Autumn Term

Year 11

#MathsEveryoneCan

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WRM – Year 11 Scheme of Learning

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Graphs**						**Algebra**					
	Gradients & lines		Non-linear graphs		Using graphs		Expanding & factorising		Changing the subject		Functions	
Spring	**Reasoning**						**Revision and Communication**					
	Multiplicative		Geometric		Algebraic		Transforming & constructing		Listing & describing		Show that...	
Summer	**Revision**						**Examinations**					

Autumn 1: Graphs

Weeks 1 and 2: Gradients and lines

This block builds on earlier study of straight line graphs in years 9 and 10. Students plot straight lines from a given equation, and find and interpret the equation of a straight line from a variety of situations and given information. There is the opportunity to revisit graphical solutions of simultaneous equations. Higher tier students also study the equations of perpendicular lines.
National Curriculum content covered includes:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- plot and interpret graphs
- interpret the gradient of a straight line graph as a rate of change
- use the form $y = mc + c$ to identify parallel **{and perpendicular}** lines; find the equation of the line through two given points, or through one point with a given gradient
- find approximate solutions to two simultaneous equations in two variables (linear/linear **{or linear/quadratic})** using a graph

Weeks 3 and 4: Non-linear graphs

Students develop their knowledge of non-linear graphs in this block, looking at quadratic, cubic and reciprocal graphs, so they recognise the different shapes. They find the roots of quadratics graphically, and will revisit this when they look at algebraic methods in the Functions block during Autumn 2, where they will also look at turning points. Higher tier students also look at simple exponential graphs and the equation of a circle. Note that the equation of the tangent to a circle is covered later when the circle theorem of tangent/radius is met. Higher students also extend their understanding of gradient to include instantaneous rates of change considering the gradient of a curve at a point.
National Curriculum content covered includes:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function 1 **{the exponential function $y = k^x$ for positive values of k}**
- plot and interpret graphs (including reciprocal graphs **{and exponential graphs}**)
- find approximate solutions using a graph
- identify and interpret roots, intercepts of quadratic functions graphically
- **{recognise and use the equation of a circle with centre at the origin;}**

Weeks 5 and 6: Using graphs

This block revises conversion graphs and reflection in straight lines. Students also study other real-life graphs, including speed/distance/time, constructing and interpreting these. Higher tier students also investigate the area under a curve.
National Curriculum content covered includes:

- plot and interpret graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- **{interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of instantaneous and average rate of change (gradients of tangents and chords) in numerical, algebraic and graphical contexts}**
- **{calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts}**

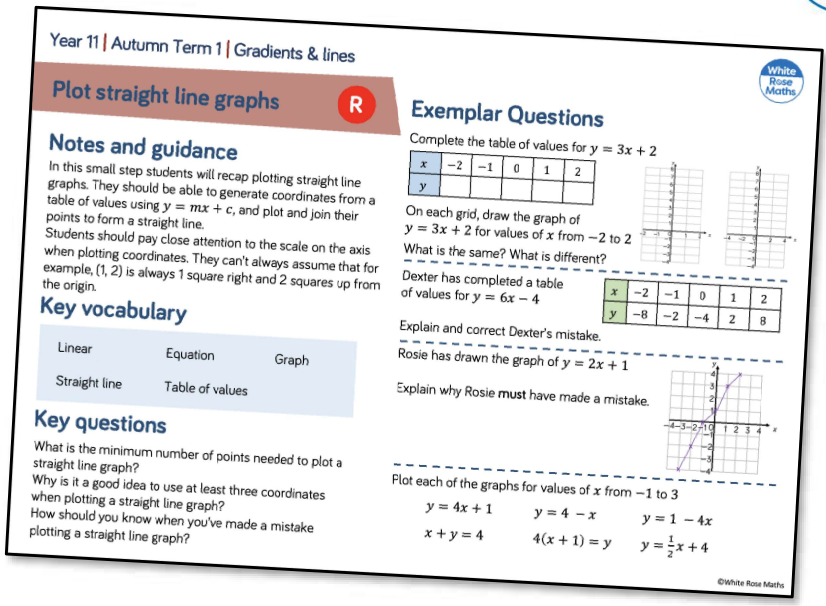
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



Year 11 | Autumn Term 1 | Gradients & lines

Plot straight line graphs R

Notes and guidance

In this small step students will recap plotting straight line graphs. They should be able to generate coordinates from a table of values using $y = mx + c$, and plot and join their points to form a straight line.
Students should pay close attention to the scale on the axis when plotting coordinates. They can't always assume that for example, (1, 2) is always 1 square right and 2 squares up from the origin.

Key vocabulary

Linear, Equation, Graph, Straight line, Table of values

Key questions

What is the minimum number of points needed to plot a straight line graph?
Why is it a good idea to use at least three coordinates when plotting a straight line graph?
How should you know when you've made a mistake plotting a straight line graph?

Exemplar Questions

Complete the table of values for $y = 3x + 2$

x	−2	−1	0	1	2
y					

On each grid, draw the graph of $y = 3x + 2$ for values of x from −2 to 2
What is the same? What is different?

Dexter has completed a table of values for $y = 6x - 4$

x	−2	−1	0	1	2
y	−8	−2	−4	2	8

Explain and correct Dexter's mistake.

Rosie has drawn the graph of $y = 2x + 1$
Explain why Rosie **must** have made a mistake.

Plot each of the graphs for values of x from −1 to 3

$y = 4x + 1$ $\quad y = 4 - x \quad y = 1 - 4x$

$x + y = 4 \quad 4(x + 1) = y \quad y = \frac{1}{2}x + 4$

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- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol 💡.
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R.

Gradients & lines

Small Steps

- Equations of lines parallel to the axis R
- Plot straight line graphs R
- Interpret $y = mx + c$ R
- Find the equation of a straight line from a graph (1) R
- Find the equation of a straight line from a graph (2)
- Equation of a straight-line graph given one point and gradient
- Equation of a straight-line graph given two points
- Determine whether a point is on a line

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Gradients & lines

Small Steps

- Solve linear simultaneous equations graphically (R)
- **Recognise when straight lines are perpendicular** (H)
- **Find the equations of perpendicular lines** (H)

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Lines parallel to the axis

R

Notes and guidance

In this small step students will revise and extend their learning from previous years. They should be able to recognise and use the equations of lines parallel to the axis. Students should understand that any point on a line satisfies the equation of that line. They should know that all lines of the form $y = a$ are parallel to the x-axis and each other, and all lines of the form $x = b$ are parallel to the y-axis and each other.

Key vocabulary

Parallel	Horizontal	Vertical	Straight line
Axis	Equation	Graph	Intercept

Key questions

Which axis is $y = 4$ parallel to? How do you know?

All of the points on the line $x = 7$ have something in common. What is it?

What is the equation of the x-axis?

What is the equation of the y-axis?

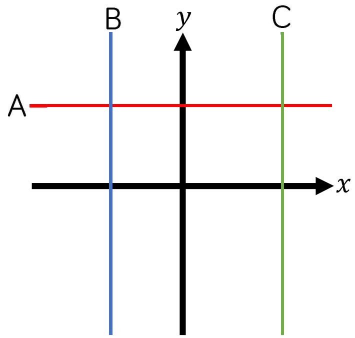
Exemplar Questions

Which of these points lie on the line $y = 9$?

$(9, 0)$ $(0, 0)$ $(0, 9)$ $(9, 9)$ $(9, 2.7)$

$(\frac{1}{2}, 9)$ $(\sqrt{81}, 3)$ $(8, 3^2)$ $(\frac{18}{2}, \frac{27}{3})$ $(1, -9)$

Lines A, B and C are all parallel to one of the axes.



Line A passes through the point $(2, 7)$

Line B passes through the point $(-3, -5)$

Lines A and C intercept at $(32, a)$

Write down the equation of each line.

What is the value of a?

Here are the equations of 8 lines, some of which need simplifying.

$y = 7$ $x + 4 = 11$ $y - 5 = 0$ $x - 5 = 0$

$x = 7$ $y + 3 = 7 - 2$ $-y = -9$ $y = 0$

- Which of these lines are parallel to the y-axis?
- Which of these lines are parallel to the x-axis?
- Which of these lines are parallel to neither the x- nor the y-axis?

Plot straight line graphs

R

Notes and guidance

This step revisits plotting straight line graphs. Students should be able to generate coordinates for a table of values using $y = mx + c$, and plot and join their points to form a straight line. Students should pay close attention to the scale on the axis when plotting coordinates. They cannot always assume that for example, (1, 2) is always 1 square right and 2 squares up from the origin, so this misconception should be challenged.

Key vocabulary

Linear	Equation	Graph
Straight line	Table of values	

Key questions

What is the minimum number of points needed to plot a straight line graph?

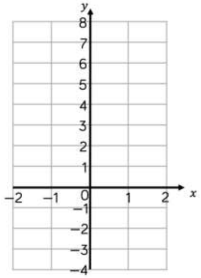
Why is it a good idea to use at least three coordinates when plotting a straight line graph?

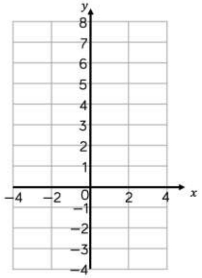
How can you tell when you've made a mistake plotting a straight line graph?

Exemplar Questions

Complete the table of values for $y = 3x + 2$

x	−2	−1	0	1	2
y					





On each grid, draw the graph of $y = 3x + 2$ for values of x from −2 to 2

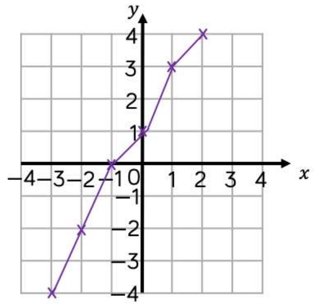
What is the same? What is different?

Dexter has completed a table of values for $y = 6x - 4$

x	−2	−1	0	1	2
y	−8	−2	−4	2	8

Explain and correct Dexter's mistake.

Rosie has drawn the graph of $y = 2x + 1$



Explain how you know Rosie **must** have made a mistake.

Plot each of the graphs for values of x from −1 to 3

$y = 4x + 1$ $\quad$ $y = 4 - x$ $\quad$ $y = 1 - 4x$

$x + y = 4$ $\quad$ $4(x + 1) = y$ $\quad$ $y = \frac{1}{2}x + 4$

Interpret $y = mx + c$

R

Notes and guidance

Students may need reminding that when the equation of a line is given in the form $y = mx + c$, m represents the gradient, and the graph intercepts the y-axis at $(0, c)$. Building on from the previous step, students could be encouraged to plot the straight lines $y = mx + a$ and $y = mx + b$ to see that they are parallel. Similarly, they could plot $y = mx + a$ and $y = nx + a$ to see that they intercept the y-axis at the same point.

Key vocabulary

Gradient	y-intercept	Equation
Parallel	Linear	Straight line

Key questions

In $y = mx + c$, what do m and c represent?

In $y = mx + c$, what do x and y represent?

