Summer Term

Year 10

#MathsEveryoneCan

White Rose Maths

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	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Similarity**						**Developing Algebra**					
	Congruence, similarity and enlargement			Trigonometry			Representing solutions of equations and inequalities			Simultaneous equations		
Spring	**Geometry**						**Proportions and Proportional Change**					
	Angles & bearings		Working with circles		Vectors		Ratios & fractions		Percentages and Interest		Probability	
Summer	**Delving into data**						**Using number**					
	Collecting, representing and interpreting data						Non-calculator methods		Types of number and sequences		Indices and Roots	

Summer 1: Delving into Data

This block builds on KS3 work on the collection, representation and use of summary statistics to describe data. Much of the content is familiar, both from previous study within and beyond mathematics (including Geography and Science) and from everyday life. The steps have been chosen to balance consolidation of existing knowledge with extending and deepening, particularly in terms of interpretation of results and evaluating and criticising statistical methods and diagrams. For students following Higher tier, there is additional content relating to continuous data including histograms, cumulative frequency diagrams, box plots and associated measures such as quartiles and the interquartile range. Again the emphasis with these topics should be on interpretation (particularly in making comparisons) and not just construction. A possible approach to teaching this unit would be project-based, where students collect primary data (or select samples from secondary data) from which they make and test hypotheses, thus giving a purpose to the creation and analysis of the diagrams and measures involved. Looking at data from other subject areas might again be useful here.

National curriculum content covered:

- consolidating subject content from key stage 3:
  - ➢ use describe, interpret and compare observed distributions of a single variable through: appropriate graphical representation involving discrete, continuous and grouped data
  - ➢ construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data
  - ➢ describe, interpret and compare observed distributions of a single variable through: appropriate graphical representation involving discrete, continuous and grouped data; and appropriate measures of central tendency (mean, mode, median) and spread (range, consideration of outliers)
- infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
- interpret and construct tables and line graphs for time series data
- **{construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use}**
- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate graphical representation involving discrete, continuous and grouped data, **{including box plots}**
- apply statistics to describe a population
- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (including modal class) and spread **{including quartiles and inter-quartile range}**

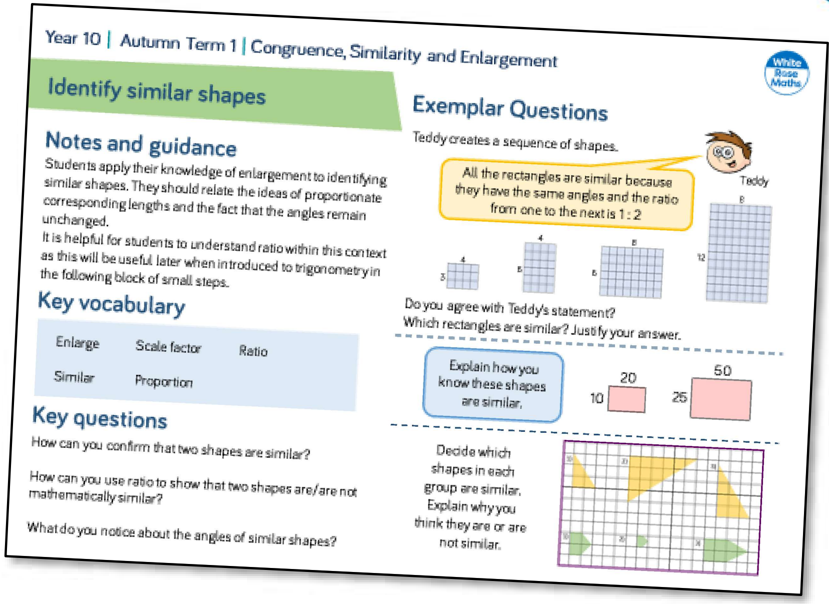
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.

It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge, Scale factor, Ratio, Similar, Proportion

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

All the rectangles are similar because they have the same angles and the ratio from one to the next is 1 : 2

Teddy

Do you agree with Teddy's statement? Which rectangles are similar? Justify your answer.

Explain how you know these shapes are similar.

Decide which shapes in each group are similar. Explain why you think they are or are not similar.

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

Delving into data

Small Steps

- Understand populations and samples
- **Construct a stratified sample** (H)
- Primary and secondary data
- Construct and interpret frequency tables and frequency polygons
- Construct and interpret two-way tables (R)
- Construct and interpret line and bar charts (including composite bar charts)
- Construct and interpret pie charts (R)
- Criticise charts and graphs

(H) denotes Higher Tier GCSE content

(R) denotes 'review step' – content should have been covered at KS3

Delving into data

Small Steps

- **Construct histograms** H
- **Interpret histograms** H
- Find and interpret averages from a list R
- Find and interpret averages from a table R
- Construct and interpret time series graphs R
- Construct and interpret stem-and-leaf diagrams
- **Construct and interpret cumulative frequency diagrams** H
- **Use cumulative frequency diagrams to find measures** H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Delving into data

Small Steps

- **Construct and interpret box plots** (H)
- Compare distributions using charts and measures
- **Compare distributions using complex charts and measures** (H)
- Construct and interpret scatter graphs (R)
- Draw and use a line of best fit (R)
- Understand extrapolation

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Population and samples

Notes and guidance

Students need to be aware that the 'population' is the whole group being studied rather than, say, the population of a city or country. They should also discuss the merits of random sampling. There is often confusion caused by the colloquial use of the word 'random' to mean haphazard or unexpected, rather than the statistical meaning that each member of the population has an equal chance of being selected.

Key vocabulary

Population	Sample	Representative
Biased	Random	

Key questions

Why do statisticians take samples rather than interview the whole population?

How could the random number generator on your calculator be used to support selecting a random sample?

Exemplar Questions

Jack wants to pick a sample of 5 students from his class to complete a survey. Which of these methods will produce a random sample?

Picking the first five names he thinks of	Picking his five closest friends in the class	Putting all the names in a hat and picking out five

---

Rosie wants to find out if people in her town are happy with the library service. She asks all of the people who live on her street their opinions.

- Explain why Rosie's survey is not reliable
- Suggest a way to improve Rosie's survey

There are about 80 000 trees in a forest.
A researcher wants to test 1% of the trees for a disease.
Compare these methods of choosing the sample.

Number all the trees and pick 800 numbers at random	Picking the first 800 at the entrance to the forest	Split the forest into 20 regions and pick 40 trees from each region

---

Alex surveys 5% of the students at her school about a proposed change to the school uniform. She talks to 25 students in Year 10 and 15 students in Year 11

- What is meant by the population in this case?
- What is the size of the population?
- Is Alex's sample likely to be representative? Why or why not?

Stratified samples

H

Notes and guidance

Rather than trying to learn a formula, it is helpful if students approach stratified sampling using proportional reasoning, finding the fraction each group/stratum is of the whole population and assigning the same fraction of the total sample size to each group. It would also be useful to discuss whether stratified samples are necessarily representative e.g. would it be useful to split each group into male/female? etc.

Key vocabulary

Population	Sample	Representative
Biased	Proportion	Stratified

Key questions

What fraction of the whole population is the sample size? How can you work out this fraction of each group/stratum?

How can we ensure the stratified sample is random?

Exemplar Questions

The table shows the number of students in each year group in a school. Brett is conducting a survey and decides to interview 10 students from each year group.

Criticise Brett's sampling method and suggest an improvement.

Year	No. of students
7	150
8	180
9	170
10	110
11	90

There are three age groups in a running club.
The table shows the number of people in each group.

Age group	12 - 14	15 - 17	18+
No of members	156	336	108

Dexter wants to survey a sample of the members.
He wants to survey 50 people altogether.
Work out the number of people he should ask from each age group.

House	Merlin	Potter	Gandalf	Glinda
No of students	92	165	83	217

The table shows the number of students in each of the four houses of a school.
In a stratified sample, 7 students are selected from Gandalf house.
How many students should be selected from each of the other houses?

Primary and secondary data

Notes and guidance

This step is probably best covered when discussing sampling, which is the most common way of gathering primary data. It is useful to discuss the pros and cons of each type of data e.g. secondary is much cheaper, but may not be as reliable. The internet is a great source of secondary data which could be useful to exploit to generate the charts and diagrams in the forthcoming steps.

Key vocabulary

Primary	Secondary	Source
Data	Questionnaire	Experiment

Key questions

Why might some secondary sources of data be biased?

Give me an example of a biased question in a questionnaire. How might you improve it?

What columns do you need on a data collection sheet?

Exemplar Questions

Would it be more sensible to use primary or secondary data to investigate these hypotheses/situations? Explain your answers.

The population is generally taller now than they were in the 1970s	Which is the wettest month of the year?
Do taller athletes perform better at the long jump?	A company wants to research the market for a new flavour of crisps
The more you revise, the better your results.	The speed at which you type text messages declines with age.

If you chose primary data, how might you go about collecting this?

Boys have faster reaction times than girls.

Jack wants to collect primary data to test his hypothesis. Suggest how he may do this.

Eva is collecting data about television and reading habits. Suggest how her questions could be improved.

- Do you watch a lot of television?
- How much do you spend on books?

£1 - £10 ☐ £11 - £20 ☐ £20 - £30 ☐

Frequency tables and polygons

Notes and guidance

Students are familiar with frequency tables for grouped data from KS3, and may recall the idea of the midpoint as used to find the estimate of the mean. Teachers could include grouped frequency diagrams here, linking to the later Higher tier topic of histograms, and explore the similarities and differences between these and frequency polygons.

Key vocabulary

Frequency polygon	Midpoint	Endpoint
Frequency	Class	Interval

Key questions

What's the difference between a midpoint and an endpoint?

Given the endpoints of a class interval, how do you work out the midpoint?

How do you choose the scales for your frequency polygon?

Exemplar Questions

The frequency polygon shows the ages of the employees at a firm.

- How many of the employees are aged less than 45?
- Why does the horizontal axis start at 30?
- Can you use the frequency polygon to work out the age of the oldest employee at the firm?

The table shows the mass in grams of 70 oranges.

Weight (w)	Frequency
$40 < w \leq 50$	7
$50 < w \leq 60$	23
$60 < w \leq 70$	26
$70 < w \leq 80$	12
$80 < w \leq 90$	2

- Draw a frequency polygon to show this information.
- What is the probability that a randomly selected orange weighs over 60 grams?

On the same axis, draw another frequency polygon to show the mass of these 70 apples.

Weight (w)	$50 < w \leq 60$	$60 < w \leq 70$	$70 < w \leq 80$	$80 < w \leq 90$
Frequency	12	26	20	12

What do you notice?

Two-way tables

R

Notes and guidance

Students have worked with two-way tables throughout KS3, so this review step is an opportunity to revisit both extracting and completing information as well as designing tables, looking at more complex tables if appropriate. There are ample opportunities to link to other areas of the curriculum that need revising, including fractions, decimals, percentages, ratios and probability.

Key vocabulary

Table	Row	Column
Total	Difference	

Key questions

How do you decide which categories to use for the rows and columns of the two-way table? Would it make a difference if the rows and columns were swapped?

What ratios can you find out from the two-way table? What probabilities can you find?

Exemplar Questions

The table shows the ages of students in a class.

- Complete the table.
- What percentage of the whole class is girls?
- A student is picked at random from the class. What is the probability the student is a 15 year old boy?

	Age 14	Age 15	Total
Girls	7	12	
Boys		8	
Total			32

The table shows the number of people who correctly identified crisp flavours in a blind taste test.

	Correct	Incorrect	Total
Cheese	25		40
Salt & Vinegar		12	30
Beef			27

Two-thirds of the people correctly identified Beef.
Complete the table.
What proportion correctly identified each of the other two flavours?

---

40 children take part in a school show.
The ratio of boys to girls is 3 : 5
$\frac{3}{5}$ of the children are dancers and the rest are singers.
There are 3 boys who are dancers in the show.

- Represent this information is a two-way table.
- How many more girls are dancers than singers?

Line and bar charts

Notes and guidance

Students will be very familiar with constructing and interpreting bar charts, and should experience them in a variety of forms – vertical, horizontal, lines instead of bars etc. They should also explore multiple and composite bar charts, as in the two exemplar questions, focusing on interpretation and what types of information it is easier to read from one type than the other.

Key vocabulary

Line/Bar chart	Frequency	Difference
Dual/Multiple	Composite	Total

Key questions

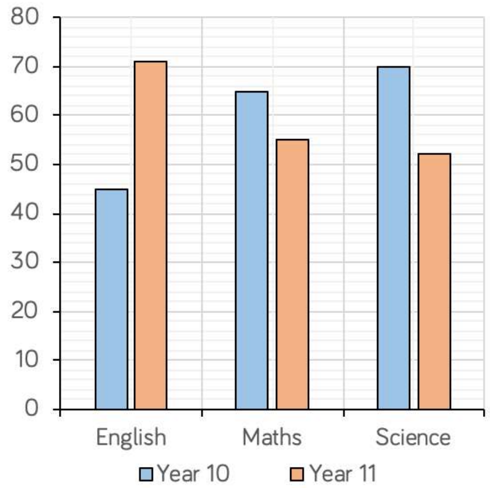
What's the difference between a vertical and a horizontal bar chart?

How thick does a bar have to be? Can you use a line?

What's the difference between a multiple bar chart and a composite bar chart? Which is easier to read?

