Autumn Term

Year 10

#MathsEveryoneCan

White Rose Maths

WRM – Year 10 Scheme of Learning

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Autumn 1: Similarity

Weeks 1 & 2: Congruence, Similarity and Enlargement	Weeks 3 to 6: Trigonometry
Building on their experience of enlargement and similarity in previous years, this unit extends students' experiences and looks more formally at dealing with topics such as similar triangles. It would be useful to use ICT to demonstrate what changes and what stays the same when manipulating similar shapes. Parallel line angle rules are revisited to support establishment of similarity. Congruency is introduced through considering what information is needed to produce a unique triangle. Higher level content extends enlargement to explore negative scale factors, and also looks at establishing that a pair of triangles are congruent through formal proof. National curriculum content covered **(Higher content in bold):** • extend and formalise their knowledge of ratio and proportion in working with measures and geometry • compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity • interpret and use fractional **{and negative}** scale factors for enlargements • apply the concepts of congruence and similarity, including the relationships between lengths, **{areas and volumes}** in similar figures • use mathematical language and properties precisely • make and test conjectures about the generalisations that underlie patterns and relationships; look for proofs or counter-examples • develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems	Trigonometry is introduced as a special case of similarity within right-angled triangles. Emphasis is placed throughout the steps on linking the trig functions to ratios, rather than just functions. This key topic is introduced early in Year 10 to allow regular revisiting e.g. when looking at bearings. For the Higher tier, calculation with trigonometry is covered now and graphical representation is covered in Year 11 National curriculum content covered: • extend and formalise their knowledge of ratio and proportion, including trigonometric ratios • apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles {and, where possible, general triangles} in two **{and three}** dimensional figures • know the exact values of $\sin\theta$, $\cos\theta$, $\tan\theta$ for required angles • **{know and apply the sine rule and cosine rule to find unknown lengths and angles}** • **{know and apply to calculate the area, sides or angles of any triangle}** • develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems • make and use connections between different parts of mathematics to solve problems • model situations mathematically and express the results using a range of formal mathematical representations, reflecting on how their solutions may have been affected by any modelling assumptions • select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems; interpret their solution in the context of the given problem.

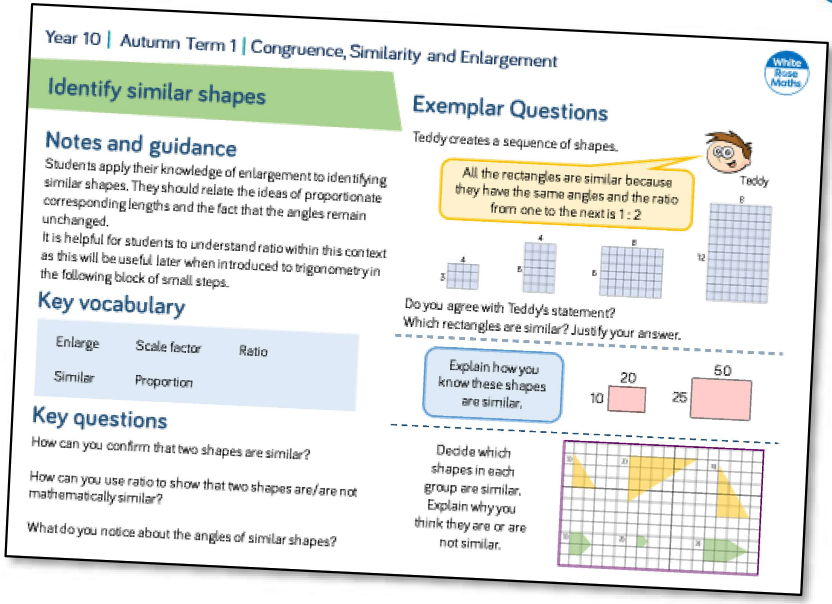
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.

It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge, Scale factor, Ratio, Similar, Proportion

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

All the rectangles are similar because they have the same angles and the ratio from one to the next is 1 : 2

Teddy

Do you agree with Teddy's statement? Which rectangles are similar? Justify your answer.

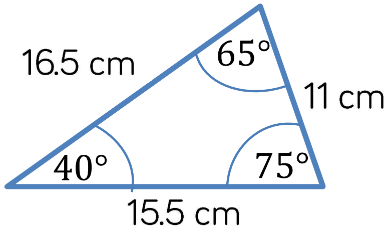
Explain how you know these shapes are similar.

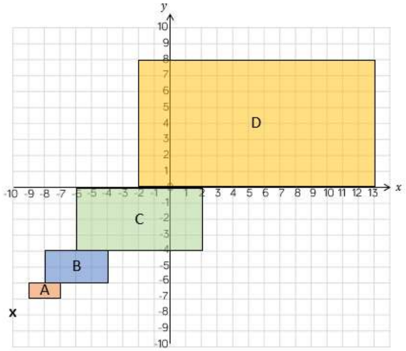
Decide which shapes in each group are similar. Explain why you think they are or are not similar.

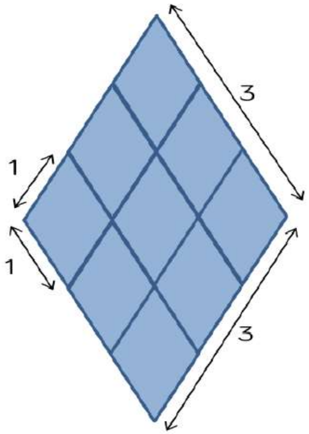
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

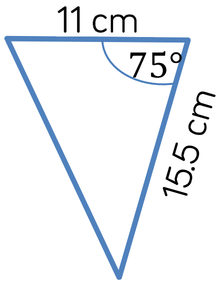
Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

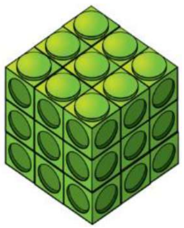
Key Representations

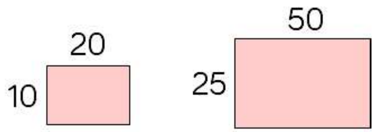


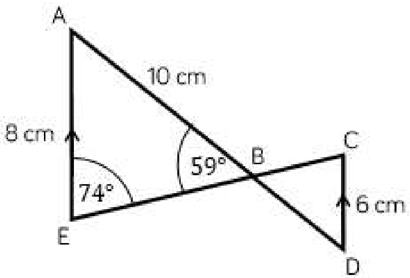


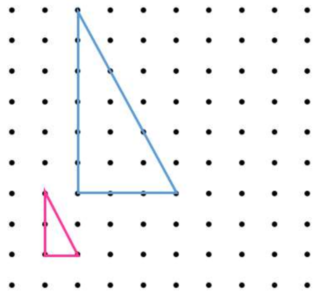


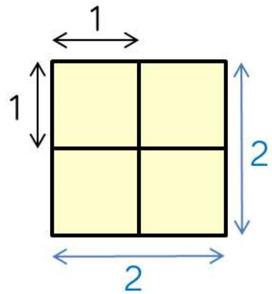












Pictorial representation is essential to support conceptual understanding of enlargement, similarity and congruence.

Students should be encouraged throughout to draw sketches or diagrams to help them visualise information. They should also be reminded to represent new information in diagrammatical form to help see further lines of enquiry that could move them towards solving the problem.

Manipulatives such as a geoboard and pattern blocks could be used to explore enlargements, similarity and area scale factor. Multi-link cubes could be useful in exploring volume scale factor.

Congruence, Similarity and Enlargement

Small Steps

- Enlarge a shape by a positive integer scale factor (R)
- Enlarge a shape by a fractional scale factor (R)
- **Enlarge a shape by a negative scale factor** (H)
- Identify similar shapes
- Work out missing sides and angles in a pair given similar shapes (R)
- Use parallel line rules to work out missing angles
- Establish a pair of triangles are similar

(H) denotes Higher Tier GCSE content

(R) Denotes 'review step' – content should have been covered at KS3

Congruence, Similarity and Enlargement

Small Steps

- Explore areas of similar shapes (1) — H
- Explore areas of similar shapes (2) — H
- Explore volumes of similar shapes — H
- Solve mixed problems involving similar shapes — H
- Understand the difference between congruence and similarity
- Understand and use conditions for congruent triangles
- Prove a pair of triangles are congruent — H

H denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Positive scale factors

R

Notes and guidance

Students start year 10 with a review of enlargement. This understanding will, in later steps, be built on as similar shapes are introduced. Therefore it would be useful to highlight the fact that angles do not change when enlarging shapes and that the ratio of lengths is the same for corresponding lengths. Dynamic geometry could be used to see what is happening when shapes are enlarged.

Key vocabulary

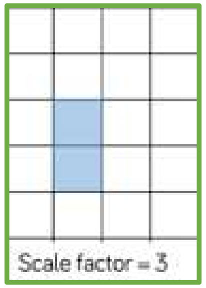
Enlarge	Scale factor	Ratio
Origin	Object	Image

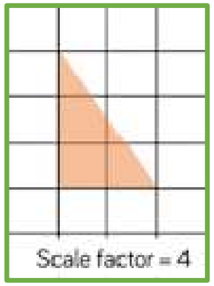
Key questions

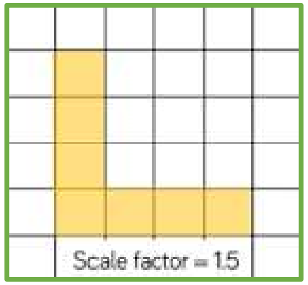
What are the size of the angles in each shape?
Do they stay the same or change when the shape is enlarged? Is this true for all shapes?
What is the ratio of sides?
Does this change depending on which lengths you are comparing?

Exemplar Questions

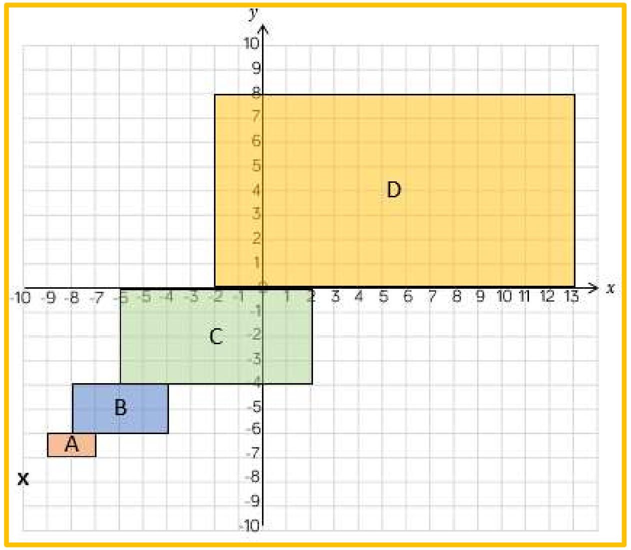
Enlarge each shape by the given scale factor.







Rectangle B, C, and D are each enlargements of A from the point marked at $(-10, -8)$.
What is the scale factor for each enlargement?



Write ratios for the following relationships.

- Width A : Width B
- Width B : Width C
- Width C : Width D
- Width A : Width C
- Width A : Width D

What is the same and what is different?
How do these connect to scale factor?

Fractional scale factors

R

Notes and guidance

Students review their understanding of enlargement in relation to fractional scale factors.

Dynamic geometry could be used to explore how the image changes in relation to the fractional scale factor (including proper and improper fractional scale factors) and in relation to the centre of enlargement.

Key vocabulary

Enlarge	Fractional scale factor	Image
Origin	Centre of enlargement	Object

Key questions

Does enlargement always make a shape bigger?

Which scale factors make the shape larger/smaller/stay the same?

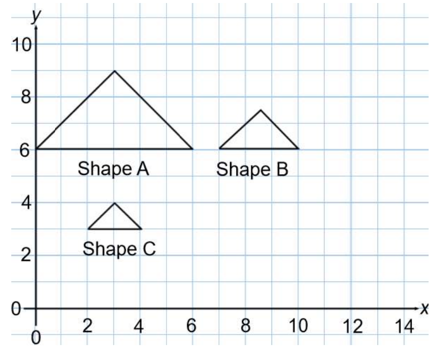
Do fractional scale factors always make the shape smaller?

Exemplar Questions

Shape A has been enlarged onto shape B.

Ron

The scale factor of enlargement is 2



Ron is incorrect.
What mistake has he made?

Write down the correct scale factor.
What is the scale factor of enlargement from Shape A to Shape C?

To find the centre of enlargement, I can join corresponding vertices on Shape A and Shape B with a straight line. The centre of enlargement is the intersection point of these lines.

Annie

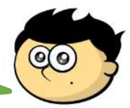
Discuss Annie's statement. Is she correct? Why?

On a set of axes, draw rectangle A with coordinates $(4, -2)$, $(8, -2)$, $(4, -4)$ and $(8, -4)$. Enlarge this shape by scale factor $\frac{3}{4}$, centre the origin. Label this rectangle B.

Dora

The ratio of sides A : B is 4 : 3

No, it isn't. The ratio is 3 : 4



Jack

Who's right, Dora or Jack? Justify your answer.

Negative scale factors

H

Notes and guidance

Dynamic software can be used to explore what happens as the scale factor moves between positive, fractional and negative scale factors. Students can explore different centres of enlargement and the effect this has on the new image. Asking students to explain their approach is helpful to aid others in strategies for visualising this transformation.

Key vocabulary

Enlarge	Negative scale factor
Reflection	Centre of enlargement

Key questions

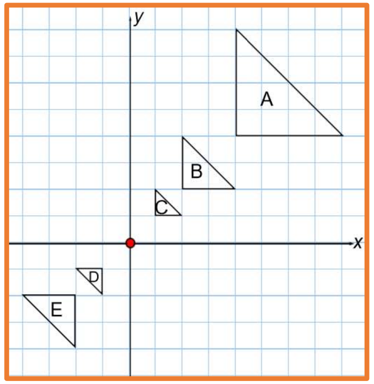
What happens to the shape using a scale factor of -1?

How would the shape change if the shape was enlarged by a negative fractional scale factor e.g. $-\frac{1}{2}$

Can you predict the position of each shape before drawing it? How could you find the centre of enlargement from a diagram?

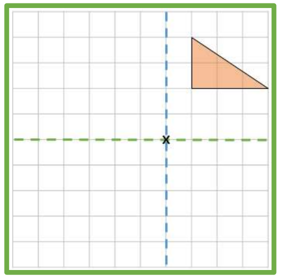
Exemplar Questions

Shape A has been enlarged onto Shapes B, C, D and E.
How do you think the scale factor will change when transforming Shape A to Shapes D and E? Complete the table.



Shape A enlarged onto:	Scale Factor
Shape B	
Shape C	
Shape D	
Shape E	

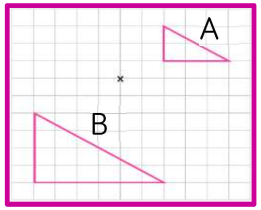
What's the same and what's different about these transformations?



Reflect the shape in the vertical line. Then reflect the shape in the horizontal line.

Enlarge by a scale factor of -1

Abdul has enlarged shape A by a scale factor of –2 to make the image B. Explain and correct Abdul's mistake.



Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.

It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge	Scale factor	Ratio
Similar	Proportion	

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

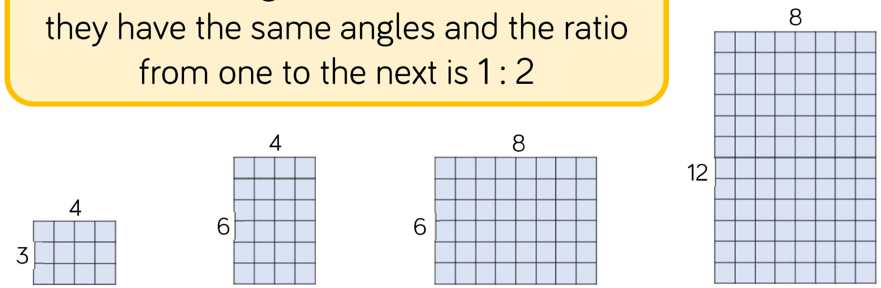
What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

Teddy

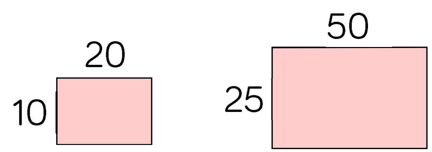
All the rectangles are similar because they have the same angles and the ratio from one to the next is 1 : 2



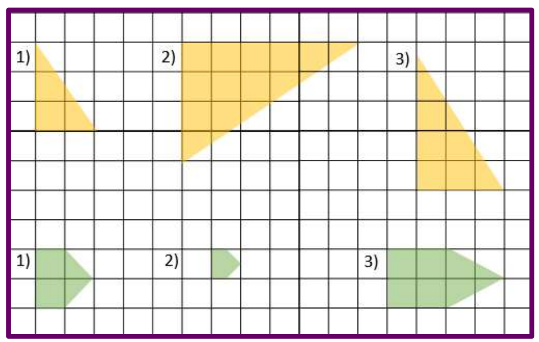
Do you agree with Teddy's statement?

Which rectangles are similar? Justify your answer.

Explain how you know these shapes are similar.



Decide which shapes in each group are similar. Explain why you think they are or are not similar.



Information in similar shapes

Notes and guidance

Students use their knowledge of similar shapes to calculate missing lengths and angles. They should be encouraged to look at scale factors both within and between shapes.

They should see similar shapes in a range of orientations and therefore have practice to ensure they correctly identify corresponding points. Careful labelling will assist this.

Key vocabulary

Enlargement	Scale factor	Ratio
Correspond	Similar	

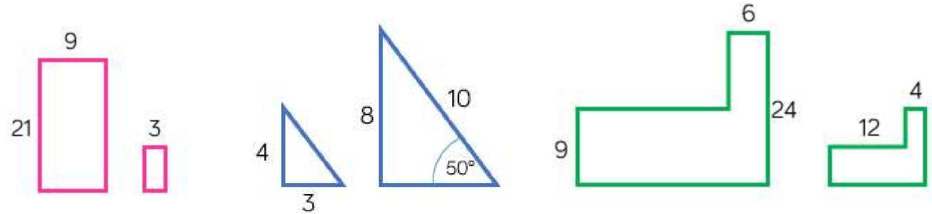
Key questions

Which angles/lengths correspond to each other? How do you know?

How does the order of the letters of the shape e.g. ABC and FGH help you decide which lengths/angles match up (correspond)?

Exemplar Questions

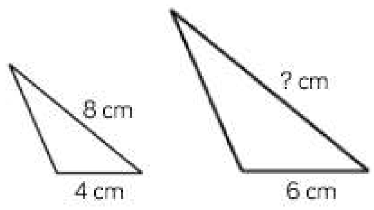
Find the missing information in the these pairs of similar shapes.



Scale factor = ____ Scale factor = ____ Scale factor = ____

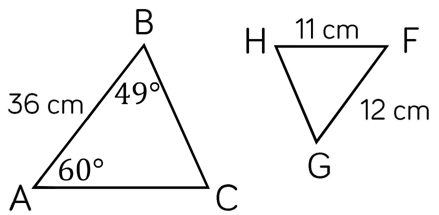
Here are two similar triangles with a missing length.
Compare the two methods. Why do they both work?

