Spring Term

Year 10

#MathsEveryoneCan

White Rose Maths

WRM – Year 10 Scheme of Learning

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Similarity**						**Developing Algebra**					
	Congruence, similarity and enlargement			Trigonometry			Representing solutions of equations and inequalities			Simultaneous equations		
Spring	**Geometry**						**Proportions and Proportional Change**					
	Angles & bearings		Working with circles		Vectors		Ratios & fractions		Percentages and Interest		Probability	
Summer	**Delving into data**						**Using number**					
	Collecting, representing and interpreting data						Non-calculator methods		Types of number and sequences		Indices and Roots	

Spring 1: Geometry

Weeks 1 and 2: Angles and bearings

As well as the formal introduction of bearings, this block provides a great opportunity to revisit other materials and make links across the mathematics curriculum. Accurate drawing and use of scales will be vital, as is the use of parallel line angles rules; all of these have been covered at Key Stage 3. Students will also reinforce their understanding of trigonometry and Pythagoras from earlier this year, applying their skills in another context as well as using mathematics to model real-life situations.

National curriculum content covered:

- interpret and use bearings
- compare lengths...using scale factors
- apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles {and, where possible, general triangles} in two dimensional figures
- {know and apply the sine rule and cosine rule to find unknown lengths and angles}
- use mathematical language and properties precisely
- reason deductively in geometry, number and algebra, including using geometrical constructions
- make and use connections between different parts of mathematics to solve problems

Weeks 4 and 5: Working with circles

This block also introduces new content whilst making use of and extending prior learning. The formulae for arc length and sector area are built up from students' understanding of fractions They are also introduced to the formulae for surface area and volume of spheres and cones; here higher students can enhance their knowledge and skills of working with area and volume ratios.

Higher tier students are also introduced to four of the circle theorems; the remaining theorems will be introduced in Year 11 when these four will be revisited.

National curriculum content covered:

- identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
- calculate arc lengths, angles and areas of sectors of circles
- calculate surface areas and volumes of spheres, pyramids, cones and composite solids
- apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results

Weeks 5 and 6: Vectors

Students will have met vectors to describe translations during Key Stage 3 This will be revisited and used as the basis for looking more formally at vectors, discovering the meaning of $-\boldsymbol{a}$ compared to $\boldsymbol{a}$ to make sense of operations such as addition, subtraction and multiplication of vectors. This will connect to exploring 'journeys' within shapes linking the notation $\overrightarrow{AB}$ with $\boldsymbol{b}-\boldsymbol{a}$ etc. Higher tier students will then use this understanding as the basis for developing geometric proof, making links to their knowledge of properties of shape and parallel lines.

National curriculum content covered:

- describe translations as 2D vectors
- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; {use vectors to construct geometric arguments and proofs}.

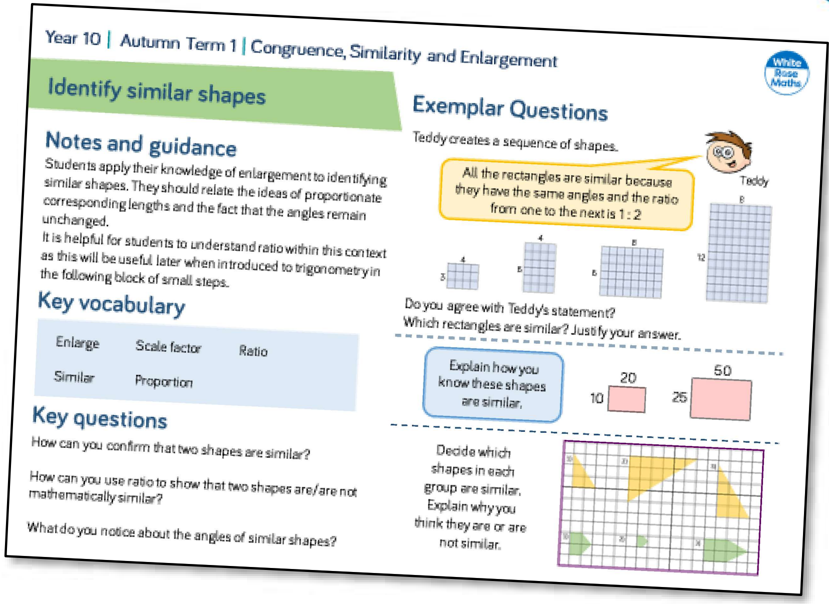
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points.
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step.
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.
It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge, Scale factor, Ratio, Similar, Proportion

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

All the rectangles are similar because they have the same angles and the ratio from one to the next is 1 : 2

Teddy

Do you agree with Teddy's statement?
Which rectangles are similar? Justify your answer.

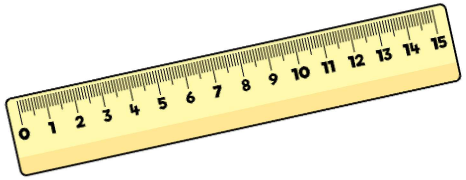
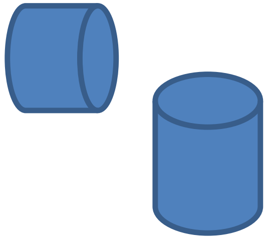
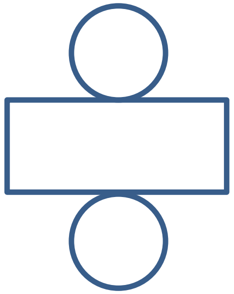
Explain how you know these shapes are similar.

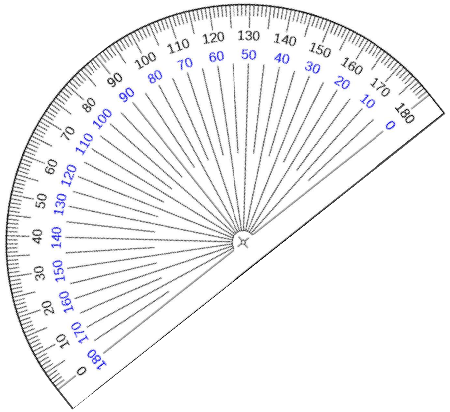
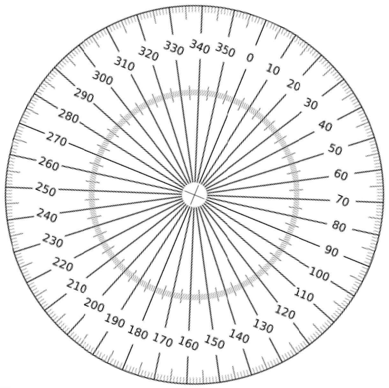
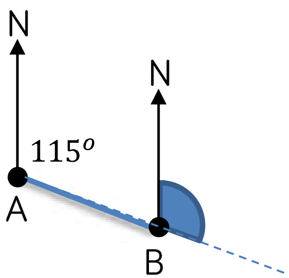
Decide which shapes in each group are similar. Explain why you think they are or are not similar.

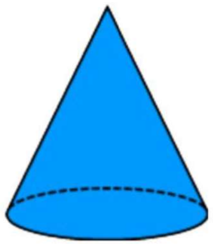
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

Key Representations




This is a very visual unit of work and students should be encouraged to make accurate diagrams and meaningful sketches throughout.

Using compasses in school or on a trip may be useful way in to bearings. Likewise, making and using actual scale drawings such as architect's drawings is helpful when looking at scale. Straws are always useful for reminding what happens with parallel and intersecting lines.

Creating nets of cylinders and cones is a good way of establishing how to find the formulae for their surface area. Likewise, sand can be used to demonstrate that the volume of a cone is one-third the volume of a cylinder of the same height.

Dynamic geometry is very useful to illustrate the circle theorems.

Angles and Bearings

Small Steps

- Use cardinal directions and related angles (R)
- Draw and interpret scale diagrams (R)
- Understand and represent bearings
- Measure and read bearings
- Make scale drawings using bearings
- Calculate bearings using angles rules
- Solve bearings problems using Pythagoras and trigonometry
- **Solve bearings problems using the sine and cosine rules** (H)

(H) Denotes Higher Tier GCSE content

(R) Denotes 'review step' – content should have been covered at KS3

Angles and compass points

R

Notes and guidance

In this small step, students will revisit their prior work on angles to prepare them for learning about bearings.

They should be comfortable with both measuring and drawing angles using a protractor and be able to identify angles using three letter notation. Familiarity with the major compass points and the angles between them will also be useful.

Key vocabulary

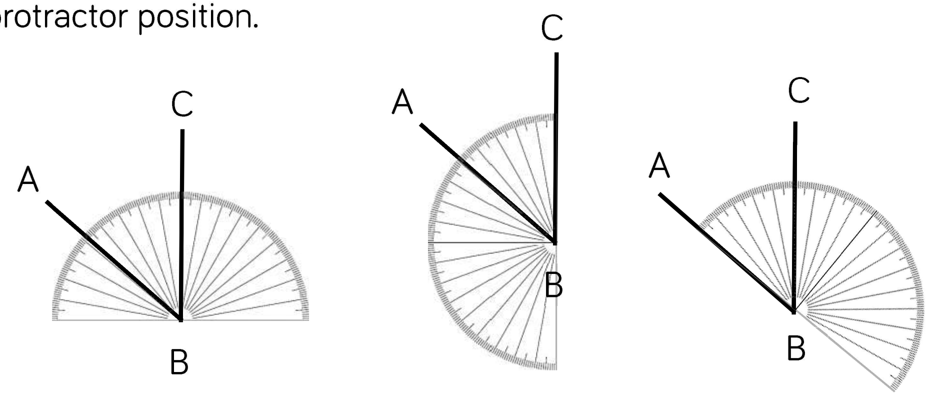
Compass	Point	Angle
Turn	Three letter notation	

Key questions

How can you draw a 200° angle using a 180° protractor?
What does due East mean?
What does the hat on $A\hat{C}D$ mean?
What else does the notation $A\hat{C}D$ tell you?
Are the angles $A\hat{C}D$ and $\angle ACD$ the same or different?

Exemplar Questions

Dora is measuring the angle $A\hat{B}C$ but isn't sure how to use a protractor. Explain how she could work out the angle using each protractor position.



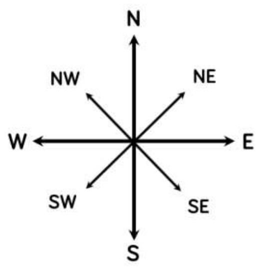
$C\hat{B}A$ is the angle extended around B from C to A. If $A\hat{B}C$ is θ, which of these expressions will find angle $C\hat{B}A$?

$180-\theta$ | $180+\theta$ | $360-\theta$ | $90+\theta$

Turning clockwise, the angle between North and South-East is $135°$

Find three pairs of compass points so that the clockwise turn between them is

- $180°$
- $45°$
- $315°$
- $225°$



Draw/interpret scale diagrams R

Notes and guidance

This review step reminds students of Key Stage 3 work on scale, constructions and ratio. Students should be able to interpret scales as well as make scale drawings. It is useful to measure and draw angles from a variety of starting points and different inclinations rather than just from the horizontal. Likewise, a variety of scales should be explored using both 1 cm = 500 m and the 1 : 50 000 formats.

Key vocabulary

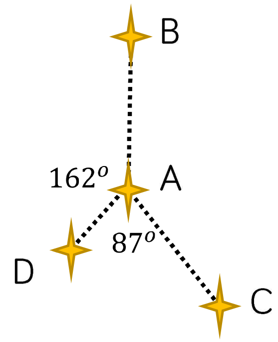
Enlarge	Scale factor	Ratio
Protractor	Convert	Similar

Key questions

Why is a scale drawing useful?
How accurate is a scale drawing?
On a scale drawing, what measurements always remain the same and which ones change? Think about lengths, areas, angles…
Which is more detailed, a 1 : 25 000 map or a 1 : 50 000 map?

Exemplar Questions

There are four stars in a constellation.
B and C are both 160 light years from A.
D is 60 light years from A.
Using a scale of 1 cm to represent 20 light years, draw a scale diagram of the constellation.
Use your diagram to find the distance between B and C in light years.



A tree is drawn to a scale of $1:30$. Explain what $1:30$ means.
Which part of the ratio represents measurements on the actual tree?
Complete the table.

	Actual Tree	Scale Drawing
Radius of trunk	69 cm	
Height of tree	2.8 m	
Width of leaf		6 mm
Acute angle between branch and trunk	7°	

Which measurement has not changed? Why?

What would be a sensible scale to use to make a scale drawing of the floorplan of your school?
Give your answer in the form $1 : n$

Understand & represent bearings

Notes and guidance

Students will learn that bearings are always measured clockwise from North and always given as 3 figures.
The wording 'of A from B' can often confuse students and is worth addressing as a class, identifying a wide variety of start and end points. It is also useful to discuss the convention that the North line is usually drawn vertically up the page, regardless of the actual direction of North.

Key vocabulary

Three-figure	North line	Clockwise
Bearing	Bearing of … from …	

Key questions

Why are bearings always given as the clockwise angle?

Is it possible to have a bearing of $400°$? Why or why not?

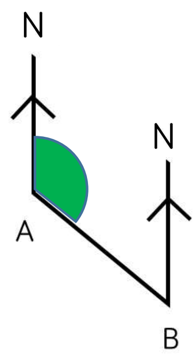
Should bearings be written to one decimal place?

Exemplar Questions

Which of these are bearings? Explain why/why not in each case.

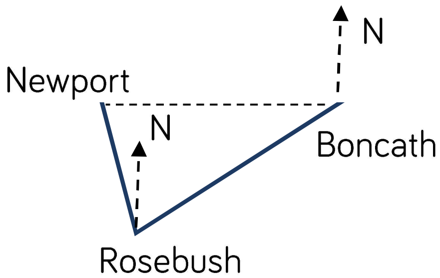
- 120°
- 24.5°
- 245°
- 12°
- 000°

Which of the statements are correct?
The highlighted angle shows,



- The bearing of A
- The bearing of A from B
- The bearing of B from A
- The bearing from A to B
- Between 090° and 180°

What other bearing could be shown on the diagram?



Newport is due West of Boncath. Amir says that the bearing of Newport from Boncath is 180° since they are on a straight line.
Is he correct?

Use the diagram to complete the sentences.
____ is due East from _____
The bearing of _____ from _____ is less than 090°
The bearing of _____ from_____ is close to 360°
The bearing of _____ from _____ is 270°

Measure and read bearings

Notes and guidance

Students need plenty of practice with the skill as confusion may arise with the wording of 'the bearing of A from B'.
Activities like the second exemplar question are very useful to let students explore and discover the relationships between angles and the relative positions of the points.
It is well worth working just with angles before proceeding to the next step and include scale as well.

Key vocabulary

Three figure bearing	Due East/West…. of
North line	Clockwise

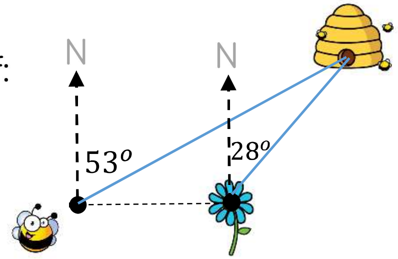
Key questions

From which compass point are bearings always measured?
Is the bearing of A from B the same as the bearing of B from A?
Why does it help to add auxiliary lines to diagrams when measuring bearings?

Exemplar Questions

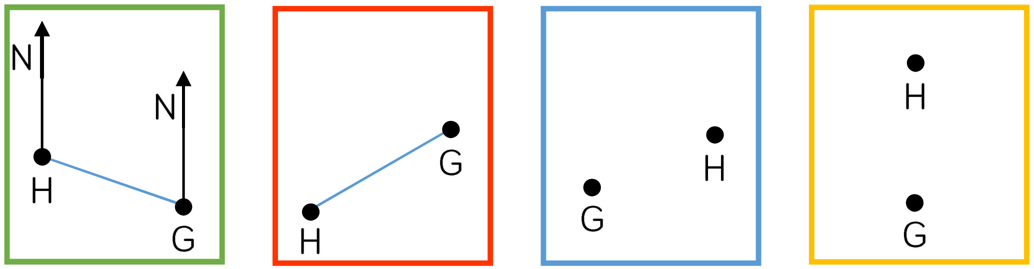
The bee is due West of the flower
From the list, choose the bearing of:

- the hive from the bee
- the flower from the bee
- the hive from the flower
- the bee from the flower



025° | 028° | 053° | 090° | 180° | 270°

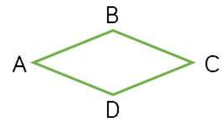
Draw the points G and H in each of the relative positions shown, including North lines for each point.



Measure the bearing of G from H and their bearing of H from G for each of your diagrams. Compare your answers with a partner's.

Construct and label a rhombus ABCD.
How many examples of the following sentence can you find?

"The bearing of __ from __ is the same as the bearing of __ from __."



Scale drawings using bearings

Notes and guidance

When confident with the measuring and direction of bearings, students can move on to more complex problems requiring them to draw scale diagrams as well. It is a good idea to use plain paper rather than squared paper (as in examinations) to promote accurate use of a protractor.

The need for accuracy can be emphasised by comparing answers.

Key vocabulary

Scale	Ratio
Bearing	Construct

Key questions

Why is a scale represented as a ratio?

What units are used in a scale?

Which part of the scale represents the actual size?

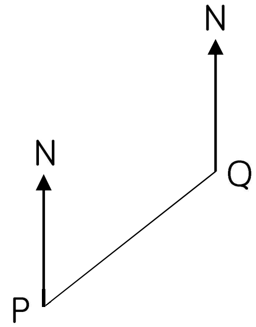
Exemplar Questions

The diagram shows the positions of two towns P and Q on a map.

- Measure the bearing of Q from P.

Another town R is on a bearing of 160° from Q, and on the map is 7 cm from Q.

- Mark and label the position of R.
- Measure the distance of R from P and the bearing of R from P.



A drone is 1.2 km from a school on a bearing of 075°
A football pitch is 500 m due South of the school.
Make a scale drawing of the drone, the school and the pitch, using 1 cm to represent 200 m.
Use your drawing to work out how far the drone needs to fly to get straight to the football pitch.

Here is a journey made by a boat.

- It sails 100 km East
- It sails 200 km on a bearing of 210°
- It sails 300 km West

Draw a rough sketch of the boat's journey.

Use your sketch to design a scale diagram of the boat's journey that fits on your page. What would be a sensible scale?

Use your scale diagram to find the distance and bearing the boat needs to sail back to get back to its starting point.

Bearings with angle rules

Notes and guidance

As well as providing a useful reminder of angle rules, this small step provides a good opportunity to allow students to compare multiple methods of solving a problem.

Adding auxiliary lines will help to emphasise where rules for angles in parallel lines might be used.

Encourage students to read the questions carefully, in particular noting where to measure the bearing from.

Key vocabulary

Parallel	Alternate	Corresponding
Co-interior	North line	Due South/West…

Key questions

Why are rules for angles in parallel lines useful for solving bearings problems?

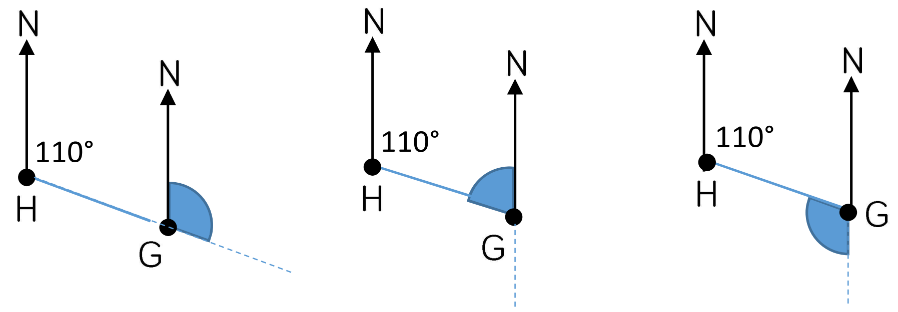
Why is the rule about co-interior angles different from the rules for alternate and corresponding angles?