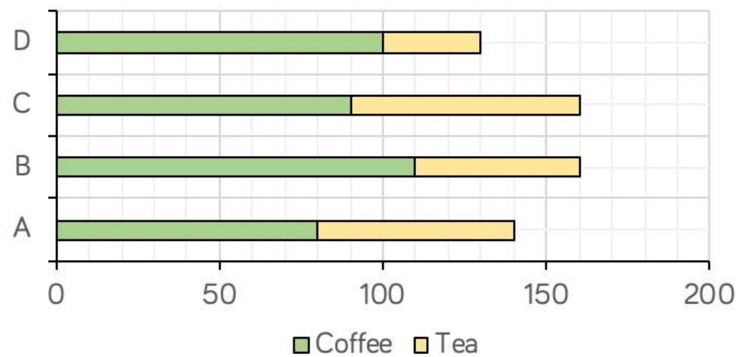
Exemplar Questions

The dual bar chart shows the numbers of students in Y10 and Y11 who chose English, Maths or Science as their favourite subject.



- How many more Year 10s chose Science than Year 11s?
- What other questions can you ask/answer?

The composite bar chart shows the sales of coffee and tea at four branches of a coffee shop between 8 am and 9 am one morning.



How much more coffee than tea was sold in total?
What other questions can you ask/answer?

Pie charts

R

Notes and guidance

Students need to be able to construct pie charts and, equally importantly, need to be able to interpret them. It is useful to look at the proportions in the chart as fractions of 360, as well as percentages and as fractions of the 'whole' that is being represented. It is also worth discussing the pros and cons of using a pie chart rather than a bar chart e.g. proportions of the whole are easier, but comparing parts is not always so.

Key vocabulary

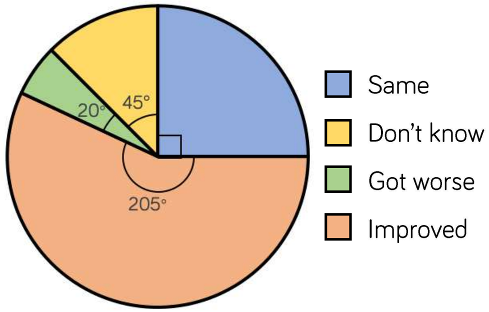
Angle
Sector
Radius
Subtend

Key questions

If you know the angle of a sector on a pie chart, how can we work out what fraction of the whole this represents?
If you know the proportion of the whole, how can we work out the angle we need for the pie chart?
Does it matter what order we present the data in a pie chart?

Exemplar Questions

The pie chart shows the results of a survey of people by a bus company to find out whether they thought the service had improved.



- What percentage of the people surveyed thought the service had improved?
- 100 people said the service had got worse. What other information can you work out?

Activity	Frequency
Dance	16
Drama	12
Football	21
Music	15
Chess	8

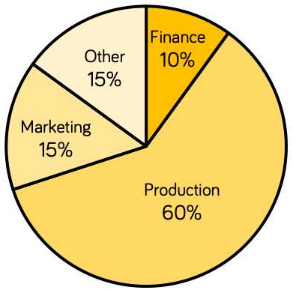
The table shows the after-school activities chosen by a group of students.

Draw a pie chart to represent the information in the table.

What fraction of the students chose music?

The pie chart shows the percentage of employees of a firm who work for each department.

- What angle represents Production?
- 24 people work in the Marketing department.
  How many work in Production or Other?



Criticise charts and graphs

Notes and guidance

Students need to look beyond the superficial criticisms of neatness, labelling of axes and titles to consider the mathematical flaws that charts or graphs may have. In particular, changes in scale, starting the axes from inappropriate points or misuse of scaling may exaggerate or minimise differences. Encourage students to find real-life examples of this – there are plenty available!

Key vocabulary

Scale

Bias

Misleading

Broken axis

Key questions

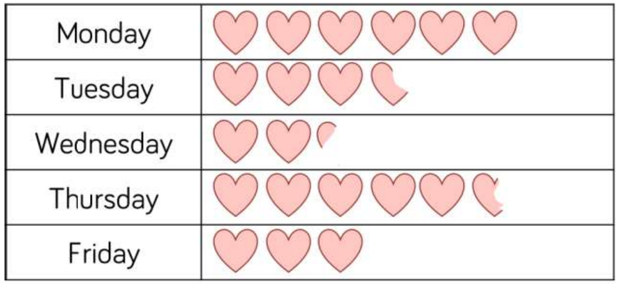
What do you notice about the scales of the axis/axes?

What is the difference between the values in ___ and ___? Does the graph show this?

Why might someone want to use a graph to make differences look bigger/smaller?

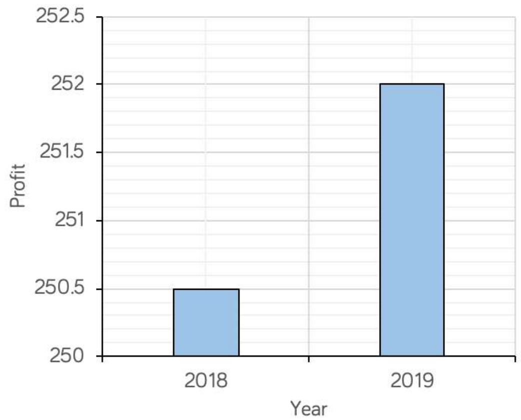
Exemplar Questions

The pictogram shows the number of heart operations carried out at a hospital over one week.



 represents 5 operations

- On which day were most operations performed?
- Comment on the design of the pictogram.

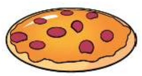


A company shows this bar chart to show the change in profit between 2018 and 2019

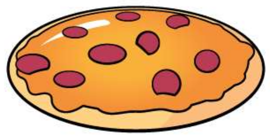
Why might this be misleading?

A pizza firm shows the change in average number of pizzas sold per week using the diagram below.
Explain why the diagram might be misleading.

2018
4000



2019
8000



Construct histograms

H

Notes and guidance

This step explores why grouped frequency diagrams are not appropriate for unequal class intervals as using height to represent frequency can be misleading – the first exemplar question addresses this. Once the idea that frequency is proportional to the area of the bar is established, the formula frequency density = frequency ÷ class width is easily established.

Key vocabulary

Histogram | Area | Frequency density | Class interval | Class width

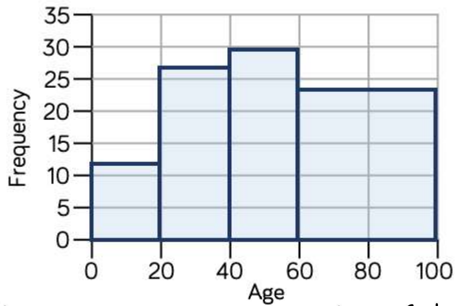
Key questions

Why do we use area to represent frequency rather than height?
What are the main differences between frequency polygons and histograms?
Do you draw histograms using midpoints or end points?

Exemplar Questions

The table and frequency diagram show the ages of people living in a small village.

Age	Number of people
Under 20	12
$20 < w \leq 40$	27
$40 < w \leq 60$	30
$60 < w \leq 100$	24



- Why is the grouped frequency diagram not representative of the data in the table?
- Complete the table and draw a histogram to show the data.

Age	Frequency	Class width	Frequency density
Under 20	12	20	$12 \div 20 = 0.8$
$20 < w \leq 40$	27		
$40 < w \leq 60$	30		
$60 < w \leq 100$	24		

- Explain why the histogram is more representative.

The times, in minutes, taken for a group of people to run a 10 kilometre race are shown in the table.

Time	$40 < w \leq 50$	$50 < w \leq 60$	$60 < w \leq 70$	$70 < w \leq 90$	$90 < w \leq 120$
Frequency	6	24	38	32	24

- Draw a histogram to represent this information.
- Find an estimate for the mean time taken.

Interpret histograms

H

Notes and guidance

As well as knowing how to construct a histogram from scratch, students should also be able to deduce frequencies from a given histogram. Exam questions often give partially completed tables and histograms for students to complete and this is explored in the second exemplar question. Students should also be encouraged to make comments about the shape of distributions.

Key vocabulary

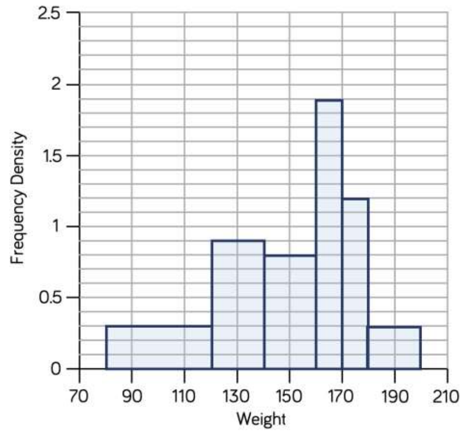
Histogram	Area	Frequency density
Class interval	Class width	Distribution

Key questions

What does the area of each bar represent?
How are frequency and frequency density connected? If we know the class width and the frequency density, how can we work out the frequencies?
What information do we know? What can we find out?

Exemplar Questions

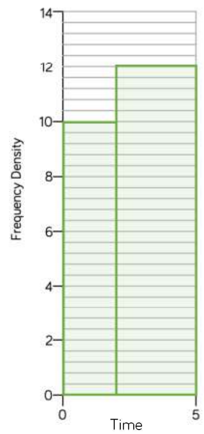
The histogram shows the weights of 87 pieces of fruit.



- Can you tell the modal class of the weights just by looking at the histogram?
- Use the histogram to complete a frequency table, and then use your table to estimate the mean weight of the pieces of fruit.

The table shows information on the amount of time, in minutes, people spend on a phone app per day.

Time (t)	Frequency
$0 < t \leq 2$	20
$2 < t \leq 5$	
$5 < t \leq 10$	20
$10 < t \leq 20$	20
$20 < t \leq 30$	8



- Use the information on the table to complete the histogram.
- Use the histogram to complete the table.

Averages from a list

R

Notes and guidance

Students will have met mean, median and mode several times and at this stage they need to be considering when each one is and isn't appropriate e.g. only the mode is possible with categorical data. Consideration of the effect of removing outliers is a good way to show distortion of the mean and why the median is often preferable.

Key vocabulary

Mean	Median	Mode
Representative	Outlier	Average

Key questions

Do you think any of these values are outliers? Which ones? Why?

What would you do if I asked you to find the average of a set of numbers? Why would you chose that one?

How much does the … give a sense of the whole set of data?

Exemplar Questions

Compare the means of the sets of numbers.

The first five positive integers	The first five square numbers
The first five multiples of 3	The smallest five factors of 40

The list shows the number of letters received in a group of ten houses on a street one Tuesday.

0, 0, 0, 3, 3, 4, 4, 5, 6, 22

- Find the mean, median and mode of the data.
- Which average is most representative of the data as a whole?
- Why do the other averages not represent the data well.
- Are any of the values outliers? Why or why not?

Here are 10 students' estimates of the size of an angle.

35°	40°	41°	30°	35°
32°	35°	125°	31°	44°

Find the mean, median and mode of the data.
Given that the angle is an acute angle, explain how this would change your calculations and your answers.

Explain whether it is possible to find the mean, median and mode of these expressions.

$3x$ $3x + 5$ $4x$ $3x - 1$ $3x - 7$

Averages from a table

R

Notes and guidance

It is useful for students to look at tables presented both horizontally and vertically when revising this KS3 topic, and then decide which is the best way to set the tables out to find averages. The term 'modal class' will need revisiting, emphasising its relationship to the mode. It is useful to consider where the median lies as an introduction to the Higher tier work on cumulative frequency graphs.

Key vocabulary

Mean	Median	Modal Class
Subtotal	Midpoint	Estimate

Key questions

What does 'modal' mean? Which average does it relate to?
Looking at the table, what is the maximum/minimum value the mean could be? Is the value you've found reasonable?
How do you work out the midpoint of a class interval?

Exemplar Questions

The table shows the number of siblings a group of students have.

Number of siblings	0	1	2	3	4	5
Frequency	7	8	6	4	2	2
Subtotal	0	8	12	12	8	10

Mo and Dora are trying to find the mean number of siblings.
Explain why both their calculations are wrong.

Mean $= \frac{50}{6} = 8.33$

Mean $= \frac{50}{22} = 2.27$

Eva is working out the modal number of siblings.

The mode is 2 because it is the frequency that occurs most.

Do you agree with Eva?

The table shows the weight of some objects.

Weight (w)	Frequency	???	???
$40 < w \leq 50$	6		
$50 < w \leq 60$	15		
$60 < w \leq 70$	14		
$70 < w \leq 80$	2		
Total			

- What is the modal class interval?
- What are the extra columns you need to work out an estimate of the mean of the weights shown in the table?
- Complete the table and find an estimate for the mean.
- Can you work out which class the median will be in?

Time series graphs

R

Notes and guidance

When reviewing these types of line graph, it is worth discussing the meaning (if any) of values between the plotted points e.g. taking a reading between May and June is meaningless and any reading between (say) 9 and 10 am is at best an estimate that assumes linear change between that period. It is also worth discussing seasonal trends and cases where there is no apparent trend.

Key vocabulary

Time Series	Quarter
Trend	

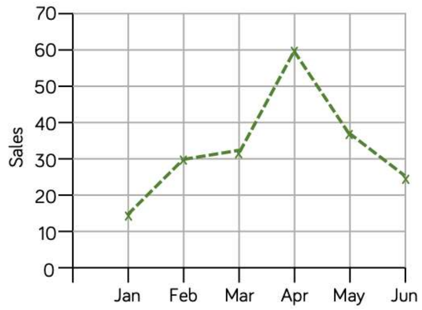
Key questions

Which of the graphs show an upward or downward general trend (or no trend)? Are there any points that don't fit the general pattern?

