$$

$$= 8: \frac{1}{4} \text{ of } 32, \frac{4}{5} \text{ of } 10$$

Fraction of an amount

Notes and guidance

In this small step, children find fractions of amounts using more pictorial and abstract methods, rather than relying on concrete resources.

Bar models are useful tools to help represent this mathematical concept and can also help to show links between finding unit fractions of amounts and non-unit fractions of amounts. Children initially use times-table facts, then move on to solve calculations that go beyond these. Once children are secure in finding non-unit fractions of amounts, they compare two calculations, for example $\frac{2}{3}$ of 30 and $\frac{4}{5}$ of 20

The learning from this step is built upon in Step 6, when children find the whole from a fractional part.

Things to look out for

- Children may divide by the numerator and not by the denominator.
- Children may find it more difficult to find non-unit fractions of amounts, as this involves more than one step and greater cognitive load.
- Children may need support to find fractions of amounts that go beyond known times-table facts.

Key questions

- How can you represent this in a bar model?
- What is the relationship between $\frac{1}{\square}$ of a number and $\frac{2}{\square}$ of a number?
- What is the first step to solve this calculation?
What is the next step to solve this calculation?
- How do you find a fraction of an amount?
- How can you find a fraction of a 3-digit number?

Possible sentence stems

- To find $\frac{\square}{\square}$ of _____, I need to divide by _____ and multiply by _____
- To find a fraction of an amount, I need to divide by the _____ and multiply the result by the _____

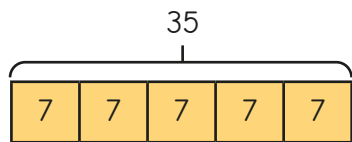
National Curriculum links

- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

Fraction of an amount

Key learning

- Esther is finding $\frac{1}{5}$ of 35



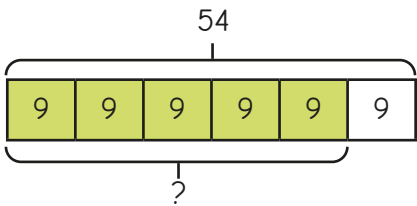
$$35 \div 5 = 7$$

$$\frac{1}{5} \text{ of } 35 = 7$$

Use Esther's method to work out the fractions of amounts.

$\frac{1}{5}$ of 45	$\frac{1}{7}$ of 35	$\frac{1}{10}$ of 80	$\frac{1}{9}$ of 81
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- Mo is finding $\frac{5}{6}$ of 54



$$54 \div 6 = 9$$

$$9 \times 5 = 45$$

$$\frac{5}{6} \text{ of } 54 = 45$$

- ▶ Complete the sentence.
To find a fraction of an amount, I need to divide by the _____ and multiply by the _____
- ▶ Use Mo's method to work out the fractions of amounts.

$\frac{2}{3}$ of 21	$\frac{4}{5}$ of 40	$\frac{3}{5}$ of 60	$\frac{5}{7}$ of 42
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- Write $<$, $>$ or $=$ to compare the fractions of amounts.

$$\frac{1}{3} \text{ of } 33 \quad \bigcirc \quad \frac{2}{5} \text{ of } 25$$

$$\frac{2}{7} \text{ of } 140 \quad \bigcirc \quad \frac{3}{4} \text{ of } 76$$

- Work out the fractions of amounts.

▶ $\frac{1}{3}$ of 30 $\frac{2}{3}$ of 30 $\frac{3}{3}$ of 30

▶ $\frac{1}{5}$ of 35 $\frac{2}{5}$ of 35 $\frac{3}{5}$ of 35 $\frac{4}{5}$ of 35 $\frac{5}{5}$ of 35

What patterns do you notice?

- Brett is finding $\frac{2}{3}$ of 618

$$618 \div 3 = 206$$

$$206 \times 2 = 412$$

$$\frac{2}{3} \text{ of } 618 = 412$$

Use Brett's method to work out the fractions of amounts.

