Autumn Block 2

Addition, subtraction, multiplication and division

Year 6 | Autumn term | Block 2 - Addition, subtraction, multiplication and division

Small steps

Step 1	Add and subtract integers
Step 2	Common factors
Step 3	Common multiples
Step 4	Rules of divisibility
Step 5	Primes to 100
Step 6	Square and cube numbers
Step 7	Multiply up to a 4-digit number by a 2-digit number
Step 8	Solve problems with multiplication

Year 6 | Autumn term | Block 2 - Addition, subtraction, multiplication and division

Small steps

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Step 10	Division using factors
Step 11	Introduction to long division
Step 12	Long division with remainders
Step 13	Solve problems with division
Step 14	Solve multi-step problems
Step 15	Order of operations
Step 16	Mental calculations and estimation

Small steps

Step 17

Reason from known facts



Add and subtract integers



Notes and guidance

This small step reviews and extends children's learning of how to add and subtract integers with any number of digits.

Children use the formal column method for numbers with the same and different numbers of digits. They also practise mental strategies with both large and small numbers, using their understanding of place value.

Children solve multi-step problems, choosing which operations and methods to use based on the context of the problem and the types of numbers involved.

The use of concrete manipulatives can support children's understanding, especially where exchanges are required.

Things to look out for

- Children may not line the numbers up correctly when setting out an addition or a subtraction.
- Children may try to use formal methods when mental strategies would be more appropriate, for example adding 999 is more easily done by adding 1,000 and then subtracting 1
- When solving multi-step problems, children may need support to choose the type and order of operations needed.

Key questions

- What is the greatest digit you can have in a place value column?
- How do you exchange when adding?
- How do you exchange when subtracting?
- Which columns are affected by the exchange?
- How do you know whether to add or subtract the numbers?
- How can you check your answer to the calculation?

Possible sentence stems

- In column addition/subtraction, we start with the ______ place value column.
- The _____ is in the _____ column. It represents _____

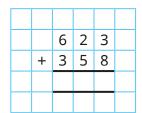
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving addition, subtraction, multiplication and division
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

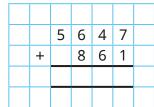
Add and subtract integers

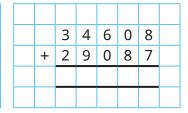


Key learning

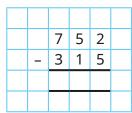
• Work out the additions.

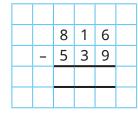


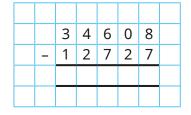




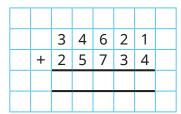
Work out the subtractions.

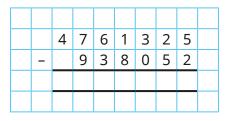






• Find the answers to the calculations.



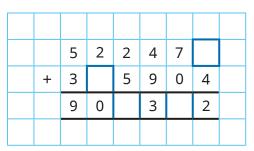


 Which calculations would you work out mentally, and which would you work out using the column method?

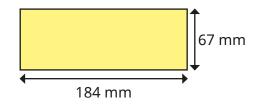
8 million subtract
$$3\frac{1}{2}$$
 million

Work out the answers to the calculations.

• Find the missing digits.



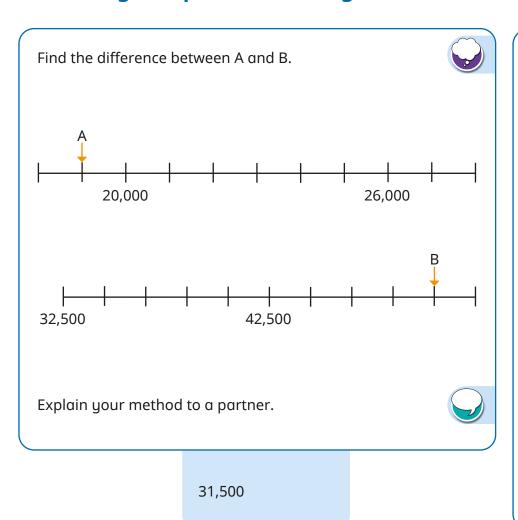
• The perimeter of the triangle is equal to the perimeter of the rectangle. Work out the unknown length of the triangle.



Add and subtract integers



Reasoning and problem solving



Here is a bar model.

A B

631,255

• A is an odd integer that rounds to 100,000 to the nearest 10,000

- The sum of the digits of A is 30
- B is an even integer that rounds to 500,000 to the nearest 100,000
- The sum of the digits of B is 10
- A and B are both multiples of 5

What could be the values of A and B?

Explain your reasoning to a partner.

multiple possible answers, e.g.

A = 99,255

B = 532,000

Common factors



Notes and guidance

This small step reinforces children's understanding of factors and common factors, introduced in Years 4 and 5 respectively.

Some children may still choose to use arrays and other representations, but knowledge of times-tables and the use of familiar rules of divisibility are to be encouraged. The rules of divisibility will be reviewed again later in the block.

Children work systematically to find the complete list of factors of a number, and learn to use their knowledge that factors usually come in pairs to spot missing factors.

Children are not required to formally identify the highest common factor of two or more numbers, but can be extended to consider this idea.

Things to look out for

- Children may confuse the ideas of factors and multiples.
- Children may not be familiar with the use of the word "common" in this context.
- Errors may be made with times-tables, resulting in incorrect factors.
- Children may forget 1 and the number itself when listing factors.

Key questions

- What are the factors of _____?
- What factors do _____ and ____ have in common?
- How can you easily tell if 2/5/10 is a factor of a number?
- If you know one factor of a number, how can you use it to find another factor of the number?
- Is 1 a factor of all numbers?
- How can you work systematically to find all the factors of a number?

Possible sentence stems

- _____ is a factor of all numbers.
- The largest factor of a number is always _____
- _____ is a factor of _____ because _____ is in the _____ times-table.

- Identify common factors, common multiples and prime numbers
- Solve problems involving addition, subtraction, multiplication and division

Common factors



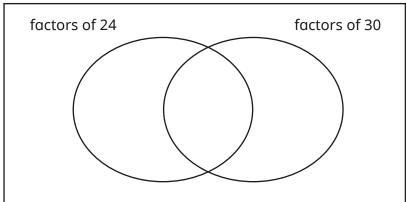
Key learning

- List the factors of 24
 List the factors of 36
 What are the common factors of 24 and 36?
- Find the common factors of each pair of numbers.



Write the numbers in the sorting diagram.

1 2 3 4 5 6 8 10 12 15 24 30



List the common factors of 24 and 30

• Decide if each statement is true or false.

5 is a factor of both 95 and 75

3 is a common factor of 45 and 54

4 is not a common factor of 56 and 80

Here is a table for sorting numbers.

Write one number in each box.

	Factor of 6	Not a factor of 6
Factor of 9		
Not a factor of 9		

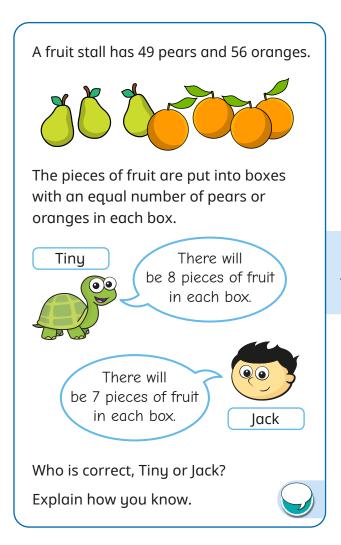
Compare answers with a partner.

- Find the common factors of 300, 400 and 500
- The common factors of two numbers are 1, 3 and 5What could the two numbers be?

