# Spring Block 1

# Multiplication and division B



# Small steps

Step 1	Factor pairs
Step 2	Use factor pairs
Step 3	Multiply by 10
Step 4	Multiply by 100
Step 5	Divide by 10
Step 6	Divide by 100
Step 7	Related facts – multiplication and division
Step 8	Informal written methods for multiplication



# **Small steps**

Step 9	Multiply a 2-digit number by a 1-digit number
Step 10	Multiply a 3-digit number by a 1-digit number
Step 11	Divide a 2-digit number by a 1-digit number (1)
Step 12	Divide a 2-digit number by a 1-digit number (2)
Step 13	Divide a 3-digit number by a 1-digit number
Step 14	Correspondence problems
Step 15	Efficient multiplication
	Emelenemanapheadon



# **Factor pairs**

### Notes and guidance

In this small step, children are introduced to factors for the first time. They learn that when they multiply two whole numbers to give a product, both the numbers that they multiplied together are factors of the product. For example,  $3 \times 5 = 15$ , so 3 and 5 are factors of 15. 3 and 5 are also referred to as a "factor pair" of 15

They then generalise this further to conclude that a factor of a number is a whole number that divides into it exactly.

Children create arrays using counters to develop their understanding of factor pairs. It is important for children to work systematically when finding the factor pairs of a number in order to ensure that they find all the factors. For example, when finding factor pairs of 12, begin with  $1 \times 12$ , then  $2 \times 6$ ,  $3 \times 4$ . At this stage, children should recognise that they have already used 4 in the previous calculation, therefore all factor pairs have been identified.

### Things to look out for

- Children may not work systematically, meaning that they could miss some factor pairs.
- Children may find it difficult to understand why not all factors come in pairs, for example  $4 \times 4 = 16$ , so this only gives 1 factor of 16, not 2

### **Key questions**

- How can you use arrays to help you find all the factors of a number?
- How do you know that you have found all the factors of \_\_\_\_\_?
- How do arrays help you to see when a number is not a factor of another number?
- Which number is a factor of every whole number?
- Do factors always come in pairs?
- Do whole numbers always have an even number of factors?

### Possible sentence stems

- \_\_\_\_ = \_\_\_ × \_\_\_\_, so \_\_\_\_ and \_\_\_ are a factor pair of \_\_\_\_
- \_\_\_\_\_ has \_\_\_\_\_ factors altogether.

### **National Curriculum links**

Recognise and use factor pairs and commutativity in mental calculations



# **Factor pairs**

### **Key learning**

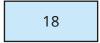
• Complete the factor pairs of 12 and the sentences.







- 12 has \_\_\_\_\_ factor pairs.
- 12 has \_\_\_\_\_ factors altogether.
- Use counters to create arrays and find the factor pairs for each number.





15

• Which of these numbers are factors of 20?

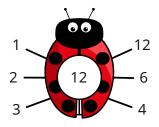
2

5

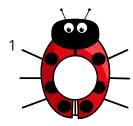
10

Use cubes or counters to show how you know.

• Here is a factor bug for 12



### Complete the factor bug for 20



Draw a factor bug for each number.





16

56

Which of the numbers has an odd number of factors?

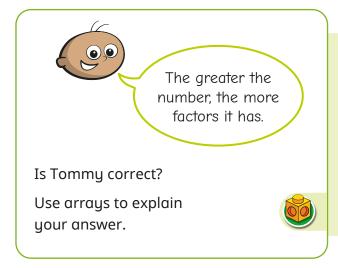
Can you find another number with an odd number of factors?

• Find all the factor pairs of 60



# **Factor pairs**

### Reasoning and problem solving



No multiple possible answers, e.g. 15 has 4 factors and 17 has 2 factors

Is the statement always true, sometimes true or never true?

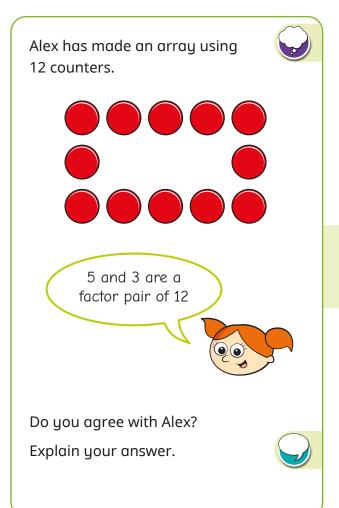


An odd number has an odd number of factors.

Explain your answer to a partner.



sometimes true



No



# Use factor pairs

### Notes and guidance

In this small step, children build on their knowledge of factor pairs from the previous step as they use them to write equivalent calculations. For example, as 3 and 4 are a factor pair of 12, this means that  $5 \times 12$  is equivalent to  $5 \times 3 \times 4$  or  $5 \times 4 \times 3$ 

Children explore equivalent calculations using different factors pairs, and then practise calculating with them to identify which factor pair produces the easiest calculation to complete mentally. The calculation that is deemed easiest will vary for different children, as they are likely to focus on using the times-tables they are most confident with.

### Things to look out for

- Children may need support finding the appropriate factor pairs that will enable them to solve the calculation mentally.
- Children may partition a number rather than finding a factor pair.

### **Key questions**

- How does knowing the factor pairs of 8 help you to find an equivalent calculation to  $7 \times 8$ ?
- For which number are you going to find the factor pairs?
- Which factor pair is the most helpful to solve the calculation?
- In what order are you going to multiply these numbers?
- Does it matter which factor pair you use?

### Possible sentence stems

- The factor pairs of \_\_\_\_\_ are \_\_\_\_
- 12 = \_\_\_\_ × \_\_\_\_, so \_\_\_\_ × 12 = \_\_\_ × \_\_\_ × \_\_\_
- I can use the factor pairs of \_\_\_\_\_ to find an equivalent calculation because ...

### **National Curriculum links**

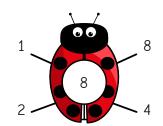
 Recognise and use factor pairs and commutativity in mental calculations



# Use factor pairs

### **Key learning**

• Rosie is working out 7 × 8



I can use a factor pair of 8 to help me.



$$7 \times 8 = 7 \times 4 \times 2 = 28 \times 2$$
  
double 28 is 56,  
so  $7 \times 8 = 56$ 

Use Rosie's method to work out the multiplications.

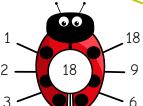
• Use your knowledge of factor pairs to complete the calculations.

Could you have used different factor pairs?

Which factor pairs are the most helpful for each calculation?

• Mo is working out 18 × 3

I can find the factor pairs of 18 to help me.



1 and 18 2 and 9 3 and 6

Mo chooses to use the factor pair 3 and 6



 $18 \times 3 = 3 \times 6 \times 3$ = 3 \times 3 \times 6 = 9 \times 6 = 54  $18 \times 3 = 54$ 

Use Mo's method to work out the multiplications.

18 × 5

14 × 3

16 × 4

There are 15 children in Class 4

Each child gets 3 sweets.

How many sweets are there altogether?



# Use factor pairs

### Reasoning and problem solving

Is the statement true or false?