What does it mean when two lines have the same gradient?

What does it mean when two lines have the same y-intercept?

Exemplar Questions

Draw each of the graphs on the same set of axis.

$y = 3x$ $y = 3x + 1$ $y = 3x + 2$ $y = 3x + 5$

What do you notice?
What do you think the graph of $y = -3x$ will look like?

Draw each of the graphs on the same set of axis.

$y = x + 1$ $y = 2x + 1$ $y = 3x + 1$ $y = 4x + 1$

What do you notice?

Write down the gradient and y-intercept of each line.

$y = 5x + 7$ $y = 5x - 7$ $y = 7 - 5x$ $y = -7 - 5x$

$y = \frac{1}{2}x$ $17 - 8x = y$ $y = 3(2x + 1)$ $2y = 10 + 6x$

$y = 9 + 2x$	$y = 8 - 2x$
$2y = 2x + 16$	$y = 4\left(\frac{1}{2}x + 9\right)$

Which lines are parallel?

Which lines have the same y-intercept?

How do you know?

Equation of a line from a graph R

Notes and guidance

Some students may need to revise finding the gradient of a line before they find its equation. This step reiterates that the gradient is m and the y-intercept is c, but sometimes students find it conceptually more difficult to 'work backwards' in this way. It is helpful to consider what information can be seen immediately from the graph (usually the y-intercept) before calculating the gradient. This step focuses on graphs will simple equal scales, with more complex scales to follow.

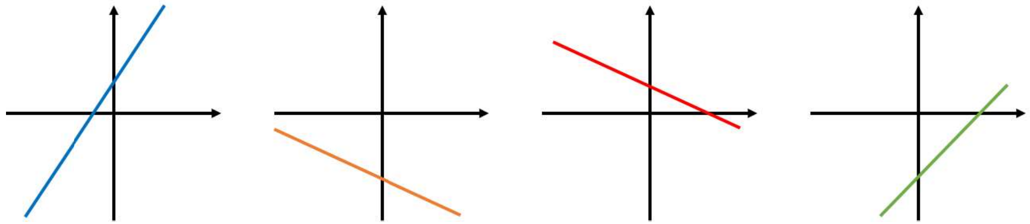
Key vocabulary

Gradient	y-intercept	Equation
Parallel	Linear	Straight line

Key questions

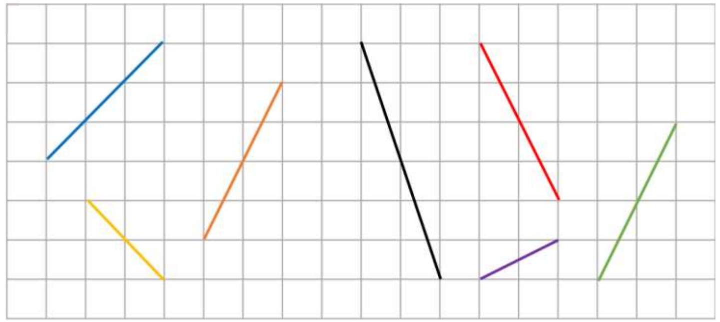
How do you know if a straight line has a positive/negative gradient?
How do you know if a straight line has a positive/negative y-intercept?
How do you calculate the gradient of a line?

Exemplar Questions

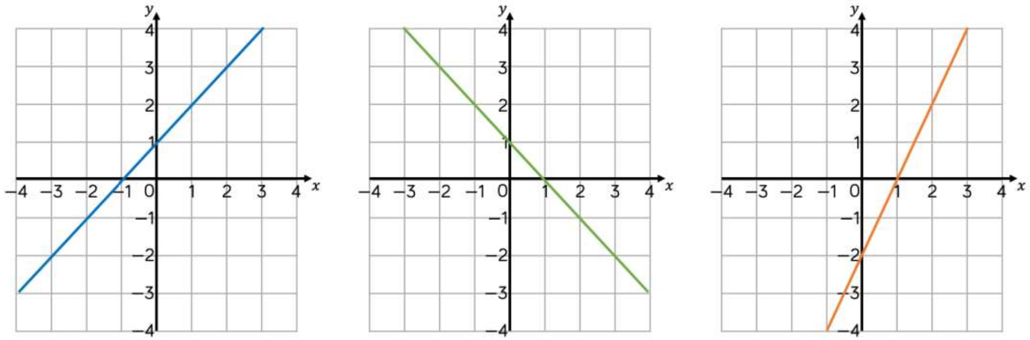


Is the gradient of each line positive or negative? How do you know?
Is the y-intercept of each line positive or negative? How do you know?

What is the gradient of each line?



What is the equation of each line?



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Equation of a line from a graph (2)

Notes and guidance

Building on from the previous small step, students will now look at finding the equation of a line from a graph where the axes are more complex. Rather than thinking of the gradient as 'for every 1 square across, how many squares up/down', students now need to shift their thinking to consider the gradient as being 'for every 1 unit across, how many units up/down' and then extending further to 'change in y divided by change in x'.

Key vocabulary

Gradient	y-intercept	Equation	Axis
Parallel	Linear	Straight line	Scale

Key questions

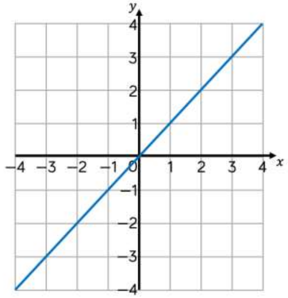
What is the scale on each axis?

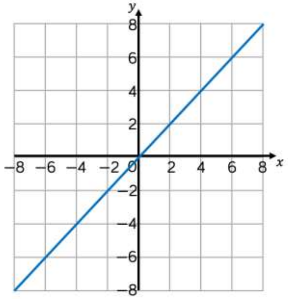
How does the scale affect the gradient?

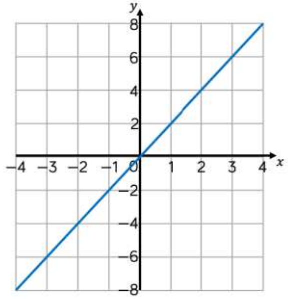
Does the scale on the axis affect how you find out the y-intercept?

Exemplar Questions

What is the equation of each line?

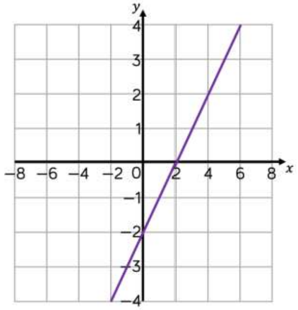






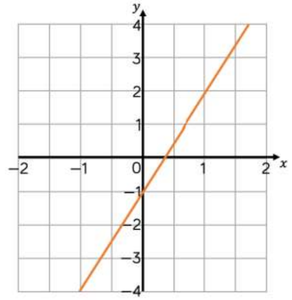
What is the same? What is different?

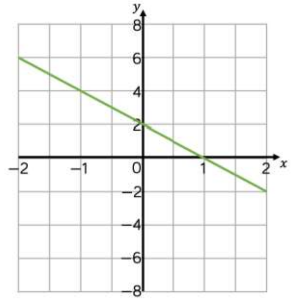
The equation of the line is
$y = 2x - 2$

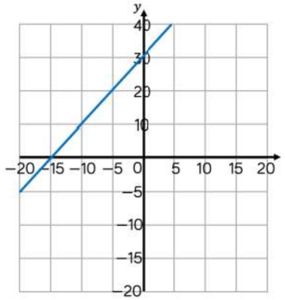


What mistake has Dora made?

Work out the equation of each line.







Equation of a line – point & gradient

Notes and guidance

Students need to be able to find the equation of a line given the gradient and a point that lies on the line. Using their knowledge of parallel lines having the same gradient, they can find the equation of a line parallel passing through a point. Students need to be exposed to examples where the point is the y-intercept and where they need to calculate the y-intercept themselves.

Key vocabulary

Equation	Line	Gradient	y-intercept
Parallel	Point	Coordinates	Substitute

Key questions

Is the point you've been given the y-intercept?

If not, how can you work out the y-intercept?

What does it mean when two lines are parallel?

Exemplar Questions

The gradient of line A is 4
Line A passes through the point (0, 5)
What is the equation of line A?

Line B is parallel to line A and passes through the point (0, −2)
What is the equation of line B?

A line has a gradient of −2 and passes through the point (1, −4)

What is the equation of the line?

A straight line has a gradient of $\frac{1}{2}$ and passes through the point (−2, 0)

The equation of the line is $y = \frac{1}{2}x - 2$

What mistake has Tommy made?

Work out the equation of a line parallel to $2y - 8 = 4x$ that passes through the point (−5, −7).

L_1 passes through the points (2, 7) and (12, 32)
L_2 is parallel to L_1 and passes through the point (4, 12)
Work out the equation of L_2

Equation of a line - two points

Notes and guidance

Students now need to be able to work out the equation of a line from two points. They should start by working out the equation of a line where one of the points is the y-intercept. They will then need to use their knowledge of substitution and solving equations to work out the y-intercept for themselves. It is essential they understand that to calculate the y-intercept of any line, they need to substitute $x = 0$

Key vocabulary

Gradient	y-intercept	Equation
Parallel	Linear	Straight line

Key questions

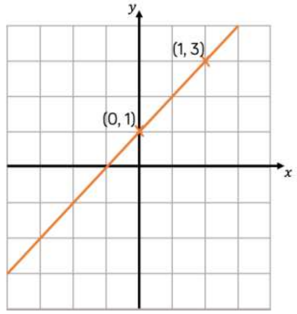
Is the gradient positive or negative? How do you know?

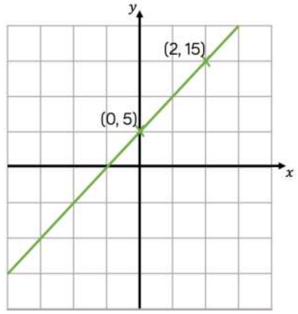
What is the gradient of the line? How do you know?

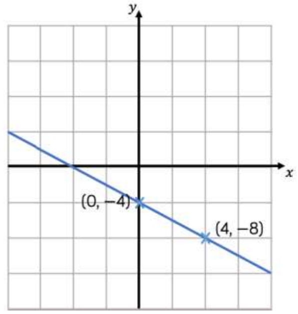
What is the x coordinate at the y-intercept? How do you know?

Exemplar Questions

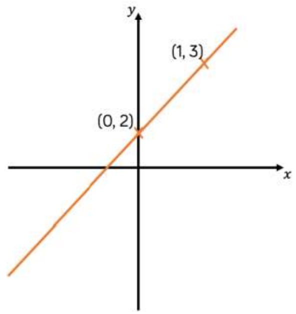
Work out the equation of each line.

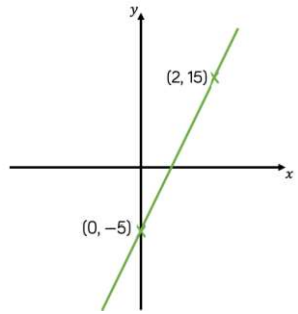


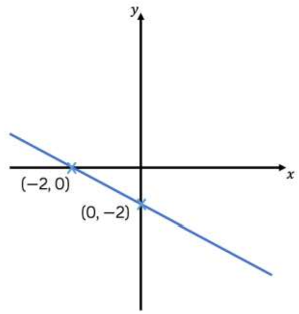




Work out the equation of each line.