When does adding the bearing of A from B and the bearing of B from A equal 360 degrees?

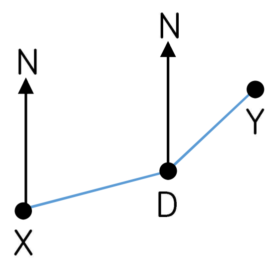
Exemplar Questions

The bearing of G from H is 110°

Explain how the diagrams help to find the bearing of H from G.

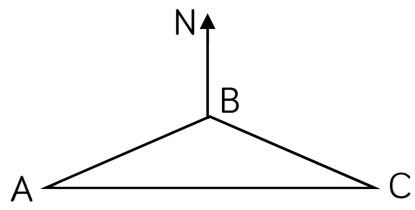


The bearing of Y from D is 055°
The angle XDY is 192°
Find the bearing of D from X.



The bearing of a ship from a lighthouse is 070°
Showing your working clearly, calculate the bearing of the lighthouse from the ship.

Triangle ABC is isosceles.
Angle ABC is 134°, C is due East of A.
Find the bearings of each vertex of the triangle from each other vertex.



Bearings & right-angled geometry

Notes and guidance

This is good opportunity to revisit the use of trigonometry studied last term, and Pythagoras' theorem. Drawing the North line is especially revealing in these questions as it introduces right angles. Adding auxiliary lines and drawing separate triangles might help students to decide which geometric methods to use. They will need support initially to form diagrams from worded questions.

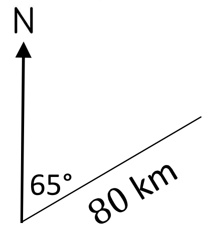
Key vocabulary

Trigonometry	$\sin\theta$, $\cos\theta$, $\tan\theta$
Due East	Perpendicular

Key questions

How do you know which trigonometric ratio to use?
Do you need to add an extra line to find a right-angled triangle in your diagram?
Do you need to use Pythagoras theorem, trigonometry or both?

Exemplar Questions



A ship sails for 80 km on a bearing of 065°
Calculate:

- How far East the ship has travelled.
- How far North the ship has travelled.

A plane flies due West for 200 km and then turns and flies due South for 90 km.

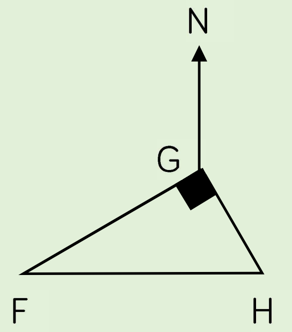
- Sketch a diagram of the plane's journey.
- Show that the plane is just over 219 km from its starting point.
- Work out the bearing the plane needs to fly on in order to return directly to its starting point.

H is due East of F.
The bearing of H from G is 158°

- Find the bearing of G from H.
- Find the bearing of G from F.

The distance from F to H is 14 km.

- What is the distance from F to G?



A hiker walks 3 km due East and then turns and walks 5 km due South. Work out the bearing of the hiker from their starting point.

Bearings: sine & cosine rule

H

Notes and guidance

This is another good opportunity to revisit and extend prior learning, using the sine and cosine rules. Teacher modelling will help students to construct their own sketch diagrams in time, but they will need support initially to correctly label information given and to identify lengths and angles needed. Scaffolding, by providing partly-drawn diagrams may be useful.

Key vocabulary

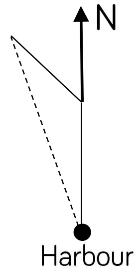
Sine Rule	Cosine Rule
Opposite	Included angle

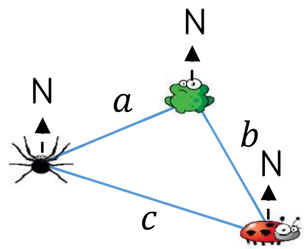
Key questions

What is the minimum amount of information required to use the sine/cosine rule?
How many angles inside the triangle can you find with the given information for your question?
How can you work out other angles?

Exemplar Questions

A boat starts from harbour and travels 30 km North, then turns to travel 12 km North West. Complete the diagram and work out how far does the boat need to travel to get back to the harbour?





The bearing of the frog from the spider is 062°
The bearing of the ladybird from the spider is 113°
The bearing of the spider from the lady bird is 295°
Draw a sketch of the diagram and find all the interior angles of the triangle in the diagram. Given that the distance from the spider to the frog is 98 cm, find lengths b and c.

A cyclist leaves a point on a bearing of 075° at a speed of 15 mph. A runner leaves the same point at a speed of 6 mph on a bearing of 318°. How far apart are they,

- after 10 minutes?
- after 1 hour?
- after x hours?

Working with Circles (1)

Small Steps

- Recognise and label parts of a circle (R)
- Calculate fractional parts of a circle
- Calculate the length of an arc
- Calculate the area of a sector
- **Circle theorem: Angles at the centre and circumference** (H)
- **Circle theorem: Angles in a semicircle** (H)
- **Circle theorem: Angles in the same segment** (H)
- **Circle theorem: Angles in a cyclic quadrilateral** (H)

(H) Denotes Higher Tier GCSE content

(R) Denotes 'review step' – content should have been covered at KS3

White Rose Maths

Working with Circles (2)

Small Steps

- Understand and use the volume of a cylinder and cone
- Understand and use the volume of a sphere
- Understand and use the surface area of a sphere
- Understand and use the surface area of a cylinder and cone
- **Solve area and volume problems involving similar shapes** R H

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Parts of a circle

R

Notes and guidance

Students will be familiar with some of the vocabulary associated with circles, but some will be new and will need regular reinforcing over the coming weeks. Showing pupils non-examples that are close in nature to the word in question will help to refine their definitions and understanding.
Regular use of geometry packages which make implicit use of the keywords will help memorising.

Key vocabulary

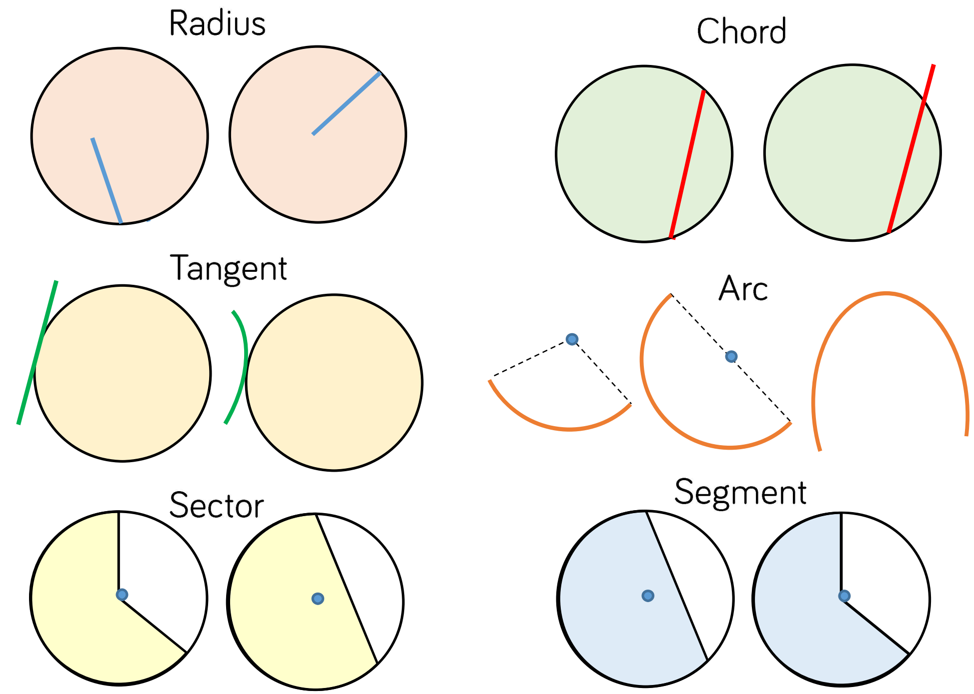
Radius	Diameter	Chord	Centre
Tangent	Arc	Sector	Segment

Key questions

Is the diameter of a circle also a chord? Why or why not?
What's the difference between a segment and a sector?
Where do sectors appear in the real world?
What about segments?
Is a tangent/chord a line or a line segment?

Exemplar Questions

Give reasons for why each diagram is/is not an example of the keyword.



Use a geometry software package to investigate:
What is the maximum number of times that three chords of a circle can intersect? How many times can four chords intersect?
Is it always possible to draw two circles that share the same chord?

Fractional parts of a circle

Notes and guidance

This step reinforces basic fraction work as well as the word 'sector'. If appropriate, the terms major and minor sector could be introduced. Looking at familiar fractions of circles such as quarters and eighths is a useful lead in to the coming steps involving working out arc lengths and areas using formulae of the form $\frac{\theta}{360} \times \cdots$ and shows that complex formulae are not always necessary.

Key vocabulary

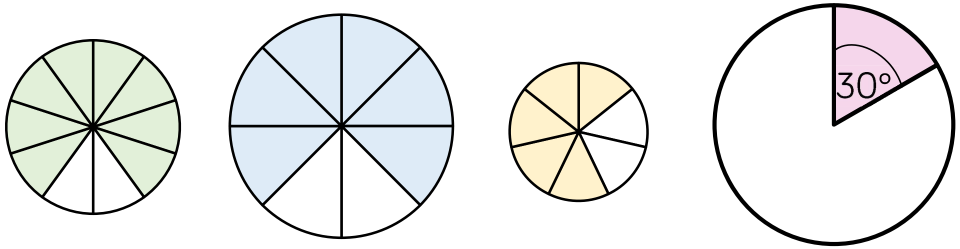
Radius	Diameter	Sector
Arc	Circumference	Area

Key questions

How many degrees are there in a full turn?
What fraction of a circle is represented by … degrees?
What are the formulae for the area and circumference of a circle? How do you remember which is which?

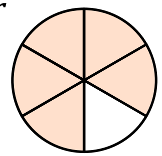
Exemplar Questions

For each circle, write down the fraction that is shaded.



Which is the correct formula for the area of a circle?

$A = \pi r^2$ $\quad A = \pi d$ $\quad A = 2\pi d$ $\quad A = 2\pi r$

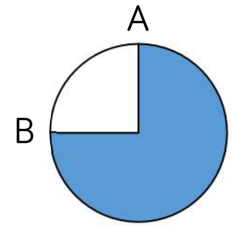


The radius of the circle is 6 cm. Find, in terms of π,
- the area of the circle.
- the area of the shaded region.

A circle has diameter 12 cm. Work out,
- the area of the circle.
- the circumference of the circle.

Tommy says the area and perimeter of a semicircle of diameter 12 cm will be half the answers you have just worked out.
Tommy is only partly right. Explain why.

The circumference of this circle is 20π cm.
What is the distance clockwise from A to B?
What assumption have you made?
Work out the shaded area.



Calculate the length of an arc

Notes and guidance

Students may need to revisit the formula of the circumference of a circle before starting this small step. Building from the previous step, they should realise that the length of an arc is the same fraction of the circumference as the fraction of a full turn given by the related angle. Angles below and above 180 degrees should be explored & both exact and rounded answers should be considered.

Key vocabulary

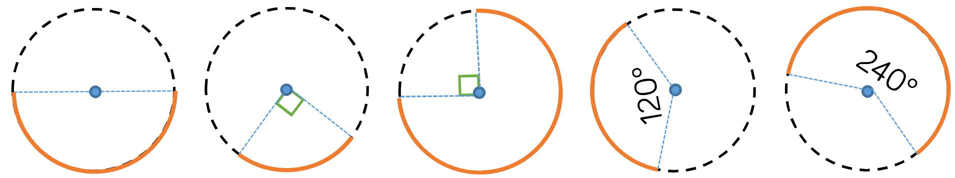
Arc	Circumference	Fraction
Minor/Major	Proportion	Sector

Key questions

What is the formula for the circumference of a circle?
Why is the perimeter of a semicircle not just half the circumference of the circle?
What fraction of a full turn is a 60 degree turn?
How does this help us to find an arc length of a 60 degree sector?

Exemplar Questions

The circumference of this circle is 24 cm. Find the arc length in each case.



Find the arc lengths if the radius of the circle were 24 cm.

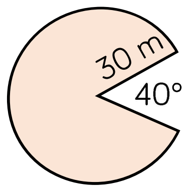
Jack thinks he can find the arc length of any circle with a radius of r and angle of θ

$\frac{\theta}{360} \times$ circumference

Is Jack correct?

Explain each term of the following calculation to find the perimeter of this shape.

$$\frac{320}{360} \times 2\pi \times 30 + 30 + 30$$



Calculate the answer to 1 decimal place.

A circle has radius 10 m. Find, in terms of π, the arc length if,

- the angle at the centre is $36°$
- the angle at the centre is $270°$

The length of an arc subtended by an angle of 120° is 8π cm. Work out the radius of the circle.

Calculate the area of a sector

Notes and guidance

Students need to be familiar with the formula for the area of a circle. Links should be made with the previous step, establishing that the proportion of a full turn taken up by the sector is identical to its proportion of the area of the circle, leading to the formula $\frac{\theta}{360} \times \pi r^2$

Again, looking at examples both in terms of π and in decimal form is useful, as is working backwards to find θ, r or d.

Key vocabulary

Arc	Area	Fraction
Subtend	Proportion	Sector

Key questions

What is the formula for the area of a circle?

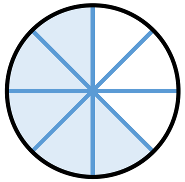
If the angle formed by a sector is e.g. 80 degrees, what fraction of the circle is the sector?

How does this help us to find the area of the sector?

Can the angle in the sector be a reflex angle?

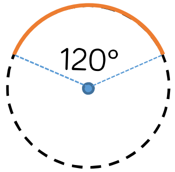
Exemplar Questions

The circle is split into equal parts.
The area of the circle is 24π cm^2.
What is the area of the blue sector?
What is the area of the white sector?



The outline for a company logo is the sector of a circle subtended by an angle of 120° as shown in the diagram.

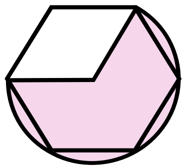
- What is the area of the logo if the radius of the circle is 10 cm?
- What is the area of the logo if the diameter of the circle is 10 cm?



Complete the table.

Radius	Diameter	Angle at centre	Area of sector	Length of arc
5 cm		100°		
	1.2 m	13°		
6 cm			3π cm^2	

Part of a circle is drawn from the centre of a regular hexagon as shown.
The sides of the hexagon are 60 cm.
Work out the shaded area.



Angles: centre, circumference H

Notes and guidance

This first circle theorem needs to be clearly understood as it is the basis of many of the other theorems that follow.
It is useful to include cases with chords/angles in many orientations, not just with the angle at the circumference at 'the top' of the circle. Students need to prove the circle theorems; the use of isosceles triangles in this case is also crucial to many other circle theorem problems.

Key vocabulary

Centre	Circumference
Angle	Isosceles

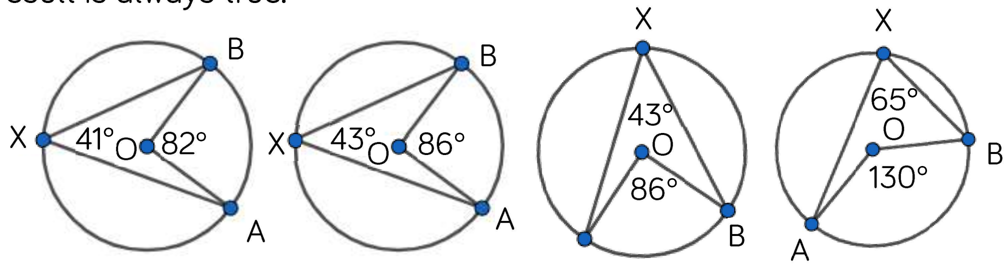
Key questions

How do you identify the 'angle at the centre' and the 'angle at the circumference'?

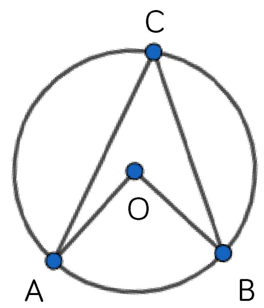
Why do we need to be careful when the angle at the centre is a reflex angle?

Exemplar Questions

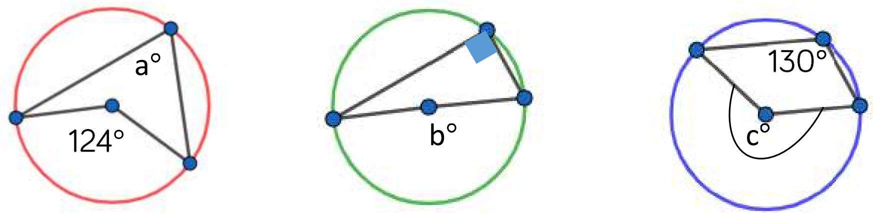
Compare angles AOB and AXB in the diagrams below.
What to you notice? Use dynamic geometry software to see if your result is always true.



Copy the diagram.
Add the radius OC to the diagram.
Write down the names of the isosceles triangles that are formed.
Which angles are equal?
Use these angles to prove that $\angle AOB = 2\angle ACB$



Work out the unknown angles.



What is the same and what is different about your calculations?

Angles in a semicircle

H

Notes and guidance

As well as the proof indicated here, this theorem can be easily proved as a special case of the previous step when the angle at the centre is 180°. Encourage the students to explore near examples of this rule, which could be constructed with geometry software packages. The use of the (unmarked) right angle to revisit Pythagoras' theorem or trigonometry could also be explored.

Key vocabulary

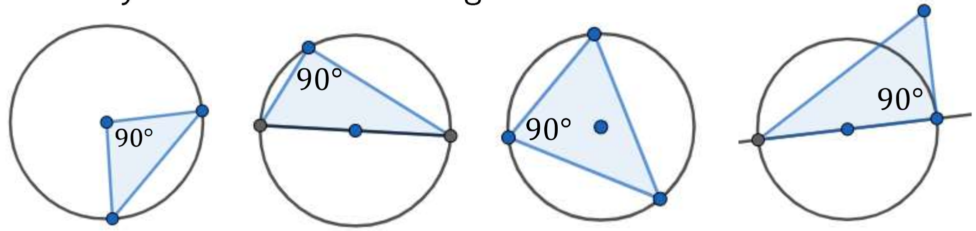
Semicircle	Diameter	Centre
Right angle	Circumference	Pythagoras

Key questions

What is the 'angle at the centre' of a circle if we have a diameter?
What does this mean about the 'angle at the circumference'?
What other rules do we know about right-angled triangles?

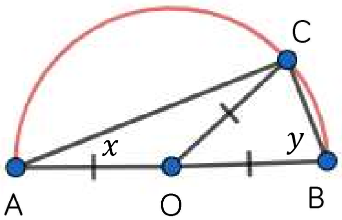
Exemplar Questions

Use dynamic geometry to investigate whether these diagrams are possible, impossible or always true?
What do you notice about the angle in a semicircle?



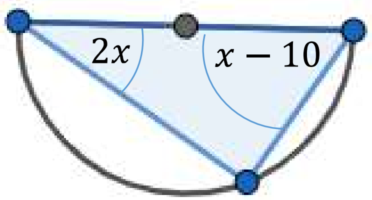
The proof that the angle in a semicircle is always $90°$ is started below.
Follow the instructions to help complete the proof.