The longest side is double the length of the smallest side, so the missing length must be $6 \times 2 = 12$ cm



The ratio of sides is 4 : 6 so there is a scale factor of 1.5
The missing length must be $8 \times 1.5 = 12$ cm

Triangle ABC is similar to FGH.



Which length in triangle FGH corresponds to AB?
Which angle in triangle FGH corresponds to ∠BAC?
Calculate the length AC.

Parallel line rules

Notes and guidance

This review of year 8 content will support students to show pairs are triangles are similar in the following step. Students are encouraged to explain their reasoning for their steps and review angle and side notation. It will be useful to distinguish between 'corresponding angles' (that are equal because of parallel lines) and 'angles that correspond' (matching pairs of angles in two shapes).

Key vocabulary

Parallel	Corresponding
Alternate	Co-interior

Key questions

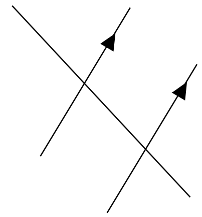
Where are the parallel lines in the diagram?

Which angles would be corresponding/alternate/co-interior?

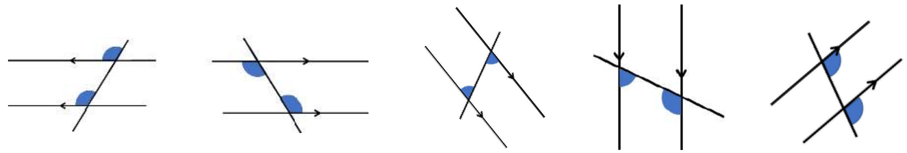
What other angle rules do you know?

Exemplar Questions

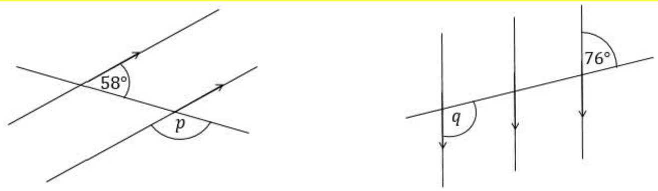
Use colour coding to identify all the equal angles.
Use colour coding to identify vertically opposite, corresponding, alternate and co-interior angles.
What are the relationships?



State whether each pair of angles are corresponding, alternate or co-interior angles. State what this tells you about the angles.

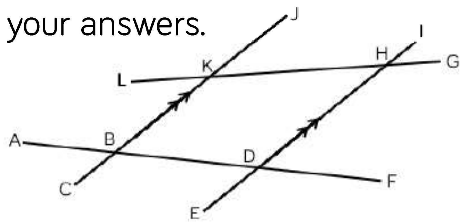


Work out the missing angles.
Hint: what other angles do you need to work out first?



Which statements are true. Justify your answers.

- $\angle CBD = \angle BDH$
- $\angle LKB = \angle KBD$
- $\angle KHD + \angle HDB = 180°$
- $\angle KBD = \angle JKH$



Establish triangles are similar

Notes and guidance

Students use their understanding of angles in parallel lines to show that a pair of triangles are similar. They may need support to work out which vertex in one triangle corresponds to which in the other and to distinguish this from 'corresponding angles' in parallel lines.
Students should also recognise that using side ratios is an equally valid method of establishing similarity.

Key vocabulary

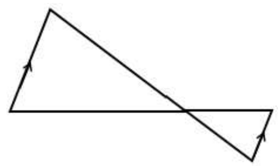
Corresponding angles	Similar
Alternate angles	Parallel

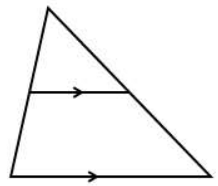
Key questions

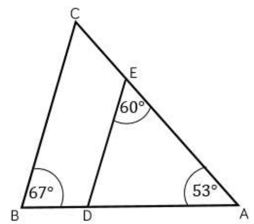
Why do you only need two pairs of equal angles to show that two triangles are similar?
What's the same and what's different about the pairs of triangles?
Which diagrams include pairs of parallel lines? How do we show they are parallel?

Exemplar Questions

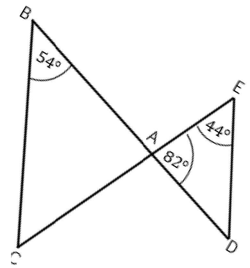
In each diagram, show that the triangles are similar.

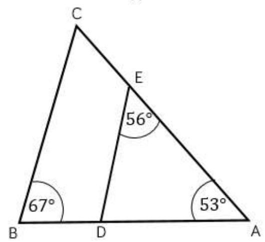


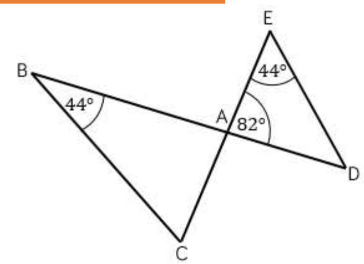




In each diagram, is ABC similar to ADE?

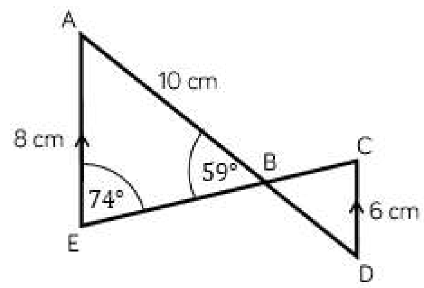






AE is parallel to CD.
Work out the following:

- ∠BDC
- Length BD



Areas of similar shapes (1)

H

Notes and guidance

Students explore how area changes as the scale factor between two shapes changes. A common misconception is that if, for example, the lengths double, the area will also double. Visual representations and use of manipulatives, such as pattern blocks or multilink cubes, are very helpful here to support student understanding of the links between the length scale factor and area scale factor. This will be revisited in the spring term of year 10

Key vocabulary

Enlarge	Length scale factor	Object
Ratio	Area scale factor	Image

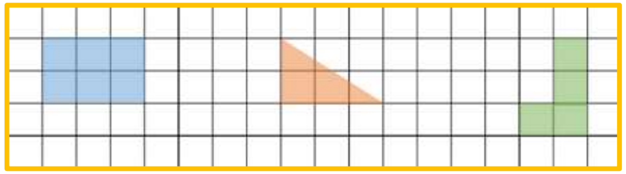
Key questions

If we know the length scale factor between two similar shapes, how can you find the area scale factor of the shapes? What about the other way round?

Can you draw a diagram to show your understanding?

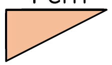
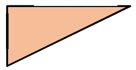
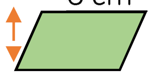
Exemplar Questions

Draw the following shapes. Then enlarge each by scale factor 2



For each enlargement, how many times does each object fit into its corresponding image?
Repeat the activity using a scale factor of 3
What does this tell you about the area of an enlarged shape compared to the area of the original shape?

Complete the table.

Original shape	Enlarged shape	Length Scale factor	Area scale factor
 5 cm, 3 cm; Area = ____ cm^2	 10 cm, 6 cm; Area = ____ cm^2		
 4 cm, 2 cm; Area = ____ cm^2	 6 cm, 3 cm; Area = ____ cm^2		
 6 cm, 4 cm; Area = ____ cm^2	 3 cm, 2 cm; Area = ____ cm^2		

Areas of similar shapes (2)

H

Notes and guidance

Teachers may decide to extend this by using area scale factors to find missing areas and to work backwards to finding missing lengths.

This is revisited in the spring term of year 10

Key vocabulary

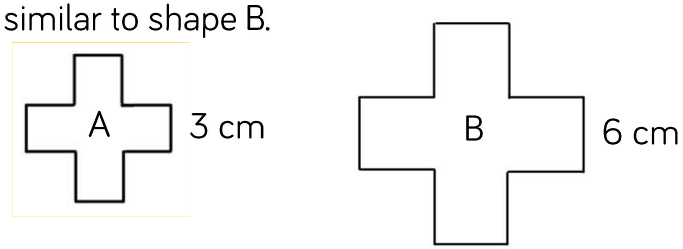
Enlarge	Length scale factor	Object
Ratio	Area scale factor	Image

Key questions

If we know the length scale factor between two similar shapes, how can you find the area scale factor of the shapes?
What about the other way round?

Exemplar Questions

Shape A is similar to shape B.



Write down the length scale factor of enlargement from Shape A to Shape B.
State the area scale factor of enlargement from Shape A to Shape B.
The area of Shape A is 45 cm^2. Find the area of Shape B?

Shape C is similar to Shape A above. The area of Shape C is 281.25 cm^2. Rosie is calculating the side length on Shape C which corresponds to the labelled 3 cm side length on Shape A. Complete her steps.

Area scale factor: 281.25 ÷ 45 = ___________.
To find the length scale factor, I need to ___________ the area scale factor. The length scale factor is __________.
To find the side length on shape C which corresponds to the labelled 3 cm on shape A, I need to multiply 3 by my length scale factor: 3 × ____ = _____ cm

Volumes of similar shapes

H

Notes and guidance

This small step leads on directly from student reasoning around area of similar shapes in the previous step.

Again, visual support will ensure students can see the link between the length, area and volume scale factors of similar shapes. Multilink cubes could be used to explore this concept.

Key vocabulary

Enlarge	Similar
Length scale factor	Volume scale factor

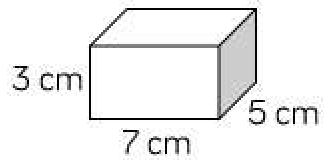
Key questions

Are the cuboids similar? How do you know?

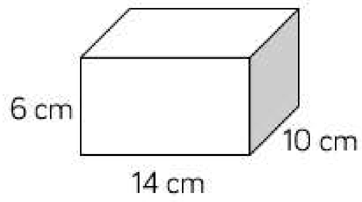
If you know the length scale factor between two similar shapes, how can you find the volume scale factor? Draw a diagram to explain why.

Exemplar Questions

Explain how the written calculations match the diagrams.



$3 \times 7 \times 5 = 105 \text{ cm}^3$



$6 \times 14 \times 10$

$= 3 \times 2 \times 7 \times 2 \times 5 \times 2$

$= 105 \times 8$

$= 840 \text{ cm}^3$

What is the length scale factor for these cuboids?
What is the volume scale factor for these cuboids? Why?

Complete:
To find the volume scale factor, we can ______ the length scale factor.

Complete the following table.

Volume of original cuboid	Length scale factor	Volume scale factor	Volume of enlarged cuboid
12 cm^3	$\times 3$	$\times 3^3$	
25 m^3	$\times 1.5$		
310 cm^3		$\times 343$	
	$\times 5$		$8\,375 \text{ cm}^3$

Similar shape problems

H

Notes and guidance

This small step brings together the previous steps to consolidate and extend student understanding of the topics while interleaving other topics.

Students should be encouraged to discuss their approaches and reasoning when solving the problems.

Key vocabulary

Length scale factor	Parallel	Similar
Alternate angles	Corresponding angles	

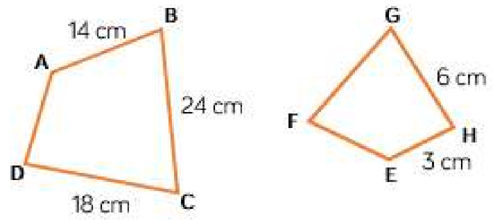
Key questions

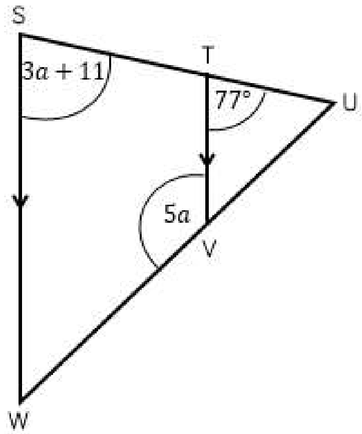
How does the order of the letters of the shape e.g. ABCD and EFGH help you decide which lengths/angles are corresponding?

If you know two shapes are similar, what do you know about those shapes?

Exemplar Questions

Quadrilateral ABCD is similar to EFGH.
Work out the length FG.
Work out the length AD.





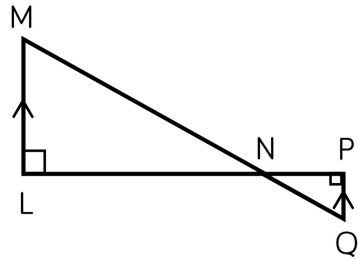
SW is parallel to TV.

- Explain why TUV and SUW are similar triangles.
- Find ∠SWV

Length UV is 50 cm and length UW is 125 cm.

- If length TV is 42 cm, what is the length SW?
- If length SU is 100 cm, what is the perimeter of triangle TUV?

Triangle LMN is similar to PQN.
The ratio of LN : NP is 3 : 1
If NP is 4 cm and PQ is 3.5 cm, what is the area of triangle LMN?
Can you find another way to calculate the answer?



Congruence and similarity

Notes and guidance

Within this small step, students bring together the ideas of similarity and congruence and through categorising, are able to distinguish between them.

By reasoning and distinguishing in this way, students will have a better idea of where the concepts overlap and what characteristics are unique to each.

Key vocabulary

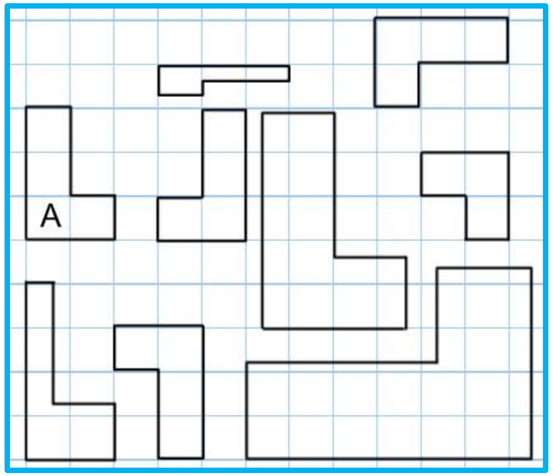
Congruent	Similar	Scale factor
In proportion	Ratio	Corresponding

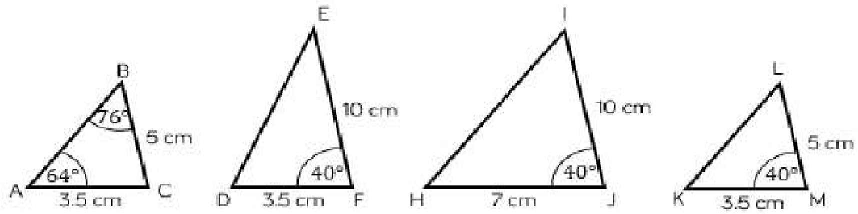
Key questions

If you know two shapes are congruent, what else do you know about the shapes?
If you know two shapes are similar, what else do you know about the shapes?
What is the ratio of corresponding lengths in a congruent shape?

Exemplar Questions

In pairs, discuss which shapes are similar to shape A, which are congruent to shape A, and which are neither similar or congruent. Label the similar shapes, S, and the congruent shapes C.





Complete the sentences.
Triangle ABC and triangle ______ are congruent.
Triangle ABC and triangle ______ are similar.
Triangle ______ is neither similar nor congruent to triangle ABC.

Conditions for congruent triangles

Notes and guidance

The conditions for congruence are formalised within this step. Students will have come across the language of SSS, ASA etc. in previous years, but will not have used them to show congruence of triangles.

Students should understand the minimum information needed to establish congruence between triangles.

Key vocabulary

Side-side-side	Angle-side-angle
Side-angle-side	Right angle-hypotenuse-side

Key questions

What is the minimum information needed for triangles to be congruent?

Does it matter which two angles and sides are given for the angle-side-angle condition to be true?

Exemplar Questions

In pairs or groups, after constructing each triangle, discuss if the triangles are congruent or not.

Construct a triangle which has one side 8 cm long and another 5 cm

Construct a triangle which has the following sides: 8 cm, 5 cm and 6 cm

Construct a triangle with one side of length 5 cm, another of length 8 cm and an angle of $50°$ between them.

Construct a triangle which has the following angles: $40°$, $80°$ and $60°$

Construct a triangle which has one side of length 6.5 cm and angles of $40°$ and $55°$ at either end.

Construct a right-angled triangle with hypotenuse of length 10 cm and base of length 6 cm

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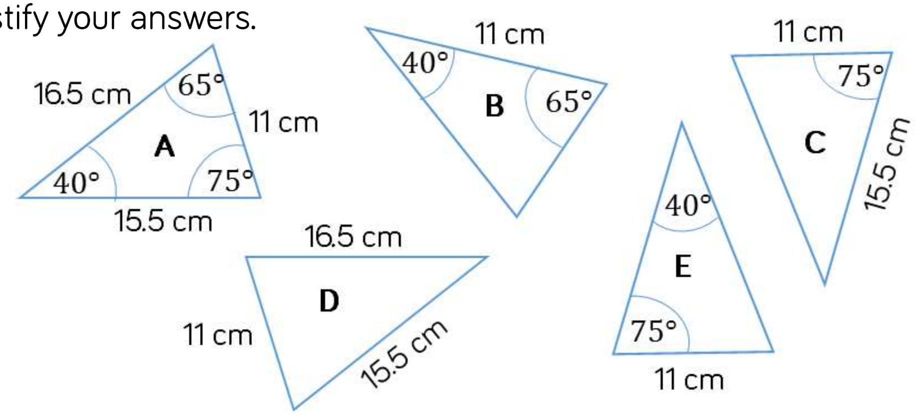
Which of the following triangles are congruent to triangle A? Justify your answers.



Prove triangles are congruent (H)

Notes and guidance

Students prove that triangles are congruent using the conditions of congruence. Teachers should model the process in the first instance and then scaffold by providing writing frames before expecting students to produce formal proofs independently. It may be useful to remind students of the properties of special quadrilaterals in preparation for this step.

Key vocabulary

SSS	ASA	SAS	RHS
Prove	Conditions of congruence		

Key questions

Would drawing a sketch help you?

What angle facts do we know about a parallelogram?

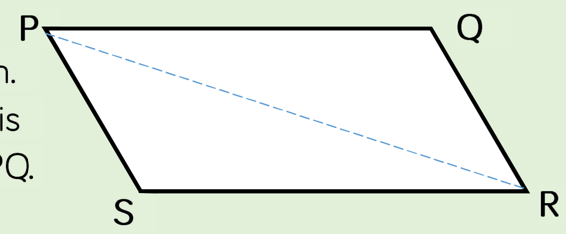
Can you prove it any other way using the conditions of congruence?

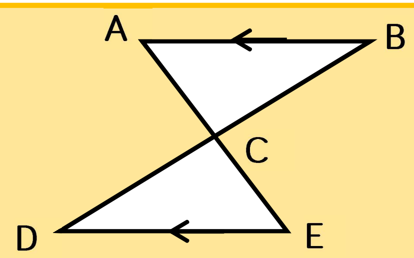
Exemplar Questions

Which two of the following triangles are congruent? Prove it.