Work out the equation of the line that passes through each pair of points.

- $(0, 5)$ and $(3, 14)$
- $(0, 2)$ and $(2, -4)$
- $(-6, -2)$ and $(0, 0)$
- $(2, 5)$ and $(3, 14)$
- $(4, 2)$ and $(2, -4)$
- $(0, a)$ and $(4, a+12)$

Determine whether a point is on a line

Notes and guidance

Students need to understand that the equation of a line is a relationship between the x and y coordinates at any point on that line. For example, on the line $y = x + 3$, every y coordinate is 3 more than the x coordinate. Any point on a grid that does not satisfy this equation, therefore does not lie on the line. Students could be extended further to explore whether a point not on the line is either above or below the line.

Key vocabulary

Equation	Satisfies	Coordinate
Below	Above	Substitute

Key questions

What is the relationship between the x and y coordinates at any point on the line $y = 2x$?

How do you know if a line passes through a point?

How does drawing the graph help you decide if a point is above or below the line? Can you tell without a graph?

Exemplar Questions

Circle the points where the y-coordinate is 3 greater than the x-coordinate.

(5, 2)   (4, 12)   (7, 10)   (1, 2) $(\frac{5}{2}, 5\frac{1}{2})$

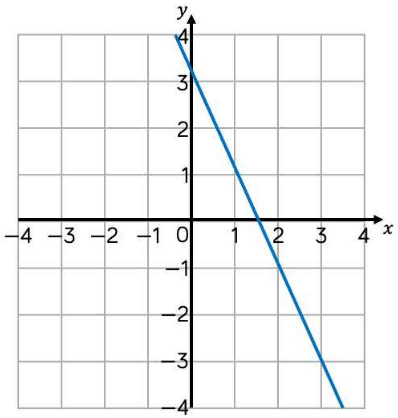
Hence, determine which points lie on the line $y = x + 3$

Does the point (7, 5) lie on the line $y = 2x - 9$?

How do you know?

Show that the point $(8, -8)$ does not lie on the line $y = -\frac{1}{2}x + 4$

The graph shows the line $y = 3 - 2x$



- Does the point $(1, -1)$ lie above or below the line?
- Does the point (3, 4) lie above or below the line?
- Does the point (17, 12) lie above or below the line?
- The point $(a, -15)$ lies on the line. Work out the value of a.

Simultaneous Equations R

Notes and guidance

This small steps provides students with opportunity to revise and extend their knowledge of both solving linear simultaneous equations and plotting linear graphs. They should understand that two straight lines will only ever intercept at a single point, and the coordinates of this point provide the solutions to the pair of simultaneous equations. Students should be aware that where the point of intersection is difficult to interpret, their solutions are estimates.

Key vocabulary

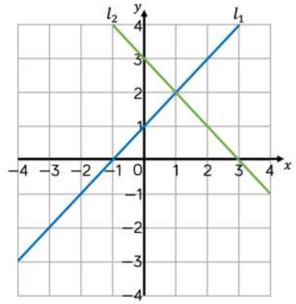
Simultaneous	Equations	Linear
Interception	Coordinates	Solutions

Key questions

How many solutions do a pair of linear simultaneous equations have?
How many points of intersection do a pair of linear graphs have? Is this always the case?
How does knowing the coordinates of a point of intersection help you solve a pair of simultaneous equations?

Exemplar Questions

Two lines, l_1 and l_2 are shown on the graph.



- What is the equation of l_1?
- What is the equation of l_2?
- What are the coordinates of the point of intersection?

Solve the pair of simultaneous equations.

$$3x + y = 8$$
$$5x + y = 14$$

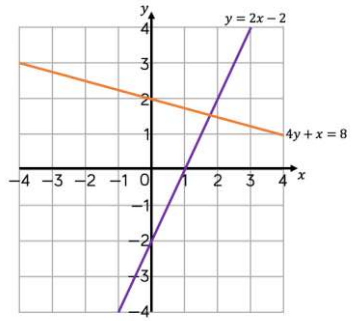
Draw the graph of each line.

$$3x + y = 8$$
$$5x + y = 14$$

What are the coordinates of the point of intersection?

What do you notice? What is the same? What is different?

The graphs of the straight lines with equations $y = 2x - 2$ and $4y + x = 8$ have been drawn on the grid.



Use the graph to estimate the solution to the simultaneous equations.

$$y = 2x - 2$$
$$4y + x = 8$$

Use an algebraic method to find the exact solution.

Recognise perpendicular lines H

Notes and guidance

Students should already be familiar with the fact that perpendicular lines intercept at right angles. They should look at lines $y = 2x$ and $y = -\frac{1}{2}x$ and recognising that the product of the gradients of a pair of perpendicular lines will always be −1. Students need to know that when two lines are perpendicular, one gradient is the negative reciprocal of the other.

Key vocabulary

Parallel	Perpendicular	Gradient
Product	Reciprocal	Negative reciprocal

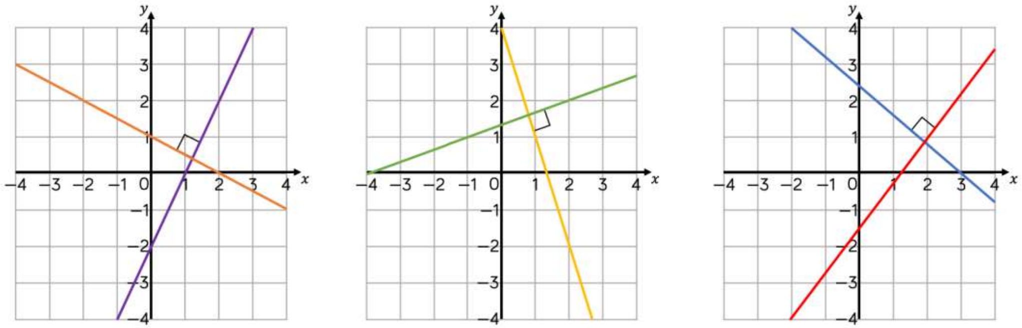
Key questions

When two lines are perpendicular, why must one gradient be positive and one be negative?

What is the product of the gradients of a pair of perpendicular lines?

Exemplar Questions

Each graph shows a pair of perpendicular lines.



Find the product of the gradients of each pair of lines.
What do you notice?

Fill in the missing numbers.

$5 \times \square = -1$ $\quad -\frac{1}{7} \times \square = -1$ $\quad \frac{2}{5} \times \square = -1$

Write down the negative reciprocal of each number.

$2 \quad -9 \quad \frac{3}{2} \quad -\frac{4}{7} \quad 2.5$

Line l_1 is given by the equation $y = 4x - 7$
Line l_2 is given by the equation $4y = 17 - 2x$

Show that l_1 and l_2 are not perpendicular.

Equations of perpendicular lines H

Notes and guidance

Students build on knowledge from the previous step and begin to find the equation of perpendicular lines. Using their understanding of the product of the gradients being −1, they first work out the gradient of a line that will be perpendicular. Once they are secure in this they can also start to calculate the y-intercept given a point on a line. Students could also find the equation of the perpendicular bisector of a given line segment.

Key vocabulary

Parallel	Perpendicular	Gradient
Product	Negative reciprocal	y-intercept

Key questions

How do you work out the gradient of a perpendicular line?
Once you know the gradient, how do you find the y-intercept?
How do you find the midpoint of a line segment? How does this help find the equation of the perpendicular bisector of the line segment?

Exemplar Questions

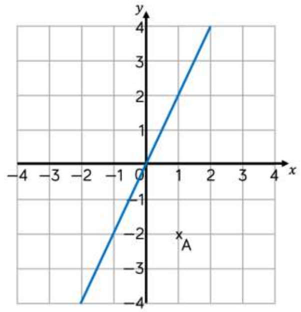
The line l_1 has the equation $y = 3x - 9$
The line l_2 is perpendicular to l_1 and passes through the origin.

What is the equation of l_2?

Point P has coordinates (3, 7).
Point Q has coordinates (9, 9).
Work out the equation of the line perpendicular to PQ that passes through the origin.

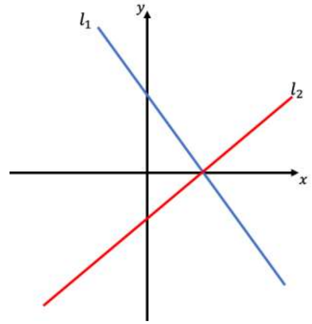
The graph of $y = 2x$ is shown on the grid.

Work out the equation of the line perpendicular to $y = 2x$ that passes through point A.



Two perpendicular lines, l_1 and l_2 are drawn on the grid.
The equation of l_1 is $y = 9 - 3x$
Work out the equation of l_2

Work out the equation of a line parallel to l_2 that passes through the point (8, 11).



Non-linear graphs

Small Steps

- Plot and read from quadratic graphs
- Plot and read from cubic graphs
- Plot and read from reciprocal graphs
- Recognise graph shapes
- Identify and interpret roots and intercepts of quadratics
- **Understand and use exponential graphs** H
- **Find and use the equation of a circle centre (0, 0)** H
- **Find the equation of the tangent to any curve** H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Quadratic graphs

Notes and guidance

Check that students can substitute a negative into an expression containing x^2 and/or $-kx$. Students may also need to revise using a calculator for this. When plotting the graph, make explicit that the points are joined with a smooth curve. In addition, students need to be aware of the shape of the curve so that they avoid just joining up two points either side of a turning point. Before reading from a quadratic graph, check they know equations of vertical and horizontal lines.

Key vocabulary

Quadratic	Parabola	Curve	Substitute
Equation	Vertical	Horizontal	Estimate

Key questions

Why is $(-3)^2$ the same as 3^2?
Is $2x^2$ the same as $2 \times x^2$ or $(2 \times x)^2$?
How could I tell if one of my coordinates was incorrect, or if I had plotted it incorrectly?
Why do we join the points with a smooth curve?
Describe the shape of a parabola.

Exemplar Questions

x^2 $2x^2$ $x^2 - x$

Eva substitutes $x = 3$ into each expression.
Jack substitutes $x = -3$ into each expression.
Jack thinks that he will get the same answers as Eva each time.
Do you agree with Jack? Justify your answer.

Complete the table for $y = x^2 - 2x + 2$

x	-3	-2	-1	0	1	2	3	4
y	17				1			10

Amir plots each coordinate and joins his points with a ruler.
Why is this incorrect?

Draw the graph of $y = x^2 - 2x + 2$ for values of x from -3 to 4

Draw the graph of $y = x^2 + x - 2$ for values of x from -3 to 3

x	-3	-2	-1	0	1	2	3
y		0				4	

On your graph, show that when $x = -0.5$, an estimate for y is -2.3
Why is there more than one answer when estimating x if $y = 1.5$? Draw the line $y = 1.5$ onto your graph and estimate the value of x.
How can you check whether your estimates are accurate?