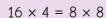
 $15 \times 4 = 10 \times 5 \times 4$ 

Explain your answer.



False

Is the statement true or false?



Use factor pairs to explain your answer.

True

 $16 \times 4 = 8 \times 2 \times 4$  $= 8 \times 8$ 

Whitney wants to use factor pairs to work out  $13 \times 8$ 



The only factor pair of 13 is 1 and 13, so I cannot use factor pairs for this multiplication.

Is Whitney correct?

Explain your answer.

Work out the multiplication.

No

104

Tiny is working out 17 × 3



I am going to use factor pairs to help me.



Will Tiny's method help?

Explain why.

No

### White Rose Maths

# Multiply by 10

### Notes and guidance

In this small step, children explore multiplying by 10. They need to be able to visualise making a number 10 times the size and understand that "10 times the size" is the same as "multiply by 10".

Children use their understanding that 1 ten is 10 times the size of 1 one and 1 hundred is 10 times the size of 1 ten to support them with this step. A place value chart is useful to show this. They recognise that when multiplying by 10 the digits move one place value column to the left and zero is needed as a placeholder in the now blank column. While children may notice a zero is always used as a placeholder when multiplying a whole number by 10, it is important that they do not develop the misconception that they just add a zero to multiply by 10, as this will cause confusion when multiplying decimals in later learning.

### Things to look out for

- Children may move only one digit and misplace the placeholder, for example  $45 \times 10 = 405$
- Children may not realise that calculations of the form 10 × \_\_\_\_\_ and \_\_\_\_ × 10 can be carried out in the same way.

### **Key questions**

- What do you notice when multiplying by 10?
- What is a placeholder? When do you use placeholders?
- What happens to the digits in a number when you multiply by 10?
- How can you use a place value chart to show multiplying — by 10?
- What is \_\_\_\_\_ multiplied by 10?
- What is 10 lots of \_\_\_\_\_?

### Possible sentence stems

- \_\_\_\_\_× 10 = \_\_\_\_\_
- 10 × \_\_\_\_ = \_\_\_\_
- \_\_\_\_\_ is 10 times the size of \_\_\_\_\_

### **National Curriculum links**

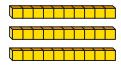
- Recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000 (Y5)



### **Key learning**

• Use the base 10 to complete the sentences.





What do you notice?

• Use base 10 to complete the number sentences.

• Mo represents 21 × 10 using place value counters.



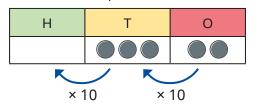
I need to exchange to find the answer.

What exchanges does Mo need to make?

What is  $21 \times 10$ ?

• Use place value counters to complete the multiplications.

Dexter uses a place value chart to work out 32 × 10



I can see that when I multiply by 10, all the counters move one place to the left on a place value chart.



Н	Т	0

 $32 \times 10 = 320$ 

What do you notice?

Use Dexter's method to work out the multiplications.



### Reasoning and problem solving

Aisha multiplies a whole number by 10



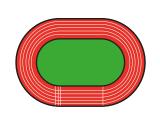
Her answer is between 440 and 540

What number could Aisha have multipled by 10?

How many possibilities can you find?



any number between 45 and 53



880 m

Filip runs 80 m.

Kim runs 10 times as far.

How far do they run altogether?

Is the statement always true, sometimes true or never true?

If you write a whole number in a place value chart and multiply it by 10, all the digits move one column to the left.

Talk about your answer with a partner.



always true

Max and Tiny have some money.



Tiny has 50p.

I have ten times as much money as Tiny.



How much money does Max have?

£5



### Notes and guidance

Building on the previous step, children learn to multiply whole numbers by 100, understanding that this is the same as multiplying by 10 and then multiplying by 10 again. They need to be able to visualise making a number 100 times the size and understand that "100 times the size" is the same as "multiply by 100".

Children use a place value chart, counters and base 10 to explore what happens to the values of the digits when multiplying by 100. Encourage children to recognise that when multiplying whole numbers by 100, the digits move two place value columns to the left and zeros are needed as placeholders in the now blank columns. As with multiplying by 10 in the previous step, it is important that they do not develop the misconception that they just add two zeros to multiply by 100, as this will cause confusion when multiplying decimals by 100

### Things to look out for

- Children may move only some of the digits and misplace the placeholder, for example  $45 \times 100 = 4,005$
- Children may need support to recognise that multiplying by 100 is the same as multiplying by 10 and multiplying by 10 again.

### **Key questions**

- What do you notice when multiplying by 100?
- How can you use multiplying by 10 to help you multiply by 100?
- What happens to the digits when you multiply by 100?
- How can you use a place value chart to show multiplying \_\_\_\_\_\_
   by 100?
- What is \_\_\_\_\_ multiplied by 100?
- What is 100 lots of \_\_\_\_\_?

### Possible sentence stems

- \_\_\_\_\_ × 100 = \_\_\_\_ × 10 × 10 = \_\_\_\_ × 10 = \_\_\_\_
- \_\_\_\_\_ × 100 = \_\_\_\_\_ , so 100 × \_\_\_\_ = \_\_\_\_
- \_\_\_\_\_ is 100 times the size of \_\_\_\_\_

### **National Curriculum links**

- Recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000 (Y5)



### **Key learning**

• Use the base 10 to complete the number sentences.



• Complete the number sentences.

• There are 8 jars.

Each jar contains 100 drawing pins.

How many drawing pins are there altogether?













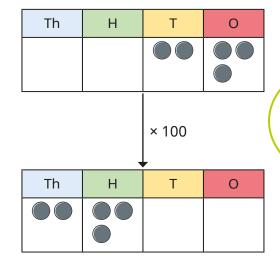




• Work out the multiplications.

What do you notice?

Dora uses a place value chart to work out 23 × 100





I can see that
when I multiply by 100,
all the counters move
two places to the left on
a place value chart.

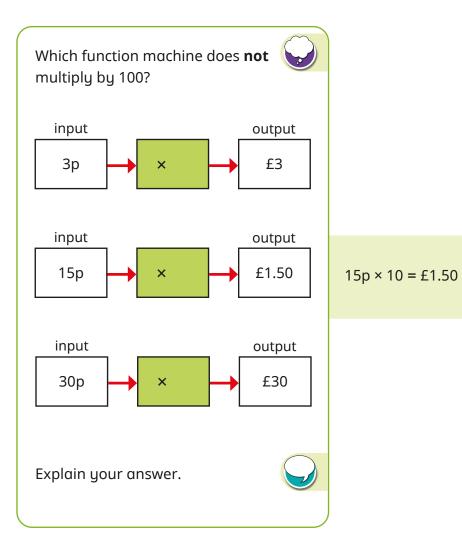
Use Dora's method to work out the multiplications.

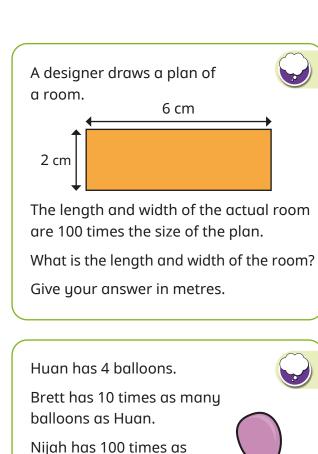
Write <, > or = to compare the multiplications.