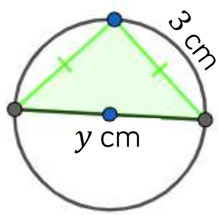
AO = OC (both radii of the same circle)
So $\angle ACO = x$ (Angles in an isosceles triangle)



- Find an expression for $\angle OCB$
- Find the angle sum of triangle ACB in terms of x and y
- Equate this to 180° and find the value of $x + y$

Work out x and y in the semicircles shown.





Angles in the same segment

H

Notes and guidance

This theorem is most commonly proved by showing that angles at the circumference from the same chord/arc share a common angle at the centre of the circle. Students may need help to reinforce the language of ‘segment’, which can often be confused with ‘sector’, and ‘subtend’.
Varying the diagram is again useful so that students do not only look for the common ‘bow-tie’ shape.

Key vocabulary

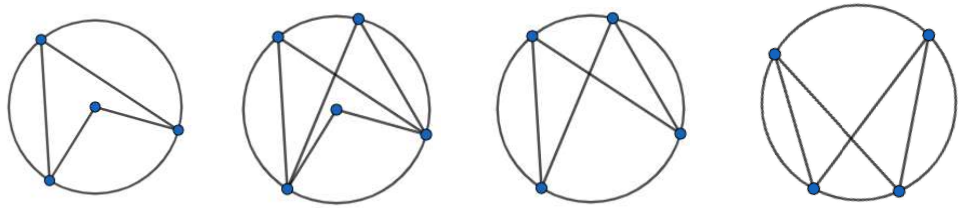
Segment	Chord
Subtend	Arc

Key questions

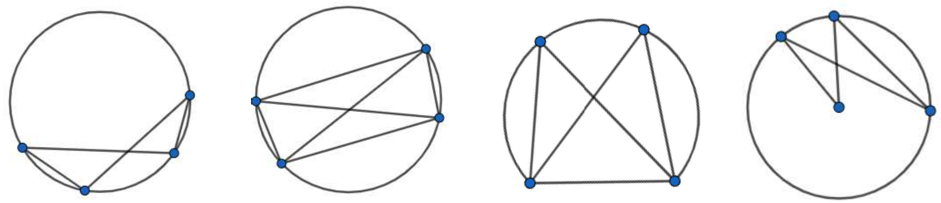
What is the difference between a sector of a circle and a segment of a circle?
How many ‘angles at the circumference’ can be drawn from a single chord? Will they all be equal in size?
Why or why not?

Exemplar Questions

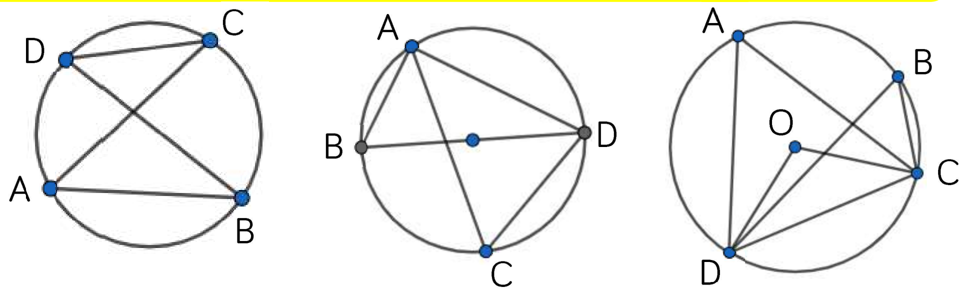
What has changed or stayed the same from diagram to diagram? Colour in angles that are equal in size.
How do you know they are equal?



Which of the diagrams below show that angles in the same segment are the same and which do not? How do you know?



In each of the circles, ∠ABD is 65°. Are any other of the angles 65°? What other angles can you find?



Angles in a cyclic quadrilateral H

Notes and guidance

This small step is an opportunity to recap students' knowledge of different types of quadrilaterals, their properties and the sum of their interior angles. The fact that opposite angles add to 180° could be explored using a geometry software package, and can be proved again by using angles at the centre and circumference. At this stage students should also explore problems that require the use of more than one circle theorem.

Key vocabulary

Quadrilateral	Cyclic	Circumference
Vertices	Opposite	

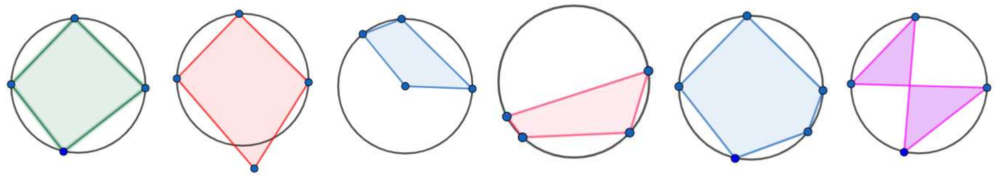
Key questions

What's the difference between opposite angles and adjacent angles? How can we identify the opposite angles of a cyclic quadrilateral?

What does cyclic mean? Are all quadrilaterals cyclic? How do you know?

Exemplar Questions

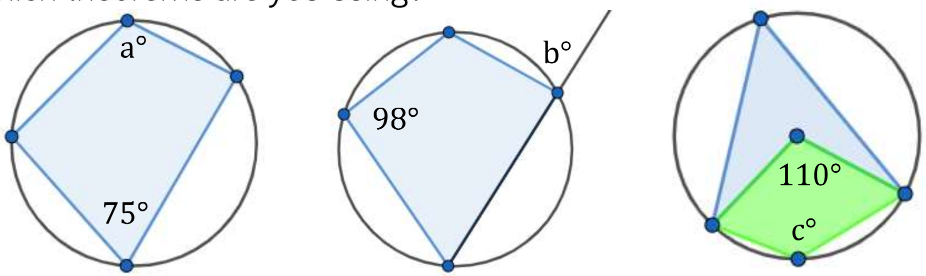
Which of the following diagrams are cyclic quadrilaterals? Explain why or why not.



Are the following statements true or false?

- Opposite angles of a cyclic quadrilateral add up to 180°
- Adjacent angles of a cyclic quadrilateral add up to 180°
- All rectangles are cyclic quadrilaterals.
- It is impossible for a parallelogram to be cyclic.
- It is impossible for a kite to be cyclic.
- A trapezium may or may not be cyclic.

Work out the lettered angles in the diagrams below.
Which other angles can you work out?
Which theorems are you using?



Volume of a cylinder and cone

Notes and guidance

It may be useful to revise the volume of a prism before moving on to the volume of a cylinder, which will have been covered in Year 9. Explore the similarities and differences between the two shapes, using physical demonstrations if possible. You could also point out that a cone is a type of pyramid with a circular base. Students do not need to learn these formulae, but should be fluent in their use.

Key vocabulary

Cylinder	Cone	In terms of π
Perpendicular height	Base	Frustum

Key questions

How do you enter the calculation for the volume of a cylinder/cone into your calculator?

What does the instruction 'leave your answers in terms of π' mean?

How can Pythagoras' theorem help us to work out the perpendicular height of a cone?

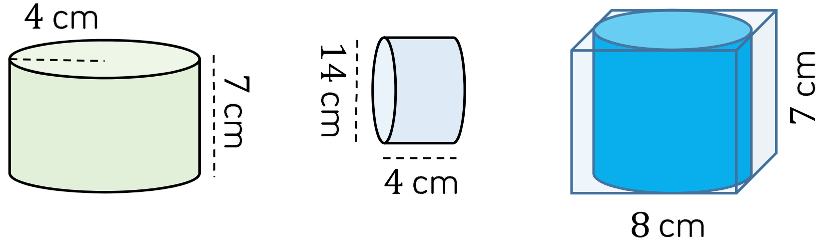
Exemplar Questions

Compare the formulae for the volumes of cones and cylinders.

$$V_{cylinder} = \pi r^2 h \qquad V_{cone} = \frac{1}{3}\pi r^2 h$$

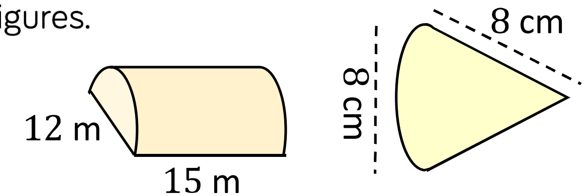
If a cone and a cylinder have the same height and radius, how many times can you empty the full cone into the cylinder?
Investigate using sand.

Which cylinder has a volume of $\pi \times 7^2 \times 4$ cm^3?



Work out the volumes of all three cylinders in terms of π and put the cylinders in ascending order of volume.

Find the volume of the shapes, giving your answers to 3 significant figures.



Hint:
Use Pythagoras' theorem to find the height of the cone.

A cone has diameter 12 cm and height 20 cm.
A smaller cone of height 10 cm is cut off the top of the cone.
Work out the volume of the remaining shape.

Volume of a sphere

Notes and guidance

Students need to be careful using this formula as both the fraction and the cubing can cause problems.
The use of a calculator could be modelled and compared with non-calculator methods. Having now looked at three shapes, students could explore the total volume of shapes made by combining these, and also look at parts of the shapes e.g. hemispheres.

Key vocabulary

Sphere	Radius	Diameter
Hemisphere	Centre	

Key questions

How many lengths do you need to know to be able to find the volume of a sphere?

How does the volume of a hemisphere compare to the volume of a sphere?

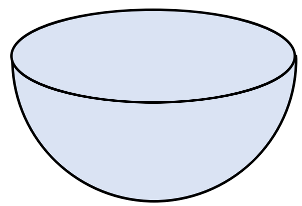
Exemplar Questions

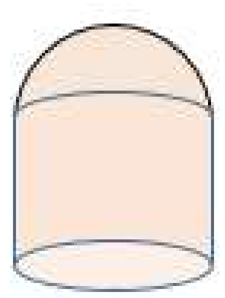
Use the formula $V = \frac{4}{3}\pi r^3$ to find the volume of,

- a sphere of radius 6 cm.
- a sphere of diameter 6 cm.

Give your answers in terms of π.

The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$
The diagram shows a hemisphere of radius 10 cm. Find its volume to 3 significant figures.





The diagram shows a hemisphere that fits exactly on top of a cylinder.
The height of the cylinder is 12 cm.
The height of the whole shape is 18 cm.
What is the radius of the sphere?
Work out the volume of the shape.

A metal cuboid measures 20 cm by 10 cm by 5 cm.
The cuboid is melted and recast into spheres of radius 3 cm.
How many spheres can be made?
Jack thinks that you can make exactly twice as many spheres of radius 1.5 cm from the same block.
Show working to show that Jack is wrong.

Surface area of a sphere

Notes and guidance

This is another given formula, and it would be useful to look at this in conjunction with either the next or previous step so that students experience making the right choice of formula to use. Again, both exact and rounded answers should be considered. You may wish to investigate and compare the structure of area and volume formula, even though dimensional analysis is not required.

Key vocabulary

Surface Area	Curved Surface	Sphere
In terms of π	Radius	Diameter

Key questions

How do you enter the calculation for the surface area of a sphere into your calculator?

How does the surface area of a sphere compare to the area of a circle?

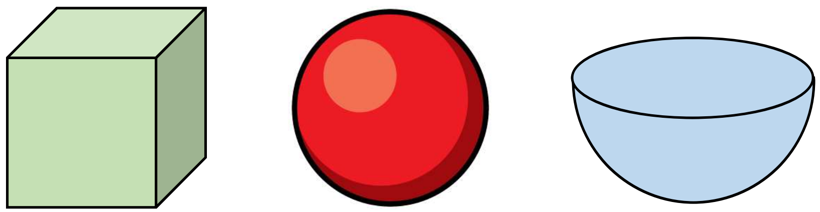
Exemplar Questions

Use the formula $A = 4\pi r^2$ to find the surface area of a sphere if,

- it has radius 10 cm.
- it has diameter 10 cm.

Give your answers in terms of π.

The radius of the small ball is 2 cm. The radius of the large ball is 6 cm.
Alex thinks that the surface area of the large ball will be three times that of the small ball. Use calculations to show that Alex is wrong.



The cube has side length 10 cm.
The sphere has radius 8 cm.
The hemisphere has radius 12 cm.
Put the shapes in order of size given by,

- their surface areas.
- their volumes.

Surface area of cylinder & cone

Notes and guidance

Students should be able to deduce the surface area of a cylinder by considering its net, whilst the formula for the curved surface area of a cone will be given. Pythagoras' theorem may be needed to calculate the slant height or perpendicular height. Allowing students to see the links between the areas by making or deconstructing cylinders and cones is highly effective.

Key vocabulary

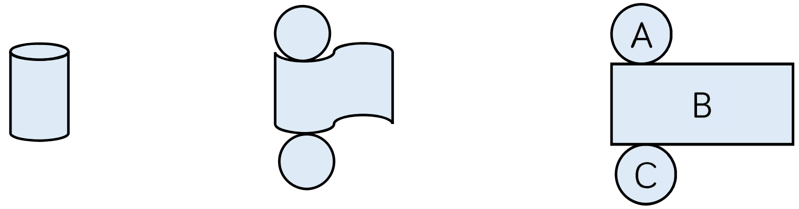
Curved surface	Base	Area
Slant height	Perpendicular height	

Key questions

How many surfaces does a cylinder have? Describe its net.
How do your calculations change if a question concerns an 'open' cylinder?
Which is longer, the slant height or the perpendicular height of a cone? Will this always be the case?

Exemplar Questions

The diagrams below show the formation of the net of a cylinder. Use the diagrams to help find the formula for surface area of a cylinder.

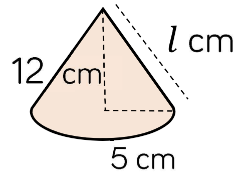


Which cylinder has the greater surface area? Use calculations to justify your answer.

Cylinder A
Height = 9 cm
Radius = 8 cm

Cylinder B
Height = 8 cm
Radius = 9 cm

The curved surface area of a cone is given by the formula $A = \pi r l$, where l is the slant height of the cone.
Work out l and the **total** surface area of the cone shown.



A sweet manufacturer is considering two types of packaging for popping candy:

- A cone of radius 2 cm and height 6 cm.
- A cylinder of radius 2 cm and height 2 cm.

Show that both packages have the same volume and compare their surface areas.

Area/Volume of similar shapes

R H

Notes and guidance

Now that students are familiar with the formulae to work with circular shapes, this is a good opportunity for higher tier students to revisit area and volume ratios. Starting with a rectangle and cuboid, students can review length, area and volume scale factor. They can then apply this to cylinders, cones and spheres. Giving opportunities to work 'backwards', by square rooting or cube rooting to find the length of a radius is useful.

Key vocabulary

Scale Factor	Ratio	Proportion
Square	Cube	Root

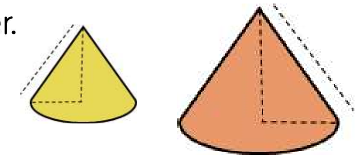
Key questions

How does doubling one length affect the area of a shape? What about doubling all of the lengths?
If one sphere has a radius half the size of another sphere, what's the relationship between their surface areas? What about their volumes?

Exemplar Questions

Two cylinders are similar. The larger cylinder is three times the height of the smaller cylinder.
200 ml of paint are needed to paint the small cylinder.
How much paint is needed to paint the large cylinder?
How many times larger is the volume of the large cylinder compared to the volume of the small cylinder?

The two cones are similar.
The smaller cone has radius 5 cm and the larger cone has radius 20 cm. Is the surface area of the larger cone four times the surface area of the smaller cone? Explain your answer.
The volume of the larger cone is 960 cm^3.
Find the volume of the smaller cone.



Two similar cones have volumes 12π cm^3 and 324π cm^3.
How many times larger is the surface area of the larger cone, compared to the surface area of the smaller cone?

Oil is stored in similar cylindrical drums. The large drum has a base area of π m^2 and the small drum has a base area of 2500π cm^2.
What's the relationship between the radii of the two drums?
How many times more oil can the large drum store than the small drum?

Vectors

Small Steps

- Understand and represent vectors
- Use and read vector notation
- Draw and understand vectors multiplied by a scalar
- Draw and understand addition of vectors
- Draw and understand addition and subtraction of vectors
- **Explore vector journeys in shapes** H
- **Explore quadrilaterals using vectors** H
- **Understand parallel vectors** H

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Vectors

Small Steps

- Explore collinear points using vectors (H)
- Use vectors to construct geometric arguments and proofs (H)

(H) Denotes Higher Tier GCSE content

(R) Denotes 'review step' – content should have been covered at KS3

Understand and represent vectors

Notes and guidance

Students have met vectors previously when translating objects, and so build on from this. A key learning point is that a vector shows both direction and magnitude. It's also important to emphasise the role of the arrow so that students get the idea of starting & end points and hence direction. Comparing vectors with the same magnitude, but different directions is very useful.

Key vocabulary

Column vector	Direction	Scalar
Size	Magnitude	

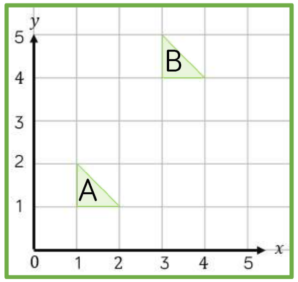
Key questions

What's the same and what's different about a translation and a drawing representing a vector?
What do the numbers in the column vector represent?
How do you know which direction they represent?
What does the arrow show?
How do you know which way to point the arrow?

Exemplar Questions

Write down the column vector that describes the translation from:

- triangle A to triangle B.
- triangle B to triangle A.

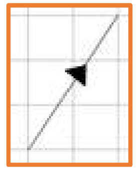


To represent the vector $\begin{pmatrix}2\\3\end{pmatrix}$ on a grid, Dora counts 2 squares to the right and then 3 squares up.

How is this the same/different from translation?

How would this representation differ for vector $\begin{pmatrix}-2\\-3\end{pmatrix}$?

Dora

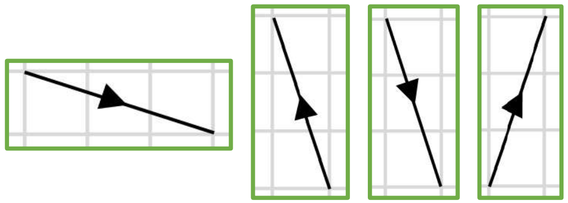


Match up the correct vectors cards. Explain your answers.

3 right and 1 down | 1 left and 3 up | 0 left and 3 up

$\begin{pmatrix}-3\\-1\end{pmatrix}$ $\begin{pmatrix}-1\\3\end{pmatrix}$ $\begin{pmatrix}3\\-1\end{pmatrix}$

$\begin{pmatrix}-1\\-3\end{pmatrix}$ $\begin{pmatrix}3\\1\end{pmatrix}$ $\begin{pmatrix}1\\-3\end{pmatrix}$



Draw a diagram to represent each column vector that doesn't have a match.

White Rose Maths

Use and read vector notation

Notes and guidance

Students are now familiar with two representations of vectors: column vector and line segment with an arrow. We can now introduce the formal notation for labelling vectors, $\overrightarrow{AB}$ and $\boldsymbol{a}$. When handwritten, $\boldsymbol{a}$ is written as $\underline{a}$. Students develop a deeper understanding of a vector representing movement from one point to another and can start comparing different representations.