Cubic graphs

Notes and guidance

Using interactive dynamic software is a powerful way of supporting students to notice features of cubic graphs. Remind students that cubing a negative gives a negative result. A common mistake is for students to multiply by 3 instead of cubing. Ensure that they use a smooth curve to join points. Students sometimes join points either side of a turning point with a flat line; to avoid this error, remind students of the shape of a cubic graph.

Key vocabulary

Cube	Cubic	Estimate
Curve	Substitute	

Key questions

What mistakes can be made when substituting?

How would these 'stick out' when you draw the graph?

Why is it important to use a smooth curve to join the points?

Exemplar Questions

$y = x^3$

x	−3	−2	−1	0	1	2	3
y		−8					27

Complete the table of values.
Draw the graph of $y = x^3$ for values of x from −3 to 3

$y = x^3 + x$

x	−3	−2	−1	0	1	2	3
y		−10					30

Ron thinks that when $x = 1$, $y = 4$, but Alex thinks that when $x = 1$, $y = 2$. Who is correct? How do you know?

- Complete the table of values.
- Draw the graph of $y = x^3 + x$ for values of x from −3 to 3
- How could you tell if one of your coordinates is incorrect?
- Describe the features of the graph.
- Use your graph to estimate the value of x when $y = 5$
- What do you notice about x when $y = -5$?

Teddy thinks that A and C are cubic graphs, but B isn't as it doesn't go through (0,0).
Why is Teddy incorrect?

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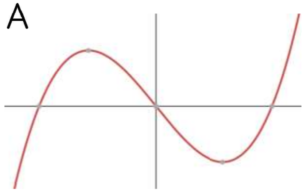

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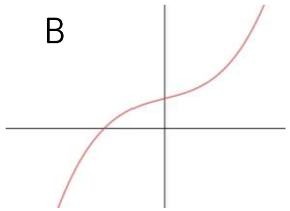

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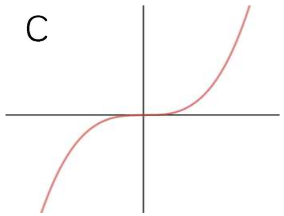


Reciprocal graphs

Notes and guidance

Again, using interactive dynamic software is a powerful way of supporting students to notice features of reciprocal graphs $y = \frac{k}{x}$ and become familiar with the concept of asymptotes. Allow students time to investigate the reciprocal function using their calculators. It is useful to introduce concepts such as infinity and negative infinity to describe the behaviour of the curves at extreme values.

Key vocabulary

Asymptote

Infinity

Reciprocal

Tends towards

Key questions

Why doesn't the graph of $y = \frac{1}{x}$ meet the axes?

What happens at $x = 0$?

What do we mean by infinity?

What are the key features of this graph?

Exemplar Questions

$y = \frac{1}{x}$

x	−4	−3	−2	−1	1	2	3	4
y								

Complete the table of values.

Investigate what happens when x is close to 0 by completing this table of values:

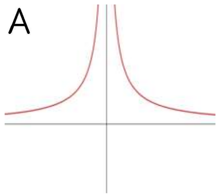
x	−0.4	−0.3	−0.2	−0.1	0.1	0.2	0.3	0.4
y								

Can $x = 0$ when $y = \frac{1}{x}$? Explain your answer.

Draw the graph of $y = \frac{1}{x}$ for x values from −4 to 4

Which of the following sketches would match the graph of $y = \frac{2}{x}$?

A



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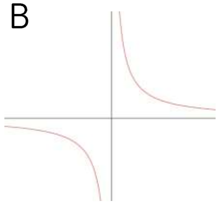

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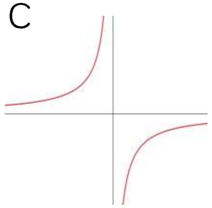

Describe the features of a reciprocal graph.

Complete the sentences about $y = \frac{1}{x}$

When x tends towards negative infinity, y tends towards _____

When x tends towards positive infinity, y tends towards _____

When x tends towards zero, y tends towards _____

Recognise graph shapes

Notes and guidance

In this small step it is important to make explicit the similarities and differences of straight line, quadratic, cubic and reciprocal graphs. Students need to consider the detail of a graph when comparing two of the same type, (e.g. $y = x^3 - 5x$ and $y = x^3$ have very different shapes). In these cases, students may need to substitute a couple of well chosen x values to check which graph matches the equation.

Key vocabulary

Gradient, y-intercept, Quadratic, Cubic, Reciprocal, Infinity, Asymptote

Key questions

What features of a graph help us to identify its equation?

Which types of graphs do you find easier to identify? Why?

If you're not sure which equation matches a graph, what could you do to find out more information?

What's different about a quadratic and a cubic graph?

Exemplar Questions

Use a dynamic geometry package to plot these graphs.
Make a sketch of each one.

$y = x$	$y = -x$
$y = x^2$	$y = -x^2$
$y = x^3$	$y = -x^3$

- What's the same and what's different about each set of graphs?
- What's the same and what's different about each pair of graphs?
- Investigate $y = \pm kx, y = \pm kx^2$ and $y = \pm kx^3$ for different values of k.

Match each graph with its equation.

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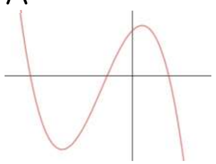

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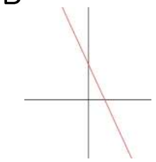

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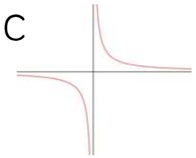

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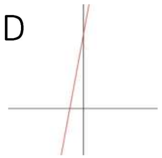

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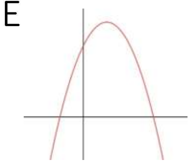

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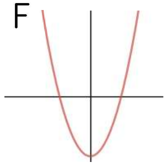


Equation	Letter	Type of Graph
$y = 10x + 10$		
$y = \frac{1}{x}$		
$y = x^2 - 10$		
$y = x^3$		
$y = -x^2 + 2x + 3$		
$y = -2x + 4$		
$y = -x^3 - 2x^2 + x + 1$		

One equation in the table doesn't have a match.
Sketch a graph to match this equation.

Roots and intercepts of quadratics

Notes and guidance

Students start by identifying a root from a graph. They understand that the root of an equation is given when $y = 0$, and should write these as $x = a$. They understand that quadratics can have 0, 1 or 2 roots. Students also locate the y-intercept from a graph and make the connection between this and substitution of $x = 0$ into the equation of the curve. It is important students write the y-intercept as a coordinate.

Key vocabulary

Quadratic	y-intercept	Coordinate
Roots	Solution	Meets

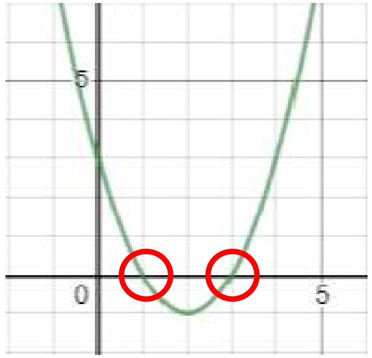
Key questions

Why do we write the y-intercept as a coordinate?
How can we locate the y-intercept from a graph?
How can we locate the roots from a graph?
Why do we write the roots as $x = a$?
How many roots is it possible for a quadratic equation to have? Can a quadratic equation have more than 2 roots? 0 roots?

Exemplar Questions

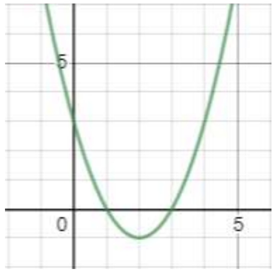
Ron circles in red where the graph intersects the line $y = 0$
One root of $y = x^2 - 4x + 3$ is $x = 1$
Write down another root.

$y = x^2 - 4x + 3$

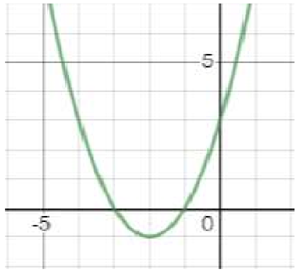


Ron checks that $x = 1$ is a root by substituting $x = 1$ into $y = x^2 - 4x + 3$
$y = 1^2 - 4 \times 1 + 3$
$y = 0$, so $x = 1$ is a root as $y = 0$
Check that the second root also gives $y = 0$

$y = x^2 - 4x + 3$



$y = x^2 + 4x + 3$



Each graph meets the y-axis at (0, 3)

Substitute $x = 0$ into each equation. What do you notice?
Explain how you can identify the y-intercept from the equation of a graph.
Annie thinks that the coordinate of the y-intercept of $y = x^2 + 3x + 4$ will be (4,0). What mistake has she made?

Exponential graphs

H

Notes and guidance

Students may need to revise negative powers and/or powers of a fraction before this step. Students will explore exponential graphs so they can spot similarities in features. Students may need strategies to draw a smooth curve such as 'keep your wrist on the table'. This step can be extended by using simultaneous equations to find a and b, given two coordinates, when the equation is in the form $y = ab^x$.

Key vocabulary

Exponential	Growth	Decay
Rapid	Tends	Infinity
Asymptote	y-intercept	

Key questions

Can you think of real-life situations that can be modelled using exponential graphs?

True/False: a graph of an equation in the form $y = a^x$ will always have a y-intercept of (0,1)

What does 'tend towards' mean? What's an asymptote?

How can I find a given the y-intercept, in an equation of the form $y = ab^x$?

Exemplar Questions

Complete the table of values for $y = 2^x$ and draw the graph for values from $x = -3$ to $x = 3$

x	-3	-2	-1	0	1	2	3
y		0.25				4	

- Find y when $x = 10, x = 20, x = 50, x = 100$
- Find y when $x = -10, x = -20, x = -50, x = -100$

Explain what happens to the graph in each case.

Alex says "y will never be 0"
Is she right? Explain your answer.

Use a dynamic geometry package to plot these graphs.
Make a sketch of each one.

- $y = 2^x$
- $y = 3^x$
- $y = 4^x$
- $y = 5^x$

Write down the coordinates of the y-intercept for each graph.
What's the same and what's different about the graphs?
Write down the coordinates of the y-intercept of $y = 10^x$
How is the graph $y = 1^x$ different to these graphs?

The sketch shows a curve with equation $y = ab^x$ where a and b are constants and $b > 0$

The curve passes through the points (0, 2) and (1, 8)

What do you know? What can you find out?

Equation of circle centre (0,0) H

Notes and guidance

Students start by finding the radius of circles with centre (0,0) and making the connection to Pythagoras' theorem. This reveals the generalised equation for a circle centre (0,0). Given an equation in the format $x^2 + y^2 = a$, students sometimes read a as the radius instead of $\sqrt{a}$. There are opportunities here to revisit simplification of surds, circumference and area of a circle.