- In triangle ABC, AB = 5 cm, ∠ABC = $40°$ and AC = 3 cm
- In triangle DEF, DE = 5 cm, ∠DEF = $40°$ and EF = 3 cm
- In triangle GHI, GH = 5 cm, ∠GHI = $40°$ and GI = 3 cm

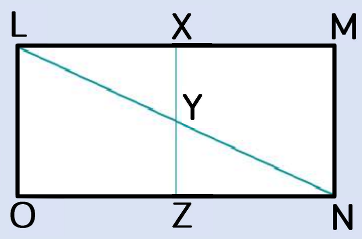
PQRS is a parallelogram.
Prove that triangle PRS is congruent to triangle RPQ.





AB and DE are parallel lines of equal length.
Prove that ABC is congruent to EDC.

LMNO is a rectangle.
X is the mid-point of LM and Z is the mid-point of ON.
Prove that triangle LXY and NZY are congruent.



Trigonometry

Small Steps

- Explore ratio in similar right-angled triangles
- Work fluently with the hypotenuse, opposite and adjacent sides
- Use the tangent ratio to find missing side lengths
- Use the sine and cosine ratio to find missing side lengths
- Use sine, cosine and tangent to find missing side lengths
- Use sine, cosine and tangent to find missing angles
- Calculate sides in right-angled triangles using Pythagoras' Theorem (R)
- Select the appropriate method to solve right-angled triangle problems

(H) denotes Higher Tier GCSE content

(R) Denotes 'review step' – content should have been covered at KS3

Trigonometry

Small Steps

- Work with key angles in right-angled triangles (1) & (2)
- **Use trigonometry in 3-D shapes** (H)
- **Use the formula $\frac{1}{2}ab\sin C$ to find the area of a triangle** (H)
- **Understand and use the sine rule to find missing lengths** (H)
- **Understand and use the sine rule to find missing angles** (H)
- **Understand and use the cosine rule to find missing lengths** (H)
- **Understand and use the cosine rule to find missing angles** (H)
- **Choosing and using the sine and cosine rules (1) & (2)** (H)

(H) denotes Higher Tier GCSE content

(R) Denotes 'review step' – content should have been covered at KS3

Ratio in right-angled triangles

Notes and guidance

Students should explore the ratio of two side-lengths in a right-angled triangle, given a specific angle. This facilitates understanding of a constant ratio between a pair of side lengths in relation to a specific angle. Teachers will need to emphasise, through class discussion, the generalisations being made. It maybe appropriate to use opposite, adjacent and hypotenuse to discuss the given side lengths.

Key vocabulary

Enlarge

Scale factor

Ratio

Corresponding

Constant

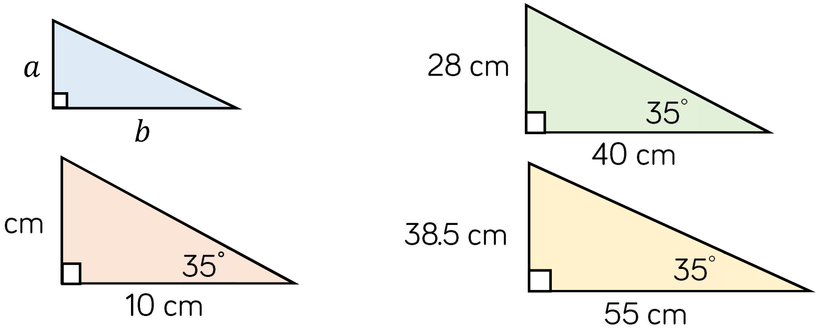
Key questions

When the side-lengths are in the same ratio, what do you notice about the position of these two-side lengths in each triangle? What do you notice about the given angle?

Will the ratio remain constant if the given angle gets bigger/smaller? Why/Why not?

Exemplar Questions

In the triangles below, a and b have been labelled with specific lengths.

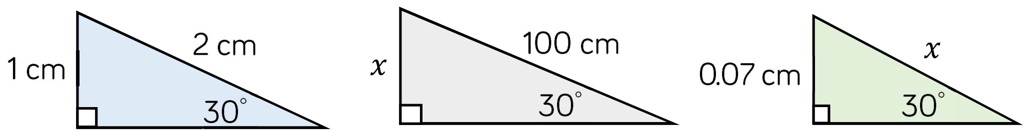


Complete the table.
What do you notice?
Will this always happen if the angle is labelled is 35°?

	a	b	$\frac{a}{b}$
Triangle 1			
Triangle 2			
Triangle 3			

Use this generalisation to find the missing value x in these triangles.

Use the information on the first triangle to find the missing values x on the following two triangles.



Hypotenuse, opposite & adjacent

Notes and guidance

Students need to be able to name the different sides of a right-angled triangle in relation to given angles. Labelling the hypotenuse first is a useful strategy. They should have opportunities to name sides in differently orientated right-angled triangles. Using both acute angles of the triangle can also assist students understanding of which sides to select in relation to an angle.

Key vocabulary

Adjacent	Opposite
Hypotenuse	Right angle

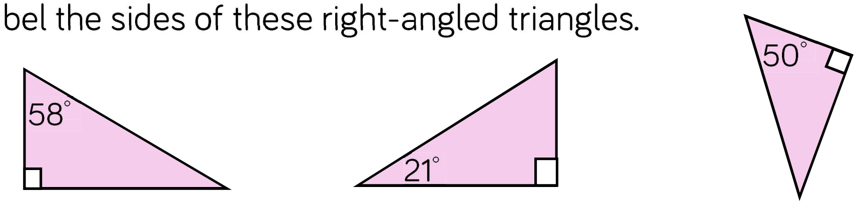
Key questions

Why can the same side on a right-angled triangle be labelled the 'opposite' on some occasions, and the 'adjacent' on others?

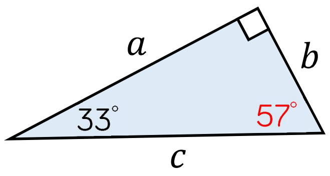
Where can you see a right-angled triangle in this shape? (Give examples of different shapes such as an isosceles triangle, a hexagon, a trapezium, a parallelogram etc.)

Exemplar Questions

Label the sides of these right-angled triangles.



Eva and Jack have been asked to label the sides in relation to the angle marked in red.

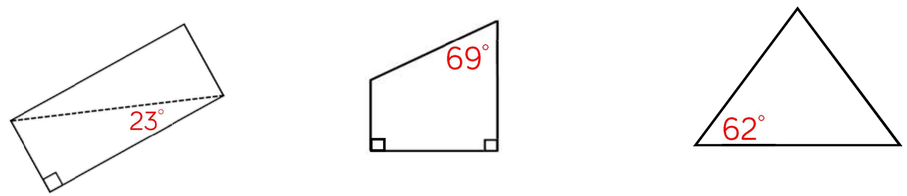


I think $a =$ opposite, $b =$ adjacent and $c =$ hypotenuse

I think $a =$ adjacent, $b =$ opposite and $c =$ hypotenuse

Who's made a mistake? What mistake have they made?

Identify a right-angled triangle which incorporates the angle labelled in red and label its sides.



A man walks a mile due East and then a mile due South. He then walks directly back to the start. Draw a right-angled triangle to represent his journey. Using the internal angle between his walk south and his walk back, label the sides of your right-angled triangle.

Tangent ratio: side lengths

Notes and guidance

Teachers should start by modelling how to solve equations of the form $a = \frac{b}{c}$

This helps students to be more confident when rearranging equations involving the tangent ratio to find missing side lengths. Teachers should model examples of finding both the opposite and the adjacent sides.

Key vocabulary

Tangent	Opposite	Adjacent	Hypotenuse
Formula	Rearrange	Subject	

Key questions

What does the 'tangent of an angle' mean?

How does it relate to similar triangles and scale factors?

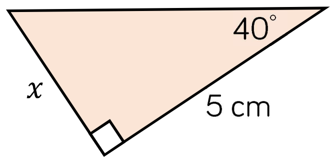
Why do we need to use division when the missing side is the adjacent side?

Exemplar Questions

Solve the following equations.

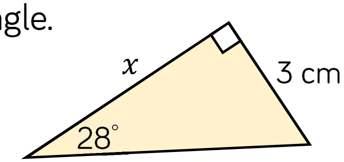
- $4 = \frac{x}{20}$
- $4 = \frac{20}{x}$
- $2.8 = \frac{5.3}{x}$
- $2.8 = \frac{x}{5.3}$

Whitney is finding length x in this triangle. Complete the stages of her workings:



$$\tan\theta = \frac{opp}{adj}$$
$$\tan\ ? = \frac{x}{?}$$
$$? \times \tan\ ? = x$$
$$? = x$$

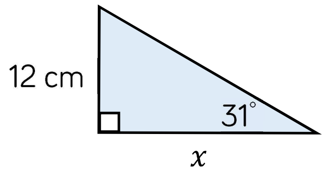
Whitney thinks that she can use the same process to find x in this triangle.



$$\tan 28 = \frac{x}{3}$$
$$3 \times \tan 28 = x$$
$$1.6 = x$$

The answer should be $x = 5.6$cm. Check her workings.

Copy and complete the following to find the missing length x in this triangle.



$$\tan 31 = \frac{?}{?}$$
$$x \times \tan 31 = ?$$
$$x = \frac{?}{\tan 31}$$

Sine and cosine ratio: side lengths

Notes and guidance

This small step starts with how to choose between sine and cosine to find a missing length. Teachers should emphasise that this is dependent on which side lengths are involved in the question. As an extension, students could explore the relationship between the sine and cosine ratios. Here, teachers could emphasise with students that the 'co' in cosine refers to the 'sine of the complement.'

Key vocabulary

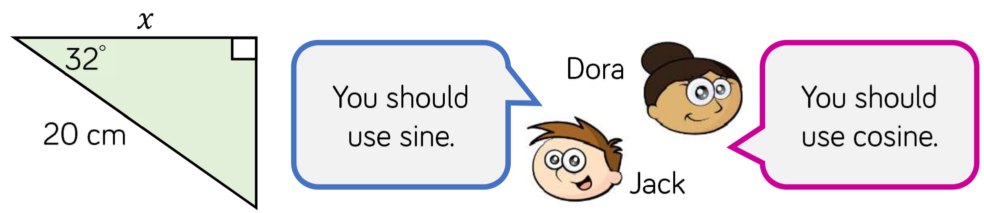
Sine	Cosine	Complement
Opposite	Adjacent	Hypotenuse

Key questions

How do we know which trigonometric ratio to use?
Why do we always label the hypotenuse first?
Why does sine 30° = cosine 60°?
Can you find other pairs of angles where sine x = cosine y? What do you notice about these pairs of angles?

Exemplar Questions

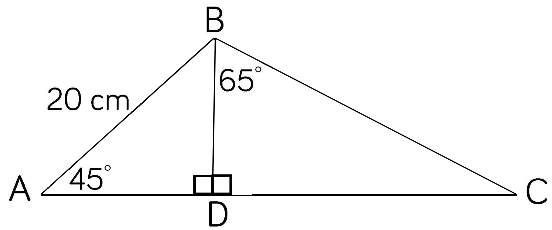
Teddy and Dora are calculating the missing length x.

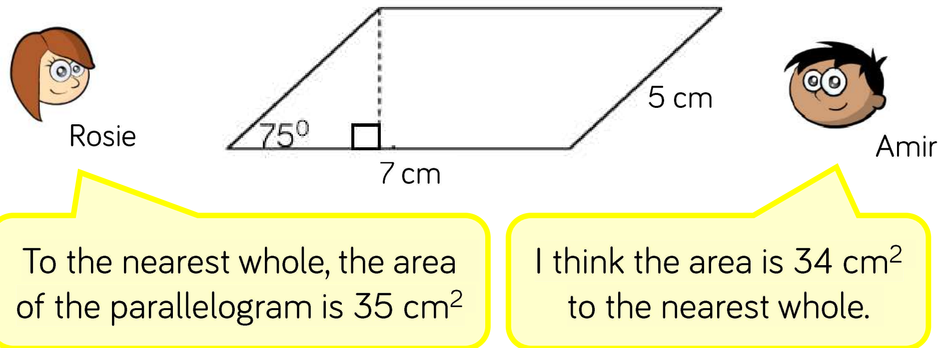


Who's right, Jack or Dora? Can they both be right? Explain your answer.
Show that the missing value x is 17.0 cm.

Calculate the length of BD.

Calculate the length of BC.





Who is correct?
Support your answer with mathematical workings.

Sin, cos and tan: side lengths

Notes and guidance

Building on previous steps, students now need opportunities to identify which trigonometric ratio to use, particularly in problems which are less structured.

By starting with an exploration of all possible trigonometric ratios in a given diagram, students begin to become more flexible in their approach to finding missing side lengths.

Key vocabulary

Sine	Cosine	Tangent	Opposite
Adjacent	Subject of formula		Hypotenuse

Key questions

How do we know which trigonometric ratio to use?

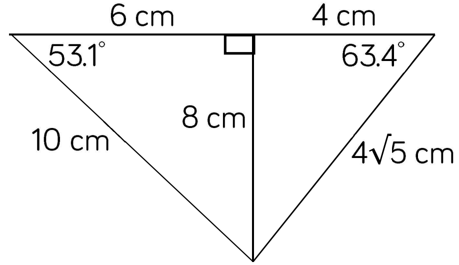
Is there more than one method of finding a missing side length? Explain your thinking.

Exemplar Questions

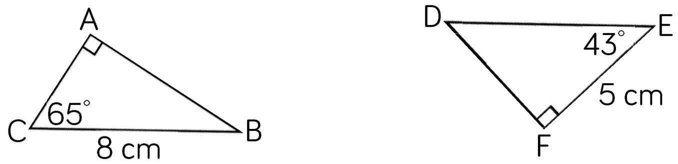
Dexter is writing down as many trigonometric relationships as he can find between the sides and angles in these right-angled triangles.
Here is the start of his list:

$$\sin 63.4 = \frac{8}{4\sqrt{5}}$$

$$\tan 26.6 = \frac{4}{8}$$

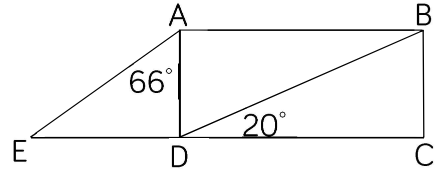


Complete Dexter's list. How many relationships did you find?
Compare your list with a partner.



Which triangle has the largest perimeter?
Calculate the difference between the perimeters.

ABCD is a rectangle.
CD = 15 cm.
CE is a straight line.
Find the length CE.



Sin, cos and tan: angles

Notes and guidance

When introducing the inverse, students might start by practising using their calculators to solve equations such as $\sin\theta = 0.33$

It's important to expose students to different notation such as angle ABC and angle x. Ensure students are given examples where all 3 lengths of a right-angled triangle are given so that they can explore different methods of finding the same angle.

Key vocabulary

Angle	Obtuse	Acute	Inverse
$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$	

Key questions

What is an inverse trigonometric function?
What's the notation for an inverse trigonometric function?
What's the difference between $\sin x$ and $\sin^{-1} x$?
Why do we need to use an inverse trigonometric function to find a missing angle?

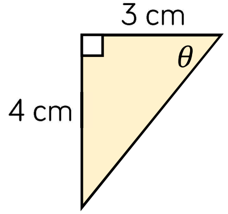
Exemplar Questions

Use your calculator to find angle θ

$\sin\theta = 0.5°$ $\quad$ $\cos\theta = 0.27$ $\quad$ $\tan\theta = 0.11$

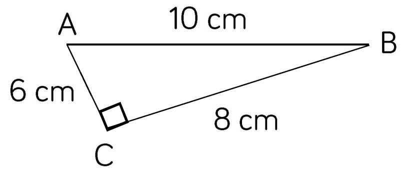
Complete:

$\tan x = \frac{?}{?}$ $\quad$ $x = \tan^{-1}\frac{?}{?}$ $\quad$ $x = ?$

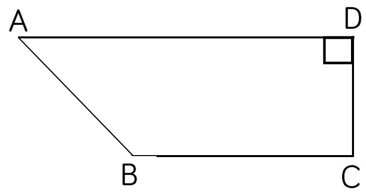


Here is Annie's method to find angle ABC.
Explain the mistake that Annie has made.

Let angle ABC $= \theta$
$\sin\theta = \frac{8}{10}$
$\theta = \sin^{-1}\frac{8}{10}$
$\theta = 53.13°$ (2 dp)



Is there more than one possible mistake? Find angle ABC.



AD = 10 cm, BC = 7 cm and CD = 5 cm.
Find angle ABC.

Pythagoras' theorem

R

Notes and guidance

Students are already familiar with Pythagoras' theorem from Year 9

This step reviews prior knowledge to ensure that students are confident in applying Pythagoras theorem.

Here, the aim is to use unfamiliar contexts to test depth of understanding.

Key vocabulary

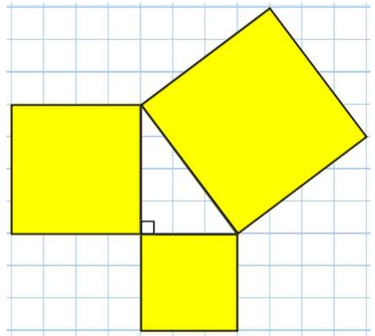
Area	Square
Square root	Right angle

Key questions

How can we use side lengths to explore whether a triangle is right-angled?
What other topics could Pythagoras' theorem link to?
(e.g. area, congruency, similar triangles, compass directions, distance between two co-ordinates.)

Exemplar Questions

Explain what this diagram tells us about the side lengths of a right-angled triangle.
If this diagram is drawn on 1 cm squared paper, work out the length of the hypotenuse.



Sketch representations of right-angled triangles, labelling all side-lengths, to match each of the following calculations.

$12^2 + 5^2 = 13^2$

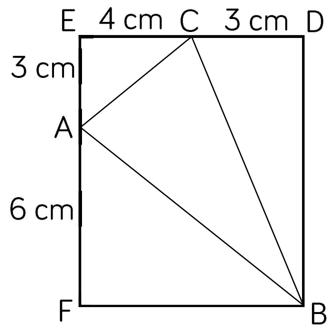
$6^2 + ?^2 = 100$

$16^2 - 12^2 = ?^2$

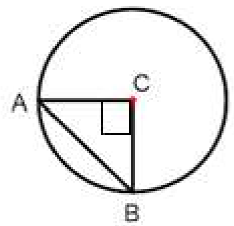
BDEF is a rectangle.

Is triangle ABC right-angled?

Show calculations to support your answer.



C is the centre of the circle.
If AB = 28 cm, find the radius of the circle.



Right-angled triangle problems

Notes and guidance

In this small step, students make decisions about when to use trigonometric ratios and when to use Pythagoras' Theorem to solve problems.

They also realise that in some situations, either can be used. Scaffolding to support students through problems needs to be reduced as they become more confident.

Key vocabulary

Pythagoras' Theorem	Similar	Hypotenuse
Adjacent	Opposite	

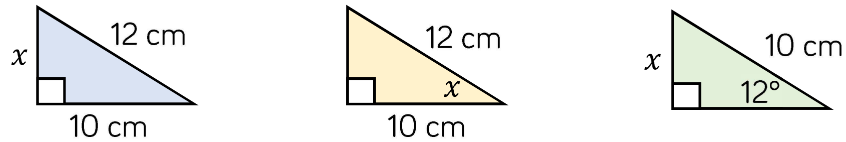
Key questions

Which calculation to solve this problem is most efficient?