Common factors



Reasoning and problem solving



Brett has two pieces of string.



One is 160 cm long and the other is 200 cm long.

He cuts them both into smaller pieces.

All the pieces are the same length.

What are the possible lengths of the smaller pieces of string?

1 cm, 2 cm, 4 cm, 5 cm, 8 cm, 10 cm, 20 cm, 40 cm

Jack

Dani has 54 red sweets and 45 green sweets.



She puts them into bags so that each bag has an equal number of red sweets and an equal number of green sweets.

What is the greatest number of bags she can make?

How many sweets of each colour will there be in each bag?

9 bags, each with6 red sweets and5 green sweets

Common multiples



Notes and guidance

Children are familiar with the idea of multiples of numbers from earlier study of times-tables. Building on this knowledge, they now find common multiples of two or more numbers.

As with factors, arrays and other representations may still be used as support, but knowledge of times-tables is key. Some multiples can be recognised using the rules of divisibility, which are explored in detail in the next small step.

Encourage children to work systematically to find lists of multiples rather than just finding the product of the given numbers, as this may miss some multiples.

Children do not need to be able to formally identify the lowest common multiple of two or more numbers, but can be challenged to consider the first common multiple of a pair of numbers.

Things to look out for

- Children may confuse the ideas of factors and multiples.
- Errors may be made with times-tables, resulting in incorrect factors.
- A common misconception is that the only common multiple of a pair of numbers is the product of the numbers.

Key questions

- How do you find the multiples of a number?
- What multiples do _____ and ____ have in common?
- What is the difference between a multiple and a factor?
- Can a number be both a factor and a multiple of another number?
- How can you tell if a number is a multiple of 2/5/10?
- When do numbers have common multiples that are less than their product?

Possible sentence stems

- The first multiple of a number is always _____
- _____ is a multiple of _____ because ____ × ____ = ____
- _____ is a common multiple of _____ and ____

- Identify common factors, common multiples and prime numbers
- Solve problems involving addition, subtraction, multiplication and division

Common multiples



Key learning

• Here is a hundred square.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Shade the multiples of 6

Circle the multiples of 5

What common multiples of 5 and 6 do you find?

Use these numbers to find other common multiples of 5 and 6

• Find the first three common multiples of each pair of numbers.

4 and 5

5 and 6

4 and 8

6 and 8

Find five common multiples of 4 and 3

• Here is a table for sorting numbers.

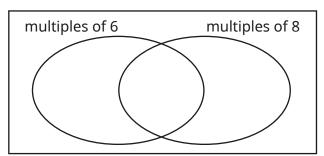
Write one number in each box.

	Multiple of 8	Not a multiple of 8
Multiple of 5		
Not a multiple of 5		

Compare answers with a partner.

Write the numbers in the sorting diagram.

12 18 40 6 48 24 16 42 56 54 30



Nijah plays football every 4 days and Kim plays football every 6 days.

They both played football today.

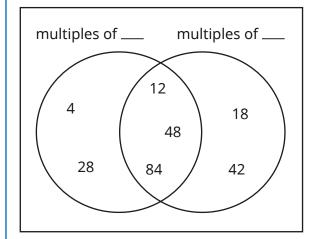
In how many days will they next both play football on the same day?

Common multiples



Reasoning and problem solving

Complete the labels of the sorting diagram.



Write another number in each section.

Find a square number that will go in the middle section.

Compare answers with a partner.



various possible answers, e.g. multiples of 4, multiples of 6

multiple possible answers, e.g. 40, 72, 66

36, 144

Ms Fisher's age is double her sister's age.

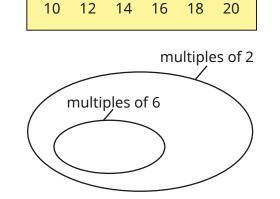
They are both older than 20 but younger than 50

Their ages are both multiples of 7

What are their ages?

Ms Fisher is 42 and her sister is 21

Write the numbers in the sorting diagram.



multiples of 2: 10, 12, 14, 16, 18, 20 multiples of 6: 12, 18

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Rules of divisibility



Notes and guidance

Children should be familiar with most rules of divisibility from looking at patterns in times-tables in their earlier learning and the previous two steps.

Children recognise divisibility by 2, 5 or 10 by looking at the ones digits of a number. They know a number is divisible by 4 if halving the number gives an even result and the corresponding rule for divisibility by 8. They know that numbers are divisible by 3 if the sum of their digits is divisible by 3, and divisible by 9 if the sum of their digits is divisible by 9

Children now learn to combine these rules to deal with other potential factors, for example to be divisible by 6 a number must be divisible by both 2 and 3

Children should recognise that a 2-digit number is divisible by 11 if the digits are the same.

Things to look out for

- Children may over-generalise rules, for example incorrectly applying the digit-sum rule for 3 and 9 or the final-digit rule for 5 to other numbers.
- Children may need support in understanding the combining of rules such as "a number is divisible by 12 if it is divisible by both 3 and 4"

Key questions

- How does the ones digit help you to decide if a number is divisible by 2, 5 or 10?
- How can you use the rule for divisibility by 2 to find out if a number is divisible by 4/8?
- What two other numbers must a number be divisible by if the number is divisible by 6/12?
- How can you tell if a 2-digit number is divisible by 11?
- Which divisibility rules are based on the sum of the digits of a number?

Possible sentence stems

- If a number is divisible by _____ and ____, then the number must also be divisible by _____
- If the sum of the digits is divisible by _____, then the number is divisible by _____
- A number is divisible by _____ if its ones digit is _____

National Curriculum links

 Solve problems involving addition, subtraction, multiplication and division

Rules of divisibility



Key learning

• Which of the numbers are divisible by 2?

 62
 901
 5,462

 10,308
 111,111
 224,528

Which of the numbers are also divisible by 4? How can you tell?

Use the digit sums to decide which numbers are divisible by 3
 and which are also divisible by 9

 78
 801
 5,460

 12,307
 555,222
 48,117

• Find a number that matches each description.

a 3-digit number that is divisible by 5

a 6-digit number that is divisible by 10

a 4-digit number that is divisible by 5 and 3

a 5-digit number that is divisible by 3 but not divisible by 5

Scott is packing cakes into boxes.

He puts an equal number of cakes into each box with no cakes left over.

He has 1,032 cakes to pack.

How many cakes can go in each box?



• Use ticks and crosses to complete the table.

		Is the number divisible by?				
	3	4	6	9	11	
87						
96						
99						
216						
702						

• The children at a school all have lunch at the same time.

There are 672 children and an equal number of them sit at each table.

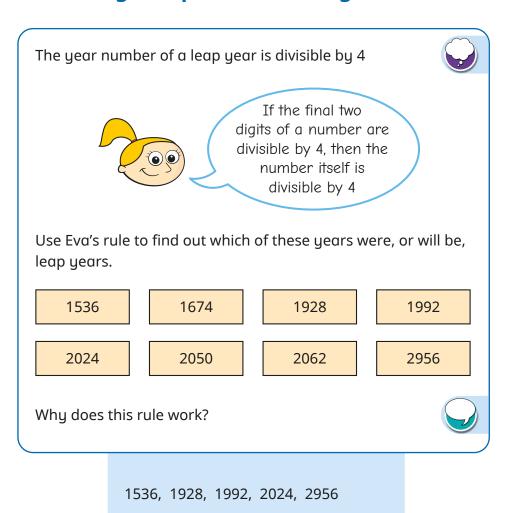
No more than 12 children sit at a table.