In this particular graph, does it make sense to read off information from points between the given points? How reliable will these readings be?

Exemplar Questions

The graph shows the sales, in £1000s, of a company in the first six months of a year. The company's target was to have mean sales of at least £35 000



- Did the company meet their target?
- Explain whether it would be sensible to estimate July's sales from the graph.

Draw a graph to represent the greenhouse gas emissions in the UK between 1990 and 2015 shown in the table.

Year	1990	1995	2000	2005	2010	2015
Emissions	794.2	744.3	705.3	681.3	597.1	492.4

Describe the trend.

The table shows the average maximum temperatures, in °C, in Halifax, UK, and Halifax, Canada, for each month of the year.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Halifax, UK	5	5	8	10	15	18	20	19	16	12	8	6
Halifax, Canada	-1	-1	2	8	13	17	22	22	18	13	7	2

On the same axes, draw a pair of graphs to represent the data. Comment on the similarities and differences between your graphs.

Stem and leaf diagrams

Notes and guidance

As with most of the diagrams in the block, interpretation is just as important as construction. When drawing stem and leaf diagrams, students need to take care to keep numbers in line so that the relative lengths of each line are meaningful.
Compare stem and leaf diagrams to horizontal bar charts where all the data is visible, and revisit averages and the range.
Include examples with decimal values e.g. 7|3 means 7.3

Key vocabulary

Stem	Leaf	Median
Range	Modal class	

Key questions

In what way is a stem and leaf diagram similar to a bar chart?
Why do we need a key for a stem and leaf diagram?
How do we work out where the median is in a stem and leaf diagram?

Exemplar Questions

The stem and leaf diagram shows the scores achieved in a test by 21 students.

2	3	7					
3	0	2	3	5	5	7	
4	1	5	5	5	6	8	9
5	0	6	7	7			
6	4	8					

Key: 4 | 1 means 41

Work out the range of the marks.
Find the median and the mode of the marks.
Put the data into groups 20 to 29, 30 to 39 etc. and find an estimate of the mean mark. Compare your answer to the actual mean.

The back-to-back stem and leaf diagram shows the IQ scores for a group of pensioners and a group of teenagers.

Pensioners					Teenagers				
	9	9	3	9	3				
7	4	4	1	10	2	4	5	8	
	8	7	6	11	1	3	6	7	
		4	0	12	0	2	2	4	8
			2	13	1	5	5		

Key: 1 | 10 | 2 means 101 for pensioners, 102 for teenagers

What was the lowest IQ score in each group?
Find the median score for each group, and state each group's modal class. What else can you find?

Cumulative frequency

H

Notes and guidance

Students can get cumulative frequency polygons mixed up with frequency polygons and it is worth investing time discussing the differences. In particular, when drawing tables for cumulative frequency polygons, it is useful to include an "upper limit" column as well as the cumulative frequency column. Although curved lines are often used to represent cumulative frequencies, these are not an expectation.

Key vocabulary

Cumulative	Frequency	Graph
Polygon	End point	Class

Key questions

What's the difference between a frequency polygon and a cumulative frequency polygon?

Why are cumulative frequency polygons plotted at the upper end points of intervals rather than the midpoints or lower class boundaries?

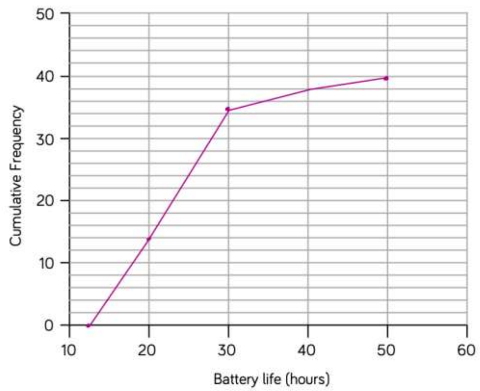
Exemplar Questions

The table shows the heights of 30 plants.

Height (h)	Frequency
$5 \leq h < 10$	4
$10 \leq h < 15$	7
$15 \leq h < 20$	9
$20 \leq h < 30$	6
$30 \leq h < 40$	4

- State the modal class of the heights.
- How many of the plants are less than 10 cm tall?
- How many of the plants are less than 15 cm tall?
- How many of the plants are less than 30 cm tall?
- In which class is the plant with the median height?
- Draw a cumulative frequency graph to represent this information.

The cumulative frequency graph gives information about the battery lives of some mobile phones.



- How many phones had a battery life of less than 20 hours?
- How many phones had a battery life of more than 30 hours?
- Complete the statement: "10 phones had a life of more than…"

Use cumulative frequency H

Notes and guidance

Building on the previous step, students look at using graphs to find the median and related measures such as the interquartile range. It is good to continue to ask questions such as, "90% of the data are greater/less than…" to ensure students are thinking about the meanings of the values and not just procedures. Drawing lines between the graph and the axes is to be advised rather than estimating by eye!

Key vocabulary

Median	Upper/Lower Quartile	
Interquartile range	Range	Outlier

Key questions

Why can't you tell the range from a grouped frequency table or a cumulative frequency graph?

Why is the value for the median only an estimate?

Given a cumulative frequency graph, how can we deduce a grouped frequency table?

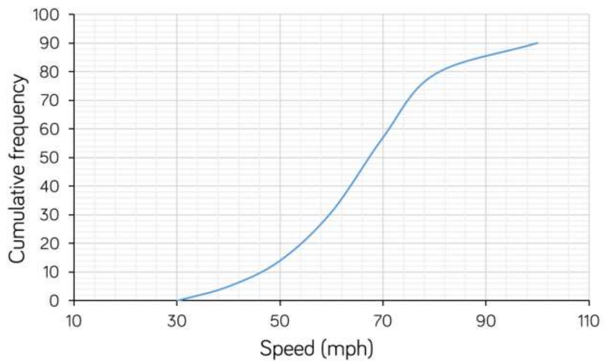
Exemplar Questions

The times taken, in minutes, for a group of people to run a 10 kilometre race are shown in the table.

Time	$40 < w \leq 50$	$50 < w \leq 60$	$60 < w \leq 70$	$70 < w \leq 90$	$90 < w \leq 120$
Frequency	6	24	38	32	24

- Draw a cumulative frequency graph to illustrate the data.
- Use your graph to estimate the median, lower quartile, upper quartile and interquartile range of the times.

The cumulative frequency graph gives some information about the speeds of 100 cars on a motorway.



Where possible, complete the cards. If it is not possible, explain why.

Median speed = ____	80% of the cars are going faster than ___
IQR of the speeds = ____	Range of the speeds = ____

Use the graph to help find an estimate of the mean speed of the cars.

White Rose Maths

Box plots

H

Notes and guidance

It is useful to tie in the teaching of box plots with that of cumulative frequency polygons, and many exam questions do this. Students need to take particular care with the maximum and minimum values, ensuring the correct end of the intervals has been used. Although many box plots are produced horizontally, some programs produce them vertically, so students should experience both.

Key vocabulary

Median	Upper/Lower Quartile	
Interquartile range	Range	Outlier

Key questions

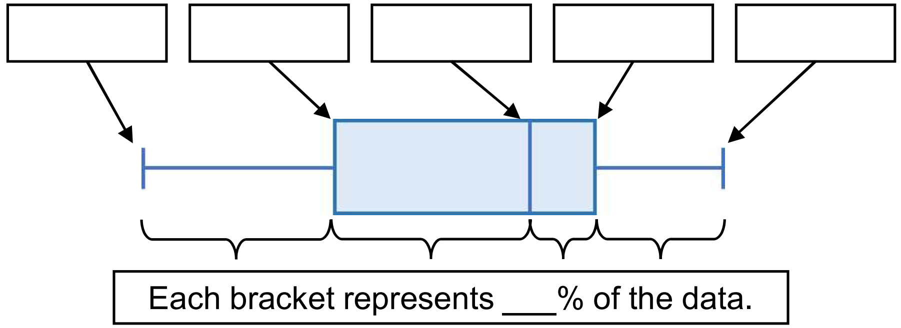
How many parts does a box plot split a distribution of data into?

What information can we read from a box plot?

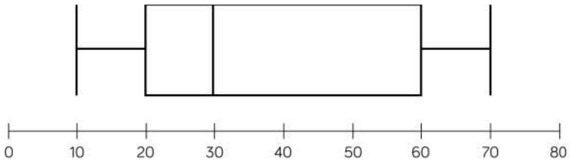
How might we be able to identify an outlier on a box plot?

Exemplar Questions

Complete the cards to describe the features of a box plot.

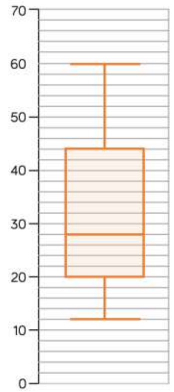


The box plot shows the times in seconds taken by a group of people to do a crossword puzzle.



What can you find out from the box plot?

The box plot shows the distributions of marks scored by students in a test.



- What was the median mark?
- What was the range of the marks?
- Describe the distribution of marks.

Comparing distributions

Notes and guidance

When comparing distributions, students should look at one of the averages and measure of spread; at Foundation level this will always be the range. The average is used as an indicator of overall performance and the range is used to describe the consistency. Students often only look at the average, so looking at data sets where the averages are equal but the ranges differ can be useful.

Key vocabulary

Median	Mean	Spread
Consistent	Range	

Key questions

What does it mean if a data set has a large/small range?
If the averages of two data sets are similar but the ranges are very different, what does that tell you?
How does the range help us decide which ___ is more reliable?

Exemplar Questions

Five students were asked to estimate when one minute had passed. They repeated the experiment five times each, recording in seconds how much time had actually passed.

Jack	55	71	58	64	49
Rosie	47	49	57	42	47
Eva	68	82	77	90	78
Dora	49	47	51	48	47
Teddy	58	65	62	57	64

- Who was the most consistent student?
- Who do you think was the best estimator?
- Justify your answers.

The cards show some information about the times taken for two different bus companies on the same route.

Rider Bus
Mean = 35 minutes
Range = 10 minutes

Speedy Bus
Mean = 32 minutes
Range = 16 minutes

Use the information to compare the performances of the two buses.

Dexter and Dora keep a record of their scores in 8 spelling tests.

Dexter's scores are: 17 18 11 16 15 10 19 12

Dora's mean score is 14 and the range of her scores is 6

Compare Dexter's and Dora's scores in the tests.

Complex comparisons H

Notes and guidance

Building on the last step, Higher tier students are expected to use box plots to make comparisons between distributions. This means there is a choice of indicators for the spread: the range or the interquartile range, but the average to consider will be the median. It is worth discussing that the range is liable to be affected by outliers. Again, students need to be encouraged to make a comparison of both aspects.

Key vocabulary

Median	Mean	Spread
Consistent	Range	Interquartile range

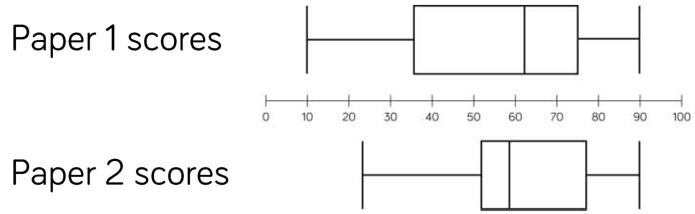
Key questions

Why is just looking at the medians not enough to give us a full picture when comparing two data sets?

What's the difference between the range and the interquartile range? Which of these measures might be affected by an outlier?

Exemplar Questions

The box plots show some information about the marks obtained by a class of students in Paper 1 and Paper 2 of an exam.



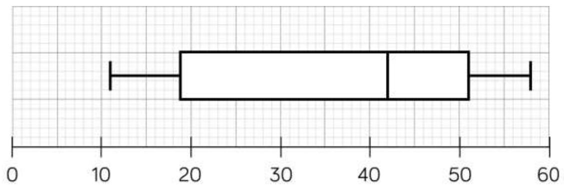
- Estimate the median score of each paper.
- Estimate the interquartile range of scores for each paper
- Use your estimates to compare the students' performances in the two papers.

The table shows some information about the lifetime, in hours, of mobile phone batteries made by company A.

Minimum	Lower Quartile	Median	Upper Quartile	Maximum
30	35	42	46	50

Draw a box plot for this information.

The box plot below shows the distribution of the lifetime, in hours, of mobile phone batteries made by company B.



Compare the distributions of the lifetimes of the batteries of the two companies.

Scatter graphs

R

Notes and guidance

Students will be familiar with correlation from KS3, but this review step is useful to remind them of the vocabulary and to practice choice of scale when plotting points. Where to start and finish axes are also good points for discussion. It is also worth reinforcing that correlation does not imply causality, and that absence of linear relationship does not necessarily mean that the variables are unconnected.

Key vocabulary

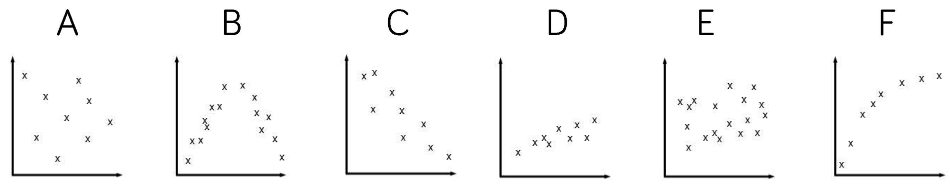
Variable	Relationship	Linear
Positive/negative correlation		Scale

Key questions

How can you tell if a correlation is positive or negative?
Describe the relationship without using the word correlation (e.g. "the taller you are…").
What are the lowest and highest values for each axis?
How do these values help us choose the scales?