Key vocabulary

Radius	Diameter	Pythagoras' theorem
Equation	Origin	Simplify

Key questions

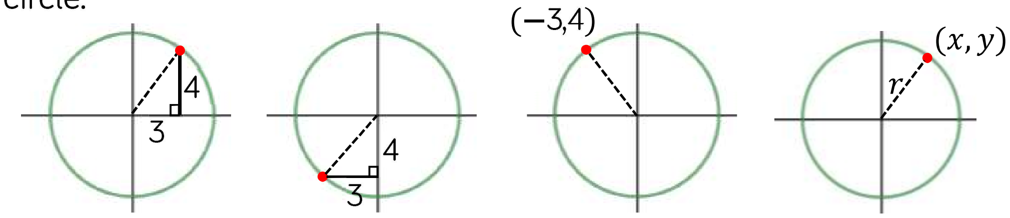
How is Pythagoras' Theorem connected to the equation of a circle?

How can I find the radius from the equation of a circle?

How can I write the equation of a circle given the diameter/circumference/area?

Exemplar Questions

Use the information given to write down the length of the radius of the circle.



Which is the correct equation of a circle, centre (0,0) with radius 5 cm?

A) $x^2 + y^2 = 5$ B) $x^2 + y^2 = 25$ C) $x^2 + y^2 = 10$

Match each equation of a circle with centre (0,0) to its radius.

$x^2 = 27 - y^2$	$r = 2\sqrt{3}$
$x^2 + y^2 + 100 = 0$	$r = 3\sqrt{3}$
$x^2 + y^2 - 100 = 0$	$r = 4\sqrt{3}$
$x^2 + y^2 - 48 = 0$	$r = 10$
$x^2 + y^2 = 12$	

Which equation doesn't have a match? Why?

The following circles all have centre (0,0).
Write down the equation of the circles.

- Radius = 0.5
- Diameter = $360^{\frac{1}{2}}$
- Radius = $\frac{1}{8}$
- Circumference 18π
- Area 40π
- Diameter = $\sqrt{20}$

Tangent to any curve

H

Notes and guidance

Using a dynamic software package and the 'zoom' function is an excellent way of highlighting why a tangent to a curve gives the gradient at a specific point on the curve. Ensure students understand the steps in drawing a tangent. They need to put their ruler on the point on the curve and adjust it so that near to the point, the ruler is equidistant from the curve on either side. Students then find the equation of the tangent using the gradient and the given point.

Key vocabulary

Tangent	Curve	Equidistant
Gradient	y-intercept	Equation

Key questions

What are the steps to draw a tangent to a curve at a given point?

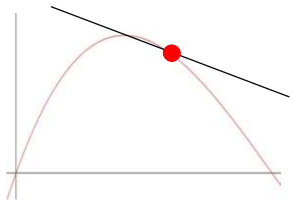
How do I find the gradient of the tangent? Why is this a good estimate of the gradient of a curve at a given point?

How do I know if the gradient is positive or negative?

Exemplar Questions

Annie is practising drawing tangents at given points on a curve.

A



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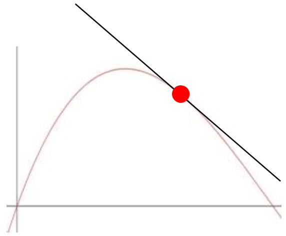
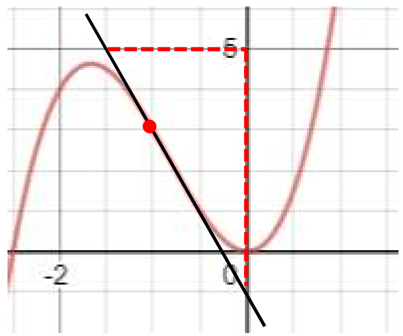


Which is her best attempt and why?



Dexter draws a tangent at $(-1,3)$.
Show that the gradient of the tangent is -4

Dexter finds the equation of the tangent.
Finish his workings.

$y = mx + c$

$y = -4x + c$

Substitute in $x = -1$ and $y = 3$ to find c.

Draw the graph $y = (x - 1)(x + 2)$ for values from $x = -1$ to $x = 4$

- Find the equation of the tangent at the point on the curve with coordinate (3, 10)
- Ron says that the gradient of the tangent to the curve when $x = -0.5$ is 0 and so the equation of the tangent at this point is $y = -2.25$
Is Ron correct? Explain your answer.

WRM – Year 11 Scheme of Learning

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Graphs**						**Algebra**					
	Gradients & lines		Non-linear graphs		Using graphs		Expanding & factorising		Changing the subject		Functions	
Spring	**Reasoning**						**Revision and Communication**					
	Multiplicative		Geometric		Algebraic		Transforming & constructing		Listing & describing		Show that...	
Summer	**Revision**						**Examinations**					

Autumn 1: Graphs

Weeks 1 and 2: Gradients and lines

This block builds on earlier study of straight line graphs in years 9 and 10. Students plot straight lines from a given equation, and find and interpret the equation of a straight line from a variety of situations and given information. There is the opportunity to revisit graphical solutions of simultaneous equations. Higher tier students also study the equations of perpendicular lines.
National Curriculum content covered includes:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- plot and interpret graphs
- interpret the gradient of a straight line graph as a rate of change
- use the form $y = mx + c$ to identify parallel **{and perpendicular}** lines; find the equation of the line through two given points, or through one point with a given gradient
- find approximate solutions to two simultaneous equations in two variables (linear/linear **{or linear/quadratic})** using a graph

Weeks 3 and 4: Non-linear graphs

Students develop their knowledge of non-linear graphs in this block, looking at quadratic, cubic and reciprocal graphs so they recognise the different shapes. They find the roots of quadratics graphically, and will revisit this when they look at algebraic methods in the Functions block during Autumn 2, where they will also look at turning points. Higher tier students also look at simple exponential graphs and the equation of a circle. Note that the equation of the tangent to a circle is covered later when the circle theorem of tangent/radius is met. Higher students also extend their understanding of gradient to include instantaneous rates of change considering the gradient of a curve at a point.
National Curriculum content covered includes:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function 1 **{the exponential function $y = k^x$ for positive values of k}**
- plot and interpret graphs (including reciprocal graphs **{and exponential graphs}**)
- find approximate solutions using a graph
- identify and interpret roots, intercepts of quadratic functions graphically
- **{recognise and use the equation of a circle with centre at the origin;}**

Weeks 5 and 6: Using graphs

This block revises conversion graphs and reflection in straight lines. Students also study other real-life graphs, including speed/distance/time, constructing and interpreting these. Higher tier students also investigate the area under a curve.
National Curriculum content covered includes:

- plot and interpret graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- **{interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of instantaneous and average rate of change (gradients of tangents and chords) in numerical, algebraic and graphical contexts}**
- **{calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts}**

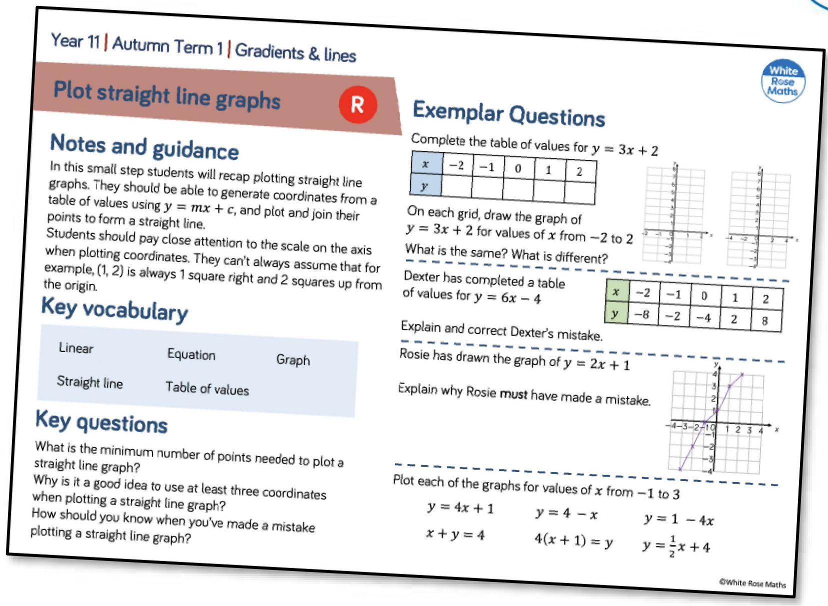
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol 💡.
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R.

Using graphs

Small Steps

- Reflect shapes in given lines (R)
- Construct and interpret conversion graphs (R)
- Construct and interpret other real-life straight line graphs (R)
- Interpret distance/time graphs
- Construct distance/time graphs
- Construct and interpret speed/time graphs
- Construct and interpret piece-wise graphs
- Recognise and interpret graphs that illustrate direct and inverse proportion

(H) denotes Higher Tier GCSE content

(R) denotes 'review step' – content should have been covered at KS3

Using graphs

Small Steps

- Find approximate solutions to equations using graphs
- **Estimate the area under a curve** H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Reflect shapes in given lines R

Notes and guidance

Students should be familiar with the equations of straight lines from the first block of the Autumn term. This step provides a reminder about lines of the form $x = a, y = a$ and $y = \pm x$ in the context of practising reflection. Students should be able to both perform and describe reflections in these lines using precise mathematical language; this key skill is revisited again in the Spring term.

Key vocabulary

Parallel	Horizontal	Vertical	Straight line
Axis	Reflection	Mirror	

Key questions

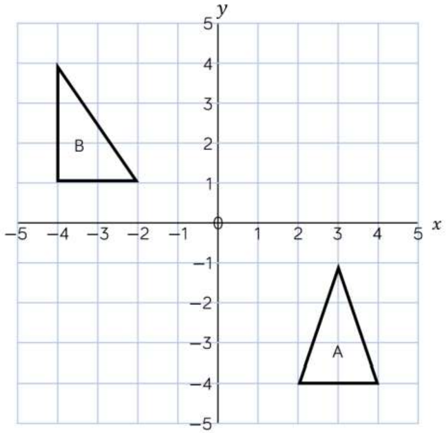
What's the same and what's different about the equations of horizontal/vertical lines compared to diagonal lines?

What's the same and what's different about reflecting a shape in horizontal/vertical lines compared to diagonal lines?

Given two shapes that have been reflected, how would you find the equation of the mirror line?

Exemplar Questions

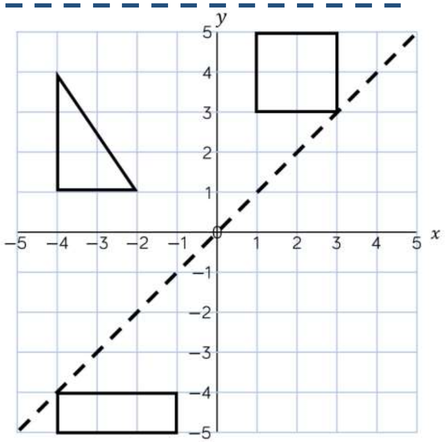
- Reflect triangle A in the line $y = -2$
Label the result X
- Reflect triangle A in the line $x = 1$
Label the result Y
- Reflect triangle B in the line $x = -2$
Label the result Z
- What is another way of saying "reflect in the line $y = 0$"?