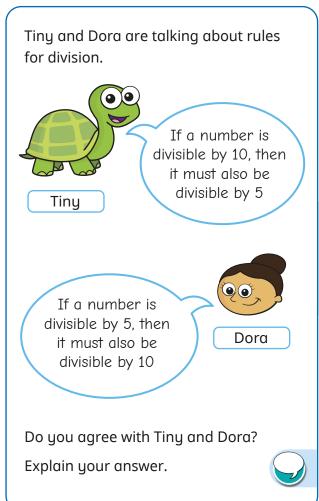
How many tables could there be?

Rules of divisibility



Reasoning and problem solving





Tiny is correct.

Dora is incorrect.

Primes to 100



Notes and guidance

Children first encountered prime numbers and composite numbers in Year 5. This small step reviews that learning and develops children's knowledge of factors so that they can deepen their understanding of prime numbers.

Children recognise that a number is prime when it has exactly two factors: 1 and itself. They also look at identifying the prime factors of a given number.

By the end of this step, children should be able to identify all the primes less than 100 and recall at least the primes to 19

Children should be familiar with square and cube numbers from earlier years, so this is something that can be revisited here, but is also covered in detail in the next small step.

Things to look out for

- A common misconception is that 1 is a prime number.
- Children may think that all prime numbers are odd and not realise that 2 is a prime number.
- Numbers that are outside times-tables knowledge (e.g. 51) may be mistakenly thought of as prime.
 Encourage children to use divisibility rules from the previous step to check these.

Key questions

- What is a prime number?
- What is a composite number?
- How many factors does a prime number have?
- Why is 1 not a prime number?
- How can you find the prime factors of a number?
- Are the multiples of prime numbers also prime?

Possible sentence stems

The factors of are	
The prime factors of are	
is prime because it has exactly factors.	
is a composite number because = ×	_

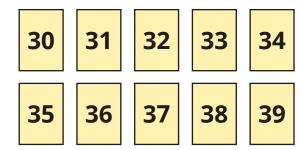
- Identify common factors, common multiples and prime numbers
- Solve problems involving addition, subtraction, multiplication and division

Primes to 100



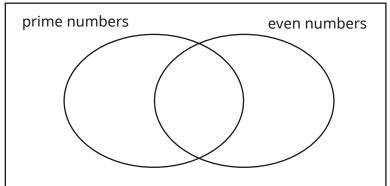
Key learning

- List all the prime numbers that are less than 20
- Which of these numbers are prime and which are composite?



- Explain how you know 51 is a composite number.
- Write the numbers in the sorting diagram.

10 13 2 12 11 6 7



- List the factors of 20Which factors of 20 are prime?
- Find the prime factors of the numbers.



- The sum of two prime numbers is 36
 What might the numbers be?
 How many different answers can you find?
- Write the three prime numbers that multiply to make 105

List the numbers from 40 to 49
 Which of the numbers are prime?
 Which of the numbers are square?
 Which of the numbers are composite?

Primes to 100



Reasoning and problem solving

Ron is thinking of a number.



15

I am thinking of a number greater than 10



Use the clues to work out Ron's number.

- It is a composite number.
- It has two prime factors.
- It is an odd number.
- It is a factor of 60

Shade the multiples of 6 on a hundred square.



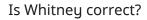
What do you notice about all the numbers either side of the multiples of 6?

> I think that there is always a to a multiple of 6

All the numbers next to a multiple of 6 are odd.

Yes

prime number next



Explain your reasoning.



Square and cube numbers



Notes and guidance

Children encountered square and cube numbers in Year 5, and this small step revisits that learning and the notation for squared (2) and cubed (3).

The concept of square and cube numbers can be supported by making links to area and volume (the formula for the volume of a cuboid will be covered next term).

Children explore the factors of square and cube numbers, noticing that square numbers always have an odd number of factors, but cube numbers can have an odd or even number of factors.

The vocabulary of earlier small steps in this block, such as "factor", "multiple" and "prime" can also be reinforced at this stage.

Things to look out for

- Children may confuse the idea of squaring/cubing with multiplying by 2/3
- Children may not realise that 1 is both a square number and a cube number.

Key questions

- How do you square a number?
- How do you cube a number?
- Are the squares of even/odd numbers even or odd?
- Are the cubes of even/odd numbers even or odd?
- Can a number be both a square number and a cube number?
- How can you use a square number to help find a cube number?

Possible sentence stems

- To square a number, you multiply the number by _____
- To cube a number, you multiply the number by _____ and then by _____ again.
- I know _____ is a square/cube number because ...

National Curriculum links

 Solve problems involving addition, subtraction, multiplication and division

Square and cube numbers



Key learning

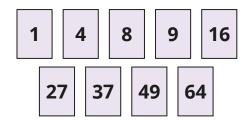
The table shows some square numbers and cube numbers.
 Complete the table and describe any patterns and connections you notice. The first row has been done for you.

1 ²	1 × 1	1	1 ³	1 × 1 × 1	1
					8
	3×3		3 ³		27
	4 × 4			$4 \times 4 \times 4$	
		25	5 ³		
				6×6×6	
8 ²					

• Write >, < or = to make the statements correct.

3 ³		42
8 ²		4 ³
1 ²		5 ³

Here are some number cards.



Which numbers are square?

Which numbers are cube?

Which numbers are both square and cube?

Which numbers are prime?

List the factors of the first five square numbers.

How many factors do they each have?

What do you notice about the number of factors a square number has?

Is the same true for cube numbers?

- **+** \triangle = 38
 - is a cube number.

 \triangle is a prime number.

Find pairs of values for \bigcirc and \triangle .

Square and cube numbers



Reasoning and problem solving

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

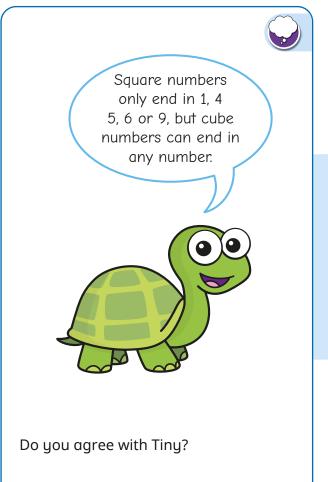
Shade all the square numbers.

Use a different colour to shade the multiples of 4 $\,$

What do you notice?



Square numbers are always a multiple of 4 or one greater than a multiple of 4



Tiny is correct about cube numbers, but square numbers can also end in zero, for example $10^2 = 100$

Multiply up to a 4-digit number by a 2-digit number



Notes and guidance

Building on their learning from previous years, children use long multiplication to multiply numbers with up to four digits by 2-digit numbers.

Children should already be aware that multiplication is commutative, so answers to calculations such as $56 \times 1,234$ can be found by rewriting as $1,234 \times 56$ and using the standard format.

Children also solve word problems and/or multi-step problems. This will be revisited in the next step, where alternative strategies are also explored, for example for multiplying by 9 or 99

Children who require additional support may benefit from revising multiplication of 2- or 3-digit numbers by a single digit before moving on to multiplication by a 2-digit number.

Things to look out for

- Children may omit the zero needed in the second line of a long multiplication.
- Children need to be secure with their times-tables, or have strategies for deriving them.
- When regrouping, children may misapply the procedure, particularly when a large number of digits are involved in the calculation.

Key questions

- How do you set out a long multiplication?
- Which number do you multiply by first?
- What is important to remember when you begin to multiply by the tens digit?
- When do you need to make an exchange? How do you do this?
- What happens if there is an exchange needed in the last step of the calculation?