Exemplar Questions

Here are six scatter graphs.



Describe the correlation in each of the graphs.

In which graphs do you think there might be a relationship between the variables?

Do you need to have strong correlation between the variables for there to be a relationship?

The table shows the heights in centimetres of 10 new mothers and the weights in kilograms of their babies.

Height	161	172	154	162	158	171	172	160	155	157
Baby's weight	3.9	3.8	4.5	3.1	4.1	3.8	3.6	3.2	2.8	4.5

Plot the information on a scatter graph, thinking carefully about the range and scales of your axes.

Tall mothers have heavier babies.

Do you agree with Tommy?

Lines of best fit

R

Notes and guidance

When using lines of best fits to make estimates, students should draw lines from/to the axes to make their intention clear and to improve accuracy. They could also link to the last steps, considering whether a line of best fit is appropriate or not, and to the next step considering the range of values over which making estimates is sensible. Be aware that in other subjects students may draw 'curves of best fit'.

Key vocabulary

Line of best fit	Origin	Estimate
Correlation	Interpolate	Interpolation

Key questions

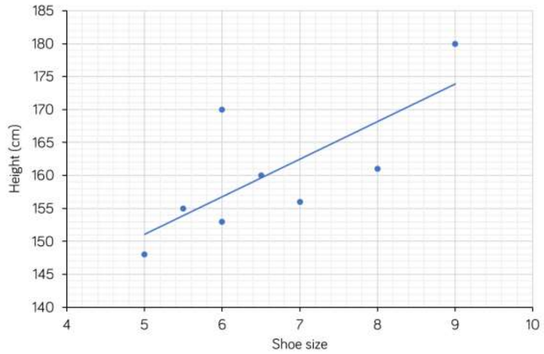
Does the line of best fit always have to go through the origin?

Is it possible to have a line of best fit with a negative gradient?

Does a line of best fit need to be straight line?

Exemplar Questions

The graph shows the shoe sizes and heights of a group of people.



Use the line of best fit to estimate the height of a person with:

- shoe size 7
- shoe size 7.5

Use the line of best fit to estimate the shoe size of someone 170 cm tall.

How reliable do you think your answers are? Why?

The table shows the scores of some students in English, Maths and Science tests.

	A	B	C	D	E	F	G	H
Maths	47	51	56	60	63	63	70	76
English	67	55	71	43	57	70	51	56
Science	51	53	67	69	66	70	69	79

On separate axes, draw the graphs of:

- Maths and English scores
- Maths and Science scores
- English and Science scores

Describe the correlation in each case.

On which graph(s) would it be sensible to draw a line of best fit?

White Rose Maths

Extrapolation

Notes and guidance

This step can be taught alongside lines of best fit, considering when it is and isn't appropriate to extrapolate outside the given data range. This can be demonstrated by looking at examples that give e.g. negative or other impractical answers. Links could be made to Science e.g. considering when relationships may work for certain intervals but not others e.g. length of an extended spring.

Key vocabulary

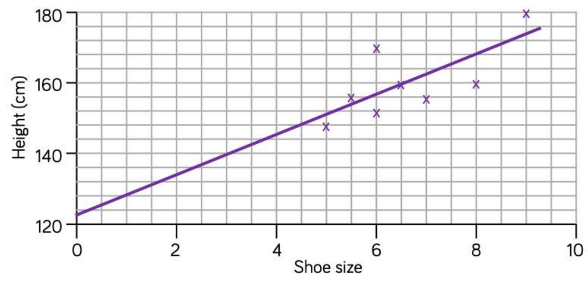
Line of best fit	Origin	Estimate
Correlation	Interpolate	Extrapolate

Key questions

When might it be appropriate to extrapolate beyond the range of values given in a data set? How reliable to do you think the estimates might be?
What difference do outliers make to where we draw the line of best fit?

Exemplar Questions

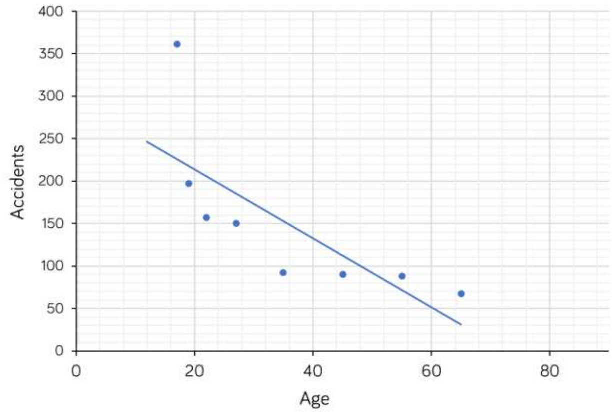
The graph shows the shoe sizes and heights of a group of people.



- Dora uses the line of best fit to estimate the height of someone with shoe size 1. Do you think her answer will be reliable?
- Can the graph be used to estimate the height of someone 1 m tall?

Justify your answers.

The graph shows the number of car accidents per million miles for drivers of different ages.



- Which point is an outlier?
- Why can you not use the graph to estimate the number of accidents for 70 or 80 year olds?

Summer 2: Using Number

Weeks 1 and 2: Non-calculator methods

This block revises and builds on KS3 content for calculation. Mental methods and using number sense are to be encouraged alongside the formal methods for all four operations with integers, decimals and fractions. Where possible this should be covered through problems, particularly multi-step problems in preparation for GSCE. The limits of accuracy of truncation are explored and compared to rounding, and Higher tier students will look at all aspects of irrational numbers including surds. This block may take longer than the following blocks and timings may need to be adjusted accordingly.

National curriculum content covered:

- consolidate their numerical and mathematical capability from key stage 3
- calculate exactly with fractions, **{surds}** and multiples of π; **{simplify surd expressions involving squares and rationalise denominators}**
- **{change recurring decimals into their corresponding fractions and vice versa}**
- apply and interpret limits of accuracy when rounding or truncating, **{including upper and lower bounds}**
- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial contexts
- make and use connections between different parts of mathematics to solve problems

Weeks 4 and 5: Types of Number and Sequences

This block again mainly revises KS3 content, reviewing prime factorisation and associated number content such as HCF and LCM. Sequences is extended for Higher Tier to include surds and finding the formula for a quadratic sequence.

National curriculum content covered:

- Consolidating subject content from key stage 3:
  - Factors, multiples, primes, HCF and LCM
  - Describe and continue sequences
- recognise and use sequences of triangular, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions ($r^n$where n is an integer, and r is a positive rational number **{or a surd}**) **{and other sequences}**
- deduce expressions to calculate the nth term of linear **{and quadratic}** sequences

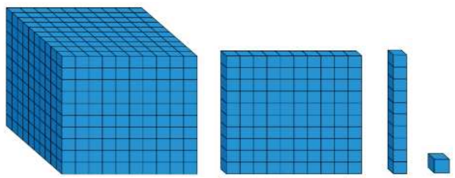
Weeks 5 and 6: Indices and roots

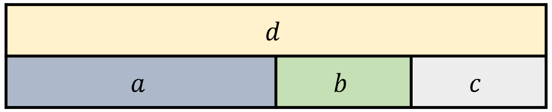
This final block of Year 10 consolidates the pervious two blocks focusing on understanding powers generally, and in particular in standard form. Negative and fractional indices are explored in detail. Again, much of this content will be familiar from KS3, particularly for Higher tier students, so this consolidation material may be covered in less than two weeks allowing more time for general non-calculator and problem-solving practice.

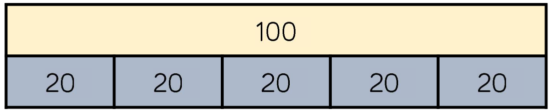
National curriculum content covered:

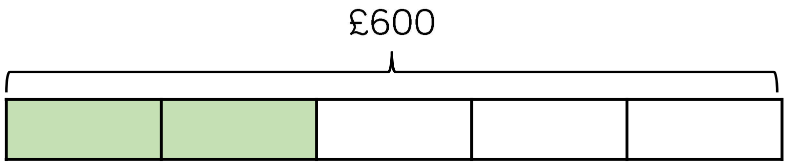
- recognise and use sequences of square and cube numbers
- **{estimate powers and roots of any given positive number}**
- calculate with roots, and with integer **{and fractional}** indices
- calculate with numbers in standard form $A \times 10^n$, where $1 \leq A \leq< 10$ and n is an integer
- simplifying expressions involving sums, products and powers, including the laws of indices

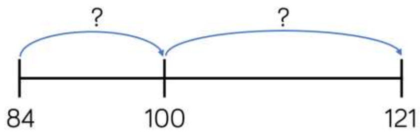
Key Representations

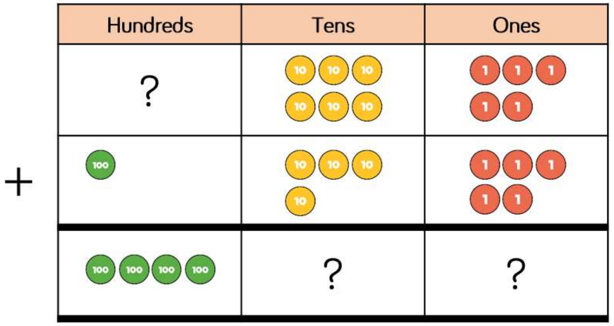


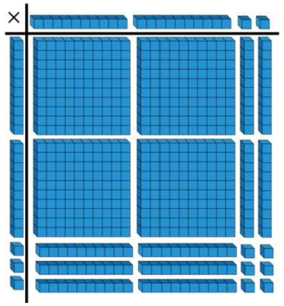


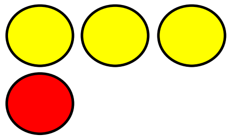


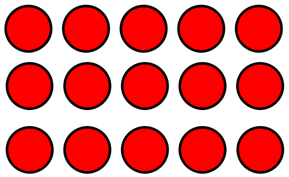


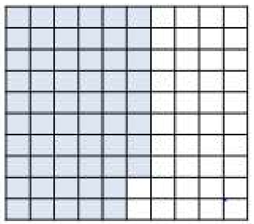


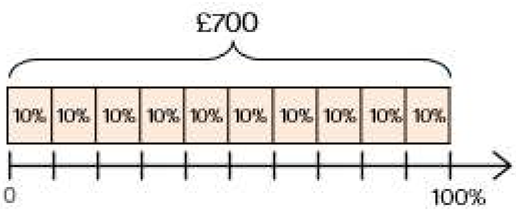












Although many students will be confident with non-calculator methods by Year 10, those who have not developed conceptual understanding will still benefit from the use of manipulatives to support the methods for the four operations of number. Many will find double-sided counters useful to support calculations involving directed number.

For all students, bar models and/or number lines will provide a “way in” to the problem solving questions in this block, helping them to decide whether an additive or multiplicative approach is appropriate, and the choice of operation.

Non-calculator methods

Small Steps

- Mental/written methods of integer/decimal addition and subtraction (R)
- Mental/written methods of integer/decimal multiplication and division (R)
- The four rules of fraction arithmetic (R)
- Exact answers
- **Rational and irrational numbers (convert recurring decimals here)** (H)
- **Understand and use surds** (H)
- **Calculate with surds** (H)
- Rounding to decimal places and significant figures (R)

(H) denotes Higher Tier GCSE content

(R) denotes 'review step' – content should have been covered at KS3

Non-calculator methods

Small Steps

- Estimating answers to calculations R
- Understand and use limits of accuracy
- **Upper and lower bounds** H
- Use number sense
- Solve financial maths problems
- Break down and solve multi-step problems

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Addition and Subtraction

R

Notes and guidance

This step looks over mental and written methods for addition and subtraction. Teachers could use this to revisit the skills in the contexts their class need to particular revise e.g. calculating angles, financial maths problems etc. Similarly, this step need not be covered as a one-off, but the content could be covered in starters or in other relevant lessons. It is useful to keep the skills covered sharp by revisiting regularly for the remainder of the course.

Key vocabulary

Add	Subtract	Balance
Adjust	Credit/Debit	Profit/Loss

Key questions

What strategies do you know to add/subtract numbers mentally?

Why is easy to add (e.g.) 99/9.9 etc. without a written method?

How do we set up written methods for addition and subtraction? What might go wrong with decimals?

Exemplar Questions

Explain how each of these calculations can be done mentally.
Compare your methods with a partner's.

142 + 99	£20 – £7.68	180 – 62	360 – 148
– 7 – 12	658 – 299	25 – 40	4.3 + 14.9

Complete the bank statement.

Date	Description	Credit (£)	Debit (£)	Balance (£)
Jun 1	Opening balance			147.52
Jun 3	Phone bill		38.65	
Jun 4	Wages	208.85		
Jun 8	Rent			171.82

Cinema Tickets

Adult	£8.90
Child	£4.65

- Dora buys one adult ticket and one child ticket. How much change should she receive from £20?
- Brett buys an adult ticket. He has a 10% student discount. How much does Brett pay for his ticket?

Multiplication and Division

R

Notes and guidance

Building on the pervious step, students can focus on multiplication and division whilst considering problems involving all four operations. Special care needs to be taken with decimals e.g. $6 \div 0.2 = 60 \div 2$ but $3.6 \times 1.7 \neq 36 \times 17$ Teachers can again choose which skills to revisit e.g. area, substitution etc. and likewise keep the skills covered sharp by revisiting regularly for the remainder of the course.