$\frac{2}{3}$ of 924	$\frac{5}{6}$ of 126	$\frac{3}{5}$ of 205	$\frac{7}{9}$ of 6,498
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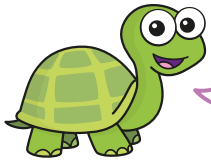
- Draw bar models to help work out the fractions of quantities.

$\frac{3}{7}$ of 21 kg	$\frac{4}{5}$ of 100 cm	$\frac{5}{12}$ of 1,440 ml
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Fraction of an amount

Reasoning and problem solving

Tiny is working out $\frac{6}{6}$ of 36



I will divide by the denominator and then multiply by the numerator.

No

Do you think that this is the most efficient method?

Explain your answer.

Find three possible ways to make the statement true.

$$\frac{3}{\square} \text{ of } \underline{\hspace{2cm}} = 15$$

Compare answers with a partner.

What do you notice?

multiple possible answers, e.g.

$$\frac{3}{4} \text{ of } 20 = 15$$

There are 32 boys and girls in a class.

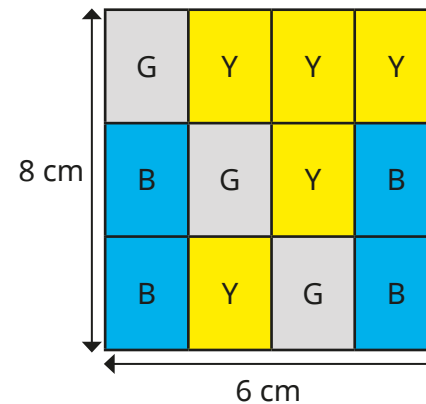
$\frac{7}{16}$ of the class are boys.

How many more girls than boys are there?

How did you work it out?

4

Find the area of each colour (grey, blue and yellow).



grey: 12 cm^2
blue: 16 cm^2
yellow: 20 cm^2

Find the whole

Notes and guidance

In this small step, children build on their understanding of finding a fraction of an amount, as they use a fraction of an amount to find the whole.

Children start with finding the whole from a unit fraction, initially using counters and bar models for support. They identify that if they know one equal part, they can use multiplication to find the whole. Once this is secure, children move on to finding the whole from a non-unit fraction. They should start by identifying what one part is to help them work out the whole.

Things to look out for

- Children may misinterpret the question by trying to find the fraction of the number given, instead of using the number to find the whole.
- Children may mix up finding one part with finding the whole.
- When dealing with a non-unit fraction, children may divide by the denominator to find one part, rather than dividing by the numerator.

Key questions

- What is the same and what is different about finding a fraction of an amount and finding the whole?
- If you know that one equal part is _____, what must all the other parts be?
- If you know one equal part, how can you work out the whole?
- If you know what _____ equal parts are, how can you find what one part is?
- Is your answer going to be greater or less than _____? How do you know?

Possible sentence stems

- If _____ is one equal part, all the parts must be _____
- If $\frac{1}{\square}$ is _____, then the whole is _____ \times _____ = _____
- If _____ is _____ parts, then one part is _____

National Curriculum links

- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams

Find the whole

Key learning

- The counters in the bar model show that $\frac{1}{4}$ of a quantity is 5

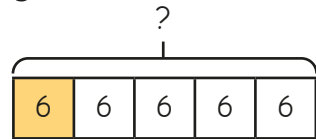


Use the bar model to work out the fractions of the same quantity.

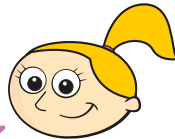
▶ $\frac{2}{4} = \underline{\hspace{2cm}}$ ▶ $\frac{3}{4} = \underline{\hspace{2cm}}$ ▶ $\frac{4}{4}$ or 1 whole = $\underline{\hspace{2cm}}$

- Eva uses a bar model to help work out the missing amount.