Possible sentence stems

- To multiply by a 2-digit number, first multiply by the ______,
 then multiply by the _____ and then find the ______
- Multiplying by _____ is the same as multiplying by _____
 and then multiplying the answer by _____

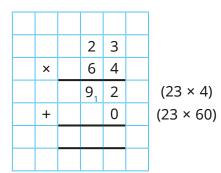
- Multiply multi-digit numbers up to four digits by a 2-digit whole number using the formal written method of long multiplication
- Solve problems involving addition, subtraction, multiplication and division

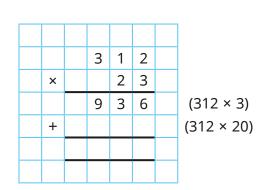
Multiply up to a 4-digit number by a 2-digit number



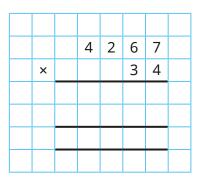
Key learning

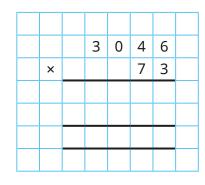
- Work out 43×6 Use your answer to find the answer to 43×60
- Complete the calculations.





• Work out the multiplications.





2,465 people buy tickets for a festival.
 Each ticket costs £48
 How much is spent altogether on the tickets?



• Work out the multiplications.

• Huan receives a new comic book every month.

Each book has 36 pages.

He reads a comic book once a month for 6 years.

How many pages does Huan read altogether?

• There are 27 classes in a school.

There are 32 children in each class.

Can all the children in the school sit in a cinema with 1,000 seats?

If yes, how many spare seats will there be?

If no, how many more seats are needed?

Multiply up to a 4-digit number by a 2-digit number

No



Reasoning and problem solving

The product of a
4-digit number and
a 2-digit number will
always have at least
six digits.

Do you agree with Dexter? Explain your answer.

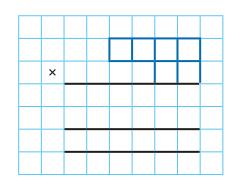
What is the product of the greatest 4-digit number and the greatest 2-digit number?

989,901

234578

Write the digits in the boxes to find the greatest product.

You can use each digit once only.



 $8,432 \times 75 = 632,400$

Solve problems with multiplication



Notes and guidance

In this small step, children use the column method for multiplication and explore alternative strategies for solving multiplication problems, including word problems.

Children use their knowledge of multiplying by powers of 10 and adjust calculations: for example, instead of multiplying a number by 99, they multiply the number by 100 and then subtract the number from the product.

Children explore using factors to find the answers to multiplication problems, such as multiplying by 5 and then by 7 as an alternative to multiplying by 35. This is a useful strategy for children who have good times-table knowledge but make errors with the algorithm for long multiplication.

Things to look out for

- Children may try to use formal methods when alternative strategies would be more appropriate.
- Children may need support to identify the most efficient method, for example × 100 subtract × 1 may be better than × 90 add × 9
- When using the factorisation method, children may forget to multiply the first product by the second factor.

Key questions

- What is the quickest way of multiplying whole numbers by 10/100/1,000?
- What number is 99 close to? How does this help you to multiply by 99?
- If you double a number and then double it again, what is the overall effect on the original number?
- What factor pairs have a product of _____? How does this help you to multiply by _____? Which factor pair is easiest to use?

Possible sentence stems

- To multiply by _____, I can multiply by _____ and add/ subtract _____ to/from the product.
- = ____ × ___, so to multiply by ____ I can
 multiply by ____ and then multiply the product by ____

- Perform mental calculations, including with mixed operations and large numbers
- Solve problems involving addition, subtraction, multiplication and division

Solve problems with multiplication



Key learning

Work out the multiplications.

Use your answers to work out these multiplications.

Office chairs cost £99

A company buys 38 chairs for its offices.

How much does the company pay altogether?

In a sale, the price of the chairs is reduced to £79

How much do 38 chairs cost at the sale price? How can you use your first answer to help you?

• Here is a strategy for multiplying numbers by 5

Multiply the number by 10 and find half of the answer.

Use the strategy to work out the multiplications.

$$84 \times 5$$

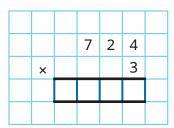
$$8,206 \times 5$$

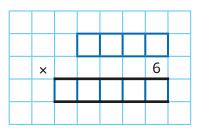
Why does the strategy work?

Explain why $83 \times 4 = 83 \times 2 \times 2$

Find the missing numbers.

Complete the calculations to work out 724×18





Find a different way to work out 724×18

Find the missing numbers.

$$63 \times 24 = 63 \times 3 \times$$

Use both factorisations to work out 63×24

Which strategy did you find easier?

Use similar strategies to work out the multiplications.

$$84 \times 15$$

$$326 \times 45$$

$$612 \times 42$$

$$3,592 \times 32$$

Solve problems with multiplication



Reasoning and problem solving

Alex is working out 6,412 × 16



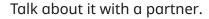
I'm going to keep doubling 6,412 until I have found 6,412 × 16



How many calculations will Alex have to do?

Use Alex's method to find 6,412 × 16

How else could Alex multiply by 16?



four calculations

 $6,412 \times 2 = 12,824$

 $6,412 \times 4 = 25,648$

 $6,412 \times 8 = 51,296$

 $6,412 \times 16 = 102,592$

35 = 1 × 35, so I can work out 832 × 35 by multiplying by 1 and then by multiplying by 35



Explain why Tiny's strategy is not a good one.

Use a different factor pair of 35 to work out 832 × 35

Tiny's strategy is not good because you still have the same calculation of 832 × 35 after multiplying by 1

 $35 = 5 \times 7$

 $832 \times 5 = 4,160$ and $4,160 \times 7 = 29,120$

or

 $832 \times 7 = 5,824$ and

 $5,824 \times 5 = 29,120$

Short division



Notes and guidance

In Year 5, children built on earlier learning of short division and learned to divide numbers with up to four digits by single-digit numbers. This small step reinforces all this earlier learning in preparation for the upcoming steps on long division.

Children perform short divisions both with integer answers and where there is a remainder. They interpret the remainder in context, for example knowing that "4 remainder 1" could mean 4 complete boxes with 1 left over so 5 boxes will be needed.

Children may need to list multiples of the number they are dividing by to help them if their times-table knowledge is not secure.

Things to look out for

- Children need to be confident with their times-tables "both ways", i.e. knowing division facts as well as multiplication facts.
- Children may not recognise sharing and/or grouping division problems when presented in words.
- Numbers with placeholders (e.g. 80,320) may cause difficulty for children.
- Children may not be able to interpret the remainder.

Key questions

- How many groups of 4 _____ are there in 40/400/4,000?
- How many groups of 4 _____ are there in 80/800/8,000?
- What do you do with any remaining ones at the end of a division?
- If you cannot make a group in a column, what do you do?
- What does the remainder mean in this question?

Possible sentence stems

•	thousands divided by is equal to thousands
	with a remainder of
	The remainder is exchanged into hundreds.
	hundreds divided by is equal to hundreds
	with a remainder of
	The remainder is exchanged into tens.

- Solve problems involving addition, subtraction, multiplication and division
- Divide numbers up to four digits by a 2-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

Short division



Key learning

• Work out the divisions mentally.

▶ 8 ÷ 2

80 ÷ 2

800 ÷ 2

 $8,000 \div 2$

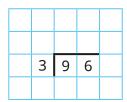
▶ 12 ÷ 4

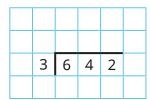
 $120 \div 4$

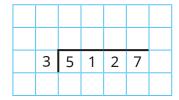
 $1,200 \div 4$

 $1,200 \div 3$

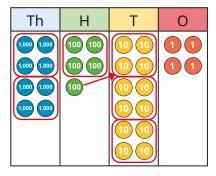
Complete the short divisions.

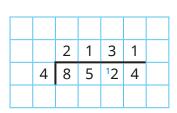






• Here is $8,524 \div 4$ shown using place value counters and short division.