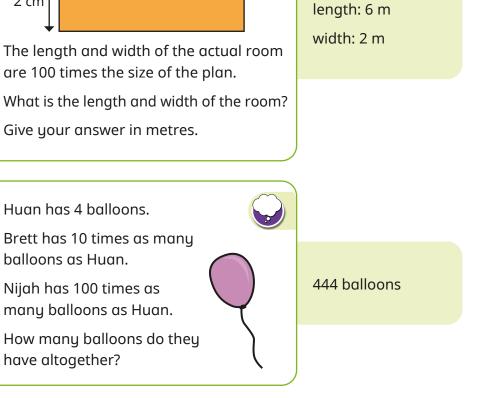
### Reasoning and problem solving





many balloons as Huan.

have altogether?





### Notes and guidance

In this small step, children divide whole numbers by 10, with questions that only have whole number answers. They need to be able to visualise making a number one-tenth the size and understand that "one-tenth the size" is the same as "dividing by 10".

Children use concrete resources and a place value chart to see the link between dividing by 10 and the position of the digits of a number before and after the calculation. They recognise that when dividing by 10, the digits move one place value column to the right. They begin to understand that multiplying by 10 and dividing by 10 are the inverse of each other.

Children may notice that in all the examples they see, they need to "remove the zero" to find the answer. Ensure that they do not generalise this too far and use it as their method, as this will cause issues in later learning when looking at decimals.

### Things to look out for

- Children may incorrectly conclude that to divide by 10, they always just remove a zero from the number.
- Children may confuse multiplying and dividing by 10, and move the digits in the wrong direction in a place value chart.

### **Key questions**

- What do you notice when dividing by 10?
- Why does this happen?
- What happens to the digits when you divide by 10?
- How can you use a place value chart to show dividing \_\_\_\_\_\_ by 10?
- What is \_\_\_\_\_ divided by 10?
- What number is one-tenth the size of \_\_\_\_\_?

### Possible sentence stems

- \_\_\_\_ ÷ 10 = \_\_\_\_
- \_\_\_\_ = \_\_\_ ÷ 10
- \_\_\_\_\_ is one-tenth the size of \_\_\_\_\_

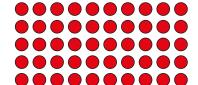
### **National Curriculum links**

- Recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000 (Y5)



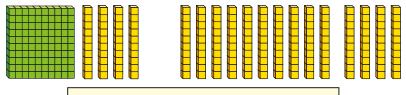
### **Key learning**

Complete the calculation shown by the array.



Draw arrays to help you complete the divisions.

Sam uses base 10 to divide 140 by 10



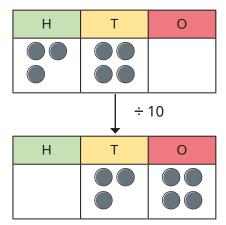
140 = 1 hundred and 4 tens 1 hundred = 10 tensThere are 14 groups of 10

 $140 \div 10 = 14$ 

Use Sam's method to complete the divisions.

• Jack uses a place value chart to work out  $340 \div 10$ 





I can see that when I divide by 10, all the counters move one place to the right on a place value chart.

$$340 \div 10 = 34$$

Use Jack's method to work out the divisions.

- Ten friends share some money equally from a money box.
  - ▶ How much would they each have if the box contained:
    - twenty £1 coins

After emptying the box and sharing the contents equally, each friend has 90p.

How much money was in the box?



### Reasoning and problem solving

Scott, Tom, Esther and Dani are in a race.



Here are the numbers on their vests.

350

35

3,500

53

Use the clues to match each vest number to a child.

- Scott's number is one-tenth the size of Tom's.
- Nobody has a number that is 10 times the size of Esther's.
- Dani's number is one-tenth the size of Scott's.

Scott: 350

Tom: 3,500

Esther: 53

Dani: 35

Mr Rose is buying furniture.



To make sure it will fit in the room, he decides to draw a plan.

The actual size of everything is 10 times the size that it is on the plan.

He makes a table to show the measurements.

Item	Actual size	Plan size
Bed length	200 cm	2,000 cm
Desk length	120 cm	12 cm
Wardrobe height	1,850 mm	185 mm

Are Mr Rose's plan measurements correct?

Explain your answers.



The length of the room is 240 cm.

How long will it be on the drawing?

bed: incorrect

desk: correct

wardrobe: correct

24 cm



### Notes and guidance

In this small step, children build on their understanding of dividing by 10 and notice the link between dividing by 10 and dividing by 100. They need to be able to visualise making a number one-hundredth the size and understand that "one-hundredth the size" is the same as "dividing by 100".

Children use concrete resources and a place value chart to see the link between dividing by 100 and the position of the digits before and after the calculation. They realise that when dividing by 100, the digits move two place value columns to the right. They begin to understand that multiplying by 100 and dividing by 100 are the inverses of each other.

Money is a good real-life context for this small step, as exchanging, for example, pounds for pence can be used for the concrete stage.

### Things to look out for

- Children may need support in recognising that onehundredth the size is the same as dividing by 100
- Children may divide by 10 instead of 100
- Children may confuse multiplying and dividing by 100, and move the digits in the wrong direction.

### **Key questions**

- What happens when you divide a number by 10 and then divide the answer by 10 again? How does the final answer compare to the original number?
- How can you use dividing by 10 to help you divide by 100?
- What happens to the digits in a number when you divide by 100?
- How can you use a place value chart to show dividing \_\_\_\_\_\_by 100?
- What is \_\_\_\_\_ divided by 100?
- What number is one-hundredth the size of \_\_\_\_\_?

### Possible sentence stems

- \_\_\_\_\_ ÷ 100 = \_\_\_\_\_ ÷ 10 ÷ 10 = \_\_\_\_\_ ÷ 10 = \_\_\_\_\_
- \_\_\_\_\_ ÷ 100 = \_\_\_\_\_ ; so \_\_\_\_ = \_\_\_\_ ÷ 100
- \_\_\_\_\_ is one-hundredth the size of \_\_\_\_\_

### **National Curriculum links**

- Recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000 (Y5)



### **Key learning**

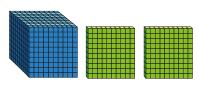
• Use the ten frame and counters to complete the sentences.

100	100	100	100	

There are \_\_\_\_\_ groups of 100 in 400

Use counters to complete the divisions.

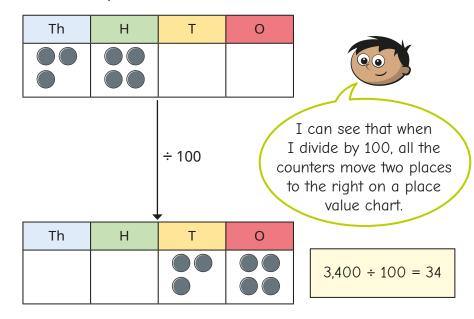
Teddy uses base 10 to work out 1,200 divided by 100



1,200 = 1 thousand and 2 hundreds 1 thousand = 10 hundredsThere are 12 groups of 100  $1.200 \div 100 = 12$ 

Use Teddy's method to complete the divisions.