Key vocabulary

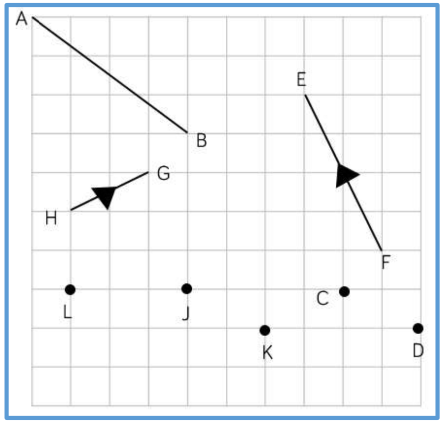
Column vector	Direction	Magnitude
Size	Arrow	

Key questions

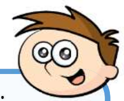
What is the significance of the order of the letters when writing $\overrightarrow{AB}$? What does the arrow tell us?
Do the letters describing a vector always have to be written in alphabetical order?
How do we write a vector using a single lower case letter?

Exemplar Questions

Add an arrow onto the line segment to represent the vector $\overrightarrow{AB}$.
Write $\overrightarrow{AB}$ as a column vector.
How would the column vector $\overrightarrow{BA}$ be different ?



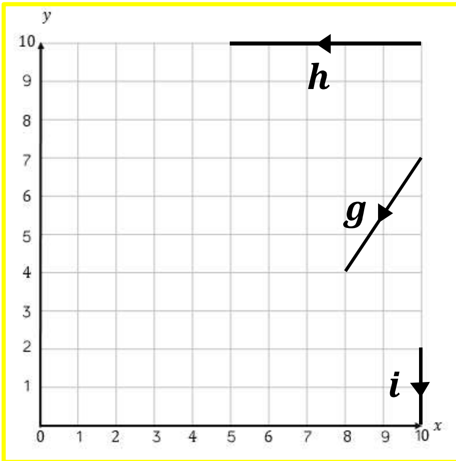
Teddy



Teddy thinks vector $\overrightarrow{EF}$ is shown on the diagram. Is he right? Explain your answer.

Represent the following vectors $\overrightarrow{CD}$ and $\overrightarrow{JK}$ on the diagram.
What do you notice about them?

Represent the vector $\overrightarrow{ML} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ on the diagram.



Write down column vectors to represent $\boldsymbol{g}, \boldsymbol{h}$ and $\boldsymbol{i}$

Copy the axes and plot the points
A (2, 6) and B (4, 3)

Plot a third point, C, to form an isosceles triangle ABC.

Write down the column vectors representing: $\overrightarrow{BA}, \overrightarrow{CB}, \overrightarrow{AC}$

White Rose Maths

Vectors multiplied by a scalar

Notes and guidance

Students explore vectors that are parallel to each other. They understand that when vectors are parallel, one is a multiple of the other and the multiplier is called a scalar. Students will need support in identifying negative multipliers where vectors are parallel, but are in opposite directions. Looking at diagrammatic and column representations of vectors will help reinforce this.

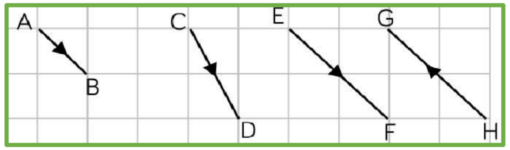
Key vocabulary

Parallel	Scalar	Multiplier
Size	Direction	Equal
Opposite		

Key questions

What's the same and what's different about parallel vectors?
How do we know if one vector is a multiple of another?
How do we know if the multiplier is negative?

Exemplar Questions



Express $\overrightarrow{AB}$ and $\overrightarrow{EF}$ as column vectors.
What do you notice?

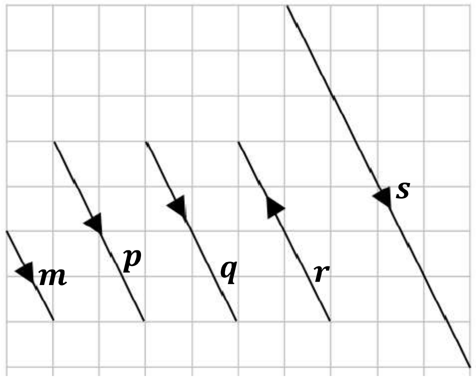
Whitney

Whitney says that $\overrightarrow{CD}$ is double $\overrightarrow{AB}$ as it is twice as long.
Explain why Whitney is incorrect.

Draw vector $\overrightarrow{JK}$ so that $\overrightarrow{JK} = 3\overrightarrow{AB}$

Complete the following statement: $\overrightarrow{GH} = \square \times \overrightarrow{HG}$
How do you know the multiplier is a negative number?

In pairs, discuss: What's the same and what's different about the vectors shown in the diagram?
Which of the following statements are true?
(for each statement, explain your answer).



- All of the vectors are parallel
- Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are all equal
- If vectors are parallel, they are always equal
- $\boldsymbol{s} = 2\boldsymbol{r}$
- $2\boldsymbol{q} = 2 \times \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}$
- $\boldsymbol{s}$ is twice as long as $\boldsymbol{p}$ and in the same direction, so $\boldsymbol{s} = 2\boldsymbol{p}$
- If you multiply vector p by -1 this gives vector q
- $\boldsymbol{r} = -\boldsymbol{p}$

Addition of vectors

Notes and guidance

The aim of this small step is for students to become confident in identifying and drawing representations of vector addition. A common misconception is thinking that the resultant vector follows on from the direction of the other vectors. To avoid this, students may need lots of practice in drawing out vector representations of addition. This can be extended to more than two vectors.

Key vocabulary

Direction	Column vector
Addition	Resultant

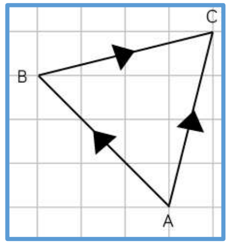
Key questions

What do we mean by the word 'resultant'?

How do we identify the resultant of two vectors? How does this relate to column vector addition?

If we are adding three vectors together, how can we identify the resultant?

Exemplar Questions



$\overrightarrow{AB} = \binom{-3}{3}$ and $\overrightarrow{BC} = \binom{4}{1}$

Complete: $\overrightarrow{AB} + \overrightarrow{BC} = \binom{-3}{3} + \binom{4}{1} = \binom{1}{?}$

Compare this to the column vector representing $\overrightarrow{AC}$

Eva and Alex are thinking about how they can find $\overrightarrow{AC}$ using $\overrightarrow{AB}$ and $\overrightarrow{BC}$. Who is right? Why?

Eva: $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{CA}$

Alex: $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

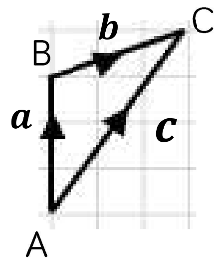
Write down vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ as column vectors.

Annie works out $\boldsymbol{a} + \boldsymbol{b}$

She writes down $\binom{3}{0} + \binom{3}{1} = \binom{6}{1}$

What mistake has she made?

Calculate, $\boldsymbol{b} + \boldsymbol{a}$ $\quad 2\boldsymbol{b} + \boldsymbol{a}$ $\quad \boldsymbol{b} + \boldsymbol{c}$



Write down possible vectors for $\boldsymbol{f}$ and $\boldsymbol{g}$ and $\boldsymbol{h}$ for each card.

$\mathbf{f} + \mathbf{g} + \mathbf{h} = \binom{10}{10}$ $\quad \mathbf{f} + \mathbf{g} + \mathbf{h} = \binom{100}{100}$ $\quad \mathbf{f} + \mathbf{g} + \mathbf{h} = \binom{1}{1}$

Can you do this so that in all three cases, all the vectors are parallel **or none of the vectors are parallel?**

Draw a diagram to show that, $\overrightarrow{EF} + \overrightarrow{FG} + \overrightarrow{GH} = \overrightarrow{EH}$

Addition & subtraction of vectors

Notes and guidance

This can be introduced by considering both $\boldsymbol{a} + (-\boldsymbol{b})$ and $\boldsymbol{a} - \boldsymbol{b}$ and allowing students to explore equivalence. Students should also be exposed to the pictorial form of this. Adding and subtracting vectors abstractly, including in situations where there are more than two vectors then builds on this. Finally, developing reasoning is key here, so activities such as ‘true/false’ or ‘always, sometimes, never’ are useful.

Key vocabulary

Addition	Subtraction	Size
Direction	Multiplying	Scalar

Key questions

What does the arrow on a vector indicate?

What’s the relationship between $\boldsymbol{b}$ and $-\boldsymbol{b}$?

How can we identify vector addition on a diagram? Which way around do the arrows go?

Exemplar Questions

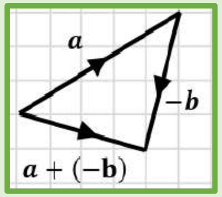
$\boldsymbol{a} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

Write down the vector $-\boldsymbol{b}$

Amir is comparing $\boldsymbol{a} + (-\boldsymbol{b})$ with $\boldsymbol{a} - \boldsymbol{b}$

$$\boldsymbol{a} + (-\boldsymbol{b}) = \begin{pmatrix} 5 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ -4 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

$$\boldsymbol{a} - \boldsymbol{b} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$



The diagram shows:

$\boldsymbol{a} + (-\boldsymbol{b}) = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

He concludes: $\boldsymbol{a} + (-\boldsymbol{b}) \equiv \boldsymbol{a} - \boldsymbol{b}$

Is he right? Use the calculations and diagram to justify your answer.

$\boldsymbol{r} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ $\boldsymbol{s} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$ $\mathbf{t} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$

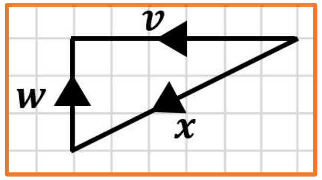
Calculate:

- $\boldsymbol{r} - \boldsymbol{s}$
- $\boldsymbol{s} - \boldsymbol{t}$
- $2\boldsymbol{r} - \boldsymbol{s}$
- $2\boldsymbol{r} - \boldsymbol{s} - \boldsymbol{t}$

Now draw a diagram to represent each calculation.

Write down the column vectors representing $\boldsymbol{v}$, $\boldsymbol{w}$ and $\boldsymbol{x}$.

Calculate the following, writing your answer as a column vector.



$\boldsymbol{v} + \boldsymbol{w}$ $\boldsymbol{v} - \boldsymbol{w}$

Vector journeys in shapes

H

Notes and guidance

Students should be encouraged to move around a shape from one vertex to the next, and to write this journey using the notation $\overrightarrow{AB}$ etc. Once they have written the journey in this format, they are then less likely to make mistakes when expressing the same journey using the notation $\boldsymbol{a}$ etc. Students will require support in identifying when to use a negative in the vector journey.

Key vocabulary

Vector journey	Express
Parallel	Negative

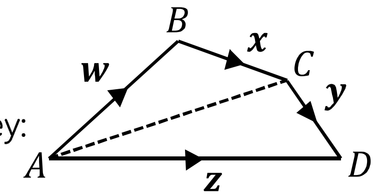
Key questions

How do we know e.g. whether $\overrightarrow{BA}$ is $\boldsymbol{a}$ or $-\boldsymbol{a}$?

If we are writing a vector journey for $\overrightarrow{AD}$, what do you know about the first letter and last letter of each part of the vector journey ($\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$)?

Why is there sometimes more than one way of writing a vector journey?

Exemplar Questions



Annie considers moving along the sides of the shape from vertex A to vertex D.
She writes down the following vector journey:

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$

Complete the following to express her journey in terms of $\boldsymbol{w}$, $\boldsymbol{x}$ and $\boldsymbol{y}$:

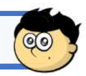
$\overrightarrow{AD} = \boldsymbol{w} + \square + \boldsymbol{y}$

Complete this vector journey for $\overrightarrow{BC}$.

$$\overrightarrow{BC} = \overrightarrow{BA} + \square + \overrightarrow{DC}$$
$$\overrightarrow{BC} = -\boldsymbol{w} + \square \ -\boldsymbol{y}$$

Jack and Dora are both writing a vector journey for $\overrightarrow{AC}$.

Dora: $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ 

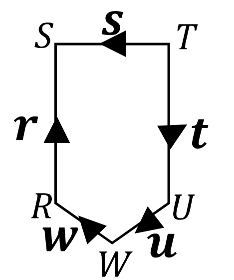
Jack: $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$ 

Express Dora's and Jack's vector journeys in terms of $\boldsymbol{w}$, $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$.

Write down two different journeys for vector $\overrightarrow{BC}$.
Now express your journeys in terms of $\boldsymbol{w}$, $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$.

$\overrightarrow{TU} = \boldsymbol{t}$ $\quad \overrightarrow{UW} = \boldsymbol{u}$ $\quad \overrightarrow{WR} = \boldsymbol{w}$ $\quad \overrightarrow{RS} = \boldsymbol{r}$ $\quad \overrightarrow{TS} = \mathrm{s}$

Are these statements true or false?
Correct any false statements.



$\overrightarrow{RT} = \overrightarrow{RS} + \overrightarrow{ST}$	$\overrightarrow{RT} = \overrightarrow{RW} + \overrightarrow{WU} + \overrightarrow{TU}$
$\overrightarrow{RT} = \boldsymbol{r} - \boldsymbol{s}$	$\overrightarrow{RT} = \boldsymbol{w} + \boldsymbol{u} + \boldsymbol{t}$
$\overrightarrow{RT} = \boldsymbol{r} + \boldsymbol{s}$	$\overrightarrow{RT} = -\boldsymbol{w} - \boldsymbol{u} - \boldsymbol{t}$

Quadrilaterals using vectors H

Notes and guidance

Students explore quadrilaterals through parallel and non parallel vectors. A common misconception is that parallel vectors are equal; teachers may need to highlight that they have the same direction, but not necessarily the same size. Throughout, students should make generalisations about different quadrilaterals.

Key vocabulary

Parallel	Vector journey	Equal
Opposite	Multiple	Magnitude

Key questions

Which quadrilaterals will have pairs of parallel vectors? How many pairs of parallel vectors will they have?

Can a square be described by four equal vectors? Why not? What is the same and what's different about the four vectors that describe a square?

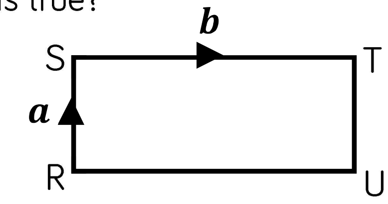
What about the vectors that describe a rectangle?

Exemplar Questions

RSTU is a rectangle. Which statement is true?

$\overrightarrow{TU} = \boldsymbol{a}$

$\overrightarrow{TU} = -\boldsymbol{a}$



Write down $\overrightarrow{UR}$ in terms of b.

The coordinate of R is (1, 5) and $\overrightarrow{RS} = \binom{0}{4}$

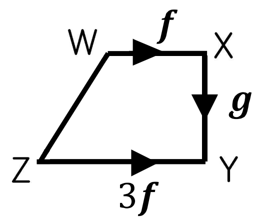
Annie works out the coordinate of S.

To find S, I need to move up 4 in the y direction, so S is (1, 9)

Annie

$\overrightarrow{ST} = \binom{6}{0}$, now find the coordinates of T and U.

What type of quadrilateral is this?
Explain how you know.
Express $\overrightarrow{WZ}$ in terms of $\boldsymbol{f}$ and $\boldsymbol{g}$.



Draw a rhombus, a parallelogram and a square. Match each statement to the shape type.

There will always be exactly 2 pairs of vectors where one vector in each pair is the multiple of the other.

All vectors have the same magnitude.

There are 2 pairs of parallel vectors, where one pair has different magnitude to the other.

Understand parallel vectors

H

Notes and guidance

Students consider parallel vectors represented in different formats e.g. column and pictorial. They appreciate that a vector is only parallel to another if one is a multiple of the other, realising that the multiplier can be negative or fractional. A common misconception can be confusing vectors of equal length with 'equal vectors' – this could be explored by considering e.g. an equilateral triangle.

Key vocabulary

Parallel	Magnitude	Direction
Column	Multiplier	Negative
Fractional		

Key questions

How do we know, without drawing them, whether column vectors are parallel to each other?
How can we identify identical vectors on a diagram?
Are two vectors which are equal in length also identical?

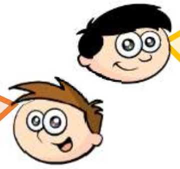
Exemplar Questions

Which five of the following vectors are parallel to each other?
How do you know?

$\begin{pmatrix}2\\4\end{pmatrix}$ $\begin{pmatrix}-2\\-4\end{pmatrix}$ $\begin{pmatrix}-2\\4\end{pmatrix}$ $\begin{pmatrix}4\\-2\end{pmatrix}$ $\begin{pmatrix}2\\-4\end{pmatrix}$ $\begin{pmatrix}8\\4\end{pmatrix}$ $\begin{pmatrix}4\\8\end{pmatrix}$ $\begin{pmatrix}-4\\-8\end{pmatrix}$ $\begin{pmatrix}3\\6\end{pmatrix}$

$3\boldsymbol{a}$ $5\boldsymbol{a}$ $6\boldsymbol{a}$

On the green cards, only vectors $3\boldsymbol{a}$ and $6\boldsymbol{a}$ are parallel as 6 is a multiple of 3 and 5 isn't.

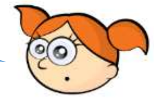


All of the vectors on the green cards are parallel as they are all a multiple of a.

Who is correct, Dexter or Teddy? Give a reason for your answer.

$3\boldsymbol{a} + \mathrm{b}$ $5\boldsymbol{a} + \mathrm{b}$ $6\boldsymbol{a} + \mathrm{b}$

If $3\boldsymbol{a}$, $5\boldsymbol{a}$ and $6\boldsymbol{a}$ are parallel, then so are the vectors on the blue cards as $\boldsymbol{b}$ has been added each time.



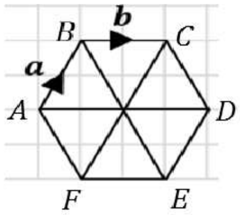
Alex

Explain why is Alex is incorrect.
Which of the following are parallel to $3\boldsymbol{a} + \mathrm{b}$?

$12\boldsymbol{a} + 4\mathrm{b}$ $12\boldsymbol{a} - 4\mathrm{b}$ $\frac{3}{5}\boldsymbol{a} + \frac{1}{5}\mathrm{b}$

ABCDEF is a regular hexagon, so AB ∥ FC.
Write a list of other pairs of parallel lines.
Express the following in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

- $\overrightarrow{FC}$
- $\overrightarrow{ED}$
- $\overrightarrow{AD}$
- $\overrightarrow{FE}$
- $\overrightarrow{AF}$



Collinear points

H

Notes and guidance

In this small step, 'collinear' is likely new language and so the meaning of this will need regular emphasis. Students sometimes struggle to give clear reasons as to why points are collinear (or not) and so may benefit from using stem sentences. It's important to emphasise that a complete reason must be given; sometimes students either state that the lines are parallel or that they share a point.

Key vocabulary

Parallel	Common point
Collinear	Same line

Key questions

Explain why parts of the same line are always parallel to each other.
How can you tell which point both lines pass through?
What does the term 'collinear' mean? When giving reasons for points being collinear, why isn't it enough to just show that two straight lines they create are parallel?

Exemplar Questions

Draw a pair of axes with $0 \leq x \leq 12$ and $0 \leq y \leq 22$

- Plot A (1, 5) and B (4, 10) and express $\overrightarrow{AB}$ as a column vector.
- $\overrightarrow{BC} = \begin{pmatrix}6\\10\end{pmatrix}$. Add the point C onto your grid.
- What do you notice about points A, B and C?
- Complete Rosie's reasoning.

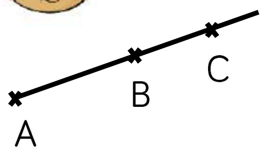
Rosie

$\overrightarrow{BC} = 2\overrightarrow{AB}$ showing that lines BC and AB are ________.
Also, both lines pass through point ___ and so must be part of the ________ line.
If all 3 points are on the same straight line then they are **collinear**.