$\frac{1}{5}$ of $\underline{\hspace{2cm}} = 6$



If one part is 6, then all the parts will be 6



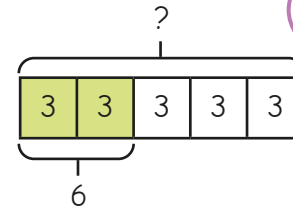
$6 \times 5 = 30$ $\frac{1}{5}$ of 30 = 6

Use Eva's method to work out the missing amounts.

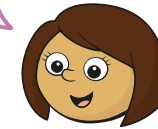
▶ $\frac{1}{5}$ of $\underline{\hspace{2cm}} = 9$ ▶ $\frac{1}{7}$ of $\underline{\hspace{2cm}} = 10$
 ▶ $\frac{1}{8}$ of $\underline{\hspace{2cm}} = 3$ ▶ $\frac{1}{12}$ of $\underline{\hspace{2cm}} = 2$

- Kim uses a bar model to help work out the missing amount.

$\frac{2}{5}$ of $\underline{\hspace{2cm}} = 6$



If 2 equal parts are 6, 1 part must be 3

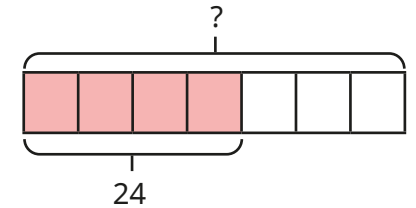
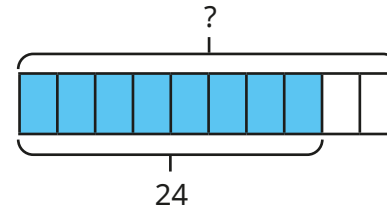


$6 \div 2 = 3$
 $3 \times 5 = 15$
 $\frac{2}{5}$ of 15 = 6

Use Kim's method to work out the missing amounts.

▶ $\frac{2}{5}$ of $\underline{\hspace{2cm}} = 8$ ▶ $\frac{3}{7}$ of $\underline{\hspace{2cm}} = 18$
 ▶ $\frac{4}{5}$ of $\underline{\hspace{2cm}} = 20$ ▶ $\frac{6}{7}$ of $\underline{\hspace{2cm}} = 54$

- Use the bar models to find the wholes.



- Jack has a bottle of juice.
 There is $\frac{3}{5}$ of a bottle left.
 There is 150 ml of juice left in the bottle.
 How much juice was in the bottle when it was full?

Find the whole

Reasoning and problem solving

Amir and Tiny are working out the missing amount.

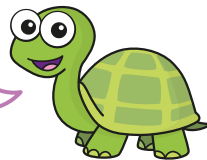
$$\frac{7}{5} \text{ of } \underline{\hspace{2cm}} = 42$$



I think the answer is 30

Amir

The answer cannot be 30. It must be greater than 42



Tiny

Who do you agree with?
Explain your answer.

Amir

Rosie takes a bottle of water to school with her.

She drinks $\frac{1}{3}$ of the water in the morning.

She drinks $\frac{1}{4}$ of the bottle at lunchtime.

So far, she has drunk 210 ml of water.

How much water was in her bottle when it was full?

360 ml

If $\frac{1}{8}$ of A = 12, find the values of A, B and C.

$$\frac{5}{8} \text{ of } A = \frac{3}{4} \text{ of } B = \frac{1}{6} \text{ of } C$$

A = 96

B = 80

C = 360

Use fractions as operators

Notes and guidance

In this small step, children revisit and compare their learning from earlier in the block as they look at fractions as operators. They should recognise the connection between finding a fraction of an amount and multiplying a fraction by an integer.

Firstly, children are encouraged to both find fractions of amounts and multiply fractions, and to identify patterns. It may be appropriate to recap converting improper fractions to whole numbers/mixed numbers. Children should also recognise that commutativity of multiplication can be used, for example $\frac{1}{3}$ of 6 is the same as $6 \times \frac{1}{3}$. They also explore when it would be more efficient to choose each method, using their knowledge of factors.