What is the equation of the dotted line?

Reflect the shapes in this line.

On another grid, redraw the shapes and reflect them in the line $y = -x$



Draw a pair of coordinate axes from −5 to 5 in both directions.
Draw the trapezium with vertices at (1, 2), (5, 4), (5, 2) and (3, 2).
Reflect the trapezium in the x-axis.
What do you notice about the coordinates of the reflection of the trapezium? State the coordinates of the point (p, q) after

- a reflection in the x-axis
- a reflection in the y-axis

Conversion graphs

R

Notes and guidance

Students may need reminding to use a ruler to draw lines to/from axes to the line rather than reading off 'by eye'. Many conversion graphs are particular examples of direct proportion, so that the point (0,0) is a point on a conversion graph line and the second point for constructing a graph should be as far from the origin as practical. Converting e.g. Fahrenheit to Celsius on a graph, would not go through (0, 0).

Key vocabulary

Convert	Axis	Gradient
Direct Proportion		

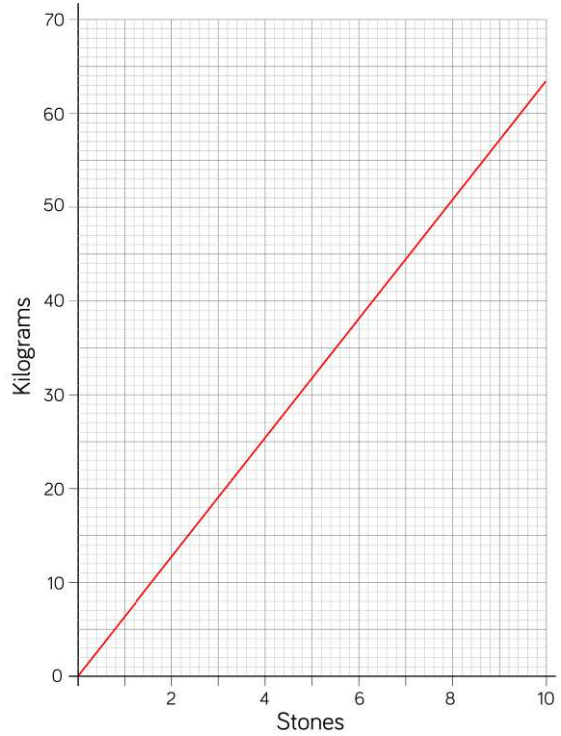
Key questions

Does it matter which axis represents which quantity when using a conversion graph?
What does the gradient of a conversion graph tell you?
Do all conversion graphs go through the origin?
How can you use a conversion graph to help work out conversions that are out of the range of the graph?

Exemplar Questions

You can use this graph to change between stones and kilograms.

- Change 5 stones to kilograms.
- Change 40 kilograms to stones.
- Explain how you could use your answers to change 40 kilograms to stones, and to change 35 stones to kg.



Use the fact that 1 inch is approximately 2.5 cm to draw a conversion graph for inches and cm. Use 0 to 12 inches on the horizontal axis and a suitable number of cm on the vertical axis.

- What two points should you plot to help draw the straight line?
- What scales should you use on the axes?
- Use your graph to convert 8.5 inches to cm and 45 cm to inches.
- Compare with answers found using a calculator.

Other real-life graphs

R

Notes and guidance

In this small step students look at linear relationships that do not go through the origin. Comparison could be made with direct proportion noting in this case that e.g. when one value doubles, the other does not. With these graphs, it is useful to consider the practical meaning of the gradient and intercept e.g. the unit increase and the fixed charge. Students could also be challenged to find the equation of the line.

Key vocabulary

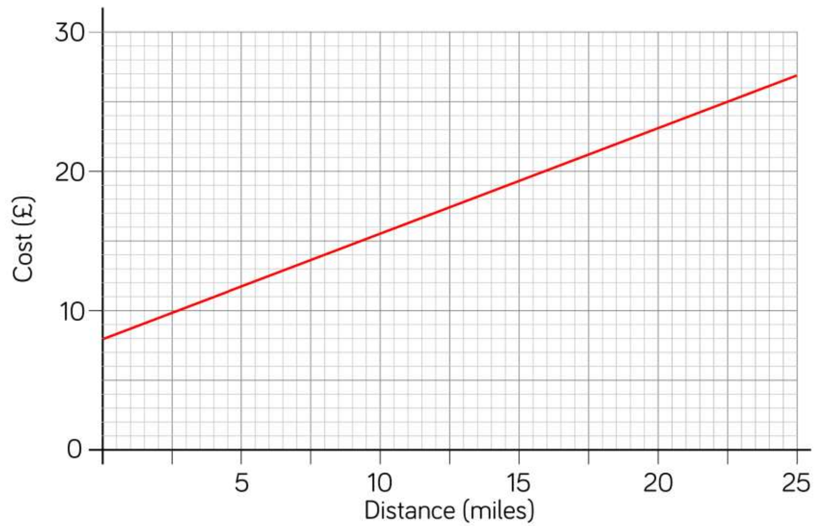
Gradient	Intercept	Interpret
Model		

Key questions

What is the value when $x = 0$? What does this mean in the context of the question?
What does the gradient of the line mean in the context of the question?
What is the same and what is different about these graphs and currency conversion graphs?

Exemplar Questions

This graph shows the cost of taxi journeys for different distances.



The taxi fare consists of a fixed charge plus a charge for each mile travelled.

- How much is the fixed charge?
- How much more does a 15 mile journey cost than a 5 mile journey?

A salesperson is paid £60 per day plus £30 for every sale they make.
Draw a graph showing how much they are paid for up to 10 sales a day.
Does this graph show direct proportion? Explain why or why not.

Another salesperson is paid £80 per day and £30 for every sale they make. Draw a graph for this salesperson on the same axes and compare the two wages.

Interpret distance/time graphs

Notes and guidance

In this step, students focus on the reading and interpretation of graphs, with construction covered in the next step. The key point is to understand that the gradient represents the speed of travel, e.g. a straight line is constant speed and a flat section implies the object is stationary. Various scales should be used, and students will need support to calculate speed in sections of less than one hour. Misconceptions about uphill and downhill direction of travel should be addressed.

Key vocabulary

Distance	Speed	Time
Gradient	Constant	Scale

Key questions

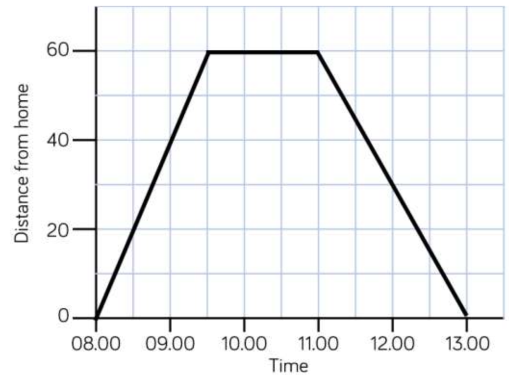
What is the connection between the gradient of a distance/time graph and the speed of travel?

Does a section of a distance/time graph with a negative gradient mean the journey is downhill? Why or why not?

What does a 'flat' section on a distance/time graph represent?

Exemplar Questions

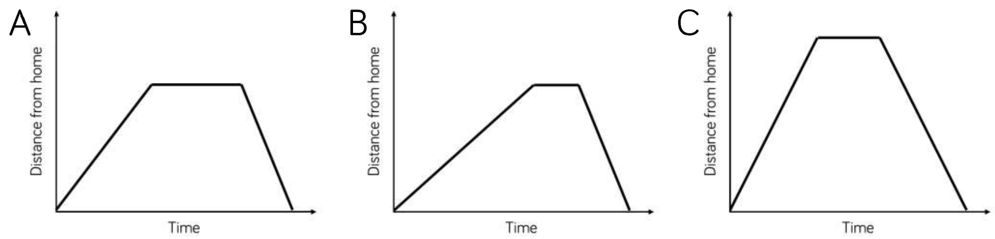
The distance-time graph shows Dora's journey to visit a friend and back.



- How long did it take to travel to see her friend?
- How long did she spend with her friend?
- How long was the journey back?

Use the formula Speed = Distance ÷ Time to work out Dora's speed for both the outward and the homeward journey.

How can you tell which part of the journey was faster just by looking at the graph?



Compare these distance/time graphs.

- Which graph shows the longest journey?
- Which graph has the fastest/slowest section?
- Which graph shows the longest break in a journey?
- Can you tell if any of the journeys were uphill/downhill?
- What else can you see?

Construct distance/time graphs

Notes and guidance

Students now move on to constructing graphs. This is relatively straightforward given times and distances, but can lead to difficulty if the speed is given, particularly if dealing with non-integer multiples of an hour. Students need to practice working out distances covered over periods of 10, 20, 30 and 45 minutes to inform their plotting of the graph. Discussion of how realistic the models are is also useful.

Key vocabulary

Distance	Speed	Time
Gradient	Constant	Scale

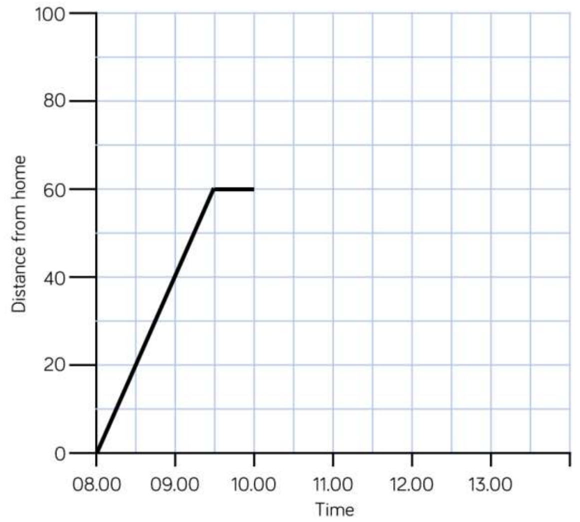
Key questions

What fraction of an hour is (e.g.) 20 minutes? If the car travels at (e.g.) 30 m.p.h., how far will it travel in 20 minutes?

What scale is the graph? How does this affect where we plot the parts of the journey?

Exemplar Questions

The graph shows part of Dani's journey to London from her home.



She takes a break, then drives the remaining 20 miles to London in half an hour. She then spends 90 minutes in London before returning directly home, arriving at 2 p.m.
Complete the graph to show this information.
Work out the speeds for each part of the journey.

Nijah goes on a cycle ride.
She sets off at 10:30 a.m., travelling at 24 k.p.h. for 45 minutes.
After a 15 minute break, she continues her journey for another 1.5 hours travelling at 16 k.p.h. She then rests for 45 minutes before returning home at a steady speed of 20 k.p.h.
Show this information on a distance/time graph.

Speed/time graphs

Notes and guidance

Students need to know the difference between speed/time and distance/time graphs, appreciating that the gradient here represents the change in speed and that this is called acceleration. They should also understand that negative gradient now represents slowing down/deceleration. Higher tier students need to be aware that the area under a speed/time is the distance travelled, both in this step and in later non-linear examples.