• Amir uses a place value chart to work out 3,400 ÷ 100



Use Amir's method to work out the divisions.

Kim has collected 800 1p coins. How much money has Kim collected altogether? Give your answer in pounds.



### Reasoning and problem solving

Alex and Tommy are dividing numbers by 10 and 100



They both start with the same 4-digit number.



My answer has 8 ones and 2 tens.

Alex

My answer has 2 hundreds, 8 tens and 0 ones.



Tommy

What number did Alex and Tommy both start with?

Who divided by what?

2,800

Alex: 100

Tommy: 10

Use the digits 1 to 9 to complete the calculations.



$$20 \times 10 = 3, 00$$

$$1.8\_0 \div 10 = 1\_6$$

$$_{9} \times 100 = 5, _{0}$$

$$6$$
\_\_ =  $6,400 \div 100$ 

 $170 \div 10 = 17$ 

 $320 \times 10 = 3,200$ 

 $1,860 \div 10 = 186$ 

 $59 \times 100 = 5,900$ 

 $64 = 6,400 \div 100$ 

Without working out the answers, use <, > or = to compare the calculations.

$$3,600 \div 10$$
  $3,600 \div 100$ 

$$2,700 \div 100$$
  $270 \div 10$ 

Explain your reasoning.



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# Related facts – multiplication and division

### Notes and guidance

In this small step, children bring together the skills learnt so far in this block as they explore calculations related to known facts.

Children explore scaling facts by 10 and 100, for example using the fact that  $4 \times 7 = 28$  to derive  $4 \times 70 = 280$  and  $4 \times 700 = 2,800$ . They then look at this relationship with division, for example using  $12 \div 3 = 4$  to derive  $120 \div 3 = 40$  and  $1,200 \div 3 = 400$ . Care should be taken to ensure that children do not also think that  $12 \div 30 = 40$ . This is a good opportunity to remind children that multiplication is commutative, but division is not.

A range of representations are used to make the link between multiples of 1, 10 and 100 that will be familiar to children from previous steps in this block and in Year 3

### Things to look out for

- Children may derive incorrect division facts by using the rules that they have learnt about related multiplication facts.
- Children may try to find results by calculation rather than recognising the relationship between one fact and another.

### **Key questions**

- What is the same and what is different about the two calculations?
- How can you represent the calculation using place value counters?
- How does knowing that \_\_\_\_\_ is 10 times the size of \_\_\_\_\_
   help you to complete the calculation?
- What calculation do you know that would help with this one?

### Possible sentence stems

- \_\_\_\_ × \_\_\_\_ ones is equal to \_\_\_\_ ones,
   so \_\_\_\_ × \_\_\_ tens is equal to \_\_\_\_ tens.
- \_\_\_\_\_ ÷ \_\_\_\_ is equal to \_\_\_\_\_,
  so \_\_\_\_\_ tens ÷ \_\_\_\_\_ is equal to \_\_\_\_\_ tens.

### **National Curriculum links**

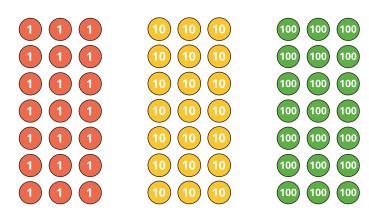
• Solve problems involving multiplying and adding, including using the distributive law to multiply 2-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects



# Related facts – multiplication and division

### **Key learning**

• Write two multiplication facts and two division facts represented by each array.

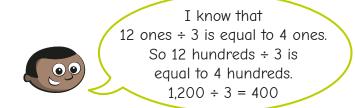


What is the same and what is different about the arrays?



Use Max's method to complete the calculations.

• Mo is working out  $1,200 \div 3$ 



Use Mo's method to work out the divisions.

- It costs £30 for one ticket to the zoo.
   How much do 7 tickets cost?
   How many tickets can you buy for £300?
- There are 120 children in Year 4
   The children are put into groups of 4
   How many groups are there altogether?



# Related facts – multiplication and division

### Reasoning and problem solving

9 friends are going to a theme park and having lunch.



Tickets to the theme park cost £30 each.

Lunch costs £10 each.

Six of the friends share the cost between them.

How much do they each pay?

£60

<

Is the statement true or false?

 $6 \times 800 = 8 \times 600$ 

Explain your answer.





I know that  $5 \times 9 = 45$ , so
I also know all these
other facts.

$$5 \times 90 = 450$$
  $450 \div 9 = 50$ 

$$500 \times 9 = 4,500$$
  $4,500 \div 9 = 500$ 

Do you agree with Tiny?

Explain your answer.



Write <, > or = to compare the calculations.



$$800 \div 2 \left( \right) 800 \div 4$$

Did you need to work them out?



Yes



# Informal written methods for multiplication

### Notes and guidance

In this small step, children use a variety of informal written methods to multiply a 2-digit number by a 1-digit number.

Children follow a clear progression of methods and representations to support their understanding. They begin by using place value charts to recognise multiples of a number and make the link to repeated addition.

The use of base 10 encourages children to partition the tens and ones and unitise the tens, laying the foundations for later work. Part-whole models are used to illustrate the informal method of partitioning. Children use number lines, along with their knowledge of multiplying by 10. For example, to work out  $32 \times 4$  they count along a number line to show  $10 \times 4 + 10 \times 4 + 10 \times 4 + 2 \times 4$ . They may also use their knowledge of factor pairs from earlier in the block to multiply.

### Things to look out for

- Children may not use the correct place value, multiplying tens as ones, for example  $34 \times 6 = 3 \times 6 + 4 \times 6$
- Children may conflate the partitioning and factorising methods, for example when calculating  $4 \times 18$ , they may do  $4 \times 9 + 4 \times 2$

### **Key questions**

- What is the same and what is different about multiplying by 1s and multiplying by 10s?
- How would you explain this method?
- What is the most efficient way to work out \_\_\_\_\_ × \_\_\_\_?
- How could you use a number line to work out this calculation?
- How could you use a part-whole model to partition into tens and ones?

### Possible sentence stems

- \_\_\_\_\_ partitioned into tens and ones is \_\_\_\_\_ and \_\_\_\_\_
- \_\_\_\_\_ × \_\_\_ = \_\_\_\_ tens × \_\_\_\_ + \_\_\_ ones × \_\_\_\_ = \_\_\_\_ tens + \_\_\_\_ ones = \_\_\_\_

### **National Curriculum links**

- Solve problems involving multiplying and adding, including using the distributive law to multiply 2-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects
- Recognise and use factor pairs and commutativity in mental calculations



# Informal written methods for multiplication

### **Key learning**

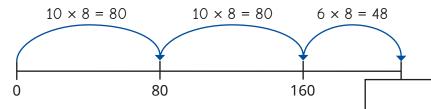
• Aisha uses base 10 to work out 3 × 26

Tens	Ones

$$3 \times 26 = 60 + 18 = 78$$

Use Aisha's method to work out the multiplications.