In this problem, is it more efficient to use Pythagoras' Theorem or trigonometry? Which has least steps?

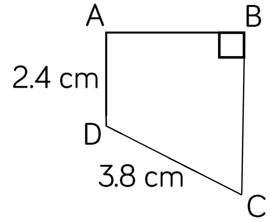
Exemplar Questions

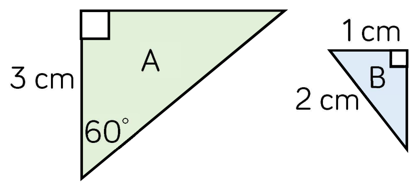
Find each value of x.



What's the same and what's different about each question?

ABCD is a trapezium. Angle BCD is 48°
What side lengths can you work out?
What angles can you work out?
Calculate the area of the trapezium.
Calculate the perimeter of the trapezium.





These triangles must be similar because triangle A is an enlargement of triangle B.

Dora is correct, but her reason is incomplete.

Show that the two triangles are similar by

- calculating the missing lengths on the two triangles.
- explaining why triangle A is an enlargement of triangle B.

Key angles (1)

Notes and guidance

In this small step, students are focusing on finding the exact trigonometric values of $30^\circ, 60^\circ$ and 45°

Students start with relevant right-angled triangles to investigate these values, comparing their answers.

Modelling how to use this information to solve right-angled triangle problems without a calculator is key. Students then use these facts to solve problems without use of a calculator.

Key vocabulary

Surds	Exact value	Simplifying

Key questions

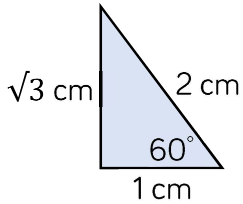
What do we mean by 'leave your answer as an exact value'?

What is a surd?

How do we simplify a fraction?

Exemplar Questions

Use the right–angled triangle to find the exact values of:



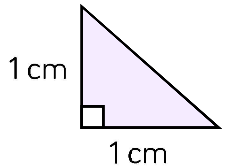
tan 30° | sin 30° | cos 30°

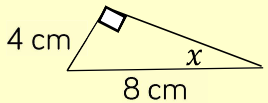
tan 60° | sin 60° | cos 60°

Look at your answers. What's the same and what's different?

Find the missing length and the missing angles in this triangle.
Use your triangle to find the exact values of:

sin 45° | cos 45° | tan 45°





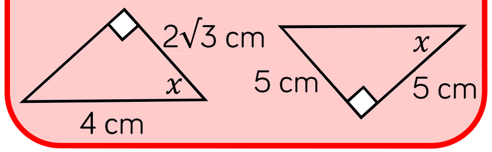
Copy and complete:

$\sin x = \frac{?}{?} = \frac{1}{2}$

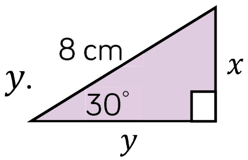
We know $\sin ? = \frac{1}{2}$

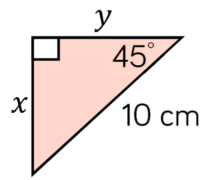
So $x = ?$

Without using a calculator, find the missing angles, x, in the following triangles.



Without using a calculator, find the exact values of x and y.





Key angles (2)

Notes and guidance

This step explores the sine, cosine and tangent of 0° and 90° and can be supported with either concrete resources and/or a dynamic geometry package. To deliver this, time will need to be allocated for student exploration and discussion, as well as pre-planned regular teacher-class discussion points where key ideas can be shared and built upon.
The concept of infinity could be explored.

Key vocabulary

Infinity	Approaching	Increasing
Decreasing	Limit	

Key questions

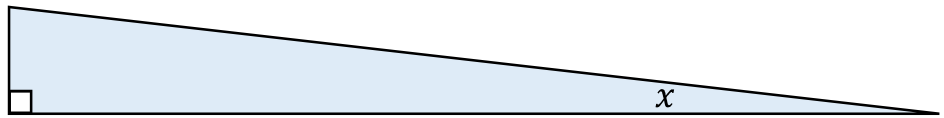
When angle x decreases in size, what happens to the length of the opposite side?

Why can't we define $\tan 90^\circ$?

Why do we say that a number divided by infinity is 0?

Exemplar Questions

Use a dynamic geometry package to draw the triangle.

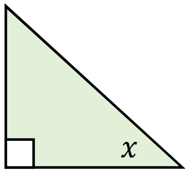


Now move the hypotenuse so that angle x is 0°
This helps to show why $\sin 0 = 0$
The opposite side is now 0 cm, so $\frac{\text{opp}}{\text{hyp}} = 0$

Discuss why $\tan 0 = 0$ and $\cos 0 = 1$

Explore what happens when angle x in the diagram approaches 90° by using your calculator to complete this table.

Round answers to the nearest whole number.



Angle (x)	89	89.9	89.99	89.999	89.9999	89.99999
Sin (x)						
Cos (x)						
Tan (x)						

Use your table to help you complete the following.
Discuss the reasons for these.

$\sin 90 =$

$\cos 90 =$

$\tan 90 =$

Use trigonometry in 3-D shapes H

Notes and guidance

Students start by recognising 3-D right-angled triangles in a 3-D shape. Using actual 3-D shapes or a dynamic geometry package is a useful way of exploring this.
The first exemplar question can be extended by adding on dimensions and/or angles and asking students to find missing dimensions and/or angles.
Also consider using cones and cylinders as examples.

Key vocabulary

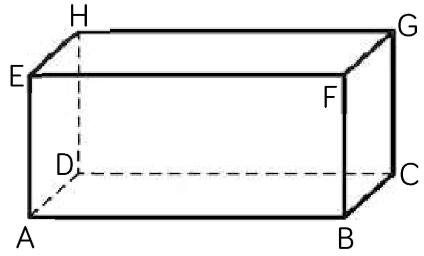
Prism	Plane	Slope	Isosceles
Midpoint	Diagonal	Square-based right pyramid	

Key questions

What is a prism? What is a plane in geometry?
What is the angle between the edge HD and the face ABCD? (see diagram of cuboid)
What is a square-based right pyramid?
What does this tell us about vertex E?

Exemplar Questions

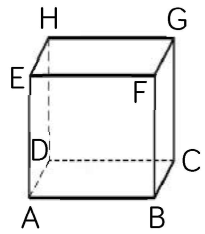
Rosie writes a list of right-angled triangles that she can see in this cuboid:
triangle ABC, triangle ACG, triangle BDF…

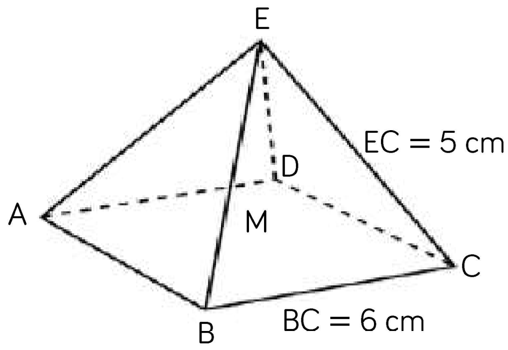


Continue her list. Can you get over 10 right-angled triangles?

The diagram shows a cube. AB = 10 cm.
Find:

- Length AH
- Length BH
- Angle ABH





This is a square-based right pyramid. M is directly in the middle of the base.
What does this tell us about vertex E and point M?
Find length AC.
Find the length MC and the height EM.
Find the angle that the sloping face BEC makes with the base.

Area of a triangle

H

Notes and guidance

Check students understand how to use standard notation to label lengths and angles of a triangle.
It is important to emphasise that the formula may 'look different', depending on which angle is given, e.g. Angle C as opposed to Angle A.
Students progress to choosing the correct angle, based on the given sides, to substitute into the formula.

Key vocabulary

Area	Perpendicular	Expression
Formula	Non-right-angled	

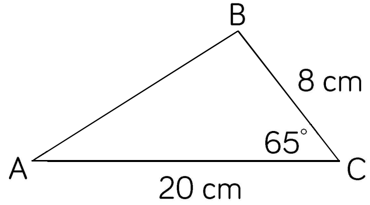
Key questions

How do we label angles and sides in a non right-angled triangles? Why is a standard format helpful?

How does this formula relate to the standard formula for finding the area of a triangle: $\frac{1}{2} \times$ base $\times$ height?

Exemplar Questions

Find the perpendicular height and the area of the triangle.



Are Tommy and Jack right? How can the triangle help you decide?

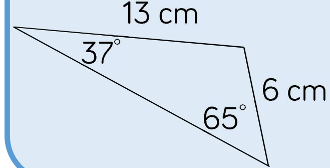
You can use algebra to find a formula for the area of a non-right angled triangle. The formula is Area $= \frac{1}{2}ab\sin C$

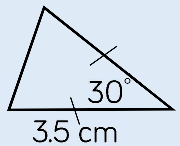
Jack

I think these will also work,
Area $= \frac{1}{2}bc\sin A$
Area $= \frac{1}{2}ac\sin B$

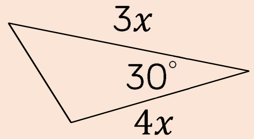
Tommy

Find the area of these triangles.

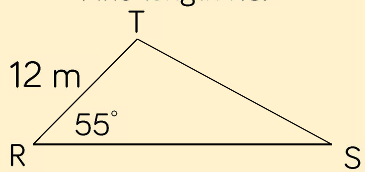




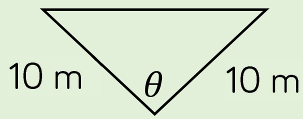
Find an expression for the area of this triangle.



Area of triangle RST $= 100\text{ m}^2$
Find length RS.



Area triangle KLM $= 25\sqrt{3}\text{ cm}^2$
Find angle θ



Sine rule: finding lengths

H

Notes and guidance

Students start by deriving the sine rule. This allows them to make connections to previous learning. They then consider correct substitution into the formula, particularly focussing on using the correct angle.

Finally, students begin to explore problems involving the sine rule. Scaffolding is provided in the exemplar questions, but this could be removed where appropriate.

Key vocabulary

Opposite	Substitute	Equation
Formula	Rearrange	

Key questions

How do we know which angle to substitute into the sine rule?
What if this angle isn't given? How can we find it?
What information do we need in a triangle in order to use the sine rule?

Exemplar Questions

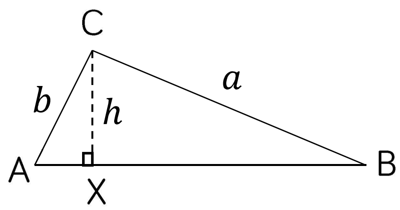
Complete the following.

In triangle BCX, $h = a \sin B$

In triangle ACX, $h =$ ______

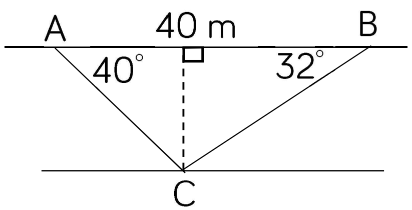
So, $a \sin B =$ ______

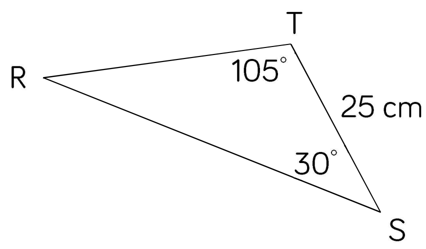
So, $\frac{a}{\sin ?} = \frac{?}{\sin B}$



Ron is calculating the width of a river. He marks two points, A and B, on the river bank so that they are 40 m apart. He then marks on point C on the opposite river bed. The river beds are parallel.

- Find angle ACB.
- Use the sine rule to find length AC.
- Now calculate the width of the river.





Annie wants to find side length RS. She uses the following method but then gets stuck as she doesn't know angle TRS.

$$\frac{t}{\sin 105} = \frac{25}{\sin\ ?}$$

Find angle TRS and then complete her workings to calculate length RS.

Sine rule: finding angles

H

Notes and guidance

Start by exploring different ways of writing the sine rule. Ensure students understand which rearrangement of the sine rule is most efficient depending on what they are trying to find.

Teachers could also consider $\frac{\sin C}{c}$ when introducing this formula. Students also need to be reminded of the need to use the inverse function when finding an angle.

Key vocabulary

Rearrange	Subject of the formula
Inverse	

Key questions

How do you know which angle to substitute into the formula?

How do you know which length to substitute into the formula?

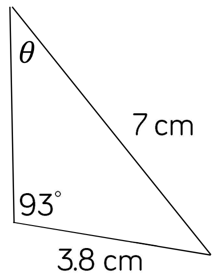
Exemplar Questions

Dora reasons that if $\frac{a}{\sin A} = \frac{b}{\sin B}$, then $\frac{\sin A}{a} = \frac{\sin B}{b}$

Show that Dora is correct by rearranging $\frac{a}{\sin A} = \frac{b}{\sin B}$

Which rearrangement of the sine rule would you use to find a missing length?

Which rearrangement would you use to find a missing angle? Why?



Amir calculates the missing angle θ

$$\frac{\sin\theta}{3.8} = \frac{\sin 93}{7}$$

$$\sin\theta = \frac{\sin 93}{7} \times 3.8$$

$$\sin\theta = 0.542113176$$

Amir finishes by stating: $\theta = 0.54$ (2d.p)

He doesn't think this answer makes sense, where is his mistake?

Sketch the following triangle:

- Length AB = 12 cm
- Length BC = 9 cm
- Angle ACB = 103°

Now use the sine rule to calculate angle BAC.

Now find the final missing angle and the final missing length.

Cosine rule: finding lengths

H

Notes and guidance

Teachers should guide the students through each step of deriving the cosine rule. Once they have derived the cosine rule, it's important that they understand that this formula can be used for any missing length (not just if it's labelled a).

Then, after practising correct substitution to find a missing length using a calculator, revisit exact values to ensure familiarity of non-calculator use.

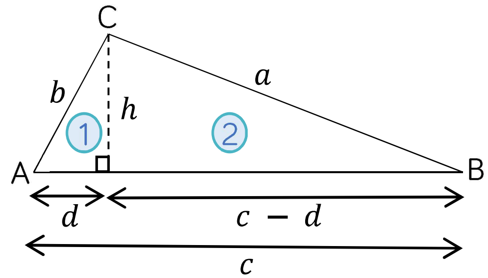
Key vocabulary

Exact value	Formulae
Substitution	Cosine Rule

Key questions

Why isn't it always possible to use the sine rule?
When finding a missing length, how do we know which angle to substitute into the formula?
How do we know when to use the sine rule?
How do we know when to use the cosine rule?

Exemplar Questions



Use Pythagoras' Theorem in triangle 1 to express h^2 in terms of b and d

Using triangle 2, we know that $h^2 = a^2 - (c - d)^2$
Simplify this to show that $h^2 = a^2 - c^2 + 2cd - d^2$

Equate the two expressions for h^2 to deduce that $a^2 = b^2 + c^2 - 2cd$

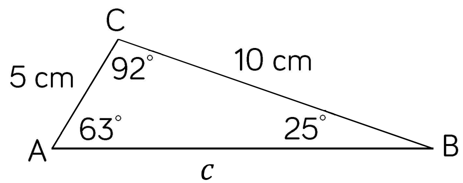
Using triangle 1, we know that $\cos A = \frac{d}{b}$, therefore $b\cos A = d$

Substitute $b\cos A = d$ into $a^2 = b^2 + c^2 - 2cd$ to deduce that:
$a^2 = b^2 + c^2 - 2bc\cos A$

If: $a^2 = b^2 + c^2 - 2bc\cos A$
Explain why: $b^2 = a^2 + c^2 - 2ac\cos B$
What will the formula involving $\cos C$ look like?

Find the exact value of length AB.

Use a calculator to find c.



Cosine rule: finding angles H

Notes and guidance

Teachers could start with an example whereby students must correctly substitute numbers into the cosine rule and then rearrange to find the angle. This particularly supports students who struggle to remember the rearranged rule where $\cos A$ is the subject of the formula. Then, teachers can help students to break down problems into stages before providing unstructured problems.

Key vocabulary

Subject of the formula	Inverse
Rearrange	

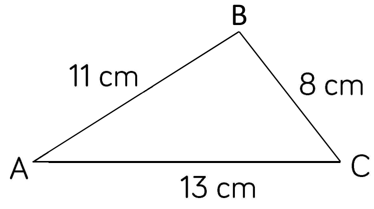
Key questions

Why can't we use the sine rule in this triangle to find the missing angle?
How do we know which numbers to substitute into the formula?
Can we find all 3 missing angles in the triangle?

Exemplar Questions

Teddy is calculating angle A in the triangle below. Complete his workings.

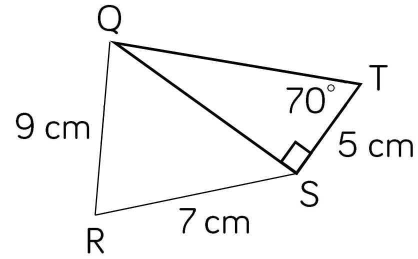
$$?^2 = ?^2 + ?^2 - 2 \times 13 \times 11 \times \cos A$$
$$= 290 - \quad ? \quad \cos A$$
$$\cos A = 290 - ?$$
$$\cos A = \frac{?}{?}$$
$$A = \cos^{-1} ?$$



Rosie is calculating angle RQS.

What length in triangle RQS does she need to find first?

- Calculate this length.
- Calculate angle RQS.



Sketch a triangle with side lengths 15 cm, 10 cm and 20 cm.

Use the cosine rule to work out the largest angle in the triangle.

Choose sine or cosine rule (1) H

Notes and guidance

Class discussion should explore which rule is most appropriate given specific information about the triangle. They then need support in unpacking problems into smaller more manageable steps.

Modelling how to break problems down, helps students to realise that problem solving is a process rather than a one-step response.

Key vocabulary

Opposite	Adjacent	Segment
Sine Rule	Cosine Rule	Included Angle

Key questions

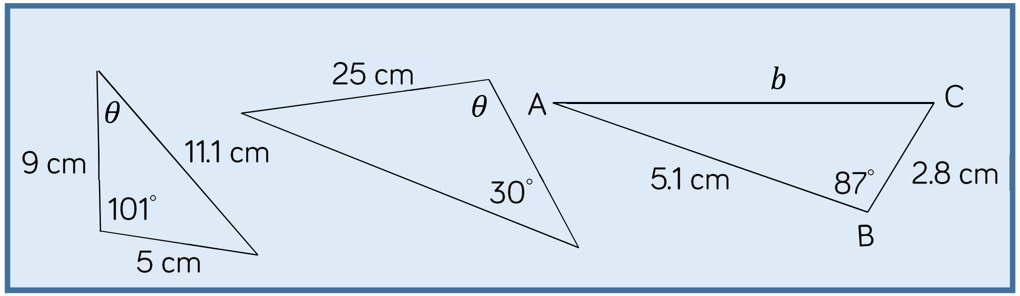
How do you know whether to use the sine or cosine rule to solve a problem?