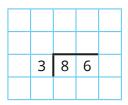
Use this method to work out the divisions.

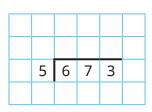
5,520 ÷ 4

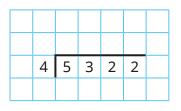
6,432 ÷ 3

2,665 ÷ 5

Complete the short divisions.







1,480 pencils are grouped into packets of 5How many groups of 5 pencils are there?



• 650 children from a school go to a theme park.

On the first ride, each car seats 4 children.

How many cars are needed for the whole school to go on the first ride?

On the second ride, each car seats 6 children.

How many cars are needed for the whole school to go on the second ride?

• Tickets to see the school play cost £9

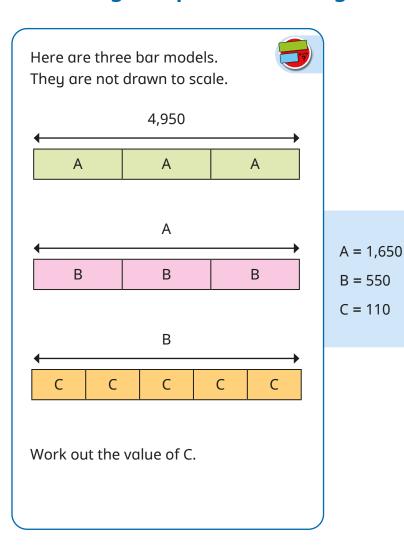
How many tickets can be bought with £100?

How many tickets can be bought with £350?

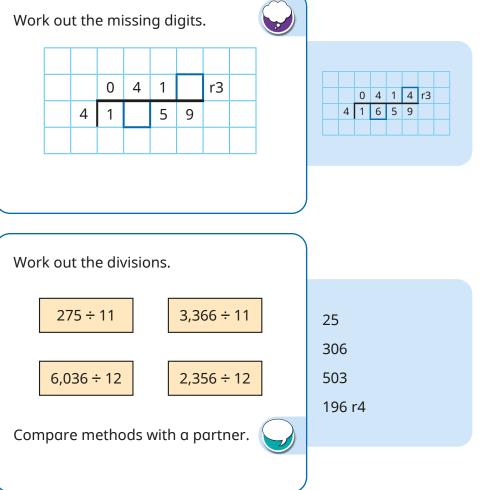
Short division



Reasoning and problem solving







Division using factors



Notes and guidance

In this small step, children build on their understanding of using factors in multiplication and learn to divide by a 2-digit number using repeated division.

Children start with the familiar strategy that to divide by 4 they can halve and halve again. They move on to dividing by multiples of 10 before looking at slightly more complex divisions using two single-digit factors. It may be worth revising what factor pairs are and practising finding factor pairs of 2-digit numbers. Children need to be aware that the divisions can be carried out in any order. This means they can choose to divide first by the factor they find it easier to work with, and then by the factor they find more difficult.

Things to look out for

- Children may partition the number they are dividing by into tens and ones instead of using factors.
- Children may factorise the number they are dividing by incorrectly.
- Children may need support identifying the most efficient pair of factors to use.
- Children may identify 1 and the number itself as a pair of factors and should recognise that this does not simplify the calculation.

Key questions

- What does the word "factor" mean?
- What are the factors of the number you are dividing by?
- What numbers do you find it easy to divide by?
- How can you check your answer?
- Which factor are you going to divide by first/second? Why?

Possible sentence stems

- Dividing by 4 is the same as dividing by _____ and ____ again.
- The factor pairs of _____ are ____
- To divide by ______, I can first divide by _____ and then divide the answer by _____
- ____ = ___ × ____, so to divide by ____ I can divide
 by ____ and then divide the answer by ____

National Curriculum links

 Solve problems involving addition, subtraction, multiplication and division

Division using factors



Key learning

Take 20 counters and share them into two equal groups.
 Share each of these groups into two equal groups.
 How many groups have you got now?
 Complete the calculation.

• Esther is working out $840 \div 4$ She knows $840 \div 2 = 420$

How can Esther use this fact to help find $840 \div 4$?

80 counters are divided into 10 equal groups.
 How many counters are there in each group?
 The counters are then shared into 2 equal groups.
 How many counters are there in each group now?

Complete the calculations.

Work out the divisions.

• Find 720 \div 15 by firstly dividing 720 by 5 and then dividing the result by 3

Why does dividing a number by 5 and then dividing by 3 give you the same answer as dividing the number by 15?

Use this strategy to work out the divisions.

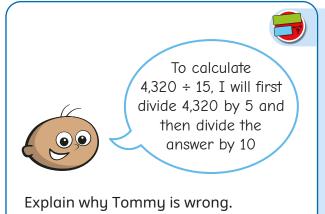
$$570 \div 15$$
 $560 \div 14$
 $720 \div 18$
 $725 \div 25$
 $560 \div 14$
 $1,176 \div 24$

Can any of the divisions be done in more than one way?

Division using factors



Reasoning and problem solving



Tommy has partitioned 15 into 5 + 10 instead of using the factor pair $3 \times 5 = 15$

Dividing by 5 and then dividing by 10 is the same as dividing by 50

Use factor pairs to work out the divisions.

1,248 ÷ 48

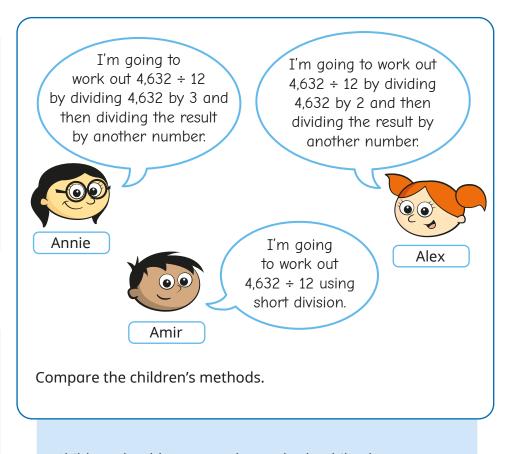
1,248 ÷ 24

1,248 ÷ 12

What do you notice about your answers?

26, 52, 104

When the number you are dividing by is halved, the answer is doubled.



Children should compare the methods while also recognising that each child gets the same answer.

Introduction to long division



Notes and guidance

In this small step, children are introduced to long division as a different method for dividing by a 2-digit number, now including numbers that cannot be factorised into single-digit numbers.

Children divide 3-digit numbers without remainders, using an expanded method that shows the multiples, before progressing to a more formal long division method. They divide 4-digit numbers, still without remainders, using their knowledge of multiplying by 10 and 100. When dividing by composite numbers, it may be worth comparing the long division method with the method of division using factors covered in the previous small step.

Long division with remainders is covered in the next small step.

Things to look out for

- Children may need support in setting out the long divisions, for example by providing the questions on pre-prepared squared grids with the questions already formatted.
- When dividing by prime numbers or large numbers, children may need support in working out the multiples of the number they are dividing by.

Key questions

- How can you use multiples to divide by a 2-digit number?
- Why do we subtract as we go along?
- What does the arrow represent in the long division?
- Can this division be done using factors instead? Why or why not?
- What is the first step when performing a long division?

Possible sentence stems

hundreds divided by is equal to hundreds
with a remainder of
The remainder is exchanged into tens.
tens divided by is equal to with
a remainder of
The remainder is exchanged into ones.

- Divide numbers up to four digits by a 2-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- Solve problems involving addition, subtraction, multiplication and division

Introduction to long division



Key learning

• Here is 432 ÷ 12 using the long division method.