Key vocabulary

Multiply	Divide	Adjust
Perimeter	Volume	Area

Key questions

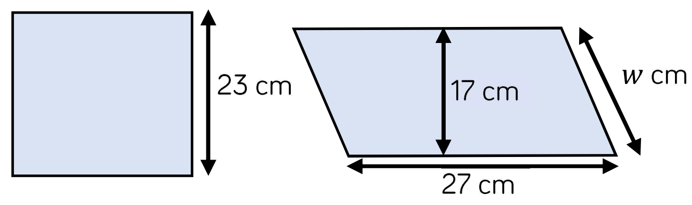
What strategies to you know to multiply numbers mentally?
How do we set up long multiplication/division?
How can we adjust calculations if the numbers involve decimals? Is it the same or different for multiplication and division?

Exemplar Questions

Explain how each of these calculations can be done mentally.
Compare your methods with a partner's.

8×0.04 | 0.48×1000 | $18 \div 0.3$ | 37×99

The perimeters of the square and parallelogram are equal.



- Find the value of w.
- Work out the difference between the areas of the shapes.

Work out the calculations on the cards.
Compare your strategies with a partner's.

36×74 | 0.36×7.4 | $89.1 \div 6$ | $360 \div 24$

Solve the problems, showing all your working clearly.

- A calculator costs £4.79
  Find the cost of 38 of these calculators.
- A two-week holiday costs £1659
  How much does the holiday cost per day?
- A metal alloy contains zinc, copper and nickel in the ratio 2 : 3 : 7
  How much nickel is there in 702 g of the alloy?
- Work out the volume of a cube of side 4.5 cm

Fraction arithmetic

R

Notes and guidance

Students often confuse the rules of fraction arithmetic so it worth revisiting the similarities and differences when working with the different operations. They have, however, been working with these for several years so at this stage it is useful to include context in practice questions whenever possible. Bar models and other pictorial support are still useful and should be encouraged, both to support choice of operation and to help to reason how to perform the calculation.

Key vocabulary

Fraction	Numerator	Denominator
Reciprocal	Mixed number	Improper fraction

Key questions

Is working with mixed numbers different from working with fractions when adding/multiplying etc.?
Can you draw a picture to show how fraction multiplication works?
How is fraction multiplication different from fraction division?

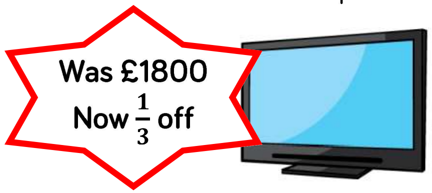
Exemplar Questions

Work out the calculations on the cards and put your answers in order of size, starting with the smallest.

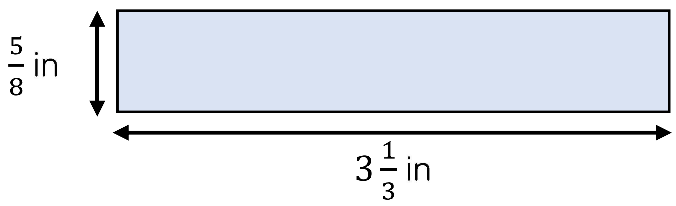
$\frac{3}{5} - \frac{1}{2}$ $\frac{3}{5} \times \frac{1}{2}$ $\frac{3}{5} \div \frac{1}{2}$ $\frac{3}{5} + \frac{1}{2}$

A company employs 160 people.
$\frac{5}{8}$ of the employees are women.
$\frac{2}{5}$ of the women work part time.
$\frac{5}{12}$ of the men work part time.
Work out the total number of people who work part time.

Work out which TV is cheaper.



Find the perimeter and the area of the rectangle.



Exact answers

Notes and guidance

This step prepares Higher tier students for the upcoming study of surds as well as reminding all of language such as “in terms of π” etc. Use of formulae for area/perimeter/volume of shapes involving circles can be revisited here and it is useful to involve “reverse questions” such as finding the height of a cone gives its volume and radius. It is also a timely opportunity to revisit exact trigonometrical values met in the Autumn term.

Key vocabulary

Exact	In terms of	Square/Cube Root
Sine	Cosine	Tangent

Key questions

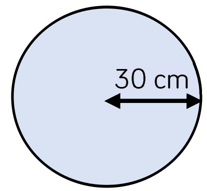
Is it okay to give the solution to an equation as a fraction rather than a decimal? Why or why not?

Can you check exact answers for (e.g.) volume of a cylinder on a calculator? What mode to you need?

What’s the difference between the sine, cosine and tangent of an angle?

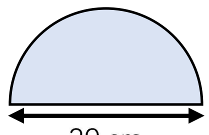
Exemplar Questions

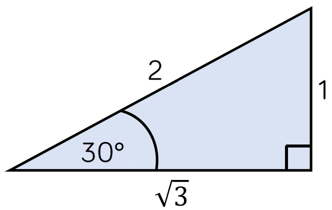
Which card shows the circumference of the circle?



30π cm | 60π cm | 900π cm

Find the area of the semicircle.
Give your answer in terms of π.

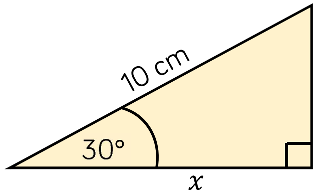




Use the diagram to write down the exact values of:

- sin 30°
- tan 30°
- cos 30°
- sin 60°

Work out the value of x.



Match the equations to the answers.

$2x^2 = 14$ | $3a + 5 = 7$ | $2b - 1 = 2$ | $12 = 5 + 3t$

$\frac{7}{3}$ | $\sqrt{7}$ | $\frac{3}{2}$ | $\frac{2}{3}$

White Rose Maths

Rational/Irrational Numbers H

Notes and guidance

Students have already met irrational numbers such as π and $\sqrt{2}$ and this step formalises this learning and the associated language. π is useful for generating other irrationals e.g. $2\pi, \pi + 3$ are both irrationals between 6 and 7. Students will learn that recurring decimals are not irrational and how to convert these to fractions. Using this technique to show that $0.\dot{9} = 1$ (or $3 \times 0.\dot{3} = 3 \times \frac{1}{3}$) is an interesting extension.

Key vocabulary

Integer	Decimal	Terminating
Recurring	Infinite	Root

Key questions

Do all linear equations have rational solutions? Why or why not?

Which of the exact trigonometric values are rational, and which are irrational?

What's the difference between a terminating decimal and a recurring decimal?

Exemplar Questions

Which of these equations have rational solutions?

$3x = 1$	$x^2 = 1$	$x^2 = 3$	$4x^2 = 1$
$4x = \pi$	$x^2 - 3 = 1$	$x^2 = 64$	$x^3 = 64$

Complete the workings to find $0.\dot{7}$ as a fraction.

Let $x = 0.\dot{7}$
So $10x = 7.\dot{7}$
So $9x = \ldots$
So $x = \ldots$

What would be the same and what would be different when writing these recurring decimals as fractions?

- $0.\dot{3}$
- $0.0\dot{3}$
- $0.1\dot{3}$
- $0.\dot{1}\dot{3}$
- $1.\dot{3}$

Decide whether these statements are always, sometimes or never true, justifying your answers.

- Terminating decimals are rational
- Recurring decimals are irrational
- Multiples of π are irrational
- $\pi + a$ is irrational
- Square roots of fractions are irrational
- Cube roots of fractions are irrational

Understand and use surds H

Notes and guidance

This first of two steps on surds, which could be introduced concurrently, looks at the definition of a surd as the irrational root of a rational number, and writing surds in simplified form. When simplifying e.g. $\sqrt{72}$ it is useful to compare using 36 and 2 as a factor pair rather than 8 and 9, demonstrating that they both lead to the same final answer but that finding the largest square factor is much more efficient.

Key vocabulary

Surd	Square root	Cube root
Simplify	Factors	

Key questions

How can you tell if the square root of an integer less than 100 will be a surd or not?
Are surds rational or irrational?
What are the factor pairs of ___? Which factor pair has the largest square factor?
Can you use your calculator to check surd simplification?

Exemplar Questions

Work out the calculations on the cards, using a calculator where necessary, giving your answers as integers, fractions or mixed numbers.

$\sqrt{36}$ $\quad (\sqrt{11})^2 \quad \sqrt{\frac{4}{9}} \quad \sqrt{7}\times\sqrt{7} \quad (\sqrt{15})^2 \quad \sqrt{6\frac{1}{4}} \quad \sqrt{196} \quad \left(\frac{\sqrt{3}}{2}\right)^2$

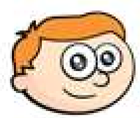
Use a calculator to match the cards that are of equal value.

$\sqrt{40}\div\sqrt{2}$ $\quad \sqrt{10}\times\sqrt{3} \quad \sqrt{4}\times\sqrt{6} \quad \sqrt{20}$

$\sqrt{2}\times\sqrt{3}$ $\quad \sqrt{30} \quad \sqrt{5} \quad \sqrt{50}\div\sqrt{10} \quad \sqrt{6} \quad \sqrt{24}$

Are these generalisations true or false?

$\sqrt{ab}=\sqrt{a}\times\sqrt{b}$ $\qquad \sqrt{a\div b}=\sqrt{a}\div\sqrt{b}$



$\sqrt{50}=\sqrt{25}\times\sqrt{2}$
$=5\sqrt{2}$

Use Ron's method to write the surds in the form $a\sqrt{b}$

- $\sqrt{75}$
- $\sqrt{27}$
- $\sqrt{98}$
- $\sqrt{63}$

Is it better to simplify $\sqrt{12}$ using $12=2\times6$ or $12=4\times3$? Why?

Calculate with surds

H

Notes and guidance

Having established the behaviour of surds when multiplied and divided in the previous step, we now investigate addition and subtraction. Students can establish rules for themselves, using calculators in both exact and decimal forms.

Sometimes students are confused that although $\sqrt{a}+\sqrt{b}=\sqrt{a+b}$ it is possible to simplify e.g. $8\sqrt{2}-3\sqrt{2}$. It is useful to compare this to e.g. $8x-3x$ or $8\pi-3\pi$. Rationalising the denominator is also explored.

Key vocabulary

Surd	Square root	Cube root
Simplify	Rationalise	Denominator

Key questions

When is it possible to simplify surd expressions involving addition and subtraction, and when is it not possible?

What's the first step when rationalising a denominator?

When expanding brackets involving surds, why might we sometimes get rational terms?

Exemplar Questions

Which of the calculations are correct and which are incorrect?

- $\sqrt{2}+\sqrt{3}=\sqrt{5}$
- $\sqrt{8}-\sqrt{3}=\sqrt{5}$
- $6\sqrt{3}-\sqrt{3}=6$
- $5\sqrt{2}+2\sqrt{2}=7\sqrt{2}$

$\sqrt{75}-\sqrt{27}$
$=\sqrt{48}$

$\sqrt{75}-\sqrt{27}$
$=5\sqrt{3}-3\sqrt{3}$
$=2\sqrt{3}$

- Who do you agree with?
- Work out $\sqrt{98}-\sqrt{50}$

Complete the steps to write $\frac{5}{2\sqrt{3}}$ as a fraction with a rational denominator.

$$\frac{5}{2\sqrt{3}}=\frac{5}{2\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\square\sqrt{3}}{2\times\square}=\frac{\square\sqrt{3}}{\square}$$

Complete the calculation.

$$(2+\sqrt{3})^2=(2+\sqrt{3})(2+\sqrt{3})$$
$$=4+\square+\square+3$$
$$=7+\square\sqrt{3}$$

Work out
- $(2\sqrt{3}-\sqrt{2})(2\sqrt{3}+\sqrt{2})$
- $(3-\sqrt{3})^3$

Rounding

R

Notes and guidance

Rounding could be required in both calculator and non - calculator papers, so although this block as a whole is focusing on the latter, it may be appropriate to use calculators for some of the questions in this step. Again, this is largely revision and can be done in the context of other areas of mathematics if appropriate, perhaps reminding students of the difference between decimal places and significant figures first if needed.

Key vocabulary

Degree of Accuracy		Decimal place
Round	Approximate	Significant Figure

Key questions

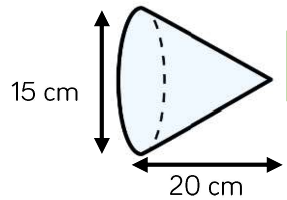
What's the difference between decimal places and significant figures?

How do you decide what degree of accuracy to give an answer to? Is the full calculator display always, sometimes or never appropriate?

Exemplar Questions

Find the volume of the cone, giving your answer

- in terms of π
- correct to 1 decimal place
- correct to the nearest integer
- correct to 3 significant figures



$V = \frac{1}{3}\pi r^2 h$

Which degree of accuracy do you think is the most appropriate?

Write each of these numbers to two significant figures.

0.0385 | 0.385 | 3.85 | 38.5 | 385

600.123 | 609.123 | 0.609123 | 0.009123

Work out $\frac{38.7 + 1.5^3}{9.2}$

Write down all the figures on your calculator display.
Write your answer correct to 2 decimal places.

---

12 357 people attended a football match.

- Write 12 357 in words
- Write 12 357 correct to the nearest thousand
- Write 12 357 correct to three significant figures

White Rose Maths

Estimating

R

Notes and guidance

Although the focus of this step will be rounding to one significant figure to support estimation, it is worth including examples where using factors might be more useful e.g. 12 on a denominator if the numerator involves multiplying 30 and 40. Students should also use their knowledge of square (and possibly cube numbers) to estimate roots. Students could also revisit fraction arithmetic by using $\pi \approx \frac{22}{7}$ in circle calculations.