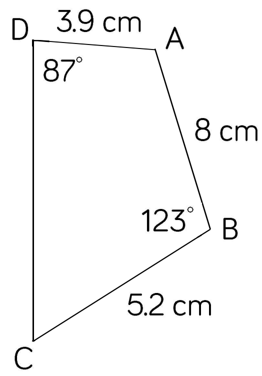
Look at the diagram. What do you know? What can you find out?

How can you break down the problem into smaller steps?

Exemplar Questions

For each triangle, decide whether you can use the sine rule, cosine rule, or either rule, to find the labelled missing length or angle.



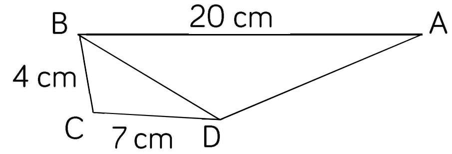


Mo is working out the area of the quadrilateral ABCD.
He starts by breaking the problem down into smaller steps:

- Find area of triangle ABC
  - Find length AC
  - Find angle ACD
  - Find angle DAC
- Find area of triangle ADC

Use Mo's steps to find the area of the quadrilateral.

Angle ADB = 120°
Angle DAB = 25°
Find length BD and angle CBD.



Choose sine or cosine rule (2) H

Notes and guidance

This continues to explore problems where students must choose between the sine rule and the cosine rule but extends to problem solving where application of other mathematical concepts, such as using ratios, is necessary.

The problems are unstructured, but teachers could add varying degrees of scaffold to support students.

Key vocabulary

Sine Rule

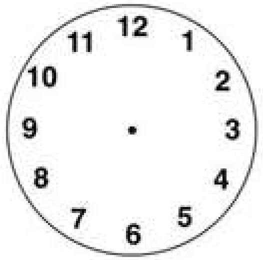
Cosine Rule

Key questions

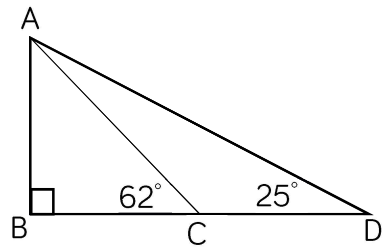
Look at the diagram. What do you know? What can you find out?
Where could you start? What might your first step be?
How can you break down the problem into smaller steps?
What might you do if you feel 'stuck'?

Exemplar Questions

A circular clock face has centre O.
The long hand is OA and 6 cm in length.
The short hand is OB and 4 cm in length.
The time is 4 o'clock.
Find the distance from A to B.



In a triangle, angle A : angle B : angle C are in the ratio 3 : 5 : 10
Find the length of side AC if side AB is 8 cm.

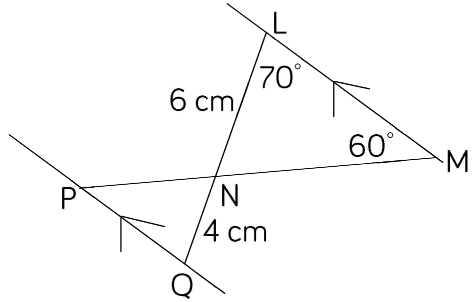


Find all the unknown angles in these triangles.

If CD = 20 m, find all the unknown lengths.

LM is parallel to PQ.

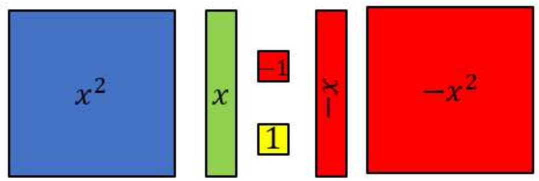
Calculate the length of PQ.

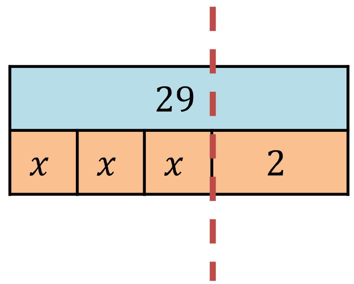


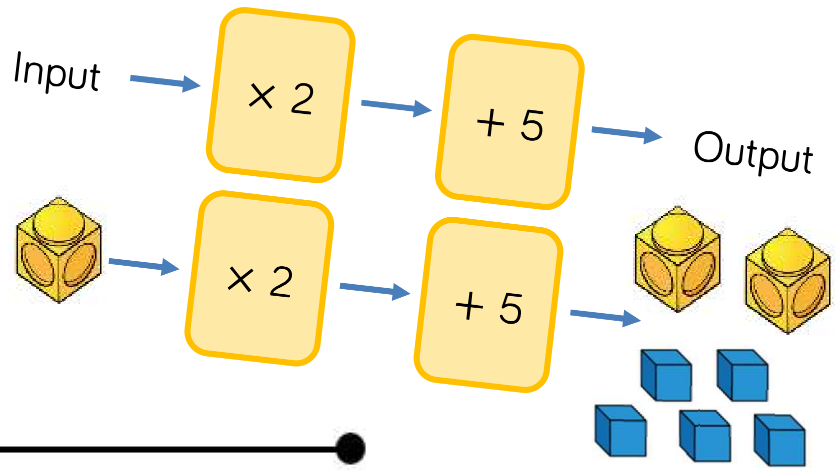
Autumn 2: Developing Algebra

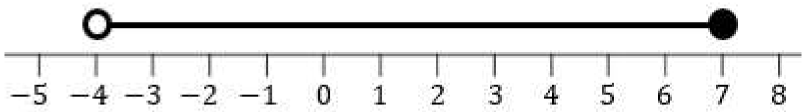
Weeks 7 to 9: Equations and Inequalities	Weeks 10 to 12: Simultaneous Equations
Students will have covered both equations and inequalities at key stage 3,and this unit offers the opportunity to revisit and reinforce standard techniques and deepen their understanding. Looking at the difference between equations and inequalities, students will establish the difference between a solution and a solution set; they will also explore how number lines and graphs can be used to represent the solutions to inequalities. As well as solving equations, emphasis needs to be placed on forming equations from given information. This provides an excellent opportunity to revisit other topics in the curriculum such as angles on a straight line/in shapes/parallel lines, probability, area and perimeter etc. Factorising quadratics to solve equations is covered in the Higher strand here and is revisited in the Core strand in Year 11 National curriculum content covered (Higher content in bold): • consolidate their algebraic capability from key stage 3 and extend their understanding of algebraic simplification and manipulation to include quadratic expressions • translate simple situations or procedures into algebraic expressions or formulae; derive an equation, solve the equation and interpret the solution • select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems; interpret their solution in the context of the given problem. • recognise, sketch and interpret graphs of linear functions, • factorising quadratic expressions of the form x^2 + bx + c (Higher only at this stage) • solve quadratic equations algebraically by factorising (Higher only at this stage) • solve linear inequalities in one **{or two}** variable**{s}**, **{and quadratic inequalities in one variable};** represent the solution set on a number line, **{using set notation and on a graph}**	Students now move on to the solution of simultaneous equations by both algebraic and graphical methods. The method of substitution will be dealt with before elimination, considering the substitution of a known value and then an expression. With elimination, all types of equations will be considered, covering simple addition and subtraction up to complex pairs where both equations need adjustment. Links will be made to graphs and forming the equations will be explored as well as solving them. The Higher strand will include the solution of a pair of simultaneous equations where one is a quadratic, again dealing with factorisation only at this stage. National curriculum content covered (Higher content in bold): • consolidate their algebraic capability from key stage 3 and extend their understanding of algebraic simplification and manipulation to include quadratic expressions • model situations mathematically and express the results using a range of formal mathematical representations, reflecting on how their solutions may have been affected by any modelling assumptions • translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution • select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems; interpret their solution in the context of the given problem. • solve two simultaneous equations in two variables (linear/linear **{or linear/quadratic})** algebraically; • recognise, sketch and interpret graphs of linear functions and quadratic functions.

Key Representations









Here are a few ideas for how you might represent algebraic expressions and the solutions of equations and inequalities.

Cups, cubes and elastic bands lend themselves well to representing an unknown, whereas ones (from Base 10) and counters work well to represent a known number. Be careful to ensure that when representing an unknown, students use equipment that does not have an assigned value – such as Base 10 equipment and dice.

Bar models are useful to support the forming of equations and also help students to make sense of the approach to a solution. Algebra tiles are also very powerful for this and help to make sense of factorising quadratics, alternate representations are very effective in ensuring all students, including higher attaining, make sense of the mathematical structures.

Equations and Inequalities

Small Steps

- Understand the meaning of a solution
- Form and solve one-step and two-step equations (R)
- Form and solve one-step and two-step inequalities (R)
- Show solutions to inequalities on a number line
- Interpret representations on number lines as inequalities
- **Represent solutions to inequalities using set notation** (H)
- Draw straight line graphs (R)
- Find solutions to equations using straight line graphs

(H) Denotes Higher Tier GCSE content

(R) Denotes 'review step' – content should have been covered at KS3

Equations and Inequalities

Small Steps

- **Represent solutions to single inequalities on a graph** H
- **Represent solutions to multiple inequalities on a graph** H
- Form and solve equations with unknowns on both sides R
- Form and solve inequalities with unknowns on both sides
- Form and solve more complex equations and inequalities
- **Solve quadratic equations by factorisation*** (*Also Foundation tier. Higher cover now, Core will cover in Year 11) H
- **Solve quadratic inequalities in one variable** H

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Understand meaning of solution

Notes and guidance

In preparation for the small steps to come, students consider what the meaning of a solution is and how they can represent this. Students consider whether a number is a solution or not by substitution and checking. They also consider how many solutions an equation could have through reasoning about different types of equations as well as why an expression would not have a solution.

Key vocabulary

Variable	Solve	Solution
Equation	Expression	

Key questions

What does the word solve mean? What connection does this have to the word solution?

Do solutions to equations have to be integers?

Can an expression have a solution?

Can an identity have a solution?

Exemplar Questions

For each of the equations, circle the correct solutions.

- $b + 8 = 25$ — $b = 17$ or $b = 33$
- $11 = 4m - 15$ — $m = -1$ or $m = 6.5$
- $10 - c = 2$ — $c = 12$ or $c = 8$
- $f + g = 10$ — $f = 7$ and $g = 3$ or $f = 11$ and $g = -1$

Is there only one solution for each of these equations? Explain how you know.

How many solutions do each of the following have? Discuss with your partner how you know.

	No solution	One solution	More than one solution
$2a + 3b = 12$			
$3c + 4c = 49$			
$3g + 4g - 5$			
$3y = 10 + y$			
$25 = t^2 + s^2$			
$p + 7 = p$			

Tom says, "in the equation $5a^2 - 3a^2 = 128$, there's only one unknown, so there's only one solution" Do you agree or disagree? Why?

Form and solve equations

R

Notes and guidance

In this review step, students practice forming and solving equations. Manipulatives such as cups and counters or algebra tiles could be useful to support students as can pictorial representations such as bar models or number lines. These can support understanding of the balance method and use of inverse relationships. Use this step to revisit other topics such as angle facts, probability etc.

Key vocabulary

Equation	Solve	Inverse
Solution	Balance	

Key questions

What does solve mean?

Does it matter which order the terms in an equation are written?

What is the same and what is different about the solution to each of these scenarios?

Exemplar Questions

Solve the equations.

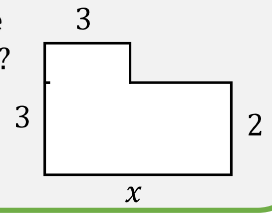
- $12g = 60$
- $5 + 3x = 44$
- $14 = \frac{t}{5} + 6$
- $12 = 60g$
- $44 + 3x = 5$
- $14 = \frac{t}{5} - 6$

Explain why each story matches with the equation $2x + 3 = 23$
What does x represent in each story?

A taxi meter starts at £3
It then costs £2 for every mile.
If the ride costs £23 altogether, how many miles is the journey?

I think of a number.
I double it and add 3
My answer is 23
What was the original number?

The area of this shape is 23 cm^2
What is the length of x?



A taxi meter starts at £3
It then costs £2 for every mile.
If the ride costs £23 altogether, how many miles is the journey?

The angles in a triangle form a linear sequence with common difference 10. If the smallest angle is $x°$, form and solve an equation to work out the angles in the triangle.
(Hint: You may use a bar model to help you).

Form and solve inequalities

R

Notes and guidance

It is useful to compare and link equations and inequalities. Beware of students changing the inequality sign to an equals sign to 'make it easier' and also assuming an integer solution is needed e.g. giving the solution $x > 5.5$ as $x = 6$ or even $x > 6$. They also need to be aware that sometimes questions will ask for smallest/greatest integer so they need to pay close attention to the question.

Key vocabulary

Inequality	Solve	Inverse
Solution set	Greater/less than (or equal)	

Key questions

What's the same and what's different about solving an equation or an inequality?

How many solutions does an inequality have?

Does an inequality still hold true if you multiply/divide both sides by a negative number? Why?

Exemplar Questions

Form an equation for each of the scenarios. What's the same and what's different?

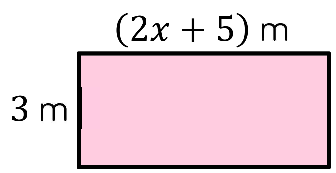
Alex is x years old. Ron is twice as old. The sum of their ages is 96

Alex is x years old. Ron is twice as old. The sum of their ages is less than 96

Alex is x years old. Ron is twice as old. The sum of their ages is at least 96

Form and solve an inequality for the following:

- I think of a number. I multiply it by 3. I then add 17. My answer is greater than 28 What is the smallest integer the number could be?
- Rosie buys 5 pens. She also buys a ruler for 70p. She pays with a five-pound note. What is the most each pen could have cost?
- The area of this rectangle is between 27 m² and 39 m² (inclusive)



What is the maximum and minimum perimeter of the rectangle?

Show solutions on a number line

Notes and guidance

It can be useful to encourage students to read the inequalities out loud to help them negotiate the meaning of the inequality symbols and how to represent them on a number line.
Students should first be introduced to conventions of this topic, such as the meaning of the shading of the circle i.e. a shaded circle means the number is included, unshaded means the number is not included.

Key vocabulary

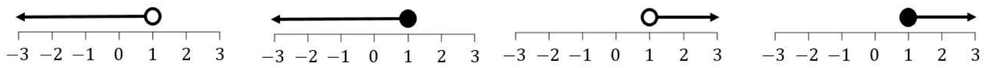
Solution set	Greater/less than (or equal to)
Inequality	Number line

Key questions

How would you read the inequality out loud?
What are the possible integer solutions for this inequality?
Is e.g. -1, 5, 0.5 a possible solution for this inequality? Why?
What does the circle mean? Which direction will the line go?

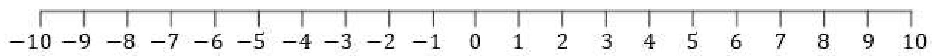
Exemplar Questions

What's the same and what's different about each of the diagrams?
Match each number line to the inequality it represents.



$x > 1$ $\quad$ $x < 1$ $\quad$ $x \geq 1$ $\quad$ $x \leq 1$

Show the solutions for these inequalities on a number line.



$x \geq 6$ $\quad$ $x < 6$ $\quad$ $-3 < x < 6$ $\quad$ $-3 \leq 3x < 6$

$x \leq 2$ $\quad$ $x + 2 \leq 2$ $\quad$ $-1 < x + 2 \leq 2$ $\quad$ $-1 \leq 3x + 2 \leq 2$

Show the possible solutions on a number line.

- I think of a number. I add 7. I then double it. My answer is less than 30
- Dexter buys 2 packs of stickers. He also buys a magazine for £4. He pays with a ten-pound note, and gets less than £3 change. What can we say about the price of a pack of stickers?

White Rose Maths

Number line representations

Notes and guidance

Now students interpret the meaning of a given number line representation and put it into an inequality format. Again, the meaning of the shading of the circle, the direction of the line and how this relates to the inequality format needs discussion. Stem sentences may be useful here to guide students. It is worth revisiting this notation regularly in starters (working both ways) to aid retention.

Key vocabulary

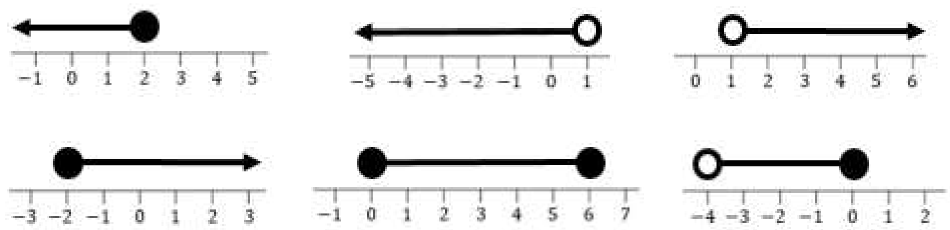
Solution set	Greater/less than (or equal to)
Inequality	Number line

Key questions

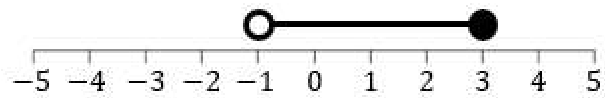
Do the solution sets contain only integers?
What's the difference in meaning if the circle is shaded or unshaded?
What direction is the line going? Are the values of the unknown smaller or greater than the number(s) shown by the circle(s)? How can you show this in an inequality?

Exemplar Questions

Write down the inequality represented by each diagram.

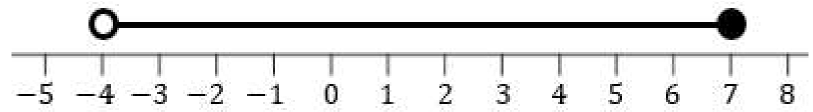


The solution for x is represented on the number line.
Are the statements below true or false? Explain how you know.



$-1 \leq x < 3$	x is greater than −1, but less than or equal to 3	x must be 0, 1, 2, or 3

The diagram shows the possible range of values for a number x



Find the single value of x if you are also given that:

- x is prime number
- $x^2 \leq 36$
- $x - 5 > -2$

Solutions using set notation H

Notes and guidance

In this higher step, students make links between the number line representation, the verbal description and formal set notation. Students will be familiar with the term 'union' from Year 7 and afterwards, but the colon notation meaning 'such that' will be new. The key aspect of this step is flexibility, so matching activities to compare representations are particularly useful here.

Key vocabulary

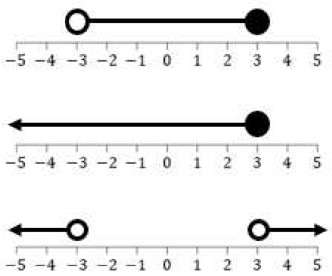
Set notation	The solution set is x such that…
Solution set	Union

Key questions

Which representation do you think is the easiest/hardest to understand?
How would you read this out loud?
What's the same and what's different about these representations?

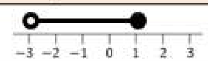
Exemplar Questions

Match each number line with its corresponding inequality and solution given in set notation.

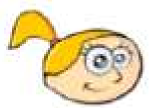


$\{x: -3 < x \le 3\}$	$7x - 13 \le 8$
$\{x: x > 3 \cup x < -3\}$	$-9 < 4x + 3 \le 15$
$\{x: x \le 3\}$	$x^2 > 9$

Complete the table.