					Multiples of 12: $12 \times 1 = 12$
	0	3	6		$12 \times 2 = 24$
12	4	3	2		12 ^ 2 - 24
	3	6	0	(12 × 30)	$12 \times 3 = 36$
		7	2		$12 \times 4 = 48$
		7	2	(12 × 6)	10 - 10
			0		$12 \times 5 = 60$
					12 × 6 = 72

Use this method to work out the divisions.

• Here is a different way of setting out a long division.

	0	3	6	
12	4	3	2	
	3	6		
		7	2	
		7	Ž 2	
			0	

Use this method to work out the divisions.

• Here is $7,335 \div 15$ using the long division method.

	0	4	8	9	
15	7	3	3	5	
	6	0	0	0	(15 × 400)
	1	3	3	5	
	1	2	0	0	(15 × 80)
		1	3	5	
		1	3	5	(15 × 9)
				0	

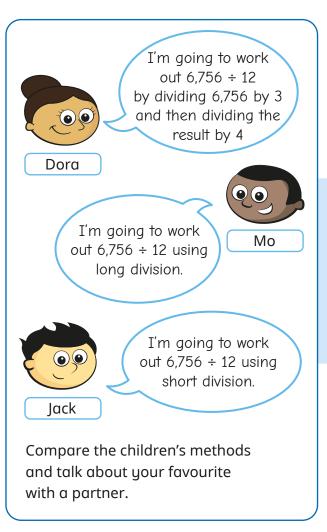
Use this method to work out the divisions.

- There are 1,989 players in a football tournament.
 Each team has 11 players and 2 reserves.
 How many teams are playing in the tournament?
- A farmer packs 8,280 eggs into cartons of 24
 Use long division to find the number of cartons needed.
 Check your answer by dividing by factors.

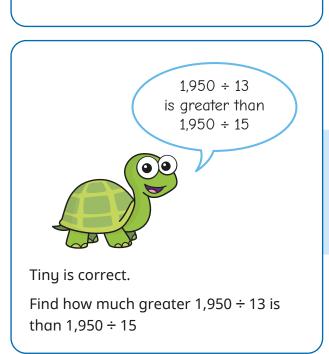
Introduction to long division



Reasoning and problem solving



Children should recognise that each child gets the same answer despite using different methods.

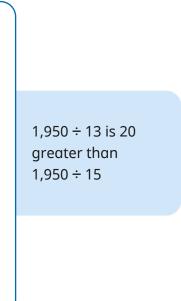


 $6.120 \div 17 = 360$

Use the given calculation to work out

the missing number.

6,480 ÷ ____ = 360



18

Long division with remainders



Notes and guidance

Now that children have learned to use the algorithm for long division with integer answers, they move on to long divisions with remainders.

This small step includes context questions where children interpret the remainder and/or adjust the number they are dividing. For example, when thinking about packing items into boxes, they consider the number of full boxes or the total number of boxes needed.

Children should always check that the remainder is less than the number they are dividing by. They can use estimation as a sense-check for their answers, for example $834 \div 18$ is close to $800 \div 20$ so the answer should be in the region of 40

Things to look out for

- Children may need support in setting out the long divisions, for example by providing the questions on pre-prepared squared grids with the questions already formatted.
- When dividing by prime numbers or large numbers, children may need support in working out the multiples of the number they are dividing by.

Key questions

- Why do we subtract as we go along?
- In a long division, what happens after the subtractions if you cannot divide exactly?
- What is the first step when performing a long division?

Possible sentence stems

1	hundreds divided by is equal to hundreds
	with a remainder of
	The remainder is exchanged for tens.
ı	cannot be divided by, so there is a
	of

- Divide numbers up to four digits by a 2-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- Solve problems involving addition, subtraction, multiplication and division

Long division with remainders



Key learning

• Filip uses multiples to help divide 372 by 15

							Multiples of 15: $15 \times 1 = 15$
	0	2	4	r	12		15 × 2 = 30
15	3	7	2				15 ^ 2 = 30
	3	0	0			(15 × 20)	$15 \times 3 = 45$
		7	2				$15 \times 4 = 60$
		6	0			(15 × 4)	
		1	2				

Use Filip's method to work out the divisions.

Here is Aisha's method for finding 1,426 divided by 13

		0	1	0	9	r	9		
	13	1	4	2	6				
		1	3	0					
			1	2	6				
			1	1	7				
					9				

Use Aisha's method to work out the divisions.

- Mrs Hall needs 380 cupcakes for a party. Cupcakes are sold in boxes of 15 How many boxes of cupcakes does she need to buy? Will she have any cupcakes spare? How do you know?
- One day, a bakery produces 7,849 biscuits. The biscuits are packed into boxes of 64 biscuits. How many full boxes can be packed?



- 576 children and 32 adults need transport for a school trip. A coach has seats for 55 people. How many coaches are needed? How many spare seats will there be?
- A portion of rice is 65 g. How many portions can be served from an 8 kg bag of rice? Will there be any rice left over? If yes, how much?



All the calculations

will have a

reminder.

Long division with remainders



Reasoning and problem solving

Which calculations will definitely have a remainder?



A 8,164 ÷ 20

B 7,836 ÷ 15

C 4,678 ÷ 18

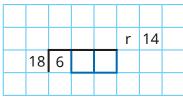
D 6,751 ÷ 12

How do you know?



the division.

Two digits are missing from



The missing digits are equal.

What must they be?

What could the digits be if they were not equal?



4 and 4

2 and 6

6 and 2

8 and 0

9 and 8

$$835 \div 17 = 48 \text{ r} 19$$

Explain why the calculation cannot be correct.

The remainder cannot be greater than 17

Solve problems with division



Notes and guidance

In this small step, children explore division problems, looking at the most appropriate strategy for finding a solution.

As well as providing an opportunity to revisit the learning of the last few steps, children look at alternative methods such as partitioning the number into appropriate multiples of the number they are dividing by. They also use counting up in multiples, for example for calculations such as $1,400 \div 200$, and compare this with other strategies.

Encourage children to think about the numbers in a division question and to consider alternative strategies before they launch into a formal method.

Later in this block, children explore using known division facts to find other division or multiplication facts.

Things to look out for

- Children may try to use formal methods when alternative strategies would be more appropriate.
- Children may try to apply strategies that work for multiplication to division situations where they do not work.
- Interpreting remainders in a given context can be challenging for children.

Key questions

- What is the most useful way of partitioning the number?
- Would you use short division or long division? Why?
- If you double a number and then double it again, what is the overall effect on the original number?
- What factor pairs have a product of _____? How does this help you to divide by _____? Which factor pair is easiest to use?

Possible sentence stems

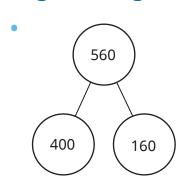
•	I will partition the number into and because
	both and are divisible by
•	= ×, so to divide by I can divide
	by and then divide the quotient by

- Perform mental calculations, including with mixed operations and large numbers
- Solve problems involving addition, subtraction, multiplication and division

Solve problems with division

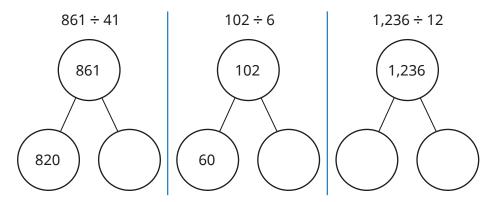


Key learning



Complete the workings for $560 \div 4$

Use partitioning to work out the divisions.



Which of the divisions can you work out mentally?

• Use your preferred method to work out the divisions.

Did you use the same method for each question?

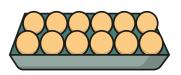
Tom has saved £8 in 20p coins.

How many 20p coins does Tom have?



Eggs are packed in trays of 12
 The trays are packed into boxes.
 Each box contains 480 eggs.