• Teddy is using a number line to work out 8 × 26

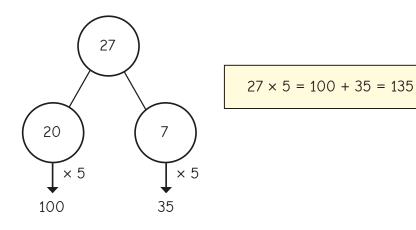


Complete the number line.

Use Teddy's method to work out the multiplications.

• Ron is working out 27 × 5

He partitions 27 into 20 and 7 and records this on a part-whole model.



Use Ron's method to work out the multiplications.

There are 7 classes in a school.

Each class has 26 children.

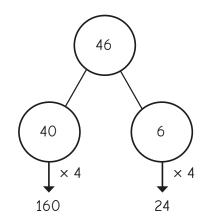
How many children are there altogether?

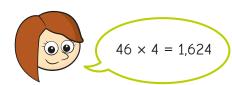


# Informal written methods for multiplication

### Reasoning and problem solving

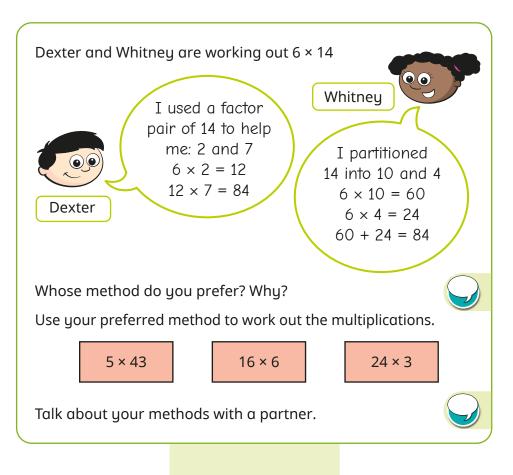
Rosie is using a part-whole model to work out 46 multiplied by 4





What mistake has Rosie made?
What is the correct answer?

She has multiplied the parts correctly, but added them up incorrectly.



215, 96, 72



# Multiply a 2-digit number by a 1-digit number

### Notes and guidance

In this small step, children progress from multiplying using informal written methods to the formal written method. The short multiplication method is introduced for the first time, initially in an expanded form and then in the formal short single-line form.

Children first do calculations where there are no exchanges, then move on to one and two exchanges. Place value counters in place value charts are used to illustrate the structure of the short multiplication by presenting the concrete model alongside the formal written method.

Concrete manipulatives alongside abstract calculations are particularly useful to support children's understanding of exchanges.

### Things to look out for

- Children may exchange ones or tens incorrectly, often by missing zeros or including zeros erroneously.
- Children may not include digits created through exchanging, either by not writing them down when completing the exchange or neglecting to include them in the calculation afterwards.
- When exchanges are performed, if digits are written in the incorrect place, this can lead to errors with the rest of the calculation.

### **Key questions**

- What is the same and what is different about multiplying by 1s and multiplying by 10s?
- How does the written method match the representation?
- Which column should you start with?
- What is the same and what is different about the different methods?

### Possible sentence stems

- ones × \_\_\_\_ = \_\_\_ ones,tens × \_\_\_ = \_\_ tens
- To multiply a 2-digit number by \_\_\_\_\_, you multiply
   the \_\_\_\_\_ by \_\_\_\_
- \_\_\_\_\_ tens multiplied by \_\_\_\_\_ plus the ten I exchange is equal to \_\_\_\_\_ tens.

### **National Curriculum links**

 Multiply 2-digit and 3-digit numbers by a 1-digit number using formal written layout

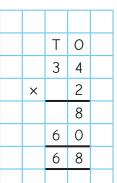


# Multiply a 2-digit number by a 1-digit number

### **Key learning**

 Dora uses place value counters alongside the written multiplication to work out 34 × 2

Tens	Ones
10 10 10	1111
10 10 10	1111

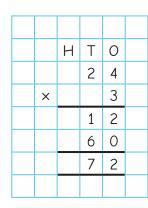


 $(4 \times 2 = 8)$  $(30 \times 2 = 60)$ 

Use Dora's method to work out the multiplications.

 Jo uses place value counters to work out 24 × 3

Tens	Ones
10 10	1111
10 10	1111
10 10	1111

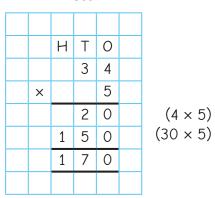


 $(4 \times 3)$  $(20 \times 3)$ 

Use Jo's method to work out the multiplications.

Brett and Scott have each worked out 34 × 5

### **Brett**



### Scott



- What is the same about their methods?
- What is different about their methods?
- Whose method is more efficient?
- Complete the multiplications.

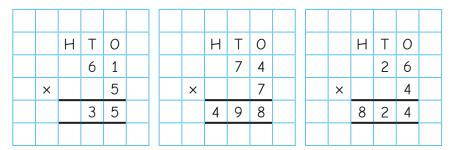
	Н	T	0			Н	Т	0			Н	T	0	
		4	3				3	6				7	4	
×			5		×			4		×			5	



# Multiply a 2-digit number by a 1-digit number

### Reasoning and problem solving

Here are three incorrect multiplications.



What mistakes have been made?

Complete the calculations correctly.



ſ		Н	Τ	0			Н	Τ	0			Н	Т	0	
			6	1				7	4				2	6	
ľ	×			5		×			7		×			4	
I		3	0	5			5	1	8			1	0	4	
		3						2					2		

Are the statements always true, sometimes true or never true?

When multiplying a 2-digit number by a 1-digit number, the product

has three digits.

When multiplying a 2-digit number by 8, the product is an odd number.

When multiplying a 2-digit number by 7, you will need to complete an exchange.

Explain how you know.



sometimes true

never true

sometimes true



# Multiply a 3-digit number by a 1-digit number

### Notes and guidance

Following on from the previous step, children extend the formal written method to multiplying a 3-digit number by a 1-digit number. They continue to use the short multiplication method, but now with more columns. Children need to be secure with the previous step before moving on to this one.

Place value counters in place value charts are again used to model the structure of the formal method, allowing children to gain a greater understanding of the procedure, particularly where exchanges are needed. They continue to use the counters to exchange groups of 10 ones for 1 ten and also exchange 10 tens for 1 hundred and 10 hundreds for 1 thousand. This is mirrored by the positioning of the exchanged digit in the formal written method.

The focus here is on the short written method, but the expanded method could be used to support understanding for children who need it.

### Things to look out for

- The use of a zero in the ones or tens column can sometimes expose misunderstandings, as children can be unsure of multiplying by zero.
- Children may omit the exchange or include the exchange in an incorrect place on the formal written method.

### **Key questions**

- How could you use counters to represent the multiplication?
- How does the written method match the representation?
- Which column should you start with?
- Do you need to make an exchange? What exchange can you make?
- What is the same and what is different about multiplying a 3-digit number by a 1-digit number and multiplying a 2-digit number by a 1-digit number?