$\overrightarrow{XY} : \overrightarrow{YZ} = 2 : 3$. If, $\overrightarrow{XY} = 2\mathbf{a} + 4\mathbf{b}$, show that $\overrightarrow{YZ} = 3\mathbf{a} + 6\mathbf{b}$.

Points X, Y and Z are **collinear**. Prove this is correct.

Dora



Dora wants to find C so that points A, B and C are collinear.

- If A (1, 2) and B (7, 4), find $\overrightarrow{AB}$.
- Write down a vector $\overrightarrow{BC}$ which is parallel to $\overrightarrow{AB}$.
- What could the coordinates of point C be?

Geometric argument & proofs H

Notes and guidance

Key command words will need explaining: 'show, justify, prove'. Start by checking confidence in finding vectors for parts of a line segment, given the vector for the whole line segment. Encourage students to draw in extra lines or to extend line segments if necessary. This is an ideal opportunity for goal free questions. Encourage students to write journeys using $\overrightarrow{AB}$ notation first.

Key vocabulary

Show	Justify	Prove
Parallel	Collinear	

Key questions

Can you draw a diagram to represent the situation?
Do you need to add on extra information e.g. points, lines, extend lines?
What do you know? What can you find out?
What do the words 'show, justify and prove' mean?

Exemplar Questions

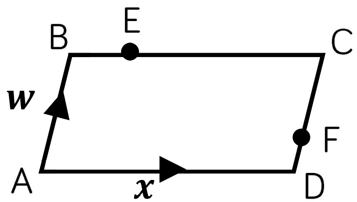
The point M is $\frac{1}{4}$ of the way along XY.
If $\overrightarrow{XY} = 8\boldsymbol{a} - 4\boldsymbol{b}$, work out $\overrightarrow{XM}$, $\overrightarrow{MY}$ and $\overrightarrow{MX}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.
If $\overrightarrow{XY} = \boldsymbol{a} + \boldsymbol{b}$, work out $\overrightarrow{XM}$, $\overrightarrow{MY}$ and $\overrightarrow{MX}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

True or false? If RS : ST = 1 : 2, then RS is $\frac{1}{2}$ of RT

If $\overrightarrow{RT} = 6\boldsymbol{x} + \boldsymbol{w}$, work out $\overrightarrow{RS}$, $\overrightarrow{ST}$, $\overrightarrow{SR}$ in terms of $\boldsymbol{x}$ and $\boldsymbol{w}$.
If $\overrightarrow{RS} = 6\boldsymbol{x} + \boldsymbol{w}$, work out $\overrightarrow{ST}$, $\overrightarrow{RT}$, $\overrightarrow{TR}$ in terms of $\boldsymbol{x}$ and $\boldsymbol{w}$.

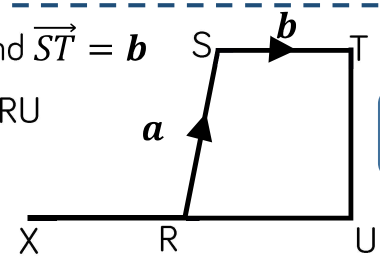
ABCD is a parallelogram.

BE : EC = 1 : 3 and CF = $\frac{3}{4}$ CD.



Express in terms of $\boldsymbol{x}$ and $\boldsymbol{w}$ $\overrightarrow{BD}$ $\overrightarrow{BE}$ $\overrightarrow{AE}$ $\overrightarrow{AF}$
Show that $\overrightarrow{BD}$ is parallel to $\overrightarrow{EF}$

RSTU is a trapezium. $\overrightarrow{RS} = \mathbf{a}$ and $\overrightarrow{ST} = \boldsymbol{b}$
ST is parallel to RU and ST = $\frac{1}{2}$ RU
M is the midpoint of RS
XR : RU = 1 : 2



Not to scale

What do you know? What can you find out? Add this to your diagram.
Prove that points X, M and T are collinear.

Spring 2: Proportions and Proportional Change

Weeks 1 and 2: Ratios and Fractions

This block builds on KS3 work on ratio and fractions, highlighting similarities and differences and links to other areas of mathematics including both algebra and geometry. The focus is on reasoning and understanding notation to support the solution of increasingly complex problems that include information presented in a variety of forms. The bar model is a key tool used to support representing and solving these problems.

National curriculum content covered:

- Consolidating subject content from key stage 3:
  - Use ratio notation, including reduction to simplest form.
  - Divide a given quantity into two parts in a given $part : part$ or $part : whole$ ratio; express the division of a quantity into two parts as a ratio.
  - Relate the language of ratios and the associated calculations to the arithmetic of fractions and to linear functions.
  - Use compound units such as speed, unit pricing and density to solve problems.
- Compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity.
- Apply the concepts of congruence and similarity, including the relationships between lengths, **{areas and volumes}** in similar figures.

Weeks 4 and 5: Percentages and Interest

Although percentages are not specifically mentioned in the KS4 national curriculum, they feature heavily in GCSE papers and this block builds on the understanding gained in KS3. Calculator methods are encouraged throughout and are essential for repeated percentage change/growth and decay problems. Use of financial contexts is central to this block, helping students to maintain familiarity with the vocabulary they are unlikely to use outside school.

National curriculum content covered:

- Consolidating subject content from key stage 3:
  - Interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%.
  - Solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics.
- Set up, solve and interpret the answers in growth and decay problems, including compound interest **{and work with general iterative processes}.**

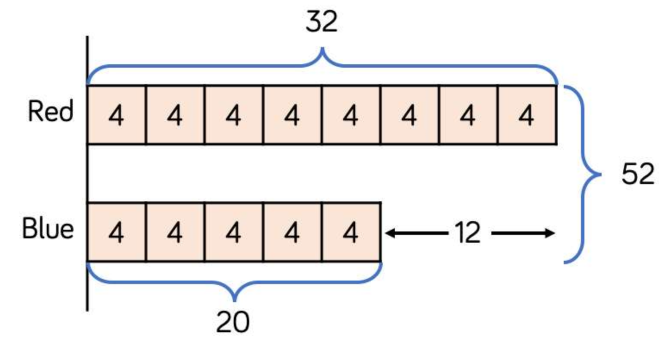
Weeks 5 and 6: Probability

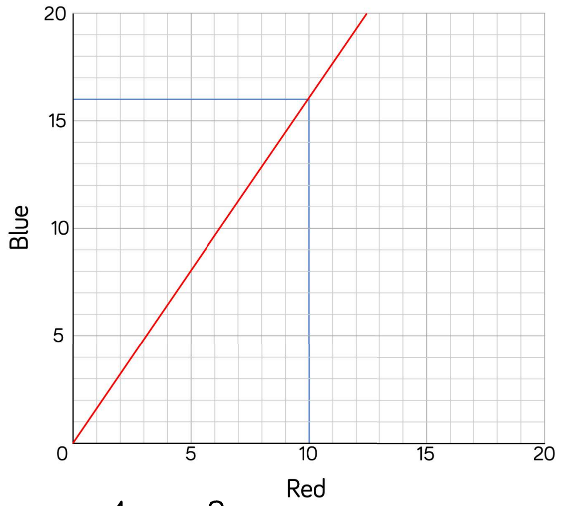
This block also builds on KS3 and provides a good context in which to revisit fraction arithmetic and conversion between fractions, decimals and percentages. Tables and Venn diagrams are revisited and understanding and use of tree diagrams is developed at both tiers, with conditional probability being a key focus for Higher tier students.

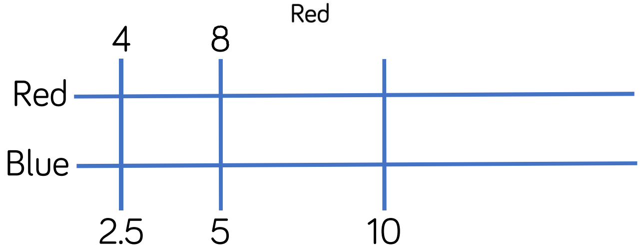
National curriculum content covered:

- Apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one.
- Use a probability model to predict the outcomes of future experiments; understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size.
- Calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions.
- **{Calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams}.**

Key Representations



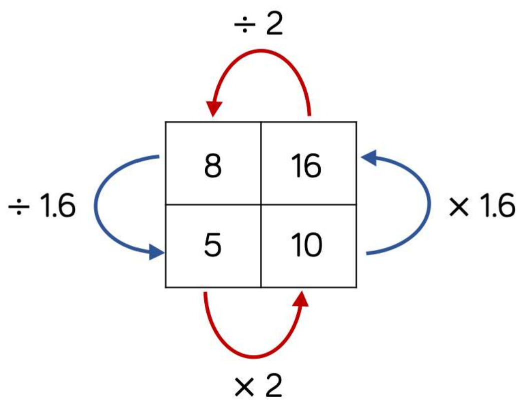




"For every 5 blue, there are 8 red."

$R : B$
$8 : 5$

$8b = 5r$



Red	8	16	4	20
Blue	5	10	2.5	12.5

Pictorial support is essential to support conceptual understanding of ratio and fractions.

The bar model is useful to visually represent ratio problems. They help students to see the equal parts and conceptually understand how to share between equal parts and more complex questions involving comparison.

Double number lines and ratio tables can be helpful tools to show proportionality. They are a structured way for students to represent their mathematical thinking when working through problems and are a consistent tool that can be used when working with proportionate reasoning.

It is important to still keep reinforcing the language of ratio and using this to help aid conceptual understanding.

White Rose Maths

Ratios and Fractions

Small Steps

- Compare quantities using a ratio R
- Link ratios and fractions R
- Share in a ratio (given total or one part) R
- Use ratios and fractions to make comparisons
- Link ratios and graphs R
- Solve problems with currency conversion
- Link ratios and scales R
- Use and interpret ratios of the form $1 : n$ and $n : 1$
- Solve 'best buy' problems
- Combine a set of ratios

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Ratios and Fractions (2)

Small Steps

- Link ratio and algebra
- **Ratio in area problems** H
- **Ratio in volume problems** H
- Mixed ratio problems

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Compare using a ratio

R

Notes and guidance

In this small step, students review expressing information in a ratio. They also encounter questions where the units are not the same and discuss why it is important to use equivalent units in these situations. Contextualising these kinds of questions aids student understanding of why units should be the same when comparing. A recap of unit conversions could be useful here.

Key vocabulary

Ratio	Simplest Form	Convert
Unit	Equivalent	

Key questions

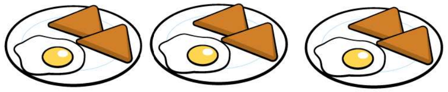
Why do the units need to be the same in order to write a ratio?

Can a ratio compare more than two quantities?

Why is (e.g.) $2 : 1$ different from $1 : 2$?

Exemplar Questions

What is the same and what is different about each of these representations?



$1 : 2$

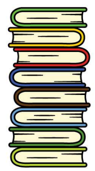
$3 : 6$

There are twice as many pieces of toast as eggs.

Eva takes 30 minutes and Teddy takes one hour to do the same homework.
Eva says the ratio of time taken to do homework is $30 : 1$
Explain why this is incorrect.

A group of children choose their favourite colour.
30% choose red, 35% choose blue, 25% choose green, and the rest choose yellow.
Express the ratio of colour choice red : blue : green : yellow in its simplest form.

There are three piles of books.
Pile 1 has twice as many books as pile 2
Pile 3 has half as many books as pile 2
Find the ratio of books in Pile 1 : Pile 2 : Pile 3



Write the ratios in simplest form.

4 kg : 500 g

25 ml : 2 litres

35 p : £4.20

3 hours : 45 mins

600 mm : 20 cm : 3 m

White Rose Maths

Link ratios and fractions

R

Notes and guidance

When looking at a ratio, it is important for students to look at both the relationships between the parts and the relationships to the whole e.g. in the ratio $a : b = 1 : 3$, a is a $\frac{1}{3}$ of b, b is 3 times the size of a, a is $\frac{1}{4}$ of the whole etc. Pictorial representations help to unpick any misconceptions as fractional relationships are clearly highlighted.

Key vocabulary

Ratio	Simplest Form	Convert
Unit	Equivalent	

Key questions

Why do the units need to be the same in order to write a ratio?

Can a ratio compare more than two quantities?

Why is (e.g.) $2 : 1$ different from $1 : 2$?

Exemplar Questions

A farmer has 20 goats, 30 sheep and 50 cows.
Decide whether each statement is true or false, and explain why.

G	G	S	S	S	C	C	C	C	C

30% of the animals are sheep.

50% of the animals are cows.

$\frac{1}{5}$ of the animals are goats.

The ratio of goats to sheep to cows is 20 : 30 : 50

Other animals	Cows

What other fractions, percentages and ratios can you write down?

Match the statements to the corresponding ratios.
You may use bar models to help you.

b is $\frac{1}{5}$ of a

$a : b = 5 : 1$

$a : b = 1 : 4$

$a : b = 4 : 1$

a is $\frac{1}{5}$ of the whole

a is $\frac{1}{5}$ of b

b is $\frac{1}{5}$ of the whole

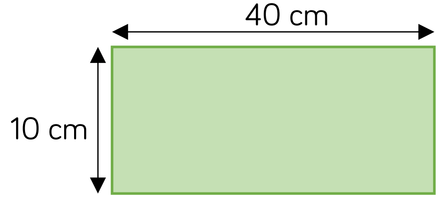
$a : b = 1 : 5$

Explain how each card relates to the rectangle.

$\times 4$

$1 : 4$

$\frac{1}{4}$



Share in a ratio

R

Notes and guidance

Students should be familiar with this step from KS3. Encouraging the use of bar models and emphasising the importance of labelling them helps students to understand the structure of ratio problems, highlighting when the total or when one of the parts is known. This also provides a good opportunity to revisit other topics such as geometry.

Key vocabulary

Ratio	Share	More/less than
Part	Whole	

Key questions

Can you represent this with a bar model? What information can you label on the diagram?

Do you always need to add the numbers of parts first when solving a ratio problem? Why or why not?

Can you tell if the answer is going to be more or less than the value(s) in the question? How?

Exemplar Questions

The ratio of pink to blue beads on a bracelet is 7 : 1
Could there be exactly 28 beads on the bracelet? Explain your answer.

What's the same and what's different about these questions?
Draw a bar model and solve each one.

Rosie, Tommy and Alex share £90 in the ratio $6 : 5 : 4$
How much more money does Rosie get than Alex?

Rosie, Tommy and Alex share some money in the ratio $6 : 5 : 4$
Rosie gets £90
How much money does Tommy get?

Rosie, Tommy and Alex share some money in the ratio $6 : 5 : 4$
Rosie gets £90 more than Alex.
How much money does Tommy get?

- The angles in a triangle are in the ratio 14 : 18 : 13
  Find the size of the largest angle.
- The exterior angles of a triangle are in the ratio 3 : 4 : 5
  Calculate the size of the interior angles of the triangle.

Dora and Amir share some money in the ratio 10 : 9

They shared £x, where x is an integer that satisfies the inequality

$$100 < x < 120$$

How much money did they share? Explain how you know.

Make comparisons

Notes and guidance

Students might need to review comparing fractions before ratios. They should be reminded that there are different ways to compare fractions (e.g. using common numerators or common denominators or decimals). Students should be encouraged to draw bar models, and to write parts of a ratio as a fraction of the whole, to support their comparisons.

Key vocabulary

Proportion	Ratio	Fraction
Convert	Compare	Equivalent

Key questions

If the numerators/denominators of two fractions are the same, how can you identify the greater fraction?

Is it more efficient/easier to use fractions to compare?

How can you decide which is the biggest and which is the smallest proportion?

Exemplar Questions

In each pair, which is the larger fraction? Justify your answer.

$\frac{5}{11}$ or $\frac{7}{11}$ $\frac{3}{5}$ or $\frac{3}{7}$ $\frac{2}{5}$ or $\frac{3}{10}$ $\frac{8}{9}$ or $\frac{7}{8}$

Compare the ratios $3 : 2$ and $3 : 4$

Blue and yellow paint are used to make tins of green paint.

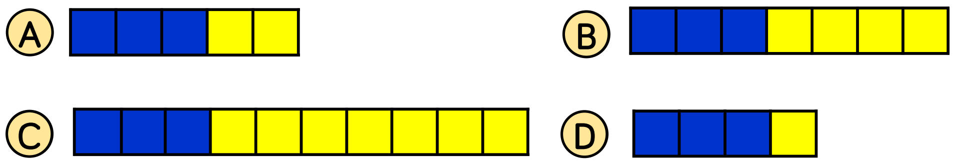
(A) $b : y = 3 : 2$

(B) $b : y = 3 : 4$

(C) $\frac{3}{10}$ of the green paint is blue

(D) $\frac{3}{4}$ of the green paint is blue

The greater the proportion of blue the darker the paint will be. Annie draws bar models to help her decide which tin will be darkest green.



Write down the fraction of blue paint in each tin.
Now put the tins in order from darkest to lightest green.

Eva, Mo and Ron make some drinks using blackcurrant and lemonade.

Eva has 2 parts blackcurrant and 6 parts lemonade.
Mo has 3 parts blackcurrant and 5 parts lemonade.
Ron has 3 parts blackcurrant and 7 parts lemonade.

Whose drink has the strongest blackcurrant flavour?
Justify your answer.

Link ratios and graphs

R

Notes and guidance

This step reviews the idea of direct proportion met at KS3, and how this links to graphical representation. Students can revisit the notion of gradient and see how this links to the ratio of the pairs of values $\frac{y}{x}$. Examples of values that are not in direct proportion are important here, observing that these do not produce a constant ratio.

Key vocabulary

Direct proportion	Ratio	Gradient
Equation	Origin	$y = mx\ (+c)$

Key questions

If two variables are directly proportional, what will the graph look like?

Can a direct proportion graph have a negative gradient?

How can you quickly find the gradient of a straight line that passes through the origin?

Exemplar Questions

1 metre of electrical cable costs £3
The table of values shows the cost, y, in pounds for some values of x metres of cable.

x	1	2	3	4
y	3	6	9	12

Plot the points given by the table of values and join them with a straight line.
What is the equation of the line?
How does this relate to the ratio $x : y$ for each pair of values?

Investigate the ratios and graphs given by these tables of values.

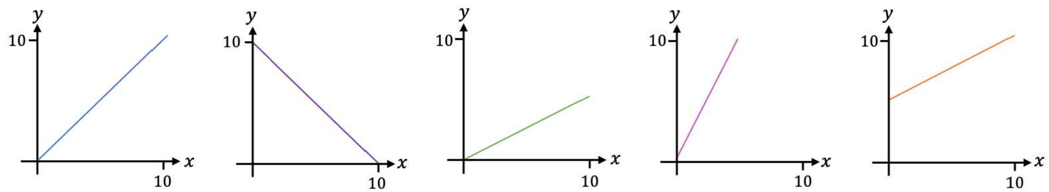
x	5	10	15	20
y	7.5	15	22.5	30

x	2	4	6	8
y	1	2	3	4

x	1	2	3	4
y	3	5	7	9

Describe how one graph and set of ratios is different to the other two.

In which of these line graphs is x **not** directly proportional to y?
Explain how you know.



Currency conversion

Notes and guidance

This small step gives students the opportunity to revisit reading information from graphs and also gives them the opportunity to reinforce their understanding and use of multiplicative reasoning. Double number lines are particularly helpful in aiding students to build up to higher quantities using multiplicative reasoning and to think about how they can use what they know to find other values, linking this to their knowledge of ratio.