Inequality	Number line representation	Solution set
		
		
$1 \ge 10x - 7$		
$9 \le 2x - 11 < 17$		
		$\{x: -3 \le x < 0\}$

Eva and Whitney have both written the solution for the inequality $\frac{3x+7}{2} \ge 35$ using set notation.



Eva

$\{x: 3x + 7 \ge 70\}$

$\{x: x \ge 21\}$

Whitney

Are they both correct? Explain why.

Draw straight line graphs

R

Notes and guidance

This review step reminds students how to draw linear graphs, making connections between the representations as a graph, an equation, a table of values and a set of coordinates. Students should be encouraged to look for errors in their table of values if their points do not form the expected straight line. Both calculator and non-calculator methods of dealing with negative numbers should be explored.

Key vocabulary

Gradient	Positive/Negative	Linear
y-intercept	Coordinate	Plot

Key questions

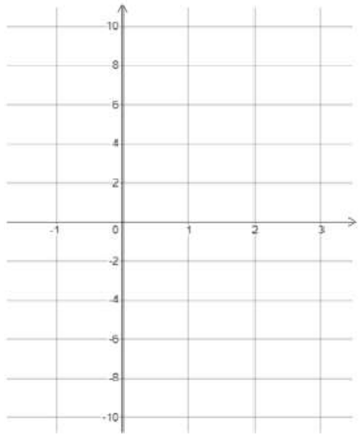
How do you decide what values of x to choose for a table of values?
What does the gradient of a graph tell you?
Why is it helpful to have the equation in the format $y = mx + c$ in order to plot a linear graph?
How many solutions does the equation of a straight line have?

Exemplar Questions

Complete the table of values for $y = 4x - 3$

On the grid, draw the graph of $y = 4x - 3$

x	-1	0	1	2	3
y					



Plot and label the following sets of graphs. What's the same and what's different?

$y = 5x$
$y = 3x$
$y = -3x$
$y = \frac{1}{2}x$

$y = 5x + 4$
$y = 3x + 4$
$y = -3x + 4$
$y = -3x - 4$

$y = 3(x + 2)$
$y = 3(x - 2)$
$y = \frac{(x + 2)}{2}$

Mo and Annie explain how they plotted the line $y = 3x + 2$
Draw the graph two times using the methods Mo and Annie describe.
Which do you prefer?

First, I plotted (0, 2)
From this point, for every one I moved across on the x-axis, I moved up 3 on the y-axis.
I then completed the line.

I completed a table of values for x for the range -2 to 2
I then plotted each coordinate pair and completed the line.

Find solutions graphically

Notes and guidance

Here students learn the connection between solving algebraically and solving graphically. It can be useful to draw attention to the fact that for a linear equation there will only be one point where the graphs meet and the x value corresponds to the solution of the linear equation. This is useful conceptual understanding that will help students later with solving simultaneous equations.

Key vocabulary

Set equal	Solution	Intersect
Solve graphically	Solve algebraically	

Key questions

Why don't we need to draw tables of values for graphs like $y = 3$ and $x = -2$?

How do we know which graphs to draw to solve e.g. $5x - 2 = 9$?

Is it always possible to solve two sets of equations graphically? Is a graphical method always going to be useful?

Exemplar Questions

Draw a set of coordinate axes from -6 to 6 in both directions.
Show these straight lines on your grid.

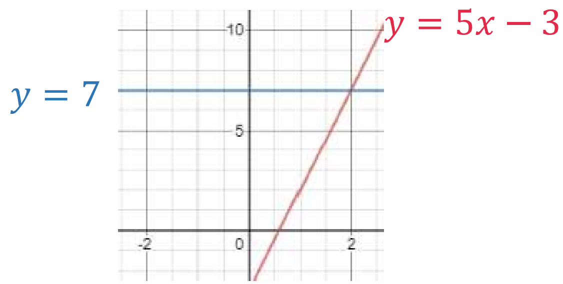
$x = 3$ $\quad$ $y = 1$ $\quad$ $x = -2$ $\quad$ $y = -4$

Where do the following pairs of lines meet?

- $x = 3$ and $y = -4$
- $y = 1$ and $x = -2$
- $x = -2$ and $y = -4$
- Explain why $x = 3$ and $x = -2$ never meet

What's the same and what's different about these representations?

$$5x - 3 = 7$$
$+3 \quad +3$
$$5x = 10$$
$\div 5 \quad \div 5$
$$x = 2$$



Which of these graphs would you draw to solve $\frac{1}{2}x + 5 = 3$?

$x = 3$ $\quad$ $y = 3$ $\quad$ $y = \frac{1}{2}x + 5$ $\quad$ $x = \frac{1}{2}y + 5$

Find where $y = \frac{1}{2}x + 3$ meets $y = 4$, $y = -1$ and $y = -1.5$

Write down the equations you can solve using your answers.

Single inequalities on a graph H

Notes and guidance

Students need to know the convention that e.g. $x \leq 3$ is represented by shading to the left of the solid line $x = 3$ whilst $x < 3$ the line would be dashed, linking to the shading of circles in number lines representing inequalities. Testing a single point above or below the line $y = 3x - 1$ is useful to decide where to shade e.g. $y > 3x - 1$; using the origin is a good strategy.

Key vocabulary

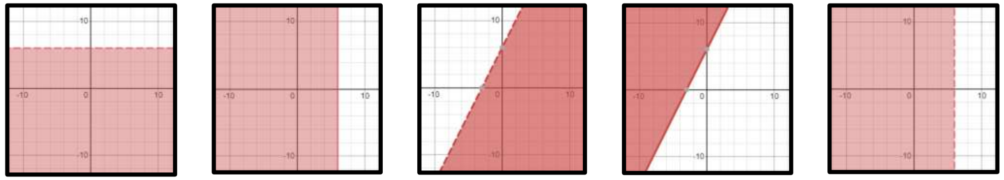
Inequality	Satisfy	Region
Dashed line	Solid line	Test point

Key questions

What's the same and what's different about the graphs?
What is the significance of the dashed line and solid line when looking at regions of inequalities?
Does the point e.g. (4, 2) satisfy this inequality? How can we find out?
Why is (0, 0) a good point to use as a test point? Why would (0, 0) not be suitable for regions like $y > -2x$?

Exemplar Questions

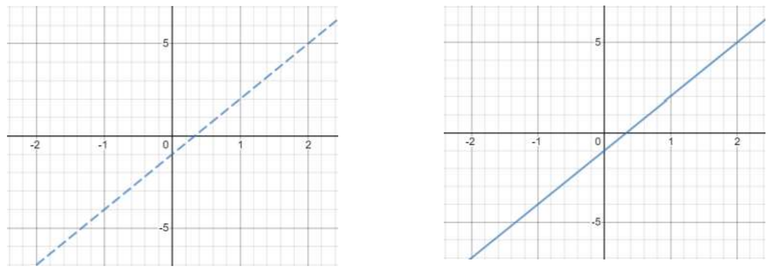
Match the inequalities to their graphical representation



$x \leq 6$ $x < 6$ $y < 2x + 6$ $y < 6$ $y \geq 2x + 6$

The line $y = 3x - 1$ is shown in both of the graphs below.

- Choose the appropriate graph and shade the region that satisfies the inequality $y < 3x - 1$
- Choose the appropriate graph and shade the region that satisfies the inequality $y \geq 3x - 1$



For each inequality draw a pair of coordinate axes going from -4 to 4 in both directions, and shade the region indicated.

- $x < 3$
- $y \geq 0$
- $y \geq \frac{1}{2}x + 1$
- $x + 3y < 9$

For which of the inequalities is (3, 0) a solution?
How can you show this graphically? Algebraically?

Multiple inequalities on graph H

Notes and guidance

Students extend their knowledge of representing inequalities on a graph to being able to shade regions defined by multiple inequalities. Students should practice both working from the graphs and writing in the inequalities and starting from the inequalities, shading the regions that are satisfied by the inequalities. Again comparing test points in/out of the regions is useful.

Key vocabulary

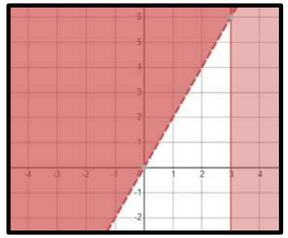
Inequality	Satisfy	Region
Dashed line	Solid line	Test point

Key questions

How can you show that the point e.g. (4, 2) satisfies each of the inequalities?
How do you decide which side of a line to shade in and which side not to shade in?
How do you show that the points on a line are included in/excluded from a solution set?

Exemplar Questions

Which inequalities satisfy the **unshaded** regions?
Give your answers in set notation.



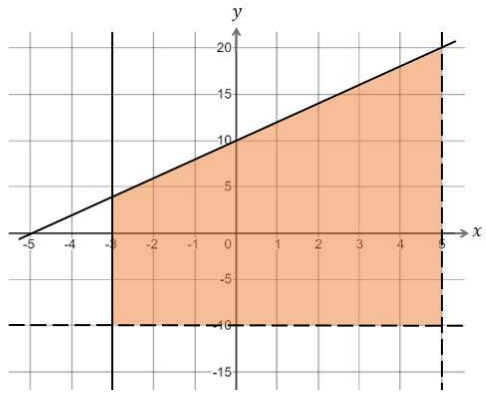
On a pair of coordinate axes going from −5 to 5 for both directions, shade the region that is satisfied by each pair or group of inequalities.

$x \geq -1$ and $y < 0$ $y \geq -2$ and $y < 2x$

$x < 0$ and $y + x > -1$ $y \geq -1, x < 2$ and $y > 2x - 1$

Find the equations of the lines that enclose the trapezium.

- Write the inequalities that are satisfied by this region
- How many solutions are there to the set of inequalities where x and y are both integers?
- Work out the area of the trapezium



Equations: unknown both sides R

Notes and guidance

Students have met equations of this form at key stage 3, so teachers will need to decide how much consolidation or practice is needed when revisiting this important topic. The use of concrete materials such as counters and cups as well as the pictorial support of bar models can be used to aid student understanding. As well as practising solving, discussion on how to form the equations is key.

Key vocabulary

Balance	Is equal to	Value
Solution	Unknown	Satisfy

Key questions

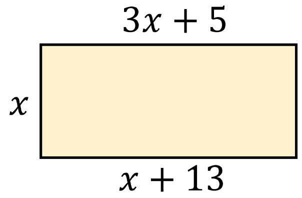
How many values of x will satisfy this equation? Why?
Explain why $8x + 3 = 8x + 9$ has no solution.
Will the solutions to $4x + 8 = 9x + 5$ be the same as the solutions to $9x + 5 = 4x + 8$?
What about $6x + 4 = 10$ and $3x + 2 = 5$

Exemplar Questions

What equation is represented by the bar model?
Solve the equation to find the value of t

t	t	t	7
t	12		

Find the perimeter and area of the rectangle.



I think of a number multiply it by 5 and subtract 12
My answer is 18 greater than my original number.
Form and solve an equation to find my original number.
Make up your own multi-step number puzzles and challenge a partner

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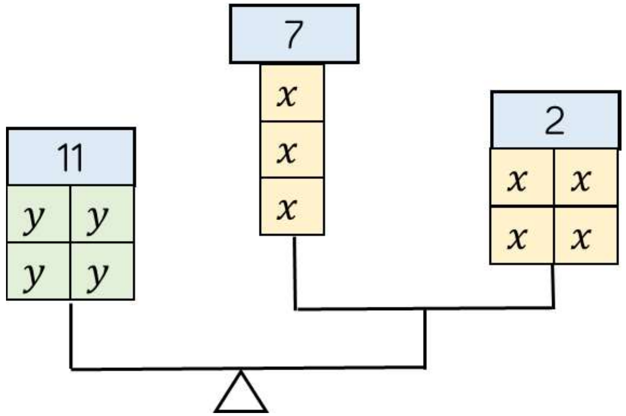
The diagram shows a balance on another balance.
Work out the values of x and y.



Inequalities: unknown both sides

Notes and guidance

This step provides consolidation of the techniques covered in the previous step and the number line notation met earlier in this block. Again teachers will need to be vigilant for students changing or omitting inequality signs.

Higher tier students should be encouraged to also give their answers in set notation to further practice this skill.

Key vocabulary

Less/greater than	Or equal to	Solution set
Linear	Inequality	Number line

Key questions

Explain the difference between an inequality and an equation.
What is the difference between $\leq$ and $<$?
Explain the difference between $x < 7$ and $7 > x$
Will the solution set of $4x + 8 > 9x + 5$ be the same as the solution set of $9x + 5 > 4x + 8$?

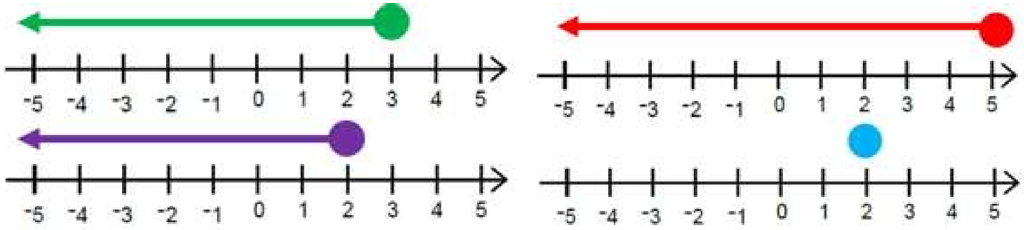
Exemplar Questions

Solve these linear inequalities.

- $3y + 12 < 4$
- $3y + 12 \leq -4$
- $3y + 12 < y - 4$
- $3y + 12 < y + 4$
- $3y - 12 \leq y - 4$
- $3y - 12 \geq y - 4$

What is the same and what is different?

Which number line represents the solution to $9x - 4 \leq 7x + 2$?

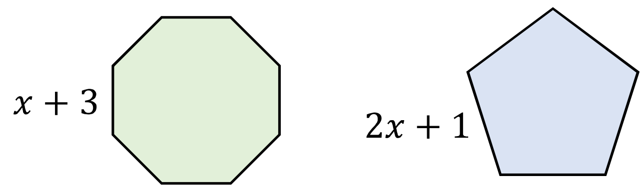


Draw a number line to show the solutions to the inequalities

$\frac{7a-5}{2} > 2a - 1$ $\quad\quad$ $2b + 7 \geq \frac{b}{2} + 1$

The perimeter of the regular octagon is less than the perimeter of the regular pentagon.

- Show this information as an inequality in terms of x
- Find the smallest possible integer value of x



Complex equations & inequalities

Notes and guidance

Students will now solve equations and inequalities where brackets may be present on one or both sides and/or more challenging contexts. The aim is to develop fluency within wider mathematics and not purely algebraic settings. Students should be exposed to different ways of answering the same question, such as multiplying the brackets out first or dividing.

Key vocabulary

Less than	Greater than	Solution(s)
Linear	Balanced	Inequality

Key questions

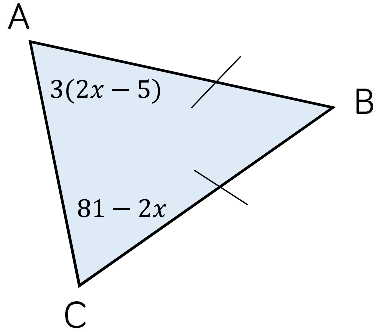
Compare (e.g.) $\frac{4x+10}{3} > 2x + 5$ and $\frac{4x}{3} + 10 > 2x + 5$
What is different about how you solve these?

Do you always need to expand brackets when they occur in an equation?

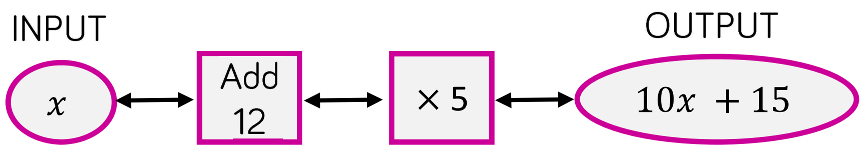
Explain the order of the steps you would take to solve…

Exemplar Questions

Calculate the size of angle ABC



Calculate the value of x.



List the integer values that satisfy both $4(x + 5) \leq 24$ and $3(3x + 1) > 7x - 9$

The angles in a triangle are $x + 50, x + 20$ and $x - 10$.
Show that the triangle is right-angled.

The solutions to the equations form a linear sequence.
Write an equation whose solution is the 4th term of the sequence.

First term: $3x + 5 = 4(x - 1.5)$

Second term: $3(2x + 1) = 5(x + 2)$

Third term: $2(x - 2) = 5 - x$

Quadratics using factorisation H

Notes and guidance

Higher tier students will have met factorisation of quadratic expressions in Year 9. Using algebra tiles and linking to area models/factors of numbers provides a solid base for making sense of factorisation. Students should also consider expressions that cannot be factorised. Making links to graphical representation of the equations is useful here and will support the next step dealing with quadratic inequalities.

Key vocabulary

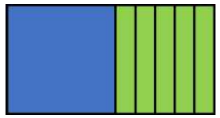
Quadratic	Roots	Solution(s)
Factorise	Brackets	

Key questions

Find some solutions to $ab = 12$ and $ab = 0$
What's the same and what's different?
How many rectangles can you make using the tiles? (e.g. 1 of x^2, 5 of x and 6 ones). Are they the same or different?
How can you tell if an equation is quadratic or linear just by looking at it? Can you think of other types of equation?
Do all quadratic equations have two solutions?

Exemplar Questions

Explain why these diagrams of algebra tiles show the given factorisations.



$x^2 + 5x \equiv x(x+5)$

$x^2 + 3x + 2 \equiv (x+1)(x+2)$

Using algebra tiles factorise the expressions, then explain why $x^2 + 3x + 5$ cannot be factorised.

$x^2 + 5x + 6$ | $x^2 + 7x + 6$ | $x^2 + 5x + 4$ | $x^2 + 4x + 4$

Which of these equations have only one solution, exactly two solutions or more than two solutions?

$2x = 0$
$x^2 = 0$
$xy = 0$
$x(x-1) = 0$
$x(x+1) = 0$
$(x+1)(x-1) = 0$

$(x+2)(x-1) = 0$
$(x+2)(x+1) = 0$
$(2-x)(1-x) = 0$
$(2-x)(1+x) = 0$
$2(2-x)(1+x) = 0$

Spot the errors in this solution.

$x^2 + 2x = 8$
$x(x+2) = 8$
Either $x = 8$ or $x + 2 = 8$
$x = 8$ or $x = 6$ ✗

Rearrange and solve the equations.

$x^2 + 2x = 8$ | $x^2 - 2x = 8$
$x^2 - 2x = 15$ | $x^2 + 2x = 3$

Quadratic inequalities

H

Notes and guidance

Using graphing software to look at multiple graphs and identifying regions is an excellent introduction to this topic. Students need to be confident in identifying which region is above or below the x axis and how this affects the solution to a quadratic inequality. Students should be encouraged to always find the critical values and draw a sketch (rather than plot the whole graph) to identify the region(s) needed.