Key vocabulary

Round	Approximate	Decimal place
Degree of Accuracy		Significant Figure

Key questions

Why do you need to be careful when rounding decimals when making estimates?

How would you estimate (e.g.) 38% of ___?

Between which two integers does the square root of ___ lie? How do you know?

Exemplar Questions

By rounding each number to 1 significant figure, estimate the answers to the calculations.

86×77 $\quad 8.6 \times 7.6$ $\quad 0.86 \times 7.5$ $\quad 0.86 \times 0.74$

Can you tell which estimates are overestimates and which are underestimates?

Annie is estimating the area of a circle of diameter 40 cm.
Is her estimate a good one?
Justify your answer.

$$3 \times 20^2 = 60^2 = 3600$$

What mistake has been made in this estimate?

$$830 \div 0.37 = \frac{830}{0.37} \approx \frac{800}{0.4} = \frac{80}{4} = 20$$

✗

Estimate the answer to: $\frac{27 \times 35}{0.056}$ $\quad 0.19^2$

$\sqrt{57} \approx 8.4$

Explain why Mo must be wrong.

Estimate $\sqrt{83}$ $\quad \sqrt{24}$ $\quad \sqrt[3]{30\,000}$

$\frac{31.9+\sqrt{97.8}}{0.191}$ $\quad \frac{\sqrt{907}}{11.6}$ $\quad \sqrt{\frac{600}{14.8+4.7^2}}$

Limits of accuracy

Notes and guidance

Students have met error intervals at KS3 and this step takes this a little further to include the effect of truncation as well as rounding. In both cases it is worth using a number line to illustrate e.g. if a number has been rounded to 4.6 to 1 decimal place then using a line from 4.5 to 4.7 helps students to see where the limits are, contrasting this with truncation (shortening) meaning the original number cannot possible be below 4.6

Key vocabulary

Limit	Error interval	Upper/Lower bound
Truncate	Round	Correct to…

Key questions

What numbers might round to give (e.g.) 4.6 to 1 decimal place? Can you think of examples both below and above 4.6? What would the limits/bounds be? Show me on a number line.

What numbers might be truncated to give 4.6 to 1 decimal place? Why is 4.599 not a possible value?

Exemplar Questions

The number of people who attended a concert is 20 000, correct to the nearest thousand.

- What is the smallest number of people who could have been at the concert?
- What is the greatest number of people who could have been at the concert?
- How would your answers change if the attendance figure had been correct to the nearest hundred?

A length l is given as 7.4 cm correct to one decimal place.
Complete the error interval for the length l.

$\_\_ \leq l < \_\_$

Complete the error interval for the length l if it had been **truncated** to one decimal place.

$\_\_ \leq l < \_\_$

Find the upper and lower bounds of the numbers given to the degrees of accuracy shown.

- £7 rounded to the nearest pound
- 7 rounded to the nearest integer
- 7.0 rounded to one decimal place
- 70 rounded to the nearest 10
- 70 rounded to the nearest integer
- 7.00 rounded to two decimal places
- 7.00 truncated to two decimal places

Upper and lower bounds H

Notes and guidance

This builds on the previous step using the limits of accuracy to find upper and lower bounds of calculations. Students often find this relatively straightforward for multiplication and addition as e.g. $(a+b)_{max} = a_{max} + b_{max}$ is quite intuitive, but subtraction and division need more careful exploration to establish e.g. $(a-b)_{max} = a_{max} - b_{min}$. Learning such formulas is not recommended here, just considering "How could we make the value greater/smaller?"

Key vocabulary

Upper/lower bound		Maximum/minimum	
Sum	Difference	Product	Quotient

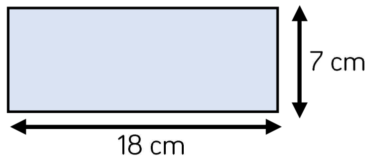
Key questions

If we want (e.g.) ab to be as large as possible, do the values of a and b also need to be as large as possible? Is it the same or is it different for $\frac{a}{b}$ to be as large a possible?

In a formula such as $V = \frac{1}{3}\pi r^2 h$, which components don't vary when looking for the minimum value of V?

Exemplar Questions

The dimensions of the rectangle are given to the nearest cm.

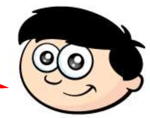


Write, the upper and lower bounds of:
- the length of the rectangle
- the width of the rectangle
- the perimeter of the rectangle
- the area of the rectangle

Give your answers to 2 decimal places.

$x = 7.4$ and $y = 3.5$, both to 1 decimal place.
Dexter is wrong.
Show that the upper bound of $x - y$ is 4

The upper bound of $x - y$ is 7.45 – 3.55 = 3.9



Find the upper bounds of $\frac{x}{y}$ $\frac{y}{x}$

Find the lower bounds of $y + 2x$ $y^2 - x$

Aisha runs 100 m in 15 seconds.
Find the upper and lower bounds of Aisha's speed if the numbers are given
- to the nearest integer
- to two significant figures

$F = 32.87$ N correct to 2 decimal places.
$A = 5.16$ m^2 correct to 3 significant figures.
Work out the value of p to a suitable degree of accuracy.
Explain your choice.

$$p = \frac{F}{A}$$

Use number sense

Notes and guidance

Building on the strategies used for the four operations in the earlier review steps, this step focuses on deriving facts from known facts. Students often struggle to decide whether e.g. $39 \times 73 = 39 \times 72 + 39$ or $39 \times 72 + 72$; area models are very useful to illustrate this. Students could also be taught to look for factor pairs such as 2 and 5, 4 and 25, 8 and 125 to make multiplications easier, and factorising to simplify division e.g. divide by 2 and then by 7 instead of dividing by 14

Key vocabulary

Adjust   Compensate   Factorise

Key questions

What are useful pairs of factors to look for in order to simplify a calculation?

If we know the value of an algebraic expression (e.g. $4xy$), what other expressions can we work out very easily?

How can we adjust (e.g. 9.9×87) to make it into easier calculations?

Exemplar Questions

Use the fact that $39 \times 72 = 2808$ to find the values of the cards. Explain how you found your answers.

$2808 \div 72$	$2808 \div 7.2$	$28.08 \div 7.2$
39×36	39×73	37×72

35% of a number is 112

What other percentages of the number can you easily find?

Work out
- $\frac{5}{8}$ of the number
- $\frac{11}{8}$ of the number

75×36
$= 25 \times 3 \times 4 \times 9$
$= 25 \times 4 \times 3 \times 9$
$= 100 \times 12$
$= 1200$

Use factorisation or other strategies to work out:
- 14×45
- 175×28
- 9.9×87
- $7665 \div 35$
- $19\,248 \div 48$
- $13\,692 \div 42$

Without calculating the answers, explain how you know:

- $96 \div 2 > 96 \div 3$
- $\frac{4}{7}$ of $420 > 200$
- $\frac{7}{12}$ of $240 > \frac{11}{20} \times 240$
- $\frac{7}{15}$ of £600 < 55% of £600
- 45% of 80 = 80% of 45

Financial Maths

Notes and guidance

Here students can revise the language and methods studied in previous units, notably within Percentages and Interest. As well as the chance to revisit compound and simple interest, students could explore personal finance such as the tax system, and utility bills. Many financial organisations provide materials to support this. As an extension, students could look at more complex products such as mortgages or investigating the meaning of terms like APR.

Key vocabulary

Credit/Debit	Profit/Loss	VAT
Standing Charge	Allowance	Tax

Key questions

What is the first step you need to take to solve the problem?

Which words tell you if you need to add/subtract/multiply or divide?

Exemplar Questions

Kim gets paid £8.20 an hour for the first 35 hours she works in a week, and any additional hours she works are paid at one and a half times her normal hourly rate.

One week, Kim works eight hours every day from Monday to Saturday.

Calculate Kim's total wage for this week.

Kim pays 32% of her wage in taxes and other deductions.

Calculate Kim's wage after these deductions.

Alex buys a house for £140 000

She spends £40 000 renovating the house.

She then sells the house for £235 000

Work out Alex's percentage profit on the sale of the house.

Pervious meter reading	3672
Current meter reading	4218
Units used	
Standing charge	£17.50
Total charges	
VAT at 5%	
Total to pay	

The table shows part of a gas bill.

The cost of each unit of gas is 84.3p.

Complete the table.

Esther earns £28 000 a year.

She pays 20% tax on earnings over £12 500

She pays 12% National Insurance on earnings over £8632

Work out Esther's monthly take-home salary.

Multi-step problems

Notes and guidance

Students often find it difficult to access GCSE questions that require several steps of working. Students can be prepared for these by practising questions that require an increasing number of steps and complexity. This is another opportunity to revisit and remind students of formulae such as those for area, volume, pressure and density, but the focus should be on looking at what information is given and what can be found out as students work towards a solution.

Key vocabulary

Force	Pressure	Area
Density	Mass	Volume

Key questions

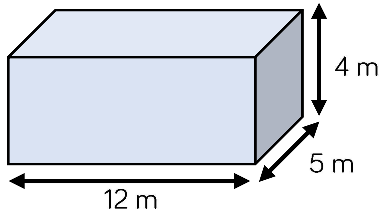
What information do we need to solve the problem?

What can we find out first? Given this new information, what can we find out next?

What formulas do we need to know to tackle this problem?

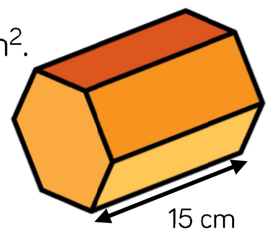
Exemplar Questions

The diagram shows a shipping container on horizontal ground.
The weight of the container is 24 000 N.
Use the formula $p = \frac{F}{A}$ to work out the pressure exerted on the ground by the shipping container.



Cat food comes in 825 g packets that cost £2.15
Huan has two cats.
Each cat eats 55g of cat food a day.
Huan wants to buy enough cat food for six weeks.
How much will the cat food cost?

The diagram shows a hexagonal prism.
The area of the cross-section of the prism is 80 cm^2.
The density of the prism is 25 g/cm^3.
1 kg of the material used to the make the prism costs £650
Find the cost of the prism.



A water tank holds 120 litres of water.
There is a hole in the water tank and water leaks out at a rate of 80 millilitres per second.
Work out the time in takes for the water tank to empty completely.
Give your answer in minutes.

Types of Number and Sequences

Small Steps

- Understand the difference between factors and multiples (R)
- Understand primes and express a number as a product of its prime factors (R)
- Find the HCF and LCM of a set of numbers (R)
- Describe and continue arithmetic and geometric sequences
- Explore other sequences
- **Describe and continue sequences involving surds** (H)
- Find the rule for the n^{th} term of a linear sequence (R)
- **Find the rule for the n^{th} term of a quadratic sequence** (H)

(H) denotes Higher Tier GCSE content

(R) denotes 'review step' – content should have been covered at KS3

Factors and multiples

R

Notes and guidance

The main emphasis of this step is to review the difference between a factor and a multiple. Building from previous years, students will explore both factors of numbers and algebraic expressions. The area model is useful in considering factors and links well to factors of algebraic terms. Ensure that students are also exposed to non-examples and non-standard examples of factors and multiples e.g. 0.5 is not a factor of 1

Key vocabulary

Integer	Factor	Multiple
Area	Factorise	Prime

Key questions

Explain why a factor is different from a multiple.

Can negative numbers be multiples/factors?

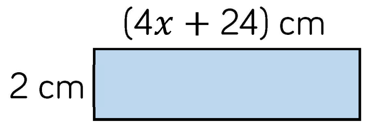
Can algebraic expressions be multiples/factors?

Can a fraction of a number be a multiple/factor?

Exemplar Questions

Using integers for length and width, find as many different rectangles with have an area of 48 cm². What are the factors of 48?

Eva draws a rectangle with an area of $8x + 48$ cm²



Draw 3 other rectangles that have the same area as Eva's.
Eva says that $x + 6$ is a factor of $8x + 48$
She is correct. Explain why.

Decide whether the statements are true or false.
Justify your decisions.

- 49 is a multiple of 7
- y^2 is a multiple of y
- y is a factor of y^2
- $8x + 48x$ is a multiple of $8x$
- $8x + 48$ is a multiple of 8
- $8x + 48$ is a multiple of $8x$

Is this statement, always, sometimes or never true? Explain why.

If a is a factor of b, then b is a multiple of a

---

500 g pack of lemons cost £1.20
1 kg pack of sugar cost £0.80

Jack buys packs of lemon and sugar for his crepe store.
He spends exactly £20
How many kg of lemons and how many kg of sugar did he buy?

Product of prime factors

R

Notes and guidance

Students should already be familiar with expressing a number as a product of its prime factors, but some of the language (e.g. express, product) requires emphasis. It's worth ensuring that students understand the concept behind the procedure and make links between the original number and the product. Students should use their reasoning skills to make connections between the prime factor decomposition of related numbers.

Key vocabulary

Factor	Prime Factor	Factorise
Product	Express	Index form

Key questions

Explain what the following terms mean:
Factor of, Express, Product, Prime Factor, Factorise

Is there more than one way to factorise 300?

If there more than one way to express 300 as a product of prime factors? Does the order of the factors matter?

Exemplar Questions

Aisha has expressed a number as a product of its prime factors.

What is the number? $2^2 \times 3 \times 5^2$

$$36 = 2^2 \times 9$$

Dexter attempts to express 36 as a product of its prime factors.
What mistake has he made?