Proportion	Convert
Double number line	Exchange rate

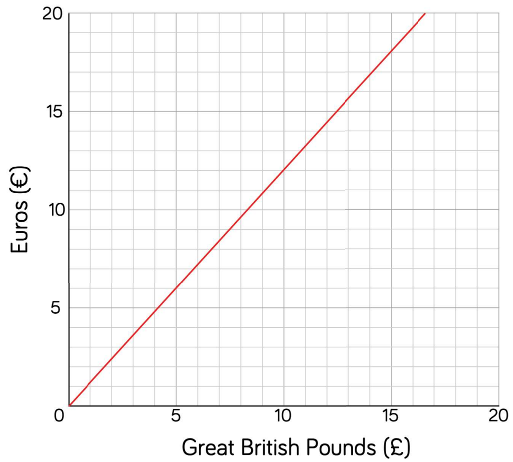
Key questions

How can you find values that cannot be read from the graph?

How can you use what you already know to build up to other values?

Are currency conversion graphs an example of direct proportion? Why/Why not?

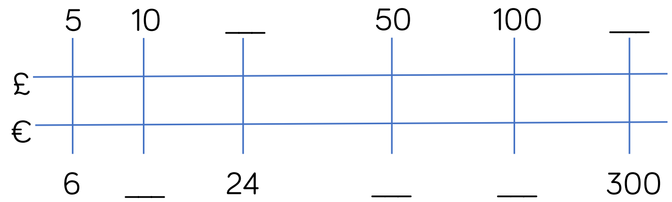
Exemplar Questions



The graph shows the conversion from pounds to euros. Use the graph to complete the sentences:

- For every £10, you get €___
- For every €15, you get £___

Complete the double number line. What connections can you see?



The exchange rate for pounds to Mexican Peso is £1 = 25 Pesos

- How many Pesos can you buy for £200?
- How many Pounds can you buy for 200 Pesos?
- Which is greater in values, £75 or 1850 Pesos?

The exchange rate for pounds to Canadian dollars is £1 = $1.70
Dora wants to buy a new tablet.
In which country is the tablet best value for money?

UK price: £299

Canada price: $525

Link ratios and scales

R

Notes and guidance

Students may need reminding about unit conversions as a precursor to this step. It is good practice to use full size maps rather than just extracts normally seen in examination and textbook questions. Using applications like Google maps to extend students' experience of different scales may also be useful. This is also a good opportunity to revisit/reinforce drawing and reading bearings.

Key vocabulary

Ratio	Scale	Map
Represent	Bearing	

Key questions

How many cm are there in a m/km?

How do you know whether to divide or multiply when doing calculations involving scales?

Why do maps have different scales?

Exemplar Questions

Match up each scale with the corresponding ratio.

1 cm represents 1 km	$1 : 100$
1 cm represents 1 m	$1 : 10\ 000$
1 cm represents 100 m	$1 : 100\ 000$

Maps A and B cover the same area.
Which map has more detail? How do you know?

Map A	Map B
Scale = $1 : 1000$	Scale = $1 : 100\ 000$

On a street map of a town, 2 cm represents 140 metres.

- Express the scale of the map a ratio is its simplest form
- Find the actual distance between two points that are 30 cm apart on the map.
- The actual distance between the town hall and the park is 595 metres. How far apart will they be on the map?

Dora is standing 600 metres away from Tom.
Her bearing from Tom is 125°
Jack is standing 400 metres away from Dora.
His bearing from Dora is 195°
Draw a diagram with a scale of 1 : 10 000 to show the positions of the three children.

Ratios of the form $1 : n$ and $n : 1$

Notes and guidance

Students sometimes find this tricky as answers do not always conform to the usual simplifying of ratios where both parts are integers. Students may need some guidance on deciding which has the highest proportion or whether a criteria is met and using stem sentences, such as 'for every 1 red, there are _____ green' can be a helpful way for students to interpret the information a bit more easily.

Key vocabulary

Ratio

For every … , there are …

Integer

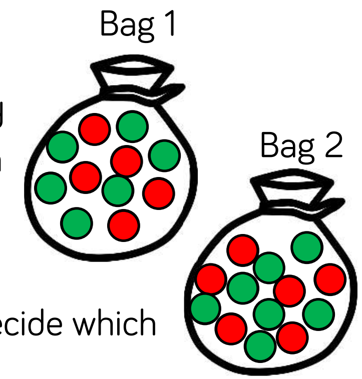
Non-integer

Key questions

How does getting the ratio into the form $1 : n$ help you to compare ratios?

What is different about reducing a ratio to the from $1 : n$ as opposed to the form $n : 1$? How do you know what to divide by?

Exemplar Questions



Alex has two bags of counters with some red and green counters. He wants to know which has the highest proportion of red counters.

- Find the ratio of red : green in each bag
- Write each ratio in the form $1 : n$ and decide which bag has the highest proportion of red

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A school is planning three school trips.
There must be at least 1 teacher for every 12 students.
Which of the trips can go ahead?

	Trip to university	Trip to museum	Trip to adventure park
Number of students	38	126	274
Number of teachers	4	10	23

Write the ratios in the form $1 : n$ and the form $n : 1$
Where necessary, give n to 3 significant figures.

40 g : 1 kg

£5 : 80 p

12 hours : 1 week

Length of the side of a square : perimeter of the square

Solve ‘best buy’ problems

Notes and guidance

In this small step, students compare prices to find best value. Students will have different methods for comparing and it is useful to share these as a group. Thinking in terms of efficiency and discussing these with students can be a powerful way to show alternative methods others may not have considered. Use of double number lines or ratio tables can be useful for structuring mathematical thinking.

Key vocabulary

Compare	Proportion	Best value
Unit cost		

Key questions

Is it the largest or smallest number that tells you which is the best value for money?

What is the difference between ‘cost per item’ and ‘number of items per £/p’?

Why might factors or multiples be useful in this problem?

Exemplar Questions

Circle which one would be the best buy for each item:

1 pen for 45p or 3 pens for £1.20

4 litres of juice for £1.80 or 3 litres of juice for £1.50

2 kg of carrots for £1.28 or 7 kg of carrots for £4.20

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9 chocolates cost £2.25

Box A

20 chocolates cost £4.00

Box B

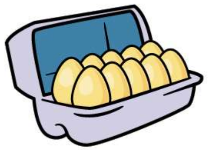
24 chocolates cost £4.30

Box C

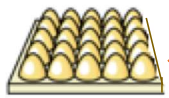
What does each of the calculations tell you?

$2.25 \div 9 = \square$	$20 \div 4 = \square$	$24 \div 4.30 = \square$
$4.30 \div 24 = \square$	$4 \div 20 = \square$	$9 \div 2.25 = \square$

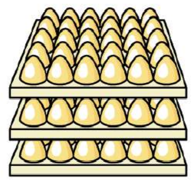
Use your answers to put the boxes in order of best value for money to worst value for money.
Check your answer by calculating the cost of 360 chocolates for each size of box.



10 eggs £1.60



25 eggs £3.70



90 eggs £13.50

Find three ways to work out which box of eggs is best value.

Combine a set of ratios

Notes and guidance

In order to combine ratios, students need to be secure in finding the lowest common multiple and in working with equivalent ratios. Pictorial methods are very helpful here and students could draw the objects (as in the sweets example), or use bar models to represent the number of parts. "Scaling up" the ratios until a common multiple is found is another very useful strategy.

Key vocabulary

Ratio	Equivalent	Combine
For every … there are …		LCM

Key questions

Why are equivalent ratios useful in this question?

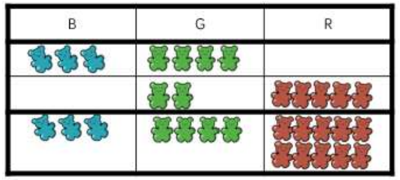
Are the parts already equivalent or do you need to use an equivalent ratio to make them equal?

How could you draw a bar model to help you?

Exemplar Questions

The ratio of the number of blue to green sweets is 3 : 4
The ratio of the number of green to red sweets is 2 : 5
Dora finds the ratio of blue sweets to green sweets to red sweets using multiples.

B : G = 3 : 4
G : R = 2 : 5 = 4 : 10
Now there are 4 red sweets in both ratios so,
B : G : R = 3 : 4 : 10



Use Dora's method to solve this problem.

The ratio of the number of cats to dogs in a pet shop is 2 : 5
The ratio of the number of dogs to rabbits in the shop is 3 : 10
Find the ratio of Cats : Dogs : Rabbits.

The ratio of strawberry muffins to chocolate muffins is 2 : 3
The ratio of strawberry muffins to blueberry muffins is 6 : 5
What is the ratio of strawberry to chocolate to blueberry muffins?
Jack says that there are 96 muffins altogether. Is this possible?

The ratio of the number of pens to pencils in my pencil case is 5 : 2
There are three times as many pens as rubbers.

- Write the ratio of the number of pens to pencils to rubbers.
- There are 30 pencils in the case. I pick one object at random from the case . What is the probability it is a pencil?

White Rose Maths

Link ratios to algebra

Notes and guidance

This step explores both the use of algebraic notation within ratios and the linking of ratio questions to problems that need to be tackled through e.g. forming and solving equations. If the ratios $a : b$ and $c : d$ are equal then the key concept that $\frac{a}{b} = \frac{c}{d}$ is often useful to solve complex looking problems.

Key vocabulary

Variable	Unknown	Equation
Equivalent	Express	

Key questions

Express a in terms of b if (e.g.) $a : b = 2 : 3$
How is this different from expressing b in terms of a?

Can you draw a bar model to represent the ratio? Is it more useful to draw a single bar or a comparison bar?

Exemplar Questions

The ratio $a : b$ is equal to $4 : 3$
Explain which of the statements are true and which are false.

$a < b$ | $\frac{a}{b} = \frac{4}{3}$ | $b < a$ | $a = \frac{4}{3}b$ | $a = \frac{3}{4}b$

Create your own true/false puzzle if $a : b$ is equal to $2 : 1$

$x : y$ is equal to $5 : 3$
Work out x and y if
- $x + y = 240$
- $x - y = 240$

---

b is 50% larger than a
Write $a : b$ in simplest form.
Write $a : b$ in the form $1 : n$

$a : b = 1 : 3$ and $b : c = 4 : 5$
Find the ratio $a : b : c$

Tom is twice as old as Kim.
Nijah is 10 years older than Tom.
The total age of all 3 people is 60 years.
Find the ratio of
Tom's age : Kim's age : Nijah's age

Amir and Mo share sweets in the ratio $5 : 3$
Amir gives Mo 5 of his sweets and the ratio is now $9 : 7$
Complete the solution to find how many sweets they shared.

At first, Amir has $5x$ sweets and Mo has $3x$ sweets
Then Amir has $5x - 5$ sweets and Mo has ☐ sweets
So $\frac{5x-5}{\square} = \frac{9}{7}$
$7(5x - 5) = 9(\square)$
etc.

Dora thinks you can solve the problem using multiples. Investigate Dora's claim.

Ratio in area problems

H

Notes and guidance

Students have explored the effect of enlargement on the areas of similar shapes in the Autumn term, looking at squaring scale factors. This is an opportunity to revisit this learning using ratio notation alongside that of scale factors. It can also be an opportunity to revisit area problems and those that involve Pythagoras' theorem and trigonometry.

Key vocabulary

Enlarge	Length/Area scale factor
Length/Area Ratio	Similar

Key questions

How can we use the ratio of the areas of two similar shapes to find the scale factor of their areas?

If we know the ratio of the areas of two shapes, how can we find the ratio of the lengths of their sides?

Exemplar Questions

Sketch a rectangle with dimensions 6 cm by 8 cm.
Enlarge the rectangle so the ratio of the side lengths of the new rectangle to the side lengths of the original rectangle are $3:2$

- Find the ratios of the areas of your rectangles
- Repeat by enlarging the original rectangle by the ratio $5:2$
- Generalise your findings

Triangle B is made by enlarging triangle A by a scale factor $\frac{1}{3}$

Which of the statements are true and which are false?

Perimeter A : Perimeter B $= 1:3$	Perimeter A : Perimeter B $= 3:1$

Area A : Area B $= 9:1$	Area A : Area B $= 1:6$	Area A : Area B $= 6:1$

Triangle C is an enlargement of triangle B by scale factor 4
Find the ratios:

Area B : Area C	Area C : Area A	Area A : Area C

The ratio of the surface area of solid X to the surface area of solid Y is $4:9$
The total length of the edges of solid Y is 180 cm.
Find the total length of the edges of solid Y

Ratio in volume problems

H

Notes and guidance

As with the previous step, students have explored the effect of enlargement on the volumes of similar shapes in the Autumn term, looking at cubing scale factors. This is an opportunity to revisit this learning using ratio notation alongside that of scale factors. It can also be an opportunity to revisit the use of volume formulae or the use of trigonometry in 3-D shapes.

Key vocabulary

Enlarge	Length/Volume scale factor
Length/Volume Ratio	Similar

Key questions

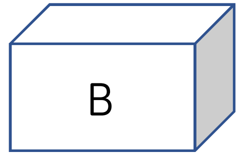
If you know the ratio of one volume to another volume and that they are similar solids, how can you use this ratio to work out missing lengths or areas?

How can you find the ratio of the volumes of two shapes if you only know their surface areas?

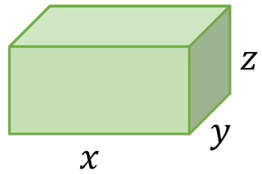
Exemplar Questions

The dimensions of cuboid A are 1 cm, 2 cm and 3 cm.
Find the volume of cuboid A.

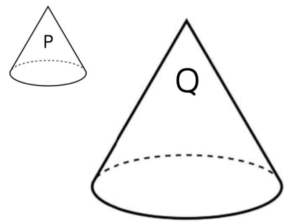
The ratio of the lengths of the sides of cuboid A to those of cuboid B is $1 : 8$
What are the dimensions of cuboid B?
What is the volume of cuboid B?
Write down the ratio Volume A : Volume B.



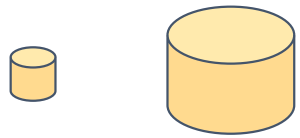
The ratio of $x : y : z = 5 : 3 : 2$
The volume of the cuboid is 240 cm^3
Find the side lengths x, y ad z.
State the volume of a cuboid of with sides $\frac{x}{2}, \frac{y}{2}$ and $\frac{z}{2}$



Cone P is similar to cone Q
The height of cone P is 4 cm.
The height of cone Q is 6 cm.
The volume of cone Q is 135 cm^3
Work out the volume of cone P.



The volumes of two similar cylinders are in the ratio $8 : 27$
If the surface area of the smaller cylinder is 40 cm^2, what is the surface area of the larger cylinder?



Mixed ratio problems

Notes and guidance

It is very useful for students to be able to reflect on a variety of topics covered rather than just see them discretely, so the purpose of this step is to provide opportunities to look again at various aspects of this unit to reinforce understanding. Teachers may use this to focus in on any areas of particular difficulty or to explore ratios in other topics that may need revision.

Key vocabulary

Enlarge	Scale factor	Ratio
Share	Similar	

Key questions

If two shapes are similar, what do we know about the ratios of the side lengths?

How could a bar model represent this problem?

Exemplar Questions

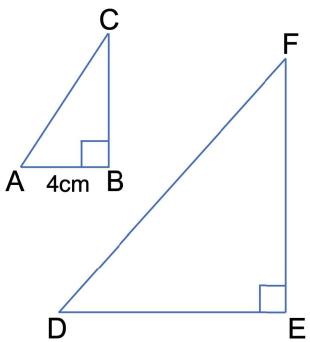
80 students study either French or Spanish.
There are 52 girls altogether.
12 of the boys study French.
Of the students studying Spanish, the ratio of boys to girls is $2 : 3$
Draw and complete a two-way table showing this information.
Find the ratio of the number of students studying French to the number of students studying Spanish.

The ratio of the angles in a triangle is $3 : 4 : 5$
Show that the triangle does not contain a right angle.

The angles in a quadrilateral are a, b, c and d.

$a = 90°$ $a : b = 3 : 5$ $c : d = 2 : 3$

Work out the sizes of angles a, b, c and d.



Triangle ABC and DEF are right-angled triangles.

AB : DE $= 1 : 3$
BC : EF $= 1 : 2$
AB : BC $= 2 : 3$

- Find the area of triangle DEF.
- Find the length AC.
- Explain why ABC and DEF are not similar triangles.

Percentages & Interest

Small Steps

- Convert and compare fractions, decimals and percentages (R)
- Work out percentages of amounts (with and without a calculator) (R)
- Increase and decrease by a given percentage (R)
- Express one number as a percentage of another (R)
- Calculate simple and compound interest
- Repeated percentage change
- Find the original value after a percentage change (R)
- Solve problems involving growth and decay

(H) Denotes Higher Tier GCSE content

(R) Denotes 'review step' – content should have been covered at KS3

Percentages & Interest

Small Steps

- **Understand iterative processes** H
- Solve problems involving percentages, ratios and fractions

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

White Rose Maths

Convert and compare FDP

R

Notes and guidance

Students will be very familiar with these conversions and the amount of time spent on this review step will be dependent on your assessment of your students' needs e.g. you may just include in starter activities or within the teaching of the other steps. It is well worth reminding students how to perform conversions on their calculators as well as through mental and written methods.

Key vocabulary

Fraction	Decimal	Percentage
Equivalent	Convert	

Key questions

Which are the most commonly used percentages? What fractions are they equivalent to?

How can you convert any decimal to a fraction?

Exemplar Questions

Match up the cards that are of equal value.
Write extra cards so that each set contains a fraction, decimal and percentage of equal value.

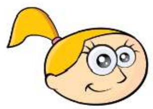
45%	0.02	0.25	$\frac{9}{20}$	$\frac{1}{50}$	$\frac{1}{4}$
20%	32%	0.32			

How can $\frac{7}{40}$ be changed to a decimal using a calculator?

Put these cards in order of size, starting with the smallest.

$\frac{1}{3}$	0.3	28%	0.33	34%	$\frac{13}{40}$	0.289

What mistake has Eva made?



$\frac{3}{7} = 3 \div 7 = 0.42 = 42\%$

$0.38 = \frac{38}{100} = \frac{19}{50}$

$0.685 = \frac{685}{1000} = \frac{137}{200}$

Use this method to convert the decimals to fractions, simplifying your answers.

- 0.16
- 0.016
- 0.0016
- 0.925
- 0.08
- 0.78
- 0.204
- 0.555

Find percentages of amounts

R

Notes and guidance

Students need to be familiar with the use of calculator as well as mental and written methods, linking multipliers to the decimals discussed in the last step. It is also worth looking at multiple methods for a series of calculations to help students decide which methods are most appropriate in a situation. Finding percentages greater than 100% is a useful lead in to reviewing percentage increase in the next step.

Key vocabulary

Fraction	Decimal	Percentage
Equivalent	Convert	Multiplier

Key questions

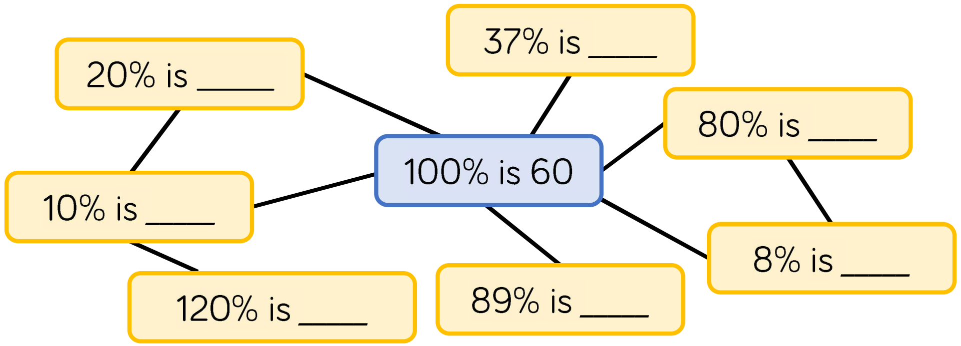
Which percentages are best worked out mentally? Give me an example of a percentage you would work out using a calculator.