Things to look out for

- Children may need support to recognise the link between “of” and \times .
- Children may make errors if their times-tables knowledge is insecure.
- Children may choose the less appropriate method and face difficult calculations as a result.

Key questions

- What is the same about _____ of _____ and _____ \times _____?
- Is the denominator of the fraction a factor of the number you are multiplying by? Why is this important?
- Which is the most efficient method? How do you know?
- How would you write this improper fraction as a whole number/mixed number?
- When is it more efficient to multiply fractions?
- When is it more efficient to find a fraction of an amount?

Possible sentence stems

- $\frac{\square}{\square} \times$ _____ is the same as $\frac{\square}{\square}$ of _____
- _____ is a factor of _____, so I can divide _____ by _____

National Curriculum links

- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
- Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number (Y4)

Use fractions as operators

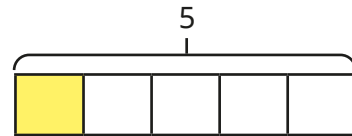
Key learning

- Use the bar models to work out the calculations.

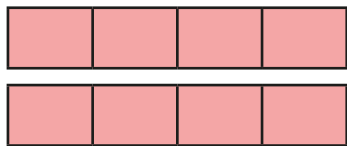
▶ 5 lots of $\frac{1}{5}$



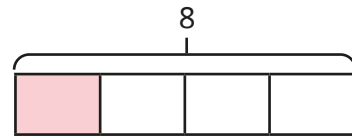
$\frac{1}{5}$ of 5



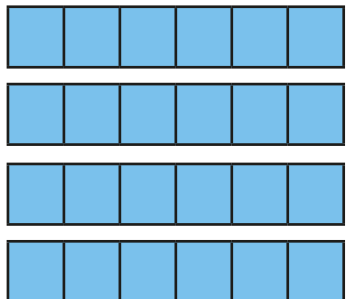
▶ 8 lots of $\frac{1}{4}$



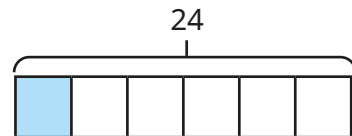
$\frac{1}{4}$ of 8



▶ $\frac{1}{6} \times 24$



$\frac{1}{6}$ of 24



What do you notice?

- Use bar models to work out the calculations.

▶ $\frac{2}{3}$ of 15

▶ $\frac{4}{5}$ of 30

▶ $\frac{3}{10}$ of 20

$\frac{2}{3} \times 15$

$30 \times \frac{4}{5}$

$\frac{3}{10} \times 20$

Which bar model did you find easiest to draw? Was this the same for each question?

- Match the calculations that give the same answer.

$\frac{4}{5}$ of 45

$10 \times \frac{2}{10}$

$\frac{2}{10}$ of 10

$\frac{1}{9} \times 81$

$\frac{7}{8}$ of 56

$\frac{4}{5} \times 45$

$\frac{1}{9}$ of 81

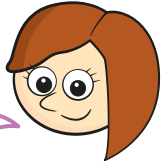
$56 \times \frac{7}{8}$

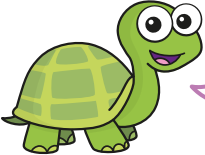
Work out the answer to each calculation.
In each pair, which was the easier calculation to work out?

Use fractions as operators

Reasoning and problem solving

Tiny is working out $\frac{4}{5}$ of 15
 Rosie is working out $15 \times \frac{4}{5}$

 Rosie: We will get the same answer.

 Tiny: No, we won't. We are working out different calculations.

Who do you agree with?
 Explain your answer.

Rosie


Is the statement true or false?

$\frac{9}{10}$ of 20 = $20 \times \frac{4}{10} + 20 \times \frac{5}{10}$

Explain your answer.

True

Max is thinking of a 2-digit number between 20 and 30
 He finds $\frac{2}{3}$ of the number.

 Max: My new number has a digit sum of 7

What number did Max start with?

24