### Possible sentence stems

- ones × \_\_\_\_ = \_\_\_ ones
   tens × \_\_\_ = \_\_\_ tens
   hundreds × \_\_\_ = \_\_ hundreds
- tens/hundreds multiplied by \_\_\_\_\_ plus the ten/ hundred from the exchange is equal to \_\_\_\_\_

### **National Curriculum links**

 Multiply 2-digit and 3-digit numbers by a 1-digit number using formal written layout

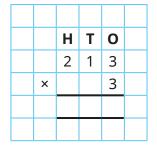


# Multiply a 3-digit number by a 1-digit number

### **Key learning**

• Use the place value chart to help you complete the calculation.

Hundreds	Tens	Ones
100 100	10	111
100 100	10	111
100 100	10	111



• Use the place value chart to help you complete the calculation.

Hundreds	Tens	Ones
100 100 100	10 10	
100 100 100	10 10	
100 100 100	10 10	
100 100 100	10 10	

	Th	Н	T	0	
		3	2	0	
×				4	

• Use place value counters and the written method to work out the multiplications.

420 × 3

4 × 601

2 × 530

A school has 4 house teams.

There are 234 children in each house team.

How many children are there altogether?

Hundreds	Tens	Ones
100 100	10 10 10	1111
100 100	10 10 10	1111
100 100	10 10 10	1111
100 100	10 10 10	1111

	Н	Т	0	
	2	3	4	
×			4	

• Complete the calculations.

	Н	T	0			Н	Т	0			Н	T	0	
	2	0	5			1	4	8			7	4	6	
×			3		×			6		×			5	

Dani reads 164 pages of a book.

Tom reads 3 times as many pages as Dani.

How many pages does Tom read?

How many pages do they read altogether?



# Multiply a 3-digit number by a 1-digit number

### Reasoning and problem solving

Sam and Jack have both completed the same multiplication.

### Sam

	Th	Н	Т	0	
		2	3	4	
×				6	
	1	2	0	4	
		2	2		

Jack

J							
		Th	Н	Т	0		
			2	3	4		
	X				6		
		1	4	0	4		
			2	2			

Who has the correct answer?

What mistake did the other child make?

Jack

Sam did not add the 2 hundreds that she exchanged from the tens column. Arrange the digit cards in the multiplication.





6





What is the greatest possible product?

Now arrange the cards to make the smallest possible product.

What strategies did you use?



 $642 \times 8 = 5,136$ 

 $468 \times 2 = 936$ 

 $321 \times 3 = 963$ 



Without working it out, which would be greater, 321 × 4 or 322 × 3?

Check your answer by working it out.

321 × 4



# Divide a 2-digit number by a 1-digit number (1)

### Notes and guidance

In this small step, children use their division facts from the Autumn term to build on their knowledge of dividing a 2-digit number by a 1-digit number from Year 3

Initially, children carry out divisions where the tens and ones are both divisible by the number being divided by without any remainders, for example  $96 \div 3$  and  $84 \div 4$ . They then move on to calculations where they need to exchange between tens and ones, for example  $96 \div 4$ . Place value counters are used to explore the sharing structure of division. Children do not need to use the formal short division method at this stage and may use informal jottings or representations such as a part-whole model to record their working instead.

### Things to look out for

- Children may partition the 2-digit number correctly, but then divide the tens as if they are ones, for example  $96 \div 3 = 9 \div 3 + 6 \div 3$
- Instead of using their times-tables knowledge, children may revert to less efficient methods such as drawing circles, then drawing dots to share between the circles.
- Children may always partition into tens and ones when other forms of partitioning are more appropriate.

### **Key questions**

- How do you partition a 2-digit number into tens and ones?
   How else can you partition a 2-digit number?
- Which is the most efficient way to partition the number so you can divide both parts by \_\_\_\_\_\_?
- If you cannot share all of the tens equally, what do you need to do?
- How can you represent the division using a part-whole model?

### Possible sentence stems

- \_\_\_\_\_ tens divided by \_\_\_\_ = \_\_\_ tens each
- \_\_\_\_\_ ones divided by \_\_\_\_\_ = \_\_\_\_ ones each
- I cannot share all of the tens equally, so I need to ...

### **National Curriculum links**

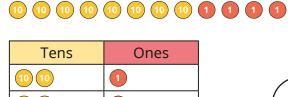
- Recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers



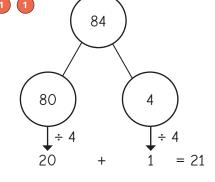
# Divide a 2-digit number by a 1-digit number (1)

### **Key learning**

Teddy uses a place value chart to divide 84 by 4



Tens	Ones
10 10	1
10 10	1
10 10	1
10 10	1



Use Teddy's method to work out the divisions.

• Complete the calculations.

Eva uses place value counters to work out 96 divided by 4
 First, she divides the tens.

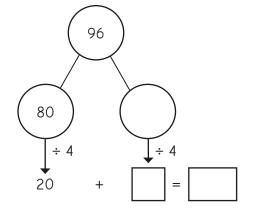
She has one ten remaining.







Tens	Ones
10 10	
10 10	
10 10	
10 10	



- What should Eva do with the remaining ten?Complete Eva's workings.
- ▶ Use Eva's method to work out the divisions.



# Divide a 2-digit number by a 1-digit number (1)

### Reasoning and problem solving





Yes

Do you agree with Tiny?

Explain your answer.



Write <, > or = to compare the calculations.

$$69 \div 3$$
  $96 \div 3$ 

$$96 \div 4$$
  $96 \div 3$ 

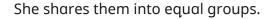
$$91 \div 7 () 84 \div ($$

<

<

<

Kim has 96 sweets.

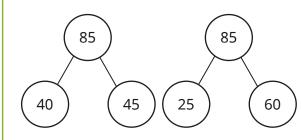


She has no sweets left over.

How many equal groups could Kim have shared her sweets into?

1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48 or 96 groups

Here are two ways of partitioning 85 to help work out  $85 \div 5$ 



What other ways could you partition 85 to help with the division?

Which way do you prefer?



multiple possible answers, e.g.

10 and 75

80 and 5

50 and 25



## Divide a 2-digit number by a 1-digit number (2)

### Notes and guidance

In this small step, children continue to explore dividing a 2-digit number by a 1-digit number, but in this step the focus is on calculations with remainders.

Children enountered remainders in Year 3, so this concept is not new but it may need reinforcing.

Using place value counters to illustrate the sharing structure of division helps children to see what is meant by the remainder. Such representations should highlight the fact that the remainder can never be greater than the number they are dividing by.

## Things to look out for

- Children may not fully divide and so will have a remainder that is greater than the number they are dividing by.
- Children may partition the 2-digit number correctly, but then divide the tens as if they are ones, for example  $95 \div 3 = 9 \div 3 + 5 \div 3$
- Children may revert to less efficient methods, such as drawing circles and then drawing dots to share between the circles.
- Children may divide the whole number rather than partitioning into tens and ones and then unitising the tens.