 How many trays are in each box?



A builder needs 8,600 bricks to build a wall.
 There are 800 bricks in a load.

How many loads must the builder buy?

Ron

by 20

400, 720

62, 20,000

Eva's strategy will

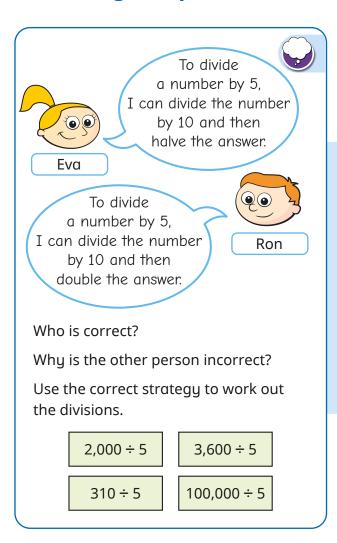
give the result for

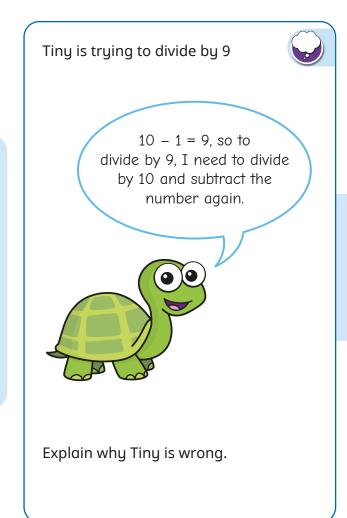
the number divided

Solve problems with division



Reasoning and problem solving





Tiny is confusing strategies for multiplication and division.

Solve multi-step problems



Notes and guidance

In this small step, children apply the skills they have developed so far in this block to solving problems in real-life contexts.

The problems involve more than one calculation and children must decide which operations they need to perform and in what order to perform them; this will need careful modelling. As the focus of the step is making the correct choice of operation, calculators can be provided or the numbers simplified if necessary. Children should be encouraged to think about the best way to perform any of the calculations and use the most appropriate written, informal or mental method. For example, this might include using a number line to work out a subtraction after a long multiplication.

Things to look out for

- In longer problems, children may find the number of words overwhelming and need encouragement to split the problem down into smaller parts.
- Children may find choosing the correct operation difficult.
- Children may need support to set out solutions with several parts clearly.

Key questions

- What can you work out first?
- Is this step an addition, a subtraction, a multiplication or a division? How can you tell?
- Could you draw a diagram to represent the problem?
- Can you work out the answer to this part of the problem mentally or do you need another method?
- What can you do next?

Possible sentence stems

- First, I need to work out _____

 The calculation I need to do is ______
- Next, I need to work out _____

 The calculation I need to do is _____

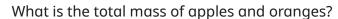
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving addition, subtraction, multiplication and division

Solve multi-step problems



Key learning

- The total mass of apples in a box is 25 kg.
 The total mass of oranges in a box is 24 kg.
 - ► There are 32 boxes of apples and 25 boxes of oranges in a supermarket.



- ► A customer orders 300 kg of apples and 600 kg of oranges. How many boxes of fruit will the customer receive?
- There are 80 g of pasta in one portion.
 How much pasta is needed for 12 portions?
 How many portions can be made from a 16 kg bag of pasta?
- At a parade, there are 25 rows of people with 8 people in each row.
 Each person holds 2 flags.
 How many flags are needed for the parade?
- A coach has 55 seats and a minibus has 17 seats.
 431 people from a school go on a trip.
 The school books 6 coaches and 8 minibuses.
 How many spare seats will there be?

Five boxes of toy trains cost £120
 Each box contains 6 trains.
 How much does each train cost?



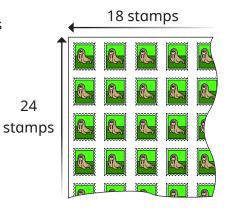
- Dr Patel can type 40 words a minute.
 How many words can she type in an hour?
 How long does it take Dr Patel to type 1,000 words?
- A headteacher has £2,000 to spend on new furniture.
 He wants to buy 15 desks for £79 each and 30 chairs for £29 each.

Does he have enough money?

A sheet of stamps has 24 rows

and 18 columns of stamps.

How many stamps are there altogether on 35 sheets?



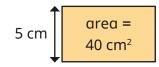
Solve multi-step problems



Reasoning and problem solving

The area of a rectangular tile is 40 cm²

The width of the tile is 5 cm.



A strip of tiles is made by laying tiles end-to-end.



How long is a strip with 15 tiles?

How many tiles are needed to make a strip 280 cm long?

How many tiles are needed to make a strip 4 m long?

120 cm 35 tiles 50 tiles





How many bottles of water can you buy for £30?

How many bottles of water can you buy for £300?

How many bottles of water can you buy for £525?

How much will 600 bottles of water cost?

48 bottles

480 bottles

840 bottles

£375

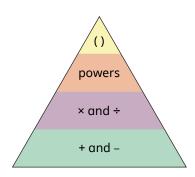
Order of operations



Notes and guidance

In this small step, children learn the order of priority for operations in a calculation: that calculations in brackets should always be done first, and that multiplication and division have equal priority and should be performed before additions and subtractions.

This image may be useful when teaching the order of operations.



Things to look out for

- If children have heard acronyms such as BIDMAS or BODMAS, they may mistakenly think that addition should be done before subtraction and incorrectly work out, for example, 10 3 + 4 as 10 7 = 3
- Similarly, children may not be aware that multiplication and division are of equal priority.

Key questions

- Does it make a difference if you perform the operations in a different order?
- What do brackets in a calculation mean? What would happen if you did not use the brackets?
- Which operation has greater priority, addition or multiplication?
- How many pairs of operations do you know that have equal priority?
- How do you find the square of a number?

Possible sentence stems

has greater priority than,	so the first part of
the calculation I need to do is	

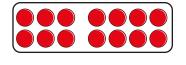
- Perform mental calculations, including with mixed operations and large numbers
- Use their knowledge of the order of operations to carry out calculations involving the four operations

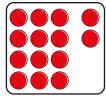
Order of operations

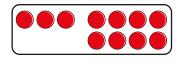


Key learning

Match the counters to the calculations.







$$3 + 4 \times 2$$

$$3 \times 4 + 2$$

$$(3 + 4) \times 2$$

• Draw counters to represent each calculation.

$$(4 + 1) \times 3$$

Work out the answers.

Work out the calculations.

$$(5 + 2) \times 3$$

$$6 + 4 \div 2$$

$$10 - 4 \div 2$$

$$5 + 2 \times 3$$

$$(6 + 4) \div 2$$

$$(10 - 4) \div 2$$

Add brackets to make the calculations correct.

$$\rightarrow$$
 6 + 4 × 3 = 30

$$\triangleright$$
 20 – 20 × 2 = 0

$$12 \times 3 - 1 = 24$$

$$\rightarrow$$
 10 ÷ 2 + 3 = 2

Work out the calculations.

$$6 \times 4 + 5 \times 2$$

$$6 \times 4 - 5 \times 2$$

$$6 \times (4 + 5) \times 2$$

• Dani has 7 bags with 5 sweets in each bag.

She adds one more sweet to each bag.