He now writes 36×24 as a product of prime factors.
Dexter considers the following two approaches:

$36 \times 24 = 864$ I can draw a prime factor tree for 864	I just need to draw a prime factor tree for 24

Which approach is the most efficient? Why?
Write 36×24 as a product of its prime factors.

Mo has written 400 as a product of its prime factors.

$$400 = 2^4 \times 5^2$$

Use this to write down the numbers as products of their prime factors:

- 200
- 50
- 100
- 80
- 16
- 800
- 4 000

$$375 = 5^x \times y$$

Show that $x \div y = 1$

HCF and LCM

R

Notes and guidance

Students may need reminders with this familiar topic as they can confuse the HCF with the LCM. They need to be careful to use prime factors when completing Venn diagrams, rather than just factors. Duplicating the common factors when filling in the intersection of the Venn diagram is a common error that may also need highlighting. Students may multiply all prime factors of both numbers when finding the LCM, so it is useful to demonstrate that this is often unnecessary.

Key vocabulary

Highest Common Factor/Lowest Common Multiple

Prime Factors   Product   Intersection

Key questions

What does common mean when looking at factors and multiples?
Why do we find the highest common factor but the lowest common multiple? Why isn't it the other way round?
What's the first step in completing a Venn diagram to find the HCF and LCM?

Exemplar Questions

Find the LCM of:

- 3 and 15
- 4 and 6
- 3 and 8
- 100 and 200
- 10 and 25
- 8 and 15

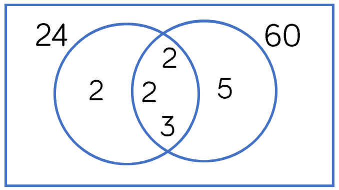
What do you notice about the LCM in each pair?

To find the LCM, multiply the numbers together

Is Ron correct? Explain your answer.

Rosie is making fried egg sandwiches.
Each sandwich contains one fried egg.
Eggs come in boxes of one dozen and bread rolls come in packs of 8
What is the least number of boxes of eggs and packs of bread rolls Rosie needs to buy so she has no ingredients left over?

Eva is using a Venn diagram to find the LCM and HCF of 24 and 60



$24 = \cancel{2} \times \cancel{2} \times 2 \times \cancel{3}$

$60 = \cancel{2} \times \cancel{2} \times \cancel{3} \times 5$

Explain why $2 \times 2 \times 3$ gives the HCF of 24 and 60

To find the LCM, Eva works out $2 \times 2 \times 2 \times 3 \times 2 \times 2 \times 3 \times 5$
What mistake has she made? Correct this and calculate the LCM.
Why does this method work?

Arithmetic/Geometric sequences

Notes and guidance

Students can have the misconception that a common ratio of a geometric sequence has to be a positive integer and so should work with examples of fractions, decimal and negatives. Revisiting compound interest is useful as students make links between this and geometric sequences. For high attainers the formula for the n^{th} term of a geometric sequence can be explored; finding the rule for arithmetic sequences is reviewed later in this block.

Key vocabulary

Arithmetic	Common difference	n^{th} term
Geometric	Common ratio	Term-to-term

Key questions

What is an arithmetic sequence? Why is a geometric sequence different?

What is a common ratio? Does this have to be a positive integer? Explain why not.

What's the difference between a term-to-term rule and a position-to-term rule?

Exemplar Questions

A: 3, 5, 7,… B: 3, 15, 75…

- What's the same and what's different about the two sequences?
- Write down the term-to-term rule for each sequence.
- Which one will have a term greater than 100 first?
- Write down the first four terms of two more geometric sequences, one of which is ascending and one descending. What's the common ratio for each sequence?

Continue each of the following sequences to generate one that is arithmetic and the other geometric.

- $\frac{1}{5}, \frac{1}{10}, \ldots$
- $-0.3, -3, \ldots$

Teddy saves £1 in January, £2 in February, £4 in March and continues to double his saving each month.

Is this model realistic for a whole year? Explain your answer.

State whether these statements are true or false. Justify your answer each time.

- It's impossible for a geometric sequence to alternate between positive and negative numbers.
- An arithmetic sequence has to be ascending or descending. It can't alternate between the two.
- If you start with a positive number, and the common ratio is 0.5, you will eventually reach 0

Explore other sequences

Notes and guidance

This small step provides an opportunity for students to explore less familiar sequences. They explore sequences such as those that oscillate, the triangular numbers and Fibonacci sequences. Square number and cube number sequences could be included but will be looked at again in the next block so could be omitted if time is short. Concrete manipulatives, such as multi-link cubes allow students to represent the sequences and support their reasoning.

Key vocabulary

Square	Triangular	Cube
Oscillate	Predict	Fibonacci

Key questions

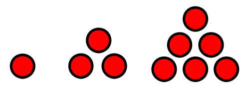
How can you represent the sequence using multi-link cubes? How does this help you justify your answer?

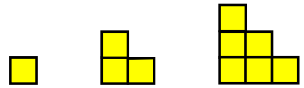
Is (e.g.) 5, 3, 5, 3, 5, 3…. a sequence? Explain the rule.

How do the square numbers differ from cube numbers?

Exemplar Questions

Here are two different ways of representing triangular numbers.





For each, draw pattern 4 What is the 4^{th} triangular number?
What is the 10^{th} triangular number? Explain how you found it.

Eva says: 20^{th} triangular number is $20 + 19 + 18 + \ldots + 3 + 2 + 1$

Explain why Eva's calculation is correct.

Eva pairs the numbers in her calculation

$$(20 + 1) + (19 + 2) + (18 + 3)\ldots$$

She says that the 20^{th} triangular number is the same as 21×10

Is Eva right? Explain your answer. Find the 100^{th} triangular number.

1, 1, 2, 3, 5

4, 3, 7, 10, 17

Look at the two Fibonacci sequences. What's the same and what's different?

- Add together the first and last term. Compare with the middle term. What do you notice? Will this always happen?
- Write down 3 Fibonacci sequences which have 7 terms. Subtract the first term from the last term. Compare this with the middle term. What do you notice? Will this always happen?
- Explore further.

$1,\ 1 + 3, 1 + 3 + 5, \ldots$

What's the 1 000^{th} term in this sequence?
What's the 1 000 000^{th} term?
Write your answers in standard form.

Sequences involving surds

H

Notes and guidance

Students have already met simplifying and calculating with surds. They practise their skills in the context of both arithmetic and geometric sequences, either starting with surds, having a common difference/ratio that is a surd or both.

Key vocabulary

Simplest form	Surd	Common ratio
Common difference	Arithmetic	Geometric

Key questions

Does the method for finding the n^{th} term of a sequence change if it involves surds?

Why is simplification important when a sequence involves surds?

How can we work out the common ratio/common difference? How can we simplify this?

Exemplar Questions

The first term in a geometric sequence is 6
The common ratio of the sequence is $\sqrt{2}$
Write down the first five terms in the sequence in their simplest form.
If there are an even number of terms in the sequence, what proportion will be integers?

$$\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots$$

Is the sequence arithmetic or geometric? Find the rule for the n^{th} term.
Calculate the 20th number in the sequence, giving your answer in its simplest form.

$$\_\_\_, \sqrt{8}+1, 3\sqrt{2}+4, \ldots$$

Simplify the second term of this arithmetic sequence.
Find the common difference of the sequence.
Show that the first term of the sequence is $\sqrt{2}-2$

Categorise the sequences into arithmetic and geometric.
For each sequence find the common difference or the common ratio, and the next term.

- $3, 3+\sqrt{5}, 3+2\sqrt{5}, 3+3\sqrt{5}, \_\_\_$
- $2, 2\sqrt{2}, 4, 4\sqrt{2}, 8, \_\_\_$
- $2\sqrt{3}, 12, 24\sqrt{3}, 144, \_\_\_$
- $5+\sqrt{7}, 1-3\sqrt{7}, -3-7\sqrt{7}, -7-11\sqrt{7}, \_\_\_$

n^{th} term of a linear sequence R

Notes and guidance

This small step reviews prior learning. Teachers might consider using sequences with decimal/fractional differences to extend this. The key here is that students understand the connection between the sequence and the associated multiplier. Use of descending sequences can also be used to prompt discussion about the multiplier.

Key vocabulary

Rule	Term-to-term	Position-to-term
Linear	Non-linear	Coefficient

Key questions

What does n represent?

How does the constant difference relate to the coefficient of n?

Why is the coefficient of n in a descending sequence negative?

Exemplar Questions

All of these sequences follow a rule in the form $5n +$ __ or $5n -$ __
Explain why this is the case.

- 2, 7, 12, 17, …
- −20, −15, −10, −5, …
- 13.2, 18.2, 23.2, 28.2, …

Find the n^{th} term for each sequence.

$f,\ f+2,\ f+4,\ f+6, \dots$

Eva

n^{th} term = +2 each time

Amir

n^{th} term $= f + 2$

Ron

n^{th} term $= f + 2n - 2$

Dora

n^{th} term $= f + 2n$

Students are finding the n^{th} term of the sequence.
Who's right? Explain why.

7, __, __, __, __, __, 31

Jack is finding the n^{th} term of this linear sequence. He starts by forming an equation for the 1st term.

n^{th} term $= an + b$
1st term $= 1a + b$
$7 = a + b$

- Write a second equation using the 7th term.
- Solve the simultaneous equations to find a and b
- Use the rule for the n^{th} term to find the missing values in the sequence.
- Show that 217 is not in the sequence. Show that 75 is in the sequence.

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n^{th} term quadratic sequence H

Notes and guidance

Students must be secure in finding the n^{th} term of a linear sequence before starting this small step. Through exploration, students should spot the link between the second difference and the coefficient of n^2. Once this is established, students can then compare an^2 with the given quadratic sequence (by finding the difference) allowing them to find $bn + c$. If appropriate, simultaneous equations and proof can be interleaved here.

Key vocabulary

Term	Difference	Linear
Quadratic	Coefficient	Show

Key questions

What's the relationship between the second difference and the coefficient of n^2?

What type of sequence is generated when comparing an^2 to the given quadratic sequence?

What are the steps in finding the n^{th} term of a quadratic sequence?

Exemplar Questions

Amir is investigating differences in quadratic sequences.

n^2	1		4		9		16
1st Difference		3		5		7	
2nd Difference			2		2		

Sequence	2nd difference	n^{th} term
1, 4, 9, 16	2	n^2
2, 8, 18, 32		$2n^2$
3, 12, 27, 48		
4, 16, 36, 64		

Follow Amir's method to complete the table. What do you notice about the relationship between the 2nd difference and the coefficient of n^2?

Mo is finding the n^{th} term of 1, 10, 23, 40

Copy and complete his steps.

Sequence				n^{th} term
2	8	18	32	$2n^2$
↓ -1	↓ 2	↓ __	↓ __	$3n$ _____
1	10	23	40	$2n^2+3n$____

2nd difference is ___ so the coefficient of n^2 is ___

Show that the 20th term is 856

Find the n^{th} term of the following sequences.

- 2, 9, 22, 41
- 2, 16, 42, 80
- 1, 8, 21, 40
- 4, 18, 44, 82

Find the n^{th} term of the sequence -2, 9, 24, 43

Which term in the sequence has a value of 241?

Indices and Roots

Small Steps

- Square and Cube numbers (R)
- Calculate higher powers and roots
- Powers of ten and standard form (R)
- The addition and subtraction rules for indices (R)
- Understand and use the power zero and negative indices
- Work with powers of powers
- **Understand and use fractional indices** (H)
- Calculate with numbers in standard form (R)

(H) denotes Higher Tier GCSE content

(R) denotes 'review step' – content should have been covered at KS3

Square and Cube numbers R

Notes and guidance

Students are usually less familiar with the cube numbers than the square numbers, so regular revisiting is important. Linking to area and volume is helpful, as is showing how the cubes can be found easily from the squares. It is helpful if students can commit the first 12 square numbers to memory and at least the first five cubes, particularly to support the finding of square and cube roots. Revisiting Pythagoras' theorem may also be useful.

Key vocabulary

Square	Cube	Root	Prime
Prime factorisation		Integer	

Key questions

What's the difference between the square of a number and the square root of a number?

How do we find squares/cubes/roots on a calculator?

What do we know about a number if its square root is an integer?

Exemplar Questions

Continue the sequences as far as you can with the square and cube numbers you know.

n	1	2	3	
n^2	1	4	9	
n^3	1	8	27	

How can you work out the next terms in the sequences

- using a calculator?
- using a written method?

Which of the numbers have integer square roots, integer cube roots or both?

81 64 125 1 1000 196

Dora says you can tell if a number is square by looking at its prime factorisation.

$$324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$$
$$= (2 \times 3 \times 2)^2$$
$$= 18^2$$

Use prime factorisation to determine if these numbers are square.

- 256
- 216
- 625
- 400
- 1024
- 729

If they are square, state their square roots.

Are any of the numbers cube numbers?

Higher powers and roots

Notes and guidance

The key point of this step is to ensure students are familiar with the notation rather than performing all the calculations by hand. It is often far more appropriate to use a calculator to work out these values and students may need to be taught how to use the x^n (or equivalent) key. The Higher tier requires students to estimate higher powers and roots, so this has been included as an extension question here. Familiarity with higher powers will help with fractional indices later.

Key vocabulary

Root	Power	Index/Indices
Fourth root	Estimate	Exponent

Key questions

What does "to the power (e.g.) 4" mean? Can you say this another way?

How do you use a calculator to quickly find a number to the 4th, 5th… power?

How do you use a calculator to find the 4th, 5th… root of a number?