### **Key questions**

- Can the counters be shared equally? If not, how many are left over?
- What does "remainder" mean?
- What is the greatest remainder you can have when you are dividing by \_\_\_\_\_\_?
- How can you partition a 2-digit number?
- If you cannot share all the tens equally, what do you need to do?
- If you cannot share all the ones equally, what happens?
- How do you know that 43 ÷ 2 will have a remainder?

### Possible sentence stems

If I am dividing by	$\_$ , then the greatest possible
remainder is	

#### **National Curriculum links**

- Recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers



# Divide a 2-digit number by a 1-digit number (2)

## **Key learning**

• Tommy uses place value counters to divide 85 by 4

10	10	10	10	10	10	10	10	1	1	1	1	1
----	----	----	----	----	----	----	----	---	---	---	---	---

Tens	Ones
10 10	1
10 10	1
10 10	1
10 10	1

First, he shares the tens.

Then he shares the ones.

He has 1 one left over.

$$85 \div 4 = 21 \text{ r1}$$

Use Tommy's method to work out the divisions.

Work out the divisions.

What do you notice?

• Alex uses place value counters to work out  $97 \div 4$ 

Tens	Ones		
10 10			
10 10	1111	1	97 ÷ 4 = 24 r1
10 10	1111		
10 10	1111		
10	- ,		

Why has Alex made an exchange?

Use Alex's method to work out the divisions.

Complete the divisions.

$$\triangleright$$
 95 ÷ 4 = \_\_\_\_ r3  $\triangleright$  \_\_\_\_ ÷ 7 = 7 r6

$$\rightarrow$$
 2 = 7 r6

There are 95 pencils.

They are shared equally between 4 pots.

How many pencils will be left over?



# Divide a 2-digit number by a 1-digit number (2)

## Reasoning and problem solving

Filip is thinking of a 2-digit number that is less than 50

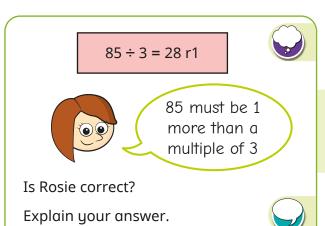


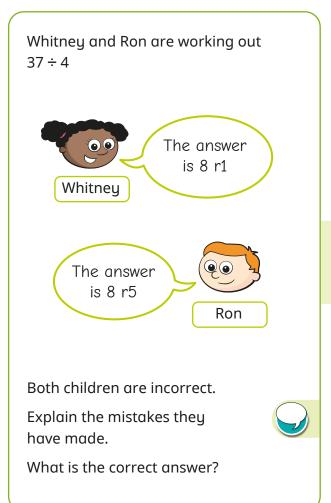
Work out Filip's number from the clues:

- When it is divided by 2, there is no remainder.
- When it is divided by 3, there is a remainder of 1
- When it is divided by 5, there is a remainder of 3

28

Yes





9 r1



## Divide a 3-digit number by a 1-digit number

### Notes and guidance

In this small step, children continue to develop their understanding of division by extending from dividing 2-digit numbers in the previous two steps to dividing 3-digit numbers.

Place value counters are again used to represent the calculations, so that children can make sense of exchanges that are needed to complete the division.

Part-whole models are also used to show how flexible partitioning can support the process of division by looking for multiples of the number being divided by.

The step starts with divisions that do not leave a remainder, before progressing to divisions with remainders.

By the end of this step, children should have a good understanding of division that will support them when they move on to the formal written method in Year 5

## Things to look out for

- Children may partition the 3-digit number correctly, but then divide the hundreds and tens as if they are ones, for example  $846 \div 2 = 8 \div 2 + 4 \div 2 + 6 \div 2$
- Children may divide the whole number rather than partitioning into hundreds, tens and ones and then unitising the hundreds and tens.

### **Key questions**

- How do you partition a 3-digit number into hundreds, tens and ones?
- How else can you partition a 3-digit number?
- What is the best way to partition the number to help you work out the division?
- If you cannot share all of the hundreds/tens equally, what do you need to do?
- How can you represent the division using a part-whole model?

#### Possible sentence stems

- \_\_\_\_\_ hundreds divided by \_\_\_\_\_ = \_\_\_\_ hundreds
- \_\_\_\_\_ tens divided by \_\_\_\_\_ = \_\_\_\_ tens
- \_\_\_\_\_ ones divided by \_\_\_\_\_ = \_\_\_\_ ones
- There is \_\_\_\_\_ left over, so I need to exchange it for \_\_\_\_\_

### **National Curriculum links**

- Recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers



# Divide a 3-digit number by a 1-digit number

## **Key learning**

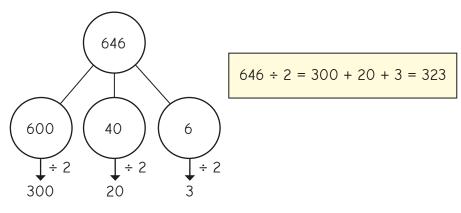
• Annie uses place value counters to divide 639 by 3

Hundreds	Tens	Ones
100 100	10	111
100 100	10	111
100 100	10	111

$$639 \div 3 = 213$$

Use Annie's method to work out the divisions.

• Mo uses a part-whole model to work out 646 ÷ 2



Use Mo's method to work out the divisions.

• Rosie uses place value counters to work out 435 ÷ 3

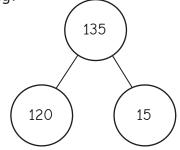
Hundreds	Tens	Ones
100	10 10 10	11111
100	10 10 10	11111
100	10 10 10	11111
100	10	

Use Rosie's method to work out the divisions.

Tiny is using a part-whole model to work out 135  $\div$  3

Why has Tiny partitioned 135 this way?

Complete Tiny's workings.



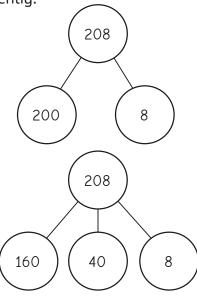


# Divide a 3-digit number by a 1-digit number

## Reasoning and problem solving

Max and Jo are working out  $208 \div 8$ 

They have each partitioned 208 differently.



Work out the division using both methods.

What do you notice?

Which method do you prefer?

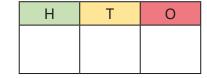
26

The answer is the same for both methods.

Use 12 counters and the place value chart to make the numbers described.

Use all 12 counters to make each number.





- a 3-digit number divisible by 2
- a 3-digit number divisible by 3
- a 3-digit number divisible by 4
- a 3-digit number divisible by 5

Is it possible to make 3-digit numbers that are divisible by 6, 7, 8 or 9?



- 2: any even number
- 3: any 3-digit number (as the digits add up to 12, which is a multiple of 3)
- 4: a number where the last two digits are a multiple of 4
- 5: any number with 0 or 5 in the ones column



## **Correspondence problems**

### Notes and guidance

In this small step, children consolidate their understanding of correspondence problems from Year 3, using multiplication to work out the number of possible combinations of sets of items.