Which calculation shows how many sweets there are in total?

$$7 \times (5 + 1)$$

$$7 \times 5 + 1$$

Work out the calculations.

$$6^2 - 3 \times 4$$

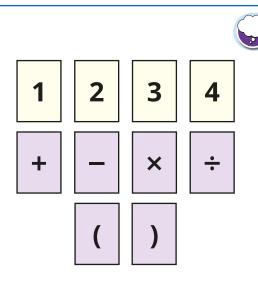
$$6^2 \div (4 + 5)$$

$$(7-4)^2$$

Order of operations

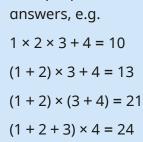


Reasoning and problem solving

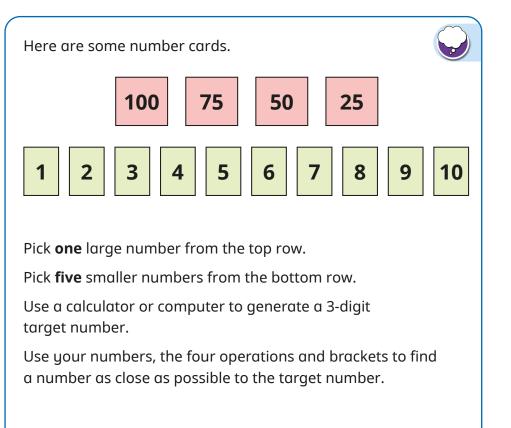


Use the digits and symbols to write as many calculations as you can that give different answers.

Is it possible to make every number from zero to 20?



multiple possible



Compare answers as a class.

Mental calculations and estimation



Notes and guidance

Children should use mental strategies and estimation whenever appropriate, and several examples have been included throughout the block. This small step reminds children of the importance of mental strategies and estimation, and gives them an opportunity to revisit and extend their learning from this block and previous years.

Children should be aware that estimating the answer of a calculation serves as a sense-check on whether their answer is correct, and this can be done either before or after a calculation. The numbers they choose when performing estimates should be simple enough for this to be done mentally.

Links should be made back to previous learning on rounding when simplifying numbers within a calculation.

Things to look out for

- Children may try to use formal methods when alternative strategies would be more appropriate.
- Children may not round numbers to an appropriate degree of accuracy. For example, 4-digit numbers should usually be rounded to the nearest 1,000 and not to the nearest 100 or nearest 10

Key questions

- Should you round the number to the nearest 10/100/1,000?
 Why?
- Are any of the numbers multiples of powers of 10? How does this help you to add/subtract/multiply/divide the numbers?
- What number is (for example) 99 close to? How does this help with the calculation? What adjustment do you need to make?
- How would partitioning/reordering the number(s) help?
- Why are estimates to the answers of calculations useful?

Possible sentence stems

- The previous multiple of _____ is _____
- The next multiple of _____ is _____
- _____ rounded to the nearest _____ is _____

- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- Perform mental calculations, including with mixed operations and large numbers

Mental calculations and estimation



Key learning

• Use rounding to estimate the answer to each calculation.

Compare answers with a partner.

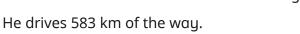
 What strategies would you use to find the exact answers to the calculations?

Compare answers with a partner.

• How could you change the order of the numbers in each of the calculations to make them easier to do mentally?

Work out the answers to the calculations.

• It is 816 km from Mr Trent's house to Glasgow.





Approximately how much further does he have to drive?

- A textbook costs £19.99
 Approximately how many textbooks can be bought for £300?
- Work out the calculations.

12,000 - 3

8,567 - 999

56 × 9

34 × 20

8,000 ÷ 20

8,204 - 6,899

Mo wants to buy a T-shirt for £9.99, a pair of socks for £2.49 and a cap for £8.99





He has £22 in his wallet.

How can he quickly check whether he has enough money?

Mental calculations and estimation



Reasoning and problem solving

Here is a number line.



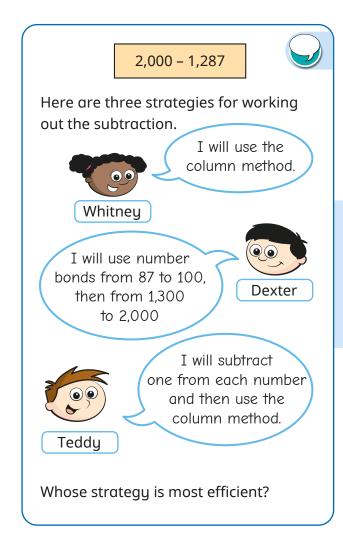


Estimate the number shown by arrow B for these values of A and C:

- A = 0 and C = 1,000
- A = 30 and C = 230
- A = 7 and C = 33
- A = 1 and C = 2
- A = 1,000 and C = 100,000

B is approximately nine-tenths of the way from A to C, so answers should be around:

- 900
- 210
- 30
- 1.9
- 90,000



Children can choose any strategy with the correct justification.

Reason from known facts



Notes and guidance

In this small step, children work out other facts from a given fact using their knowledge of place value, inverse operations, commutativity and the mental strategies practised in this block, particularly in the previous small step. Using diagrams, including area models and number lines, can help children to see the links between the different calculations. They need to be confident in multiplying and dividing by powers of 10. Children also explore the idea of doubling and halving.

It is important that children can not only work out an answer of a related fact, but also explain the connections between calculations that helped them arrive at this answer.

This small step will focus on integers, and decimal calculations will be covered in Spring Block 3

Things to look out for

- Children may try to calculate the answers instead of looking at the relationships between the calculations and using reasoning.
- Children may over-generalise and try to use multiplication strategies that do not work for other operations.
- Children may need support to see the connections between the given fact and the adjusted calculation.

Key questions

- What is an inverse operation?
- How can you use an inverse operation to find related facts?
- What is the same and what is different about the numbers in the given calculation and the numbers in the calculation you want to work out?
- How will the answer change if you increase/decrease/ multiply/divide one/both of the numbers by ______?

Possible sentence stems

- If I add/subtract _____ to/from one of the numbers in the calculation, then the answer will change by _____
- If I multiply/divide _____ one of the numbers in the calculation by _____, then the answer will change by _____

- Perform mental calculations, including with mixed operations and large numbers
- Solve problems involving addition, subtraction, multiplication and division

Reason from known facts



Key learning

• Write four facts shown by each bar model.

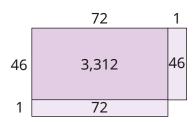
	503		222						
168	335	37	37	37	37	37	37		

• Use the fact that 327 + 482 = 809 to work out the answers to the calculations.

• Use the fact that $11,832 \div 29 = 408$ to work out the answers to the calculations.

• Use the fact that $46 \times 72 = 3{,}312$ to work out the multiplications.

You can use the area model to help you.



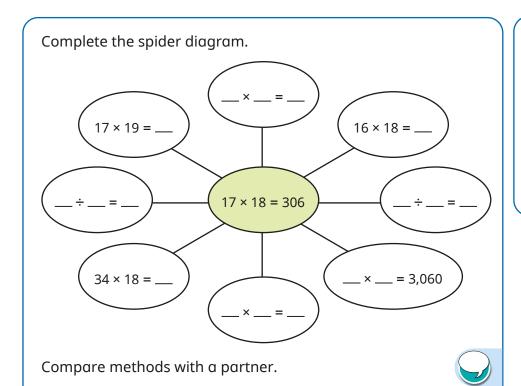
- Use the fact that $5,138 \div 14 = 367$ to work out 15×367
- Use the fact that $14 \times 16 = 224$ to work out the calculations.

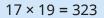
• Work out the missing numbers.

Reason from known facts



Reasoning and problem solving





 $34 \times 18 = 612$

 $170 \times 18 \text{ or } 17 \times 180 = 3,060$

Without working them out, which calculation has the greater answer?

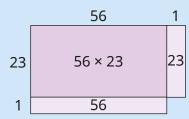


57 × 23

56 × 24

Draw a diagram to explain how you know.

Compare both calculations to 56×23



 56×24 is 56 greater than 56×23

 57×23 is only 23 greater than 56×23

So 56×24 is greater.