Key vocabulary

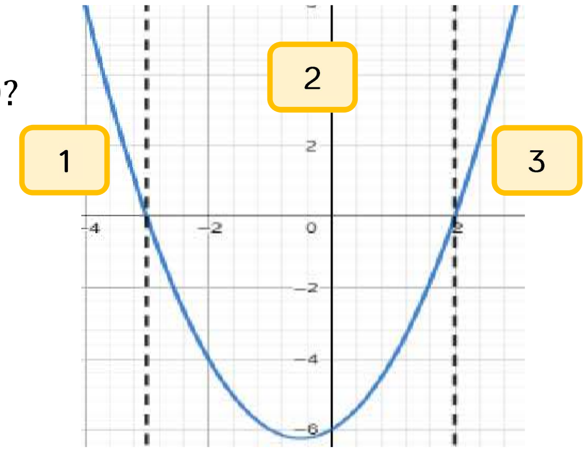
Roots	Solutions	Intercept
x-axis	Factorise	Sketch

Key questions

How does $\leq$ or $\geq$ change the solution?
How does the solution set of …> 0 differ from the solution set of …. ≥ 0?
How do we know whether to look above or below the x-axis?
What is the first step we need to take to solve a quadratic inequality.?

Exemplar Questions

Which of the region(s) 1, 2 or 3 on the graph represent(s) the solution of $(x-2)(x+3) > 0$?



Solve the inequality $(x-2)(x+3) > 0$, showing your answer on a number line, in set notation and as a pair of inequalities.

Sketch the graph of $y = x^2 + 4x - 5$, showing where the curve meets the axes. Use your graph to solve the inequalities.

$x^2 + 4x - 5 > 0$ | $x^2 + 4x - 5 \leq 0$

$x^2 + 4x \geq 5$ | $x^2 + 4x < 5$ | $x^2 > 5 - 4x$

Find the set of values that satisfies both $x^2 - 3x - 10 < 0$ and $7x + 5 > 8 + 4x$, showing your answer on a number line.

Explain why $x^2 + 4 < 0$ has no solutions.

Simultaneous Equations

Small Steps

- Understand that equations can have more than one solution
- Determine whether a given (x, y) is a solution to a pair of linear simultaneous equations
- Solve a pair of linear simultaneous equations by substituting a known variable
- Solve a pair of linear simultaneous equations by substituting an expression (1) & (2)
- Solve a pair of linear simultaneous equations using graphs
- Solve a pair of linear simultaneous equations by subtracting equations
- Solve a pair of linear simultaneous equations by adding equations
- Use a given equation to derive related facts R

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Simultaneous Equations

Small Steps

- Solve a pair of linear simultaneous equations by adjusting one equation
- Solve a pair of linear simultaneous equations by adjusting both equations
- Form a pair of linear simultaneous equations from given information
- Form and solve pair of linear simultaneous equations from given information
- **Determine whether a given (x, y) is a solution to both a linear and quadratic equation** H
- **Solve a pair of simultaneous equations (one linear, one quadratic) using graphs** H
- **Solve a pair of simultaneous equations (one linear, one quadratic) algebraically** H
- **Solve a pair of simultaneous equations involving a third unknown** H

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

More than one solution

Notes and guidance

Students explore equations that have more than one possible solution. Students should use different types of numbers when finding these solutions (e.g. negatives, decimals, fractions). Building on this, students think about what else is needed to reduce to just one solution. This leads into the idea of requiring two equations and hence into the concept of simultaneous equations.

Key vocabulary

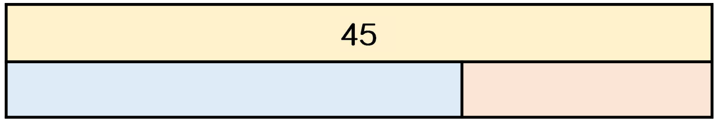
Possible	Solutions	Infinite
Finite	Variables	Equation

Key questions

What possible solutions are there? Are there an infinite number of solutions? Why/Why not?
What else do we need to determine just one solution?
Can you think of a second equation that would help?
Why do you need 2 equations to find each variable? How many equations would be needed if you had 3 variables?

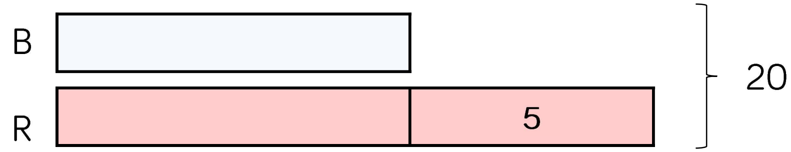
Exemplar Questions

Two numbers add together to give 45. One number is bigger than the other. List some possible solutions. Compare these as a class. Is there one pair of values that must be true? Explain your answer.



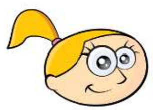
There are 20 red and blue counters in a bag. There are  more red counters than blue counters.

Explain why this bar model must be incorrect.



How many more red counters than blue counters could there be?

If $x + y = 10$, list all possible positive integer values for x and y.



If we also know that the x is 4 more than y, then we can only find one pair of values for x and y.

Eva is correct. Why?
What must the values of x and y be?

Is (x, y) a solution?

Notes and guidance

Students may need practice substituting (including with negative numbers) before attempting this small step. Use of formulae for area and perimeter can be interleaved here. Students then substitute values into equations to work out whether or not they have a possible solution. They understand that there is one possible solution when two equations are given in terms of two variables.

Key vocabulary

Solution	Substitute	Equation
Variable	Verify	

Key questions

Are there any other possible dimensions for a (e.g.) rectangle, parallelogram, trapezium with area……?
What if I now tell you that the perimeter is…..?

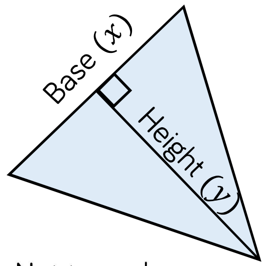
Why can two variables be the same value?

Why is there only one solution to two equations containing two variables?

Exemplar Questions

A straight line has the equation $y = 3x + 5$
Show that the point (1, 8) lies on this line.
Show that the point (2, 7) does not lie on this line.
Show that $x = 3$ and $y = 14$ is one solution to the equation.
Does the point (3, 14) lie on this line?

The area of the triangle is 50 cm^2
Complete the table exploring possible values for x and y.



Not to scale

Base (x)	Height (y)	True/False
10	5	
10	10	
25	2	
2.5	40	
−10	−10	

$$j^2 + k = 6$$

Mo thinks that $j = 2$ and $k = 2$ is a solution for this equation.
Annie says that can't be right as j and k can't be the same value.
Who do you agree with and why?
Show that j $= -3, \quad k = -3$ is also a solution to this equation.

Is $x = 5$ and $y = 3$ a solution to both equations?
Show workings to justify your answer.
Does the point (5, 3) lie on both of the lines?

$$y = 13 - 2x$$
$$3x - 2y = 9$$

Substituting a known variable

Notes and guidance

Before starting, students need to review solving equations. Modelling substitution and solving equations is key. There is opportunity to interleave aspects of measure (e.g. $P = 2l + 2w$). Using bar models to begin with will support algebraic thinking. The students go onto realise that there may be more than one way of finding a solution if presented with two related equations.

Key vocabulary

Substitute	Solve	Solution
Variable	Unknown	Inverse

Key questions

Which variable can we substitute for?

Can you write an equation with one unknown variable?

What are the steps in solving this equation?

If I have two related equations, does it matter which one I substitute into and then solve?

Exemplar Questions

Three hops (h) and two jumps (j) have a total length of 39 m.

39				
h	h	h	j	j

A hop is 6 m in length. How long is a jump?
How long would two hops and three jumps be?

A straight line has equation $2x + 10 = y$

Amy wants to find possible solutions to this equation.
She knows that $x = 5$ Use this information to find the value of y.
Does the point with coordinates (5, 20) lie on this straight line?
At another point on the line, $y = 30$ Use this information to find x.

Teddy and Whitney are working out the value of h. Whose method do you like best? Why might it be useful to do both?

$h + j = 25$
$3h - j = 27$
$j = 12$

Teddy

If we know $j = 12$, then we can use the first equation to work out h.
$h + 12 = 25$
$h = 13$

Whitney

If we know $j = 12$, then we can use the second equation to work out h.
$3h - 12 = 27$
$3h = 39$
$h = 13$

Substituting into an expression (1)

Notes and guidance

This small step introduces the idea of substituting one equation into a second equation and is split into two parts. Double-sided counters could be used so that students can physically make the substitution. Students might then use pictorial representations before attempting the abstract substitution. At this stage, students are not rearranging in order to make the substitution.

Key vocabulary

Substitute	Substitution	Solve

Key questions

What object/letter can I substitute for?

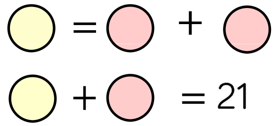
What can I replace it with?

Why does this help me to find the value of one of the variables?

How can you check your answers?

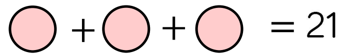
Exemplar Questions

A
B



Annie looks at equation **A** and says: "Every yellow counter is worth two red counters"

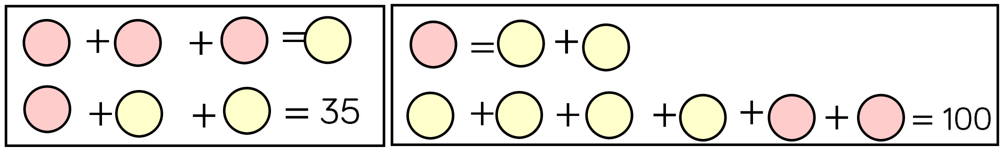
Annie looks at **B** and says: "I can swap one yellow counter for two red counters"



Annie

Annie says "If 3 red counters are worth 21, then each red counter is worth 7". Using **B**, work out the value of the yellow counter.

Using the same method, find the value of the red and the yellow counter in each of these cases.



Equation 1	□	=	△ + 3
Equation 2	□ □ △ △ △	=	26
In equation 2 I can substitute △ + 3 for each □			
	△ + 3 △ + 3 △ △ △	=	26

Rosie

How did Rosie know what to substitute?

Simplify the new equation, to work out the value of △

Use another equation to work out the value of □

Which equation did you use to find □ ? Why?

Substituting into an expression (2)

Notes and guidance

In this second part of the step, higher tier students might now explore rearranging an equation to make a substitution. Finding the subject of the formula may need revising. Teachers might then emphasise the importance of checking directed number when substituting (particularly when substituting an expression for x into $-x$), and the choice of which letter to substitute for.

Key vocabulary

Subject of the formula	Rearrange
Simultaneous equations	Substitute

Key questions

What happens if we substitute (e.g.) $x = 7 - y$ into $y - x = 5$?
Which equation is easiest to rearrange?
Which variable is easiest to make the subject of the formula?
Which equation do we now need to substitute into?

Exemplar Questions

Rearrange each of the equations to make x the subject.

$x - 3y = 6$ | $2x - 3y = 6$ | $3y - 2x = 6$

$$x + y = 7$$
$$y - x = 5$$

Rearranging the 1st equation:
$x = 7 - y$
Substitute into $y - x = 5$

Rearranging the 2nd equation:
$y = 5 + x$
Substitute into $x + y = 7$

Which method is correct? Explain your answer.
Try both methods. Which was easiest to do?
Choose a method and complete it to find the value of x and y.

Jack 

$$k = -9 - 2m$$
$$k = 3m + 11$$

Jack starts solving this by rearranging the first equation.

$$k + 2m = -9$$
$$2m = -9 - k$$
$$m = \frac{-9 - k}{2}$$

Is there an easier first step that Jack could have taken?
Solve the simultaneous equation using a more efficient method.
How can you check your answers?

Solve by using graphs

Notes and guidance

Students learn that the intersection point of two straight lines represents the solution to a pair of linear equations, comparing graphical and algebraic methods. It's important that teachers emphasise that it is the value of x and the value of y that give the solution, rather than the coordinate. Teachers could extend this by exploring why some pairs of linear equations do not have any solutions (parallel lines).

Key vocabulary

Intersect	Coordinate	Solution
Substitution	Meet	

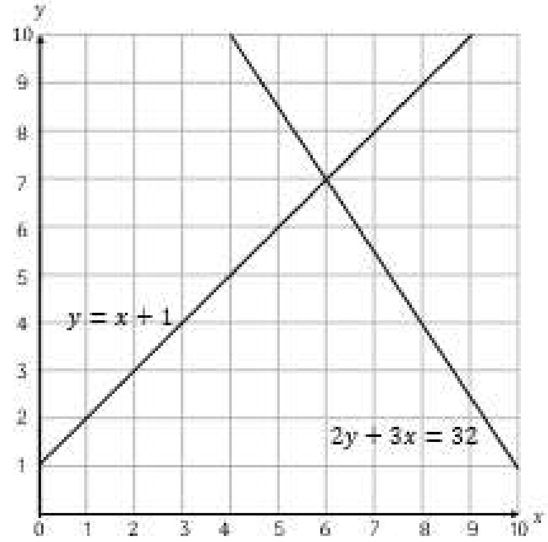
Key questions

What does intersect mean? Where go graphs 'meet'?
What is true about the coordinates of the point where two lines meet? How do they relate to the equations?
Explain why the intersection point represents the solution to a pair of equations.
Can there be more than one pair of solutions to this pair of simultaneous equations?

Exemplar Questions

Tommy draws the graphs of

$$y = x + 1$$
$$2y + 3x = 32$$



a) Write down the coordinate of the point where the lines meet.

b) What is the value of x? What is the value of y?

c) Solve the simultaneous equation using substitution.

d) What do you notice?

Complete the tables.

$y = 2x$

x	0	3	5
y			

$x + y = 9$

x	0	3	5
y			

Use your tables to draw the graph of $y = 2x$ and $x + y = 9$

Use your graph to solve:

$$y = 2x$$
$$x + y = 9$$

There might be more than one possible solution

Ron

Explain why Ron is incorrect for this pair of equations.
Could there be a pair of equations with more than one solution, or no solutions? Explain you answer.

Solve by subtracting equation (1)

Notes and guidance

This step is split into two; firstly looking at subtracting positive number. Teachers might introduce this using concrete resources allowing students to physically subtract two equations. Bar models also clearly show the difference between two equations. Once students understand why subtracting eliminates a variable they can attempt abstract simultaneous equations, include answers which are zero, negative or non-integer.

Key vocabulary

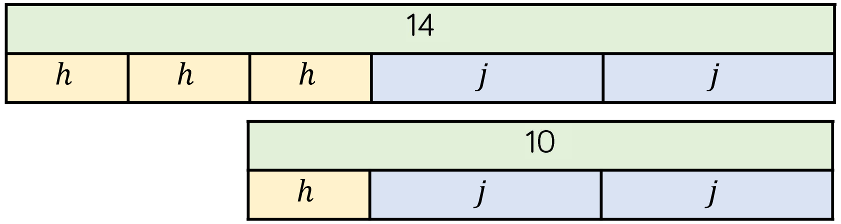
Subtract	Negative	Eliminate
Variable		

Key questions

Why is it useful to 'eliminate' one of the variables?
Which equation do we substitute into? Does it matter? Why/why not?
Does it matter if we subtract equation 1 from equation 2 or equation 2 from equation 1? Which is easier to do?
How can we check our answers?

Exemplar Questions

Amir is comparing the lengths of hops (h) and jumps (j)



$3h + 2j = 14$

$h + 2j = 10$

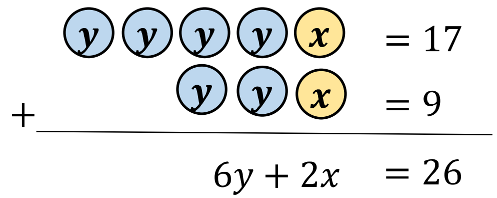
From the bar models, Amir works out,

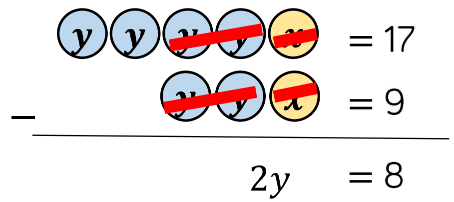
$$2h = 4$$

How did he work this out? Now work out the values of h and j

$4y + x = 17$ A
$2y + x = 9$ B

How do the diagrams below link to equations A and B?





Which would be more beneficial to help us solve these equations, to add the two equations or to subtract the two equations? Why? Now find the value of y and use this to find the value of x.

Solve the following simultaneous equations by subtracting the equations.

$2x + y = 17$
$x + y = 10$

$3x + y = -2$
$3x + 2y = 2$

$4x - 2y = 11$
$4x + y = 18.5$

White Rose Maths

Solve by subtracting equation (2)

Notes and guidance

Careful revision of subtracting a negative will be required at the start of this continuation of the step. Revising solving single equations that involve negatives will also be useful. Students then consider why subtraction of the two equations eliminates a variable, before considering the pitfalls of subtracting equations containing negatives. Students will be able to generalise (when do we subtract the two equations – what do you notice?)

Key vocabulary

Expression	Equation	Eliminate
Subtract	Negative	Solve

Key questions

What happens when we subtract a negative number?

What do you notice about the type of coefficients in these simultaneous equations, where the first step is to subtract?

How can we check our answers for both original equations?

Exemplar Questions

Simplify the following expressions.

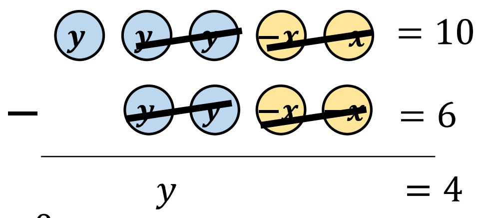
$-3x - (-3x)$

$-5y + 3x - (-5y) + 6x$

$4y - x - 2y - (-x)$

The diagram represents the two equations in the purple box.

$3y - 2x = 10$
$2y - 2x = 6$



Explain why $-2x - (-2x) = 0$

Is it useful to add the two equations together. Why or why not?

Now solve the simultaneous equations and check your answers.

Annie, Tommy and Dexter are attempting to solve this pair of simultaneous equations. They all start by subtracting the two equations. **Two** of them have made a mistake.

Who has made mistakes? What mistake did they make?

$-6x - 7y = 9$

Annie 

$3y = 9$

Tommy 

$7y = 63$

Dexter 

One person has started correctly.
Continue their solution to find the values of x and y.

Solve by adding equations

Notes and guidance

By considering the simplification of expressions, students understand how to make zero using addition. They build on this to solve simultaneous equations involving negative or non-integer solutions. They progress to consider which equation is more efficient when substituting to find the second solution. It's also important to consider equations where it might be easier to rearrange before adding.

Key vocabulary

Expression	Equation	Eliminate
Subtract	Negative	Solve

Key questions

What happens when we add with negative numbers?

Which equation should we now substitute in to? Why?

Does it matter which equation we substitute into?

Would it help to rearrange the equations first?

Exemplar Questions

Simplify the following expressions.

$-2x + 2x$

$5y + 3x + (-5y) + 6x$

$4y - x - 2y + (-x)$

$3x + 2y = 16$ A
$6x - 2y = 2$ B

Which variable, x or y, would you try to eliminate? Why?

Ron decides to eliminate y.

Should he add equations A and B or subtract them? Why?

Show that $x = 2$

Ron substitutes this into equation A. $6 + 2y = 16$

Show that $y = 5$

Ron wants to check his answer. He substitutes his values for x and y back into equation A. Why isn't this sensible?

Check Ron's values by substituting into equation B.