Children use a range of representations and contexts to support them. Using tables helps to encourage children to adopt a systematic approach to finding all of the possible combinations in a given context. Children then generalise to make the link between the number of possibilities for each item and using multiplication to find the total number of combinations.

Once confident with finding all possible combinations for two sets of items children may begin to explore finding all possible combinations for three sets of items.

## Things to look out for

- Children may see the same choices in a different order as a different choice.
- Children may need support to work systematically when listing all possibilities.
- Children may add instead of multiply the number of possibilities for each item.

### **Key questions**

- How can you use a table to help you find the possible combinations?
- How can you be sure that you have listed all the possibilities?
- How could you use a code to help you list the combinations?
- What do you notice about the number of choices for each item and the total number of combinations?
- How can you check your answer?
- Does the order in which you make your choices matter?

### Possible sentence stems

- For every \_\_\_\_\_, there are \_\_\_\_\_
- Altogether, there are \_\_\_\_\_ × \_\_\_ = \_\_\_\_ possible combinations.

#### **National Curriculum links**

• Solve problems involving multiplying and adding, including using the distributive law to multiply 2-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects



## **Correspondence problems**

## **Key learning**

A cafe has 4 flavours of ice cream and 2 choices of toppings.

Ice cream flavours	Toppings
vanilla	
chocolate	sauce
strawberry	wafer
lemon	

Complete the table to show the 8 possible combinations of flavours and toppings.

	Sauce	Wafer
Vanilla		VW
Chocolate		
Strawberry		SW
Lemon	LS	

What multiplication could you use to work out the total number of combinations?

How do you know?

- ► How many combinations would there be if the cafe also offered mint ice cream?
- ► How many combinations would there be if there were 6 ice cream flavours and 3 different toppings?

Huan has two piles of coins.
 He chooses one coin from each pile.





- List all the possible combinations of coins Huan could choose.
- ▶ How many different combinations of coins are there?
- List all the possible total amounts of money Huan can make.
- ▶ How many different total amounts of money are there?
- Esther is choosing what to wear on a snowy day.

Hat	Scarf	Gloves
4444	Will may will simble the simple that	

- How many different ways can Esther choose a hat and a scarf?
- How many different ways can Esther choose a hat and a pair of gloves?
- ► How many different ways can Esther choose a hat, a scarf and a pair of gloves?

How can you check your answers?



## **Correspondence problems**

## Reasoning and problem solving

Here are the meal choices in the school canteen.

Starter	Main	Dessert
soup garlic bread	pasta chicken	cake ice cream
garne	beef	fruit salad
	salad	

Children can make one choice from each section.

How many possible combinations of starters, mains and desserts can be chosen?

If there were 20 possible meal combinations, how many starters, mains and desserts could there be?



multiple possible answers, e.g.

1S, 1M, 20D

1S, 2M, 10D

1S, 4M, 5D

2S, 2M, 5D

1S, 20M, 1D

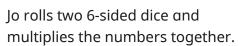
Brett has 6 T-shirts and 4 pairs of shorts.

Dani has 12 T-shirts and 2 pairs of shorts.

Who has the most combinations of T-shirts and shorts?

Explain your answer.







There are
6 × 6 = 36 different
possible answers Jo
could get.

Explain why Tiny is wrong.

How many different answers could Jo get?



18





# **Efficient multiplication**

### Notes and guidance

In this small step, children consolidate their knowledge and understanding of multiplication and begin to make decisions regarding the most efficient or appropriate methods to use in a range of contexts.

Children look at times-tables facts, building strategies for finding unknown facts that will support them to strengthen their fluency of times-tables. They then examine a range of strategies for multiplying a 2-digit number by a 1-digit number. Finally, they use arrays to explore multiplicative structure, in particular the associative law and distributive law.

## Things to look out for

- Children may conflate different methods, leading to misunderstanding.
- Children may partition the numbers correctly, but then multiply the tens as if they are ones, for example  $34 \times 6 = 3 \times 6 + 4 \times 6$
- Children may attempt to learn the different methods procedurally. It is vital that children understand how they are manipulating the numbers, rather than try to remember a long series of instructions.

### **Key questions**

- Which method do you find most efficient? Explain how this method works.
- What is the most efficient way to work out \_\_\_\_\_ × \_\_\_\_?
- What happens if you double one factor and halve the other?
- How could you use factor pairs to help you calculate?

### **Possible sentence stems**

#### **National Curriculum links**

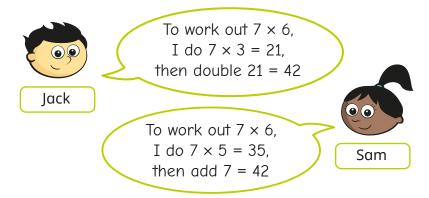
• Solve problems involving multiplying and adding, including using the distributive law to multiply 2-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects



## **Efficient multiplication**

## **Key learning**

• Jack and Sam are working out 7 × 6



- ▶ Use Jack's method to work out 8 × 6
- ▶ Use Sam's method to work out 9 × 6
- For each calculation, show two ways that you could find the answer if you do not know the times-table fact.

9×7

4 × 7

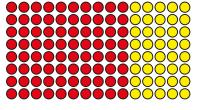
7 × 8

- Work out the missing numbers.
  - ► 5 × 8 = 5 × 4 × \_\_\_\_\_
- ▶ 16 × 5 = 16 × 10 ÷ \_\_\_\_\_
- > 7 × 4 = 7 × 2 × \_\_\_\_\_
- ▶ 19 × 7 = 20 × 7 \_\_\_\_ × 7

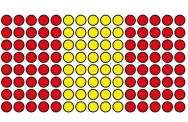
Here are four different ways of working out 15 × 8 mentally.
 Complete the calculation in each method.

#### Method 1

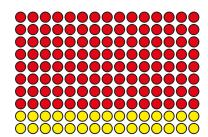
$$15 \times 8 = 10 \times 8 + 5 \times 8$$
  
=  $80 +$ \_\_\_\_\_



#### Method 2



#### Method 3



#### Method 4

$$15 \times 8 = 30 \times 8 \div 2$$
  
= \_\_\_\_ \div 2  
= \_\_\_\_



## **Efficient multiplication**

## Reasoning and problem solving

Find four different ways to work out  $18 \times 5$ 

Compare methods with a partner.



multiple possible answers, e.g.  $(18 \times 10) \div 2$ 

Kim uses an array to help her work out 19 × 3



$$20 \times 3 = 60$$

$$60 - 1 = 59$$

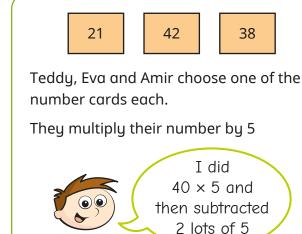
$$19 \times 3 = 59$$

What mistake has Kim made?

Draw or make the array correctly.



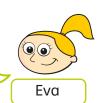
Kim has subtracted one counter, rather than one group of 3 counters.



I multiplied my number by 10

Teddu

and then divided 210 by 2



Which number card has Amir got?

Talk about the different methods Amir could have used.



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