How do you work out 10% of a number? How do work out 1% of a number? How are these connected?

Exemplar Questions

Dani is working out 40% of £30 000
Which of these methods will give the correct answer?
Explain why or why not.

$0.4 \times 30\,000$	$30\,000 \div 10 \times 4$	$30\,000 \div 4 \times 10$
$30\,000 \div 5 \times 2$	$30\,000 \div 100 \times 40$	$\frac{40}{100} \times 30\,000$



Complete the spider diagram. Which calculations are best done mentally, and which with a calculator?

Show that 60% of 40 $= \frac{2}{3}$ of 36 $= 0.05 \times 480$

- Is it true that 45% of 80 is the same as 80% of 45?
- Which is greater 34% of 200 or 20% of 350? By how much?
- Find three ways to work out 85% of 90

Increase/decrease by a %age

R

Notes and guidance

This review step can also be used to explore different methods and compare their efficacy. Some students get confused when reducing by a given percentage and use the wrong multiplier; the use of estimation is a good strategy here. Confidence with using multipliers is essential for the following steps so it is worth exploring changes of e.g. 3% or 2.7% to avoid over-reliance on mental "build-up" methods.

Key vocabulary

Increase	Decrease	Reduce
Interest	Convert	Multiplier

Key questions

How do you find the multiplier to increase/decrease by __%? How is this different from finding out __% of the number?

What words in a question might mean you need to increase the quantity? What works indicate a decrease?

Exemplar Questions

Esther has £20 000 and she invests £12 000 of this in a bank.
After a year, her investment grows by 5%.
Which of these give the value of her investment after a year?

12 000 × 1.5 | 12 000 + (12 000 ÷ 100 × 5) | 12 000 × 0.05 + 12 000 | 12 000 × 1.05

She buys a car with the remaining £8 000
After a year, the value of the car decreases by 15%.
Which of these give the value of her investment after a year?

8 000 – 0.85 × 8 000 | 8 000 – 0.15 × 8 000 | 0.85 × 8 000 | 8 000 ÷ 100 × 85 | 8 000 × 0.15

In a shop where Nijah works, the cost of a phone is the list price plus VAT at 20%. The list price of a phone is £480
Work out the cost of the phone.
Nijah gets an 18% staff discount on anything she buys from the shop.
How much does it cost Nijah to buy the phone?

Which multiplier increases a number by 3.5%?

× 1.35 | × 3.5 | × 1.035 | × 0.35

Brett earns £36 000 a year.
Calculate his monthly wage after a 3.5 % pay rise.

Express as a percentage

R

Notes and guidance

Students are sometimes challenged when asked to express something as a percentage, rather than the more regular finding of a percentage. A good strategy is to visit this step little and often (perhaps in starters) and to mix the questions students are set rather than treating them discretely. Encouraging students to express as a fraction first and then considering how to convert is also useful.

Key vocabulary

Fraction	Decimal	Percentage
Numerator	Denominator	Express

Key questions

How can I convert any fraction to a percentage using a calculator? If I don't have a calculator, what denominators are useful for converting fractions to percentages?

How can I find a relevant fraction in this question? How can I identify the numerator and denominator?

Exemplar Questions

Convert these test scores into percentages without using a calculator.

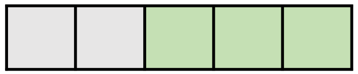
The teacher realises there were only 38 marks on the test, not 40
Use a calculator to work out the new percentage scores, giving your answers to two significant figures.

In a metal bar, the ratio of aluminium to other metals is 2 : 3

- What percentage of the bar is aluminium?

The bar contains 12 kg of aluminium.

- What is the total weight of the bar?



A TV advert for cat food claims that "8 out of 10 cat owners prefer to buy our brand"
In a survey of 72 cat owners, 59 said they preferred to buy the brand.
Use percentages to decide whether you think the claim is true.

	Red	Green	Total
Squares	7	9	16
Circles	18	4	22
Total	25	13	38

The table shows the type and colour of shapes in a game.

- What percentage of the circles are green?
- What percentage of the green shapes are circles?
- What percentage of the shapes are red circles?

Simple and compound interest

Notes and guidance

A useful strategy for helping students to distinguish and remember the difference between simple and compound interest is to compare them alongside each other rather than just looking at them independently. The strategy for compound interest is identical to that of all repeated percentage changes and so will be revisited in many of the upcoming steps.

Key vocabulary

Simple	Compound	Interest
Repeated	Power/Index/Exponent	

Key questions

What is the difference between simple and compound interest? Which one is most common in real life?

What is a quick way of writing (e.g.) 1.07 × 1.07 × 1.07? What buttons do you press on your calculator?

What is a sensible degree of accuracy to use in interest questions? Why?

Exemplar Questions

Alex invests £2 000 at 3% simple interest.

- How much interest will she earn in three years?
- What will her investment be worth after five years?

Fill in the missing numbers.

$6^4 = 6 \times \square$

$1.05 \times 1.05 \times 1.05 = 1.05^{\square}$

$800 \times 1.2 \times 1.2 \times 1.2 \times 1.2 \times 1.2 = 800 \times \square^{\square}$

Annie invests £3 000 at 2% compound interest.
Compare these different ways of calculating the value of her investment after 3 years.

£3 121.20 × 1.02^3
= £3 183.62

£3 000 × 1.02 = £3 060
£3 060 × 1.02 = £3 121.20
£3 121.20 × 1.02 = £3 183.62

How much interest has she earned?
Explain why this is not the same as 6% of £3 000

Find the difference between the interest earned on an £8 000 investment at 4% if the interest is simple or compound.

Tommy is working out formulae for the amount of interest, £I, that £P earns at r% over n years. Are his formulae correct?

Simple Interest

$$I = \frac{Pnr}{100}$$

Compound Interest

$$I = P\left(1 + \frac{r}{100}\right)^n$$

Repeated percentage change

Notes and guidance

This builds on the previous step, generalising the method for compound interest to any repeated percentage change situation, including repeated reduction. Students may not be aware of the term "depreciation". It is worth considering cases of e.g. an increase of x% followed by a decrease of x% and showing that this does not return to the original value. This is also a good preparation for the next step.

Key vocabulary

Compound	Repeated	Change
Depreciate	Power/Index/Exponent	

Key questions

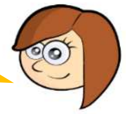
Why is it that increasing a quantity by (e.g.) 10% twice in a row is not the same as increasing it the quantity by 20%?

What is the overall effect of increasing a number by a percentage and then decreasing it by the same percentage? Why don't we get back to the original number?

Exemplar Questions

A shop reduces prices by 20% and then by a further 10%

Prices have now been reduced by 30%



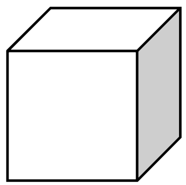
Use calculations to show that Rosie is wrong.

A car costs £15 000 new and loses 18% of its value every year.
Which calculation shows the value of the car in 5 years' time?

$15\,000 \times 1.18^5$ | $15\,000 \times 0.18^5$ | $15\,000 \times 0.82^5$

There are 800 000 bacteria in a jar.
The number of bacteria is increasing at a rate of 20% every hour.
How many bacteria will there be in two hours' time?
How many bacteria will there be in one day's time?
Give your answer in standard form and to 3 significant figures.

The population of an island is 62 000
It is predicted that the population is decreasing at a rate of 2% a year.
What will the population of the island be in 20 years' time?
Comment on the accuracy of your answer.



The lengths of the sides of a cube are increased by 20%.
Find the percentage increase in its surface area
Find the percentage increase in its volume

Find the original value

R

Notes and guidance

The exemplar questions show bar models as a way of accessing these problems. Although this will have been covered in KS3, it is worth revisiting as students often make errors such as taking the required percentage off the final value. It is worth looking at multiple methods such as finding 10% or 1% from the given value or using equations of the form "Original × Multiplier = Final Value".

Key vocabulary

Fraction	Decimal	Percentage
Reverse	Original	Multiplier

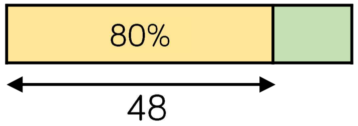
Key questions

If we know (e.g.) 40% of a number, what else can we find?

If an amount is the result of a (e.g.) 20% decrease/increase, what percentage do we know? What other percentages can we find out easily? What percentage is the original value? How can we find this?

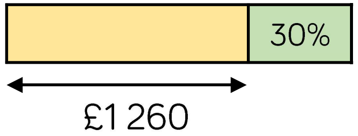
Exemplar Questions

In a test, Whitney answered 80% of the questions correctly.
She answered 48 of the questions correctly.



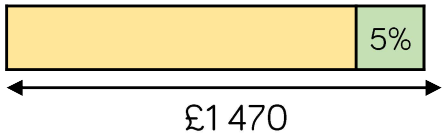
Work out the total number of questions on the test.

In a sale, the price of a TV set is reduced by 30% to £1 260
What percentage of the original price is the sale price?



Work out the original price of the TV.

After a 5% pay rise, Huan earns £1 470 a month.



How much did Huan earn before the pay rise?

Aisha invests £6 000 for 3 years in a saving account.
She gets x% compound interest each year.
Aisha has £6 749.18 at the end of 3 years.
Work out the value of x

Growth and decay

Notes and guidance

This step builds on repeated percentage change, again looking at a variety of contexts. There are no new techniques but students may need to be directed to the links with compound interest and depreciation using the vocabulary of "growth" and "decay". Higher tier students could also consider "working backwards", finding the original value after repeated percentage changes, combining the last two steps.

Key vocabulary

Growth	Decay	Multiplier
Repeated	Compound	

Key questions

If you reduced a number by 50% twice a row, why is the answer not 0?

How can you tell from a question whether the multiplier should be more or less than 1?

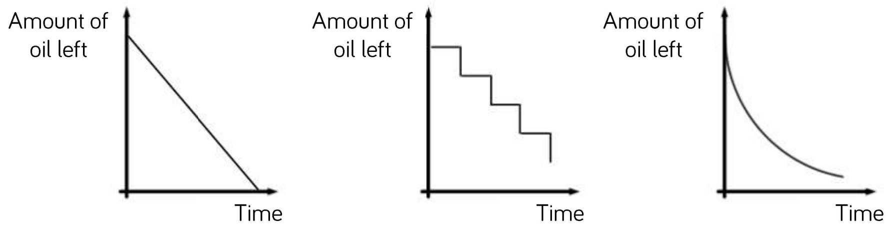
Exemplar Questions

Tom and Dani start work on a salary of £20 000

Tom	***Dani***
4% pay rise every year	£900 pay rise every year

Who will be earning more in five years' time?
Who will be earning more in ten years' time?

There is a hole in an oil tank.
Every hour, 10% of the oil that is left in the tank leaks out.
Which graph shows this?



Describe a situation the other graphs might show.

There are only 500 of a rare species of bird left in the world.
The number of birds is expected to reduce by 10% a year.
If a preservation order is introduced, the number of birds is expected to increase by 5% each year.
Find the difference between the number of birds in 4 years' time depending on whether the preservation order is introduced.

Iterative processes

H

Notes and guidance

Iterative methods for solving equations are covered in Year 11, but if time allows this represents a good opportunity to introduce the notation in the context of repeated change, and also links to the vocabulary of sequences. The differences between linear and geometric sequences and the alternative forms of the rules for the n^{th} term or formalised term-to-term rules can also be explored.

Key vocabulary

Repeat	Iterate	Subscript
$u_n, u_{n+1},$	Term	Geometric

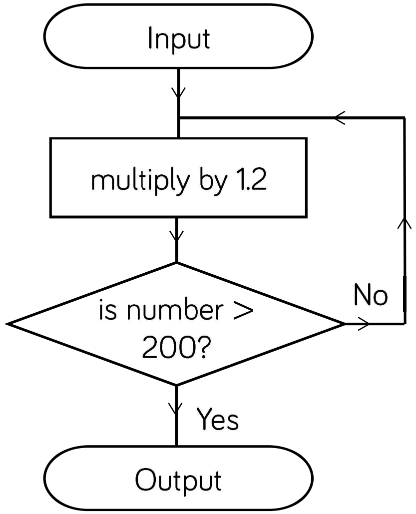
Key questions

What does u_n mean?

Given u_1 and a rule, how many times do I need to iterate in order to find the value of (e.g.) u_5?

Given (e.g.) u_5 and a rule, can I work backwards to find u_1?

Exemplar Questions



- If 100 is input into the flowchart, how many times do you go round the flowchart before getting an answer over 200?
- Investigate the number of iterations for different inputs and different multipliers m if $1 < m < 2$
- Investigate $0 < m < 1$? Will the output ever be 0?

A sequence is given by the rule:

$$u_1 = 10, u_{n+1} = u_n + 6$$

Work out u_2, u_3, u_4 and u_5

- Describe the rule in words.
- Find the rule for the n^{th} term of the sequence.

Repeat for the rule $u_1 = 10, u_{n+1} = 2u_n$

Kim says the rule $u_1 = 1, u_{n+1} = 3u_n$ gives the same sequence as the n^{th} term rule 3^{n-1}

Is she right? Justify your answer.

Problems with FDP and ratio

Notes and guidance

This step provides a nice link with the previous block of learning and can be used to explore examination-style questions that feature a combination of FDP as well as ratio. Bar models and tables are key ways to represent problems to enable students to access the questions which may at first appear overwhelming; teacher modelling of extracting/organising information is extremely helpful.

Key vocabulary

Fraction

Decimal

Percentage

Ratio

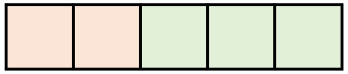
Key questions

What information do you already know? What other information can we work out? How does this help us to solve the given problem?

Is the ratio (e.g.) 2 : 3 the same as the fraction $\frac{2}{3}$? Why or why not?

Exemplar Questions

In a school, the ratio of boys to girls is 2 : 3
What percentage of the students are girls?
10% of the boys wear glasses.
What fraction of the students in the school are boys who wear glasses?



	City	Seaside	Total
Boys			
Girls			
Total			60

A group of 60 children get to choose a school trip.
$\frac{7}{12}$ of the children pick a city trip and the rest pick a seaside trip.
19 boys pick the city trip and 20% of the girls choose the seaside.
Find the ratio of boys to girls in the group.

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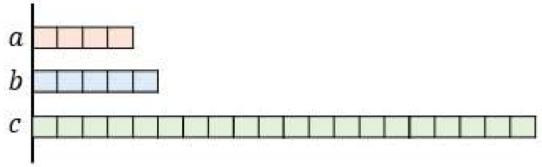
a, b and c are integers.
$a : b = 4 : 5$ and b is 25% of c
Show that a is one-fifth of c



If $a + b + c = 435$, work out the values of a, b and c

Eva, Jack and Rosie share £620
The ratio of Eva's amount to Jack's amount is 8 : 5
The ratio of Jack's amount to Rosie's amount is 2 : 1
How much does Eva get?
What percentage of all the money is Eva's share?

Probability

Small Steps

- Know how to add, subtract and multiply fractions R
- Find probabilities using equally likely outcomes R
- Use the property that probabilities sum to 1 R
- Using experimental data to estimate probabilities
- Find probabilities from tables, Venn diagrams and frequency trees
- Construct and interpret sample spaces for more than one event R
- Calculate probability with independent events
- Use tree diagrams for independent events

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Probability

Small Steps

- Use tree diagrams for dependent events
- **Construct and interpret conditional probabilities (Tree diagrams)** H
- **Construct and interpret conditional probabilities (Venn diagrams and two-way tables)** H

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Add, subtract and multiply fractions (R)

Notes and guidance

Students need a conceptual understanding of adding, subtracting and multiplying fractions before exploring probability. Returning to pictorial representations may be necessary. There is then an opportunity to interleave many previously taught topics such as order of operations, area of polygons, volume, similar shapes, sequences (linear and geometric) and algebra.

Key vocabulary

Numerator	Denominator	Exact value
LCM	Simplest form	

Key questions

Why do we ensure fractions have a common denominator before adding or subtracting?

How do we multiply together two fractions? Explain why this procedure works.

What is meant by 'exact value'?

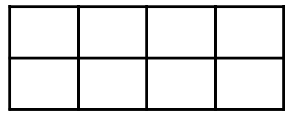
Exemplar Questions

Here is part of Dexter's homework:

$$\frac{1}{5} + \frac{1}{15} = \frac{15}{75} + \frac{5}{75} = \frac{20}{75} = \frac{4}{15}$$

Write down a more efficient method to calculate $\frac{1}{5} + \frac{1}{15}$

Use the diagram to justify why $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$



Which of the following is larger:

$\frac{1}{2} \times \frac{3}{4}$ $\quad$ $\frac{8}{9} \times \frac{15}{32}$

Show your working and then explain your answer.

The following is a linear sequence: $\frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \square, \square, \frac{9}{10}$

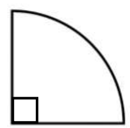
Calculate the missing terms in the sequence.

Simplify the following:

$(x^{\frac{2}{7}})^{\frac{3}{8}}$ $\quad$ $g^{-\frac{1}{3}} \times g^{\frac{5}{6}}$ $\quad$ $h^{\frac{3}{4}} \div h^{-\frac{1}{2}}$ $\quad$ $\dfrac{d^{\frac{4}{5}} \times d^{\frac{2}{3}}}{d^{\frac{1}{5}}}$

The radius of a circle is $\frac{3}{10}$ cm.

Find the area and circumference a quadrant of this circle, leaving your answers in terms of π.



Equally likely outcomes

R

Notes and guidance

This step supports students to become conceptually fluent in using equally likely outcomes to find probabilities.
Misconceptions should be highlighted here, particularly considering factors such as 'size' of spinner and whether this impacts on probability of outcomes. Reminding students that they can write probability as a fraction, decimal or percentage is also useful.

Key vocabulary

Equally likely	Outcome	Event
Denominator	Numerator	

Key questions

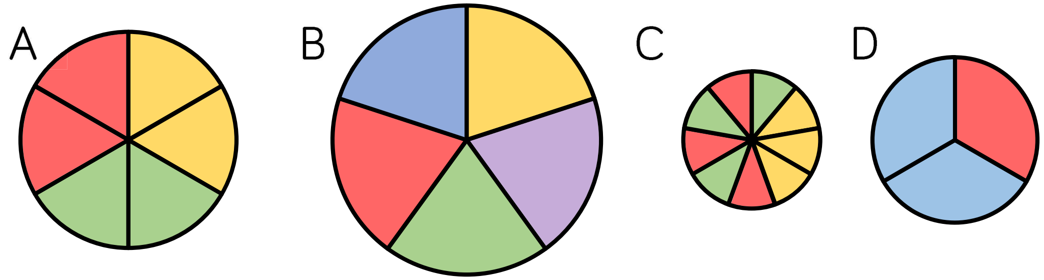
What makes events equally likely to occur?

If it (e.g.) might rain, or might not, are these events equally likely?

How can we use the fact that events are equally likely to find probabilities?

Exemplar Questions

Which of these spinners have equally likely outcomes?



Which spinners have an equal chance of landing on any of the colours?

What's the probability of landing on red when using
- Spinner B?
- Spinner C?

Tickets with the numbers 1 to 9 are put into a bag. One ticket is randomly selected from the bag.