For each of the pairs of simultaneous equations., decide whether you would add or subtract the equations and then solve each pair of simultaneous equations.

$3x + 2y = 24$
$3x - 5y = -18$

$6x + 2y = 12$
$6x - 2y = 0$

$3w + 2v = 2$
$w = -6 + 2v$

Related facts from an equation R

Notes and guidance

Teachers might start this small step by reviewing where students have met the concept of equivalence before (for example, ratios, fractions). It's important to ensure that students understand that equivalent equations have the same solutions. This step relates closely to deriving related number facts e.g. working out 4×17 from doubling 2×17, and this makes a good introduction.

Key vocabulary

Equivalent	Solution	Coefficient
Variable	Multiplier	

Key questions

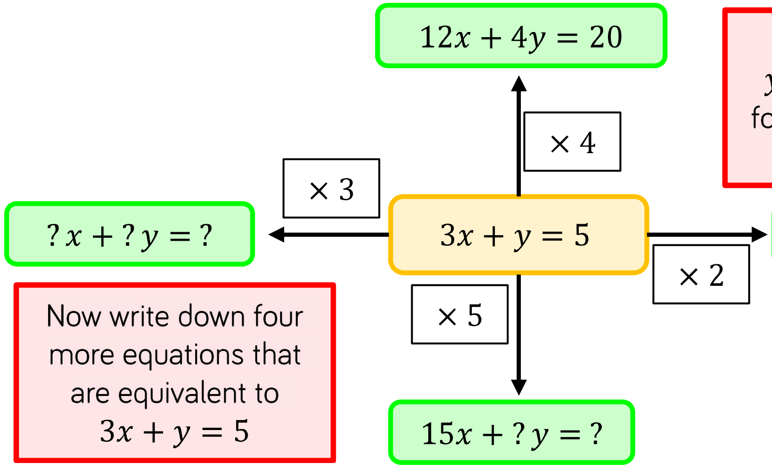
What happens when we substitute our original solutions into the equivalent equations? Why does this happen?

How can I generate an equivalent equation?

What happens if I divide the equations by a number instead of multiplying? Do the solutions change now?

Exemplar Questions

Complete the diagram by replacing the questions marks.



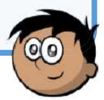
Is $x = 1$ and $y = 2$ a solution for each equation? Why?

Now write down four more equations that are equivalent to $3x + y = 5$

Amir and Dora are discussing the following pair of equations.

$4x + 3y = 23$ $\quad$ $8x + 6y = 46$

The equations look different so x and y must be different in each equation.



Amir

The equations are equivalent. This means that the value of x is the same in both equations.

Dora

Who is correct? Why? If $x = 5$, work out y. Are these values of x and y solutions to both equations? Explain your answer.

Alex uses the equation $6s - 2t = 4$ to form three other equations. She's made some mistakes. Find her mistakes and correct them.

$60s - 20t = 4$ $\quad$ $18s - 6t = 8$ $\quad$ $12s - 2t = 8$

White Rose Maths

Solve by adjusting one equation

Notes and guidance

Bar models are a good way of demonstrating why equal coefficients of one of the variables is necessary when we are solving by elimination. It is useful to provide the abstract equation alongside each bar model to support conceptual understanding of the method. Teachers should also discuss whether to make the coefficients of x the same, y the same, and whether it matters.

Key vocabulary

Coefficient	Variable
Multiplier	Solve

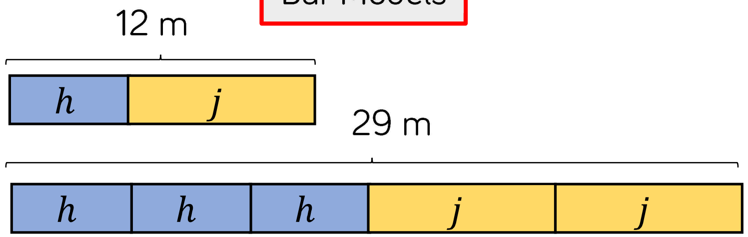
Key questions

Why do we need the coefficient of one of the variables to be the same in both equations? How does this help us to solve the equations?
Once we know one variable, how do we find the other?
Which equation do we multiply and why? Is there more than one way of making the coefficients the same?

Exemplar Questions

Whitney draws a bar model to represent the following problem.
A hop and a jump have total length 12 m
Three hops and two jumps have total length 29 m

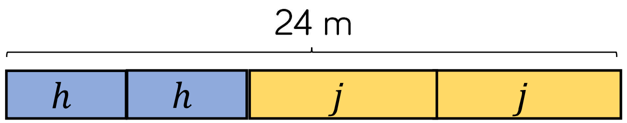
Bar Models | Equation



$h + j = 12$ (A)

$3h + 2j = 29$ (B)

Whitney doubles the length of her first bar.



$2h + 2j = 24$ (C)

How do these bar models show that $h = 5$?
Show that (B) − (C) also gives $h = 5$
Now find the length of a jump.

Another way to find the length of a hop is to multiply equation by 3
Then you have the same number of hops.

Draw a bar model to represent this. Compare your bar model with B.
Find the length of a hop and a jump using this method.

In each of the following, multiply one equation to make the coefficient of x or y the same.

$3x + 2y = 4$
$4x + y = 9$

$3x + 2y = 14$
$x + y = 5$

$x + y = 6$
$3x - 3y = 0$

Solve by adjusting both equations

Notes and guidance

To begin with, teachers may need to guide students in choosing appropriate multipliers; a review of LCM will assist this. Students will realise that it doesn't matter which variable they focus on when making coefficients the same, but should consider which variable is easier (for example, avoiding negatives). Choosing whether to add or subtract should again be reinforced.

Key vocabulary

Lowest Common Multiple	Coefficient
Variable	Multiplier

Key questions

When making the coefficients the same, which variable should we choose?
Why would we choose to multiply by smaller numbers wherever possible?
How do we know whether to add or subtract? If we have a choice, which is easier?

Exemplar Questions

Find the lowest common multiple of the following.

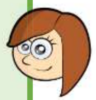
8 and 12 | 6 and 10 | 4 and 5

Amir and Rosie are solving this pair of simultaneous equations.

$2x + 3y = 39$ (A)
$5x - 2y = -7$ (B)

Multiply equation **A** by 2
Multiply equation **B** by 3
Now add the two new equations.

Multiply equation **A** by 5
Multiply equation **B** by 2
Now subtract the two new equations.



Explain why both Amir's and Rosie's methods work.
Find the solutions using both methods. Which did you find easier?

Tommy multiplies equation **A** by 8 and equation **B** by 12
Write down two smaller multipliers he could have used.
To eliminate x, should Tommy add the two equations or subtract them?
Explain your answer.
Solve the simultaneous equations.

$12x - 5y = -22$ (A)
$-8x + 4y = 16$ (B)

Dexter wants to eliminate t. What should he multiply by? He now subtracts the two equations. Will this eliminate t?
By eliminating t, solve the simultaneous equation.
Check your answer by substituting your values back into one equation.

$-6s + 10t = 11$ (A)
$10s - 6t = 3$ (B)

Form a pair of linear equations

Notes and guidance

Students might start by using counters to solve wordy problems. This will help them to formulate the algebraic equations. Students often get confused about forming equations involving 'more than' or 'doubling', placing the addition/multiplication on the wrong side of the equation. This will need exploring by testing values. Students must give final answers in the context of the question.

Key vocabulary

Formulate	Variable	Context
Equation		

Key questions

Tell me one thing that you know. How could we write this as an equation? What could we use to represent the variable?
How could we check whether the equation we have written down is correct?
Does your answer relate to the context of the question?

Exemplar Questions

In the car park, there are 18 vehicles. Some are cars and some are bikes. There are 60 wheels in total in the car park.
Dora decides to use algebra to solve the problem.

Let c = number of cars and b = number of bikes

Dora

Complete the equations.

$b + c = \square$

$2b + \square c = 60$

Alex and Jack have £10.00 between them. Alex has £1.80 more than Jack. Let a = amount of money Alex has and let j = amount of money Jack has.
Write an equation to show how much money they have in total.
Using the given information, Alex and Jack have written another equation:

Alex

$a = j + 1.8$

$a + 1.8 = j$

Jack

Who's right and why?

Miss Rose is twice the age of her little brother.
If x = age of Miss Rose and y = age of her brother, which of these equations is correct?

$2x = y$

$x = 2y$

Check your answer by substituting in a value for Miss Rose's age.
The total of their ages is 48. Write down a second equation.

Form & solve two linear equations

Notes and guidance

Building from the last step, students continue to develop their skills in forming equations in conjunction with practising solving them. Students may need to be provided with scaffolding when first attempting to form and then solve two linear equations. This should be gradually removed.
Interleaving other topic areas, such as shape, works well in this small step.

Key vocabulary

Formulate	Variable	Context
Equation	Solve	Solution

Key questions

Tell me one thing that you know. How could we write this as an equation?
When making the coefficients the same, which variable should we choose? How do we know whether to add or subtract the equations?
Does your answer relate to the context of the question?

Exemplar Questions

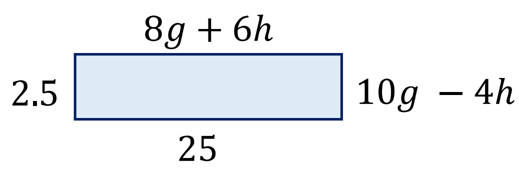
1000 tickets are sold for a concert.
Adult tickets are £10 and Child tickets are £6
£7304 was collected through ticket sales.

Ron writes down these two equations.
What do you think x and y represent?
What mistakes has Ron made?
Correct the equations.

$$10x + 6y = 1000$$
$$x + y = 7304$$

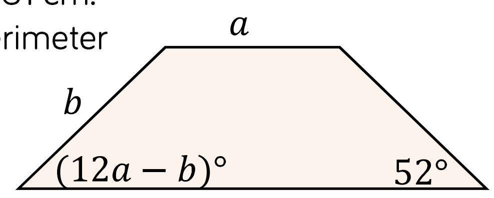
How many adult tickets and how many child tickets were sold?

What do you know about the side lengths of a rectangle?
Use this to form two equations involving g and h.



Solve this simultaneous equation to find g and h.

The base of an isosceles trapezium is double the length of side a.
The perimeter of the trapezium is 31 cm.
Write down an equation for the perimeter of the trapezium.
Look at the marked angles.
Write down another equation relating a and b.
Find the length of the base of the trapezium.



Is (x, y) a solution? H

Notes and guidance

Teachers might start with a quick review of substituting negative numbers into quadratic expressions. Students also need to be aware that when substituting into expressions such as $2x^2$, they calculate x^2 before multiplying by 2
Recognising linear and quadratic equations may also need reviewing. It's then important to make the link between coordinates that are on both curve and line & the solution to the simultaneous equations.

Key vocabulary

Quadratic	Curve	Linear	Coordinate
Solution	Substitute	Square	

Key questions

What's the same and what's different about the equations of a straight line and the equations of a curve?

How can we recognise which equation will produce a curve?

Exemplar Questions

Whitney is thinking about this equation of a curve. $y = x^2$

She uses substitution to find out whether $(-1, 1)$ is on the curve.

If $x = -1$ then $y = -1^2$ and so $y = -1$

She concludes that $(-1, 1)$ isn't on the curve.
Whitney is wrong. Why?

- Is $(1, 1)$ on the curve?

Whitney looks at this pair of equations:

$$y = x^2$$
$$y = x$$

- Is $(1, 1)$ on both the curve and the line? What about $(-1, 1)$?

Whitney thinks that $(0, 0)$ is a solution to both equations.
Show that she is correct.

State which of the following coordinates are,

- On the curve only
- On the line only
- On both the curve and the line
- On neither the curve nor the line

Show your workings to justify each answer.

$$y = 2x^2$$
$$y = 6x - 4$$

$(1, 2)$	$(-1, -2)$	$(2, 16)$	$(2, 8)$
$(1, 4)$	$(-1, 2)$	$(-1, -10)$	$(-1, 10)$

Write down the values of x and y that are solutions to both equations.

Solving graphically

H

Notes and guidance

This is a good opportunity to revise how we know whether an equation represents a curve or a straight line. Students may need reminding about how to draw a smooth curve.
Students then make the link that the solution is represented by the intersection point. It's important that teachers emphasise the difference between how the solution is presented, $x = \_\_ , y = \_\_$ and the intersection point (x, y)

Key vocabulary

Intersection	Solution	Linear
Non-linear	Quadratic	Curve

Key questions

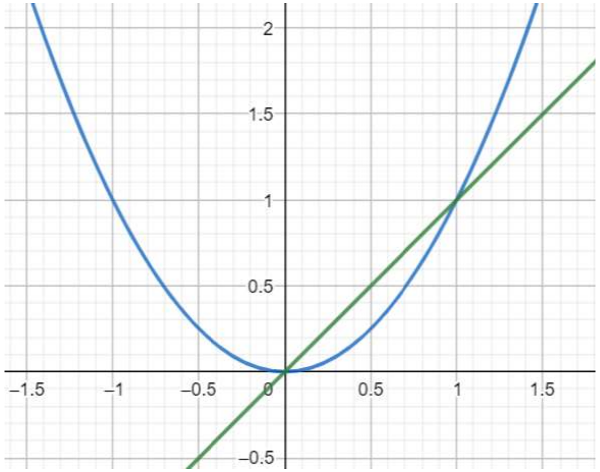
How can we recognise a quadratic equation?
What does this look like on a graph?
Why don't we use a ruler to sketch the curve?
How do we represent the point of intersection ?
How is this presentation different to the presentation of the solution of the pair of equations?

Exemplar Questions

Tommy draws the graph of:

$$y = x^2$$
$$y = x$$

Label the curve and the line with the correct equations.



- Write down the coordinates of the point where the curve and the line intersect. What are the values of x and y?
- Are these pairs of values solutions to both equations? Substitute the values into the equations to check.
- How do you know that there will only be two solutions?

Complete both table of values.

$y = x^2 + 2$

x	−3	−2	−1	0	1	2	3	4	5
y	11					6			

$y = 2x + 10$

x	−3	0	3
y			

Use your tables to draw the graphs of $y = x^2 + 2$ and $y = 2x + 10$
Use your graphs to solve,

$$y = x^2 + 2$$
$$y = 2x + 10$$

Write your solutions in the form $x = \_\_, \ y = \_\_$

Solving algebraically (1)

H

Notes and guidance

This step is split into two parts. It is useful to start by revising factorising and solving quadratics. Students consider simultaneous equations where both are in the form $y =$ This allows them to gain confidence in manipulating and solving quadratics, before going onto more complex equations. Finally, to find the corresponding value of y, teachers should emphasise that it is easier to substitute back into the linear equation.

Key vocabulary

Factorise	Rearrange	Solve
Linear	Quadratic	

Key questions

Why does it make sense to substitute for y in these cases?
Could I substitute for x instead? Why is this less efficient for these equations?
Why is it easier to substitute back into the linear equation to find the value of y?
How can we check the answers?

Exemplar Questions

Factorise

$x^2 - 2x - 3$ $\quad x^2 - 6x + 8$ $\quad x^2 + 5x - 24$

Solve

$x^2 - 2x - 3 = 0$ $\quad x^2 - 6x + 8 = 0$ $\quad x^2 + 5x - 24 = 0$

Mo is working on this pair of simultaneous equations.

$y = x^2$ (A)
$y = 2x + 3$ (B)

I can substitute y for x^2 in equation B:
$x^2 = 2x + 3$
I can rearrange this so that I can solve it:
$x^2 - 2x - 3 = 0$

Complete Mo's solution by finding the values of x.

Find the corresponding values of y by substituting your values of x into the linear equation (B).

Write down two pairs of solutions.

By equating the two expressions for y, show that the pair of solutions to this simultaneous equation are:,
$x = 4, y = 20$ and $x = 2, y = 8$

$y = x^2 + 4$
$y = 6x - 4$

Solving algebraically (2)

H

Notes and guidance

Teachers may wish to start this continuation of a small step by expanding brackets e.g. $(y-3)^2$

Students are now encouraged to think whether it is easiest to rearrange the linear equation first, or whether they can make a direct substitution. They understand that they can substitute for either x or y and decide which is the most efficient.

Key vocabulary

Rearrange	Linear	Substitute
Quadratic	Solve	Solution

Key questions

Could I substitute for x? How could I do this?

Could I substitute for y? Do I need to rearrange the linear equation first. How could I do this?

Which method is most efficient?

Exemplar Questions

Dora is solving the simultaneous equations.

Ⓐ $x = y - 3$
Ⓑ $x^2 + 2y = 9$

Here is Dora's first step: $(y-3)^2 + 2y = 9$

Explain what Dora has done in this first step.

$$y^2 - 4y = 0$$

Now solve this equation to show that $y = 0, y = 4$

Complete the solution by substituting these values into equation **A**.

Tommy is solving the simultaneous equations.

Ⓐ $y = 2x - 5$
Ⓑ $x^2 + y^2 = 10$

He substitutes $y = 2x - 5$ into equation **B**: $x^2 + (2x-5)^2 = 10$

Tommy simplifies this: $x^2 + 4x^2 - 25 = 10$

Where has Tommy gone wrong?

Correct Tommy's work and go on to solve the simultaneous equation.

Is (3, 1) an intersection point of the two graphs? How do you know?

Alex is solving this simultaneous equation.

Ⓐ $y = x^2$
Ⓑ $5x = 24 - y$

She starts by rearranging equation **B**:

$$y = 24 - 5x$$

She substitutes this into equation **A**: $24 - 5x = x^2$

Dexter thinks it's easier to substitute $y = x^2$ into equation **B**:

$$5x = 24 - x^2$$

Which approach do you prefer? Why?

Solve the simultaneous equations.

Solve with a third unknown

H

Notes and guidance

A good starting point would be to ask what's the same and what's different when comparing the usual simultaneous equations with two variables to these questions involving a constant. They may need guidance in understanding how to write x and y 'in terms of' a constant. They can check by substituting a values for the constant and verifying their solutions work in the original equations.

Key vocabulary

Variable	Constant	Simplest Form
In terms of		

Key questions

What is a constant?

If I replaced the constant with a number would you be able to solve the pair of equations?

What does 'give your answers in terms of…' mean?
What is meant by 'simplest form'?

Exemplar Questions

In the simultaneous equations, a is a constant.

$$x + 4y = 22a \quad (A)$$
$$y = 2x + a \quad (B)$$

Complete the solution.

Step 1: Substitute equation **B** into equation **A**:

$$x + 4(\square) = 22a$$

Step 2: Find x in terms of a:

$$x + 8x + 4a = 22a$$
$$\square x + 4a = 22a$$
$$9x = \square a$$
$$x = \square a$$

Step 3: Substitute $x = 2a$ into equation **A**:

$$\square + 4y = 22a$$

Step 4: Solve this to find y:

$$4y = \square a$$
$$y = \square a$$

In the simultaneous equation, p is a constant.
Solve the simultaneous equations, giving your answers in terms of p in their simplest form.

$$4x + y = 5p \quad (A)$$
$$y = 2x + 2p \quad (B)$$

When Dexter attempted this question, he left his answer as $2x = p$

Why is this incorrect?

Check your solutions in the equations if $p = 6$