Exemplar Questions

Match the cards of equal value.

8^2 2^6 4^4 16^2 2^8 4^3 64^1

Using a calculator, or otherwise, work out the values.

- 4^5
- 5^4
- 2^{10}
- $(-2)^6$
- 0.1^3
- $\sqrt[4]{6\,561}$

Using a calculator, or otherwise, work out the values.

- $(-1)^1$
- $(-1)^2$
- $(-1)^3$
- $(-1)^4$
- $(-1)^5$

What do you notice?

Write down the value of

- $(-1)^6$
- $(-1)^{11}$
- $(-1)^{204}$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

The last digit of powers of 2 follow a repeating pattern: 2, 4, 8, 6, 2, 4 …

Investigate the pattern formed by last digits of

- powers of 5
- powers of 10
- powers of 3
- powers of 8

$\sqrt{620\,000} = \sqrt{62} \times \sqrt{10\,000}$

$\approx 8 \times 100 = 800$

Use Whitney's strategy, or another method, to estimate:

- $\sqrt{890}$
- $\sqrt{39\,000}$
- 611^3
- $\sqrt[3]{28\,000\,000}$
- 18.5^4

White Rose Maths

Standard Form

R

Notes and guidance

As students have been working with numbers in standard form since Year 8, the conversions chosen for the exemplar questions are in the context of simple calculations rather than just "convert" questions. It is always useful to look at numbers in context e.g. populations, land areas, atoms etc. to provide meaning. This is also a good chance to revisit words like million, billion etc. Some students may need to be supported with a place value chart.

Key vocabulary

Standard form	Power	Index/Indices
Exponent	Million/Billion	

Key questions

How can you tell if a number is written in standard form or not?

How can you convert a number greater than 1 /less than 1 to/from standard form?

What numbers do you look at first when comparing numbers written in standard form?

Exemplar Questions

Which of these numbers are not in standard form? Explain why.

3.7×10^{-2} | $8.5 \times 10^{0.5}$ | 0.5×10^{8} | 11×10^{4}

Work out the calculations, giving your answers in standard form.

- $3.7 \times 1\,000$
- 26×100
- $10 \times 1.4 \times 1\,000$
- 0.3×100
- $0.16 \times 10\,000$
- $35.7 \times 100\,000$
- $1.5 \div 10$
- $6.8 \div 1\,000$
- $1.407 \div 100$
- $93 \div 1\,000$
- $204 \div 10\,000$
- $0.55 \div 1\,000$

The mass of Saturn is 5.6×10^{26} kg.

The mass of Uranus is 8.7×10^{25} kg.

Which planet is heavier? Explain how you know.

Work out the calculations, giving your answers in standard form.

- $3\,000 \times 200$
- $60\,000 \div 20$
- $5\,000 \times 4\,000$
- $400 \div 2\,000$
- $400 \div 20\,000$
- $40\,000^2$
- 70×0.000120
- 0.001 0.3^3

Identify the larger number in each pair.

$30\,000$	3×10^{5}
0.006	5×10^{-3}
1 billion	8×10^{8}
6.1×10^{-2}	0.016
0.0001	1×10^{-5}
4×10^{17}	7×10^{14}

Addition/Subtraction indices R

Notes and guidance

Students have met the rules of indices at key stage 3, so this review step is designed to reinforce their prior learning. It is helpful to look at questions with both numerical and algebraic bases, and also to include questions that involve both the addition and subtraction of indices. Negative results could be included here if appropriate, but these are covered in detail in the next step. It is always worth reminding students that a and a^1 are equivalent.

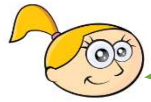
Key vocabulary

Base	Index/Indices	Simplify
Power	Exponent	

Key questions

What is the difference between a base and an index?
How can you simplify the multiplication of two terms involving indices if they have the same base?
Can you use the same rule if the bases are different?
Why is (e.g.) $a^6 \div a = a^5$ when there is no index on the second term?

Exemplar Questions



$3^4 \times 3^5 = 3^{20}$

By writing out the calculation in full, show that Eva is wrong.

Write these expressions as a single power of 3

- $3^5 \times 3^2$
- $3^5 \times 3^{12}$
- $3^5 \times 3$
- $3^5 \times 3^5 \times 3^5$

$$2^6 \div 2^3 = \frac{2^6}{2^3} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{\square} = 2^{\square}$$

Complete the calculation.
Which is the correct generalisation?

$a^m \div a^n = a^{m \div n}$ or $a^m \div a^n = a^{m-n}$

Write these expressions as single powers of a.

- $a^{10} \div a^2$
- $a^6 \div a$
- $a^7 \div a^5 \times a^2$
- $a^7 \div (a^5 \times a^2)$

Find the values of the letters.

- $2^5 \times 2^a = 2^{10}$
- $2^8 \div 2^b = 2^5$
- $2^5 \times 2^c \div 2^3 = 2^2$
- $2^d \times 2^d = 2^8$
- $2^9 \div 2^e = 2^8$
- $2^3 \times 2 \times 2^f = 2^8$

Do you agree?
Why or why not?

$2^3 \times 5^4 = 10^7$

Zero and negative indices

Notes and guidance

The common misconception that a number raised to the power zero gives the result zero needs to be addressed and revisited often. Similarly, students often confuse negative indices with negative numbers, so deriving the rules to provide meaning is a helpful strategy, as is comparing with earlier experience of standard form. Using a calculator to verify results is useful here, and it is useful to link to the previous step, particular using $a^m \div a^n = a^{m-n}$ where $m - n \leq 0$

Key vocabulary

Base	Index/Indices	Negative
Power	Exponent	Simplify

Key questions

What is the result when you divide a number by itself?
What is the value of a^0 for any value of a?
Can you use negative numbers when finding powers? Is this the same as, or different from negative indices?
How do negative powers of 10 connect with standard form?

Exemplar Questions

$5^3 \div 5^3 = 1$ as any number divided by itself is 1

But $5^3 \div 5^3 = 5^{3-3} = 5^0$ So $5^0 = 1$

Use Rosie and Mo's reasoning to write down the answers to

- 6^0
- 7^0
- $\left(\frac{1}{2}\right)^0$
- 37^0
- $2^3 \times 2^0$

I know from standard form that $10^{-1} = \frac{1}{10}$ and $10^{-2} = \frac{1}{10^2}$, so in the same way $3^{-1} = \frac{1}{3}$

Amir is correct. Use his reasoning to match the cards of equal value.

3^{-2}	3×-2	3^0	3^2	3^{-3}	0^3	-3^3
-6	0	$\frac{1}{9}$	-27	1	$\frac{1}{27}$	9

All the statements are wrong. Correct them.

$5^0 = 0$	$3 \times 2^{-1} = \frac{1}{6}$	$4^{-1} = -4$	$2^3 \times 2^{-2} = 2^{-6}$
$4 \times 8^{-1} = \frac{1}{32}$	$8^9 \div 8^3 = 1^6$	$6^3 \times 6 = 6$	$6^{-2} = \frac{1}{12}$

Powers of powers

Notes and guidance

Some students will have met $\left(a^b\right)^c = a^{bc}$ at KS3, but this may well be new to Foundation tier students. Again, deriving the law from writing calculations in full helps understanding and retention. Students should also use this law in conjunction with those from the previous step, deciding which rule to use in which situation. This is a good point at which to consider questions of the form $(3x^5)^4$ as well as e.g. $5x^3y^2 \times 3x^4y^5$ and divisions.

Key vocabulary

Base	Index/Indices	Negative
Power	Exponent	Simplify

Key questions

Will $\left(a^b\right)^c$ be the same as, or different from $(a^c)^b$? Why?

Why do we need to be careful with expressions like $(6x^2)^3$?

How would you start solving an index question that involves more than one operation?

Exemplar Questions

$(2^5)^3 = 2^5 \times 2^5 \times 2^5 = 2^{5+5+5} = 2^{15}$

Use the same reasoning to work out

- $(4^3)^2$
- $(6^2)^4$
- $(9^4)^3$

What is the "quick way" of finding the answers to the questions?

Generalise $\left(a^b\right)^c = a$

Write down the answers to

- $(5^6)^4$
- $(6^8)^5$
- $(7^6)^a$
- $(b^9)^3$
- $(2^{-2})^5$

Find the values of the letters.

$(3^{10})^4 = 3^a$	$(6^a)^4 = 6^{16}$	$(4^5)^3 \times 4^b = 4^{27}$
$(5^c)^2 \times 5^4 = 5^{10}$	$7^d \div (7^3)^4 = 1$	$(7^{-2})^e = 7^8$

The answer to a question is a^{20}.
Find some different possible questions, using as many indices rules as you can.

Which is the correct answer to $(2c^5)^3$?

- $6c^{15}$
- $8c^{35}$
- $8c^{15}$
- $6c^{53}$

Write these expressions as single powers of 2

- 4
- 4^3
- 8^5
- 8^{-1}
- 16^{-2}
- $4^2 \times 8^{-2}$
- 3^0

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Fractional Indices

H

Notes and guidance

As well as covering the meaning of indices that are unit fractions, this step extends understanding to look at non-unit fractions though 'reversing' the powers of powers law. Familiarity with square, cube and higher roots is vital here. As students have already met irrational numbers, this is a good point at which to revisit surds e.g. by linking $2^{\frac{1}{2}}$ and $\sqrt{2}$ or $8^{\frac{1}{2}} = \sqrt{8} = 2\sqrt{2}$ etc.

Key vocabulary

Power	Index	Root
Unit-fraction	Non-unit fraction	

Key questions

What's the difference between "finding one half" and "raising to the power one half"?

If you know the value of (e.g.) $x^{\frac{1}{3}}$, how can you find the value of $x^{-\frac{1}{3}}$?

What are the steps in finding e.g. $125^{-\frac{2}{3}}$?

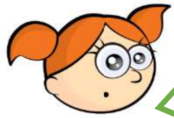
Exemplar Questions

$9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9^1$

So $9^{\frac{1}{2}} = \sqrt{9} = 3$

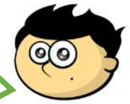
Use Annie's reasoning to work out

- $81^{\frac{1}{2}}$
- $64^{\frac{1}{2}}$
- $64^{\frac{1}{3}}$
- $64^{-\frac{1}{3}}$
- $16^{\frac{1}{4}}$



$16^{\frac{3}{4}} = \left(16^{\frac{1}{4}}\right)^3 = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$

$16^{\frac{3}{4}} = (16^3)^{\frac{1}{4}} = \sqrt[4]{16^3} = \sqrt[4]{4\,096} = 8$



Which method do you prefer?

Showing all your working, find the values of

- $81^{\frac{3}{2}}$
- $64^{\frac{2}{3}}$
- $100^{-\frac{5}{2}}$
- $125^{-\frac{2}{3}}$
- $32^{0.8}$
- $16^{\frac{5}{4}}$
- $\left(\frac{64}{125}\right)^{\frac{2}{3}}$
- $\left(\frac{64}{125}\right)^{-\frac{2}{3}}$
- $\left(\frac{81}{16}\right)^{\frac{3}{4}}$
- $\left(\frac{16}{25}\right)^{-1.5}$

Find the values of the letters.

- $16^{\frac{3}{4}} \times 64^{\frac{2}{3}} = 2^a$
- $100^{\frac{3}{2}} \times 1\,000^{\frac{2}{3}} = 10^b$
- $72^{\frac{1}{2}} = c\sqrt{d}$
- $9^{-\frac{1}{2}} \times 27^{\frac{2}{3}} = 3^{e+2}$
- $8^{\frac{3}{2}} = f\sqrt{2}$

Standard Form Calculations R

Notes and guidance

This step revises KS3 work, and includes solving problems both with and without a calculator. Non-calculator work in particular is useful in reinforcing the laws of indices from earlier in this block. Care needs to be taken to establish the different ways of working when adding and subtracting rather than multiplying or dividing. It may be useful to remind students how to round to significant figures and how this works with numbers given in standard form.

Key vocabulary

Standard form	Power	Index/Indices
Exponent	SCI/EXP	Scientific Notation

Key questions

How do you input a number in standard form in your calculator? Is it the same or different if the power of 10 is negative?

What's the same and what's different about adding/subtracting and multiplying/dividing standard form numbers without a calculator?

Exemplar Questions

Correct the errors in these calculations.
Give all answers in standard form.

- $(3 \times 10^5) \times 4 = 12 \times 10^{20}$
- $(8 \times 10^{10}) \div 2 = 4 \times 10^5$
- $(7 \times 10^4) \times (6 \times 10^3) = 42 \times 10^7$
- $(3 \times 10^4) + (2 \times 10^5) = 5 \times 10^9$
- $(8.4 \times 10^6) \div (2 \times 10^3) = 4.2 \times 10^2$
- $(6 \times 4) \times 10^0 = 0$

Solve the problems without using a calculator, giving all answers in standard form unless otherwise stated.

- What number is one million times bigger than 9×10^7?
- What number is one million times smaller than 9×10^7?
- What number is one million greater than 9×10^7?
- What number is one million smaller than 9×10^7?
- Show that $4 \times 10^{-2} + 5 \times 10^{-3} = \frac{9}{200}$
- Work out $(3.16 \times 10^4) \times (4 \times 10^{-2})$
- Work out $(3.16 \times 10^4) \div (4 \times 10^{-2})$

The mass of the Sun is 1.989×10^{30} kg.
The mass of the Earth is 5.792×10^{24} kg.
How many times heavier than the Earth is the Sun?
Give your answer in standard form, correct to 3 significant figures.