Key vocabulary

Distance	Speed	Time
Gradient	Constant	Acceleration

Key questions

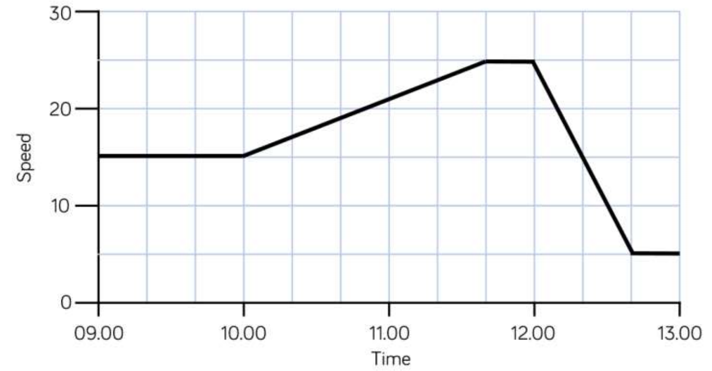
What is the difference between speed and acceleration?

What does a negative section mean on a speed/time graph? Why is it different from a distance/time graph?

What does the area under the graph represent?

Exemplar Questions

The graph shows the speed of a boat over a four hour period.



- What was the maximum speed of the boat?
- For how long altogether was the boat travelling at constant speeds?
- Between which times was the boat accelerating? Find the acceleration at this time
- Between which times was the boat decelerating?

The speed of boats at sea is measured in knots

- Given that 1 knot = 1.15 m.p.h., what was the fastest speed of the boat in knots over the four-hour period?

The area under a speed-time graph is the distance travelled.
Work out the total distance travelled by the boat.

A car accelerates from 0 m/s to 15 m/s in 12 seconds.
The car maintains this speed for 40 seconds before decelerating to rest in a further 20 seconds.
Represent this information in a speed/time graph.
Work out the acceleration and deceleration of the car.

Piece-wise graphs

Notes and guidance

Students now look at piece-wise graphs, which are discontinuous. These will be less familiar and may require careful explanation. Students can make links to the solutions of inequalities represented on number lines, as in this topic they again need to be careful when considering what values are included and not included. Piece-wise graphs could also be represented algebraically for each section, but if there are several sections this can become overwhelming.

Key vocabulary

Piece-wise

Key questions

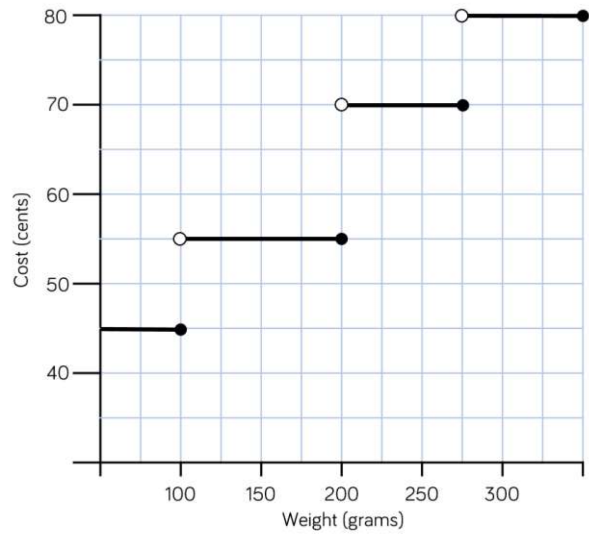
What is the gradient of each of the sections of the piece-wise graph?

When does the graph 'jump'? Is the boundary point included or excluded?

How does this compare to writing and solving inequalities?

Exemplar Questions

The graph shows the cost, in cents, of posting letters of different weights in a country.



Letters up to and including 100 g cost 40 c to send.
How much will a letter weighing 220 g cost to send?
Find the total cost of sending two letters weighing 150 g and a letter weighing 300 g.

The table shows the costs of using a car park in a town centre.
Represent this information on a piece-wise graph.

Duration	Up to 1 hour	Up to 2 hours	Up to 3 hours	Up to 5 hours	Up to 8 hours
Cost	50p	£1.00	£2.00	£3.50	£5

Rewrite the table using inequality notation e.g. $2 \leq t < 3$

Direct and inverse proportion

Notes and guidance

Direct and inverse proportion calculations using formulae are studied in depth next term under Multiplicative Reasoning, although the idea of constant multiplier and constant product could be explored here. Students explore the graphs of both types of proportion, with direct being more familiar. Teachers may wish to compare the graphs of inverse proportion relationships with that of the reciprocal function covered in the previous block.

Key vocabulary

Direct | Inverse | Proportion

Speed | Pressure

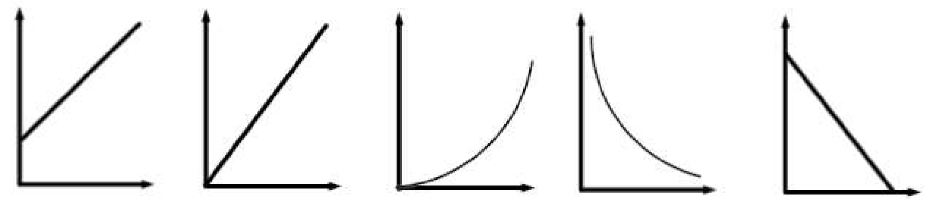
Key questions

How is the graph of an inverse proportion relationship different from the graph of a direct proportion relationship?

If you double the value of the x-axis quantity, what happens to the y-axis quantity? Is this the same for both direct and inverse proportion?

Exemplar Questions

Do the graphs show direct proportion, inverse proportion or neither? Explain your answers.



The table shows how long a journey takes at different speeds.

Speed (m.p.h.)	5	10	15	20	30	40	60	80	90	100
Time (hours)	24	12	8	6	4	3	2	1.5	1.333	1.2

Work out the length of the journey, and using this information or otherwise, find the time taken for the journey at each of 4, 3, 2, 1 m.p.h.
Draw a graph of time taken against speed, joining the points with a smooth curve.
Explain why the graph shows an inverse proportion relationship.

Sketch the shapes of the graphs for each of the situations.

- The time taken to complete a project against the number of people working on the project
- The distance travelled by a cyclist travelling at constant speed against the time spent cycling
- The number of biscuits each person has from a packet against the number of people sharing the packet

Approximate solutions

Notes and guidance

Students are familiar with finding the roots of a quadratic graphically from the previous block; their learning is reinforced both here and in later blocks when algebraic methods are considered. Students now also explore finding the approximate solutions of other equations by looking at the points where a graph and line intersect and can check answers by substitution. Higher tier students can also check their answers for quadratics that factorise, and will study other methods next term.

Key vocabulary

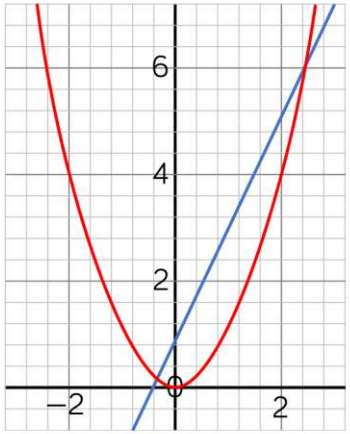
Quadratic	Solutions	Root
Estimate	Approximate	

Key questions

How can we check if the approximate solutions are close to the actual solutions? What values should we get when we substitute into the original equation?

What do we mean by the roots of a quadratic? Is this the same as, or different to, solutions?

Exemplar Questions



The diagram shows the graphs of $y = x^2$ and $y = 2x + 1$

By looking at the where the graphs intersect, find the values of x for which $x^2 = 2x + 1$

Draw suitable straight lines on the graph to find approximate solutions to the equations

- $x^2 = x + 3$
- $x^2 = 3 - x$

Complete the table of values for $y = x^2 - x - 6$

x	−3	−2	−1	0	1	2	3
y			−4				

On a grid with x from −3 to 3 and y from −7 to 7, draw the graph of $y = x^2 - x - 6$. (Use a scale of 2 cm to 1 unit on the x-axis and 1 cm to 1 unit on the y-axis)

Use your graph to state the roots of the equation $x^2 - x - 6 = 0$

Use your graph to estimate the solutions of the equations

- $x^2 - x - 6 = 4$
- $x^2 - x - 6 = -3$
- $x^2 - x - 5 = 0$

Draw the graph of $y = x^3 - x^2 - 4x + 4$ for values of x from −2 to 2

Use your graph to estimate the solutions of $x^3 - x^2 - 4x + 4 = 3$

Estimate area under a curve H

Notes and guidance

As a preparation for A level mathematics, students use trapezia to estimate the area of a curve. They may need reminding of the formula for the area of a trapezium, particularly as the trapezia are right-angled and usually in a less familiar orientation. Students also revisit the fact that the area under a speed/time is the distance travelled as an application of this process. Similarly, they could also revisit finding the tangent at a point of the curve.

Key vocabulary

Trapezium	Area	Approximate
Estimate	Speed/time graph	

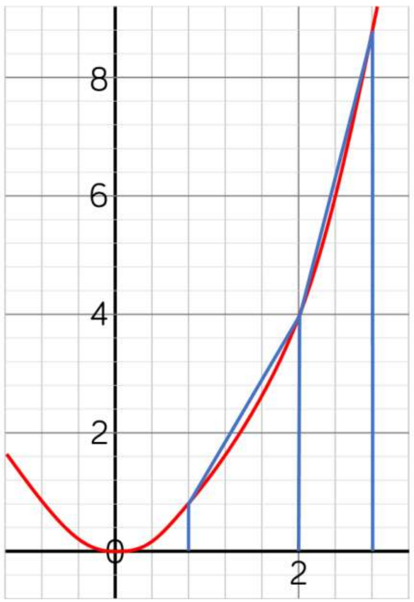
Key questions

How do you find the area of the trapezium?

Which dimension is the height of the trapezium? Which are the parallel sides?

If the trapezia all lie below the curve, why is the estimate for the area an underestimate?

Exemplar Questions



The diagram shows the graph of $y = x^2$ for values of x from -1 to 3

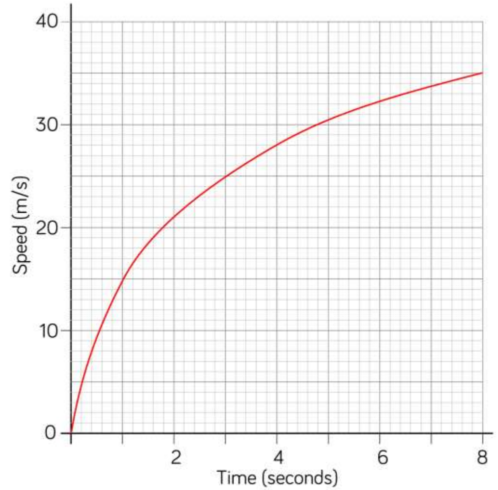
Use the two trapezia drawn on the graph to estimate the area of under the curve of $y = x^2$ for $1 \leq x \leq 3$

Is the area an underestimate or an overestimate?