- Write down the probability that the number on the ticket is even.

Decide if the probability of selecting an even number

A: Stays the same B: Increases C: Decreases

when,

- 5 more even and 4 more odd numbers are added to the bag
- 8 more even and 10 more odd numbers are added to the bag
- 1 odd and 1 even number are removed from the bag

Justify your answers.

Probabilities sum to 1

R

Notes and guidance

Students use the fact that probabilities sum to 1 (or 100%) to calculate missing probabilities. Students should have opportunities to work with percentages, fractions and decimals when finding probabilities. This step is also an opportunity to revisit Venn diagrams, set notation and forming/solving linear equations

Key vocabulary

Event	Complement	Venn diagram
Intersect	Union	

Key questions

What types of number can we use to represent probabilities? Can we use a ratio? Why or why not?

How do we know that for these events probabilities must add up to 1? Why can't they add up to more/less than 1?

What does the word 'complement/union/intersect' mean? Where is this represented on the Venn diagram?

Exemplar Questions

Only one of these statements is true.
Which one is it? Explain why the other statements are false.

It's a million percent certain that I'll sleep this week

In a bag, there are only red, blue and yellow marbles. The probability of getting a red is 0.15, a blue is 0.45, and a yellow is 0.35

In a bag of chocolates, 2 are dark and 8 are milk. The probability of getting a dark chocolate is $\frac{1}{5}$, and a milk chocolate is $\frac{4}{5}$.

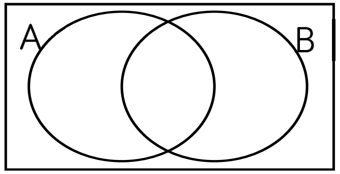
A large number of people are asked their favourite of five sports. The table shows the probabilities that certain sports are picked. Given that it is twice as likely that someone chose hockey rather than running, complete the table.

Sport	Netball	Swimming	Running	Football	Hockey
Probability	0.3	0.05		0.35	

Red, blue, yellow and green balls are in a bag in the ratio $7 : 2 : 1 : x$
The probability of selecting a red ball is 50%.
Find the value of x

If there are 42 balls in the bag, how many of each colour are there?

$P(A \cup B) = 0.6$
Find the probability that neither A nor B occur.
Given that $P(A) = 0.3$ and $P(A \cap B) = 0.2$, complete the Venn diagram.



Experimental data

Notes and guidance

Students consider why experimental and theoretical probability are different. They learn that the more trials completed, the closer experimental probability is likely to be to the theoretical probability. They consider how they can use relative frequency to predict future events, by calculating expected values. Students could be supported to find experimental probabilities from a variety of sources.

Key vocabulary

Relative frequency	Estimate
Expectation	Expected value

Key questions

Why is experimental probability different from theoretical probability?

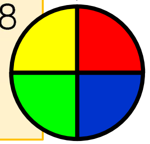
What happens to experimental and theoretical probability when a very large number of trials have been completed?

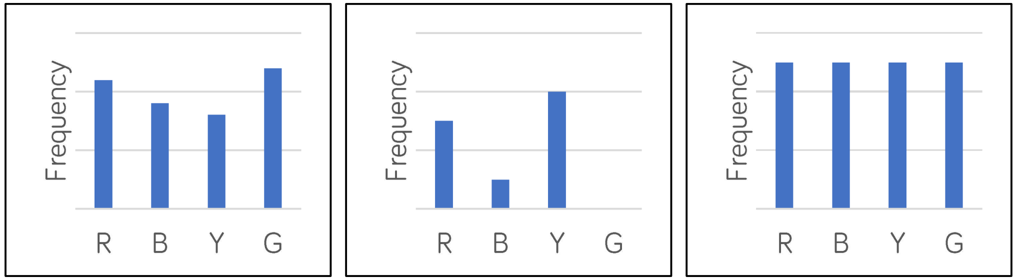
Is the 'expected value' the exact number of times you would expect an event to occur?

Exemplar Questions

Mo uses a computer simulation of a spinner.
The bar charts show the frequency of each colour after 8 spins, 40 spins and 10 000 spins. Match each graph to the number of spins, explaining your reasoning.

Spinner





Rosie and Eva are flipping a coin.

Rosie says: My coin must be biased

	Heads	Tails	Total
Rosie	3	7	10
Eva	53	47	100
Total	56	54	110

Explain why she is incorrect.

Teddy estimates the probability of getting a head.

P(Head) = 0.3 P(Head) = 53% P(Head) = $\frac{56}{110}$

Explain how he got each estimate. Which is most accurate? Why?

How many times is a head expected if the coin is flipped 220 times?

Dora notes down how many times a netball team wins, loses and draws over 60 games:

Win	Lose	Draw
31	x	y

If the expected number of losses in 900 games is 330, find x and y.

Tables, Venn diagrams, frequency trees

Notes and guidance

This is an opportunity for students to revise key ways of representing information. When working with Venn diagrams, students might need reminding that P(A) includes P($A \cap B$). Higher students should also consider more advanced notation such as P($A \cap B'$). When working with two-way tables, students might need support in choosing the correct cell value for the denominator.

Key vocabulary

Two-way tables
Venn diagrams
Frequency trees
Universal set

Key questions

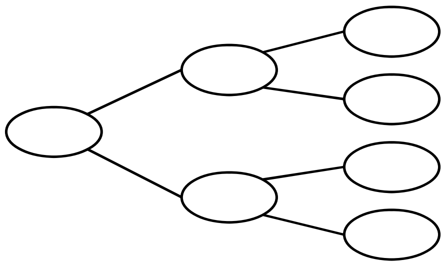
How do we know which cell value is the denominator when calculating a probability from a two-way table?

Where do you start when filling in a Venn diagram?

What's the same and what's different about frequency trees and two-way tables?

Exemplar Questions

A local council believe that girls are more likely to walk to school than boys. They conduct a survey of 800 children to find out.

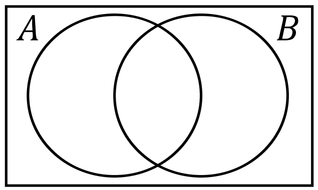


48% of the children are girls

Girls 'walk to school' or 'don't walk' to school in the ratio 7 : 5

104 boys 'don't walk' to school. The rest of the boys 'walk to school'.

Copy and complete the frequency tree. Is the council right that girls are more likely to walk to school than boys? Justify your answer.



ε = { integers from 1 to 12}

A = {even numbers}   B = {multiples of 3}

Complete the Venn diagram and then find:

a) P(A)   b) P(B)   c) P(A')

d) P(B')   e) P($A \cup B$)   f) P($A \cap B$)

If 60 is added to the universal set, how does this affect your answers?

The table shows sandwich choices made a group of children. Find:

	Mustard	Chutney	Total
Ham	28	22	50
Cheese	10	49	59
Total	38	71	109

a) P(Cheese and chutney)
b) P(Chutney)
c) P(Ham)

Sample spaces

R

Notes and guidance

Students construct sample spaces from a variety of sources so that they know whether a list, or grid is most efficient. Discuss how to be systematic and the different ways of being systematic. A misconception is to add the total number of possible outcomes from each event and use this as the denominator when calculating probabilities (e.g. thinking the total number of possible outcomes when rolling two dice must be 12).

Key vocabulary

Sample Space	Systematic	Outcome
Event	Array	

Key questions

How can we present the outcomes?

What does 'systematic' mean?

How can we be systematic? Is there more than one way?

How many outcomes will there be in total? How do we know?

Exemplar Questions

BREAD:	FILLING:	TOPPING:
White	Cheese	Chutney
Brown	Ham	Mayo
	Chicken	Mustard

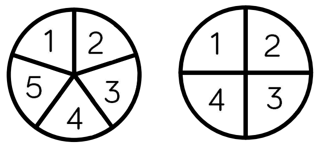
A sandwich shop owner needs to work out how many different choices his customers have.

Bread	Filling	Topping
Whi	Che	Chu
Bro	Che	Chu
Whi	Che	May
Bro	Che	May

He systematically begins to write a list. Explain how he is being systematic. What is the benefit of this?

Complete his list.

In a game, the outcome is the product of the score on Spinners A and B.



Spinner A Spinner B

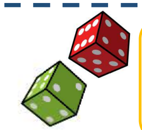
Complete the sample space

Product	1	2	3	4
1				
2		4	6	
3				
4	4			
5				20

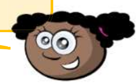
Show that P(product is an even number) $\neq 0.5$

Find P(product is a prime number) .

How many times would you expect a prime number in 100 spins?



There are 6 possible outcomes on each dice, and that makes 12 overall. Therefore P (total score is 12) $= \frac{1}{12}$



Use a sample space to show that Whitney is wrong.

What is the probability of getting a total score of 12?

Independent events

Notes and guidance

Before working with tree diagrams, students need to understand that for independent events, P(A and B) = P(A) × P(B). They also need to be clear that the outcome of one event has no bearing on the outcome of the other. This can be demonstrated using sample spaces. Examples and non-examples of independent events supports understanding of this term.

Key vocabulary

Independent events	Tree diagrams
Outcomes	Product

Key questions

Give an example of a pair of independent events. Give an example of a pair of events that aren't independent.

Do you add or multiply to find the probability of two independent events both happening?

Exemplar Questions

Which of the following events are independent?

Rolling a 6 on a dice and then rolling another 6

Rolling a 6 on a dice and getting a head when flipping a coin

Rolling a 6 on a dice and it raining tomorrow

Selecting a red sweet at random from a bag of sweets, eating it, and then selecting another red sweet.

Getting a tail on a coin happens half the time.
So, P(Tail and a 1 on a dice) = half of $\frac{1}{6}$
P(Tail and a 1 on a dice) = $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

Use a sample space to convince your partner that this is true.

Complete the statement by adding in the correct operation:

For independent events, P(A and B) = P(A) ◯ P(B)

The probability that Dora is late for school is 0.1
The probability that Ron is late for school is 0.2
The probability that Eva is late for school is 0.15
The events that any of the students are late are independent.
Find the probability that

- Dora and Ron are both late for school
- Ron and Eva are both late for school
- All three students are late for school

Tree diagrams for independent events

Notes and guidance

Sample spaces alongside the tree diagram can provide a helpful transitionary step. Initially scaffolding by providing students with the tree diagrams to enable all to access this concept. Teachers might include tree diagrams where there are more than two outcomes in each trial. Students may need support in identifying 'pathways' and what final outcome each shows.

Key vocabulary

Independent events	Tree diagrams
Outcomes	At least one

Key questions

Explain why getting (e.g. a red and a blue) means getting (e.g. colour of the counters) in any order.

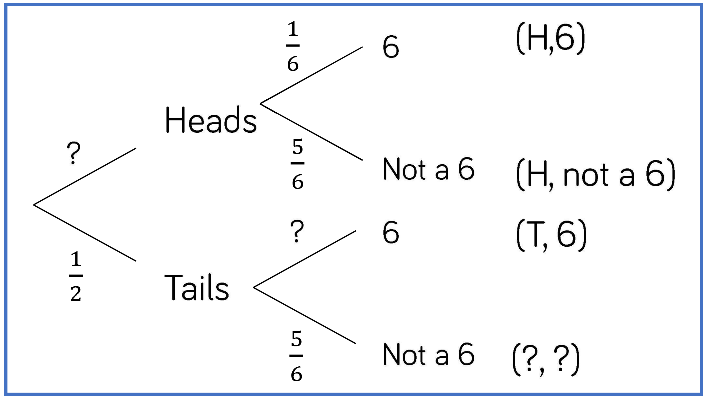
What are the different methods for finding the probability of 'at least one'? Which is the most efficient?

How do you draw a tree diagram if the first event has three possible outcomes and the second event has two?

Exemplar Questions

Mo flips a coin and rolls a dice. He is working out P(Head and a 6).
The tree diagram shows possible outcomes.

Complete the tree diagram.



Work out the probabilities of all four possible outcomes.

A bag contains 3 blue and 2 red counters. A counter is randomly selected and replaced. A second counter is then randomly selected.
Draw a tree diagram to show this.
Alex and Tommy calculate P(red and a blue counter)

Alex: $P(R \text{ and } B) = \frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

Tommy: $P(R \text{ and } B) \text{ or } P(B \text{ and } R) = \left(\frac{2}{5} \times \frac{3}{5}\right) + \left(\frac{3}{5} \times \frac{2}{5}\right) = \frac{6}{25} + \frac{6}{25} = \frac{12}{25}$

Tommy is correct. What mistake has Alex made?
Alex and Tommy now calculate P (at least one red)

Will they both get the same answer?

Alex: P(at least one red) = 1 – P (no reds) = 1 – P(B,B)

Tommy: P(at least one red) = P(R,R) + P(R,B) + P(B,R)

Tree diagrams for dependent events

Notes and guidance

Prior to this small step, it is useful to generate examples of dependent events with students, to ensure that they understand what these are. Again, scaffolding by providing incomplete information on a tree diagram or in a method provides a starting point. Working with probability in percentages, decimals and fractions and then discussing which is easer to calculate with can also be helpful.

Key vocabulary

Dependent events

Independent events

Tree diagram

Key questions

Give me an example of two events which are dependent.

Is it easier to calculate mentally with decimals, fractions or percentages?

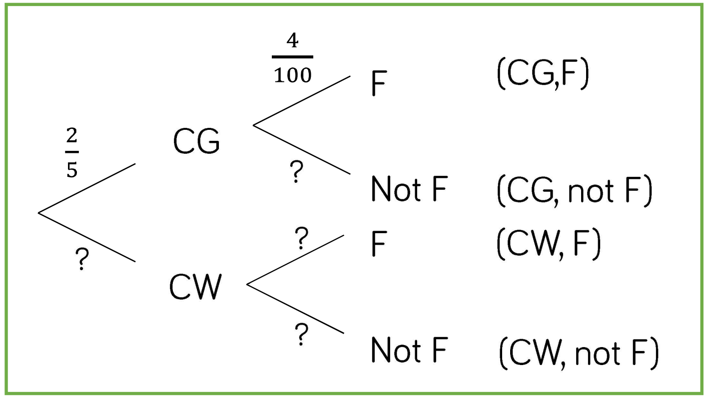
How do you know which "branch" or "branches" to use in a tree diagram?

Exemplar Questions

A garage orders two-fifths of its car parts from 'Car Giant' (CG) and the remainder from 'Car World' (CW).
4% of the parts from 'Car Giant' are faulty and 3% are faulty from 'Car World'.

Complete the tree diagram:

Write down the question that each of these calculations answer.



$$\left(\frac{2}{5}\times\frac{4}{100}\right)+\left(\frac{3}{5}\times\frac{3}{100}\right)$$

$$1-\left\{\left(\frac{2}{5}\times\frac{4}{100}\right)+\left(\frac{3}{5}\times\frac{3}{100}\right)\right\}$$

In order to test the best revision technique in a group of students:

- 40% are randomly selected to revise on their own (O)
- 30% to revise in a group (G)
- 30% to revise with a teacher (T)

All students then took the same test, which they could either pass (P) or fail (F). The table shows the results:

	O	G	T
Pass	0.7	0.5	0.8

Construct a tree diagram to show all possible outcomes.

Teddy estimates P(Fail test) $\approx \frac{1}{3}$ Is he right? Justify your answer.

Conditional (Tree diagrams) H

Notes and guidance

Teachers should introduce this using concrete or pictorial resources, so that students can see how the probabilities in the second trial are affected by the first. Students will practise constructing tree diagrams and assigning probabilities. This should include situations where there are more than two possible outcomes for each trial. Students should also practise expressing probabilities algebraically, and manipulating these.

Key vocabulary

Conditional probability	Given
Show	Outcomes

Key questions

What does 'given' mean? Which part of the tree diagram, does this refer to?

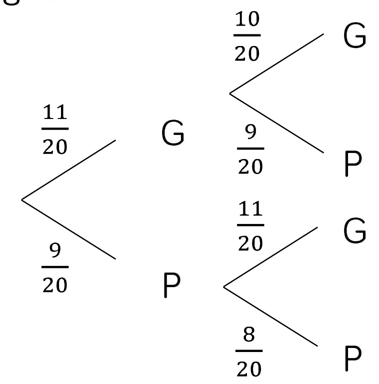
Why do the probabilities change between trials? How do they change?

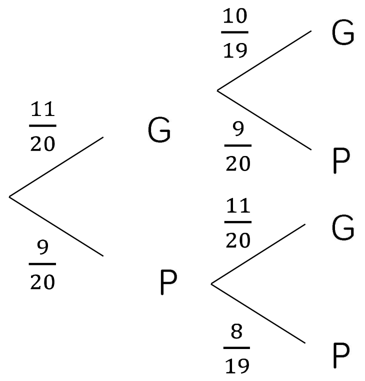
How can you work out the probabilities associated with the second trial if algebra is involved?

Exemplar Questions

There are 20 sweets in a bag.

11 of the sweets are green. The rest are purple.
Jack randomly takes a sweet and eats it. He repeats this a second time.
Both tree diagrams contain errors. Identify these and draw a correct tree diagram.





There are n socks in a drawer.
5 of the socks are red. The rest of the socks are black.
Dexter takes a sock at random from the drawer and puts it on his foot.
He then takes at random another sock from the drawer and puts it on his other foot.

The probability he is wearing two red socks is $\frac{1}{12}$

- Show that $n^2 - n - 240 = 0$
- Solve $n^2 - n - 240 = 0$ to find the value of n
- Calculate the probability that Dexter wears a matching pair of socks.

Conditional (Other)

H

Notes and guidance

The key concept is understanding that the term ‘given’ means that only one set of outcomes are relevant when selecting the event. Students need to identify which set of outcomes they are selecting from. They should be confident in using Venn diagrams and two-way tables to find conditional probabilities. Conditional probability applies to both dependent and independent events.

Key vocabulary

Conditional probability	Given	Intersection
Outcomes	Set	Venn

Key questions

Why do we use the term conditional probability? (e.g. on the condition that…)

What does ‘given’ mean? Which part of the Venn diagram/two-way table does this refer to?

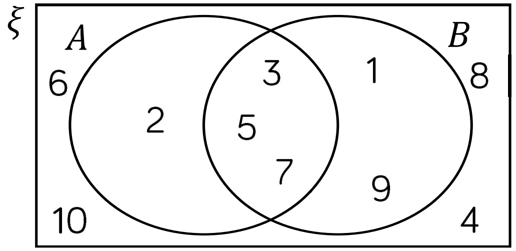
Exemplar Questions

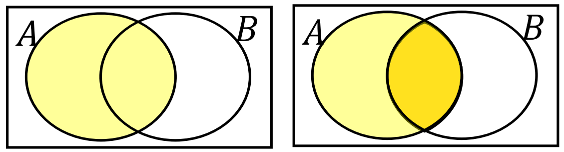
The Venn diagram shows:

ξ = {integers from 1 to 10}

A = {prime numbers}

B = {odd numbers}





How do the two Venn diagrams help to show the probability of a number being odd, given it’s prime? Calculate this probability.

Use a similar approach to show that:

The probability of a number being prime, given that it’s odd is $\frac{3}{5}$

---

60 children get to choose a school trip.

The ratio of boys to girls is 2 : 3

$\frac{7}{12}$ of the children pick a city trip, the rest pick a seaside trip.

19 boys picked the city trip.

- Construct a two-way table to show this information.

Copy and complete:

- P(given the child is a ____, they chose a _____ trip) $= \frac{19}{24}$
- P(given the child chose a_____ trip, they are a_____) $= \frac{16}{35}$

Work out

- P(the child is a boy, given they chose a seaside trip)
- P(the child chose a city trip, given they are a girl)