The graph shows the speed of a cyclist in the first 8 seconds of a journey.

Work out an estimate for the distance travelled by the cyclist in the first 8 seconds.

Use 4 strips of equal width.



Autumn 2: Algebra

Weeks 1 and 2: Expanding and factorising

This block reviews expanding and factorising with a single bracket before moving on to quadratics. The use of algebra tiles to develop conceptual understanding is encouraged throughout. Context questions are included to revisit e.g. area and Pythagoras' theorem.
National Curriculum content covered includes:

- know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**
- simplify and manipulate algebraic expressions by: factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares; **{factorising quadratic expressions of the form $ax^2 + bx + c$}**
- know the difference between an equation and an identity; solve quadratic equations **{including those that require rearrangement}** algebraically by factorising, **{by completing the square and by using the quadratic formula}**
- identify and interpret roots; deduce roots algebraically **{and turning points by completing the square}**
- solve two simultaneous equations in two variables (linear/linear **{or linear/quadratic}**) algebraically; find approximate solutions using a graph

Weeks 3 and 4: Changing the subject

Students consolidate and build on their study of changing the subject in Year 9. The block begins with a review of solving equations and inequalities before moving on to rearrangement of both familiar and unfamiliar formulae. Checking by substitution is encouraged throughout. Higher tier students also study solving equations by iteration.
National Curriculum content covered includes:

- solve linear inequalities in one variable
- know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**
- translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- **{find approximate solutions to equations numerically using iteration}**

Weeks 5 and 6: Functions

As well as introducing formal function notation, this block brings together and builds on recent study of quadratic functions and graphs. This is also an opportunity to revisit trigonometric functions, first studied at the start of Year 10.
National Curriculum content covered includes:

- where appropriate, interpret simple expressions as functions with inputs and outputs; **{interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function'}**
- solve two simultaneous equations in two variables (linear/linear **{or linear/quadratic}**) algebraically; find approximate solutions using a graph
- identify and interpret roots; deduce roots algebraically **{and turning points by completing the square}**
- solve linear inequalities in one **{or two}** variable**{s}**, **{and quadratic inequalities in one variable}**; represent the solution set on a number line, **{using set notation and on a graph}**
- recognise, sketch and interpret graphs of quadratic functions
- apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles **{and, where possible, general triangles}** in two **{and three}** dimensional figures

Expanding and factorising

Small Steps

- Expand and factorise with a single bracket (R)
- Expand binomials (R)
- Factorise quadratic expressions
- **Factorise complex quadratic expressions** (H)
- Solve equations equal to 0
- Solve quadratic equations by factorisation
- **Solve complex quadratic expressions by factorisation** (H)
- **Complete the square** (H)

(H) denotes Higher Tier GCSE content

(R) denotes 'review step' – content should have been covered at KS3

Expanding and factorising

Small Steps

- **Solve quadratic equations using the quadratic formula** H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Single bracket

R

Notes and guidance

This reviews concepts covered in Key Stage 3. Illustrate expanding a single bracket using the area model (e.g. rectangle, length of 5 and width of $x + 3$) or by using algebra tiles. Factorise numbers (e.g. $24 = 12 \times 2$) before algebraic expressions to make the link between factors and factorising. Students need to be careful to find the highest common factor of the terms in an expression in order to factorise fully.

Key vocabulary

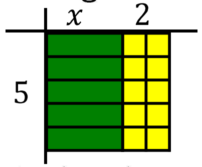
Expand	Factorise	Multiply out
Coefficient	Bracket	Identity
HCF	Factorise fully	

Key questions

What is the link between multiplication and repeated addition?
Is it possible to have three or more terms inside a bracket?
What do you look for to find the HCF of a set of terms?
Is it always true that if you can't halve an expression then the expression doesn't factorise?

Exemplar Questions

Aisha uses algebra tiles to expand $5(x + 2)$.



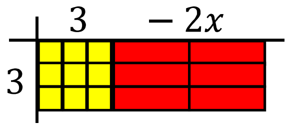
$$5(x + 2) \equiv 5x + 10$$

Expand the brackets.

- $5(2 - x)$
- $-5(2 + x)$
- $-5(2 - x)$
- $-(2 - x)$
- $x(x + 2)$
- $x^2(2 + x)$

Show that $5(2 - x) - (2 - x) \equiv 8 - 4x$.

Dani factorises $9 - 6x$ using algebra tiles.



$$9 - 6x \equiv 3(3 - 2x)$$

Factorise.

- $9 + 6x$
- $12x + 4$
- $-15 - 10x$
- $x^2 + 6x$

Find the highest common factor of each pair.

- 3 and 15 $3a$ and $15b$
- 24 and 36 $2rs$ and $3sr$
- 4 and 16 x and x^2
- 5×5×3 and 5×3×3 r^2s and rs^2

Factorise.

- $3a + 15b$
- $x^2 - x$
- $2rs - 3sr$
- $r^2s + rs^2$

$$12a^2 + 18ab - 24a \equiv 2(6a^2 + 9ab - 12a)$$

Why doesn't this show the full factorisation of $12a^2 + 18ab - 24a$?
Fully factorise the expression.

Expand binomials

R

Notes and guidance

Here we revisit the meaning of binomial and quadratic, and use the area model as a visual prompt for discussion on how to expand binomials. Concrete resources such as algebra tiles are useful in supporting student confidence in this step. Students need to be confident with simplification and dealing with negative numbers. Where appropriate, extend to contexts where students generate the binomials and then manipulate them.

Key vocabulary

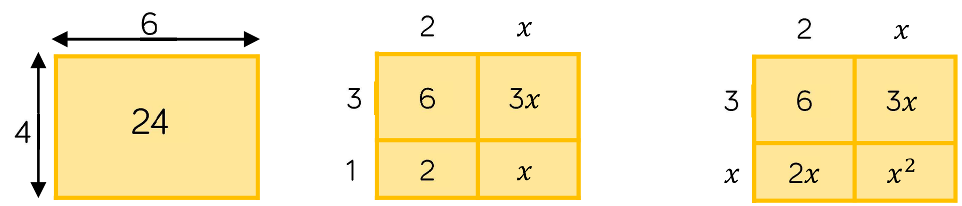
Binomial	Simplify	Like/unlike terms
Expand	Quadratic	Difference of two squares

Key questions

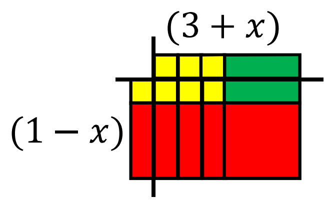
Why do you get four terms when you multiply two binomials?
Why can you simplify some quadratic expressions to three or fewer terms, but not others?
Do simplified quadratics always have three terms?
What happens when a single bracket is squared?

Exemplar Questions

Compare the diagrams. What is the same and what is different?



Explain how the algebra tiles show that.



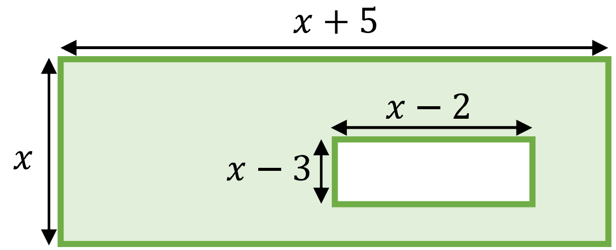
$(3+x)(1-x) \equiv 3 + x - 3x - x^2$
$(3+x)(1-x) \equiv 3 - 2x - x^2$

Use algebra tiles to expand the expressions.

- $(x+2)(x+3)$
- $(2x+2)(x+3)$
- $(x+2)(x-3)$
- $(x-2)(x-3)$
- $(x+2)^2$
- $(x-3)^2$

Expand and simplify $(x+3)(x-3)$ and $(y-5)(y+5)$.
What do you notice? Can you generalise?

Find an expression for the shaded area.



Factorise quadratic expressions

Notes and guidance

High attaining students may have covered this step in Year 10. Here students need to link finding factors with factorisation. Students should understand that a quadratic expression has a maximum of two binomial factors. Students consider how the factors of the constant terms relate to the coefficient of the x term. Again, algebra tiles can be used. Finally, students should factorise quadratics with negative x terms or a negative constant.

Key vocabulary

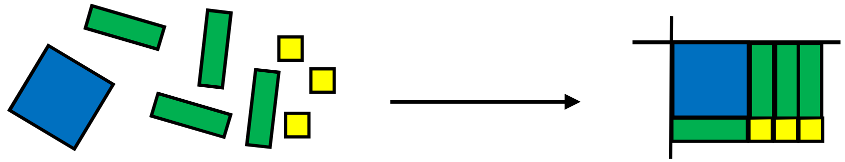
Expression	Quadratic	Term
Coefficient	Factor	Factorise

Key questions

How can algebra tiles be used to show that a quadratic expression only has two factors?
How do the factors of the constant term relate to the coefficient of x?
Why is factorising e.g. $x^2 + 4x + 3$ different from factorising $x^2 + 4x$?

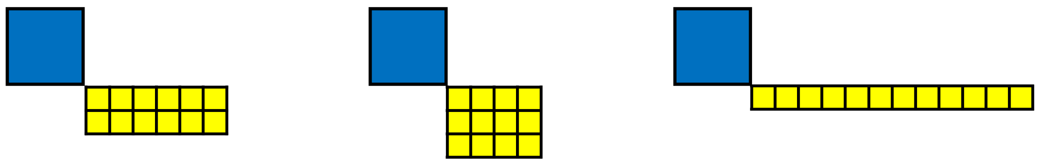
Exemplar Questions

Annie factorises $x^2 + 4x + 3$ using algebra tiles. What is her answer?



Can you make any different rectangles using Annie's algebra tiles?
List the factors of $x^2 + 4x + 3$

Use x tiles to complete each diagram to make a rectangle.
Write down the expression, and the factorisation that each represents.



What's the same and what's different about each one?
Mo says: "To factorise $x^2 + 8x + 12$ I need to think about which factors of 12 will give the x term a coefficient of 8".
Use the diagram to explain his thinking.

List the factors of −12
Factorise the expressions.

- $x^2 + x - 12$
- $x^2 - x - 12$
- $x^2 - 4x - 12$
- $x^2 + 4x - 12$
- $x^2 - 11x - 12$
- $x^2 + 11x - 12$

Complex quadratic expressions H

Notes and guidance

In this Higher tier step, students realise that both the factors of the coefficient of x^2 and the factors of the constant term need to be considered when factorising. Algebra tiles support this thinking. Students could then consider a more abstract approach to complex factorising by using trial and improvement to establish the correct combination of pairs of factors. Encourage students to expand the brackets to check the factorisation.

Key vocabulary

Term	Quadratic	Trial and improvement
Coefficient	Factor	Factorise

Key questions

Why is it efficient to start with the tiles representing x^2 and ones when forming the rectangle?

Why is the coefficient of x dependent on the factors of x^2 and the constant term?

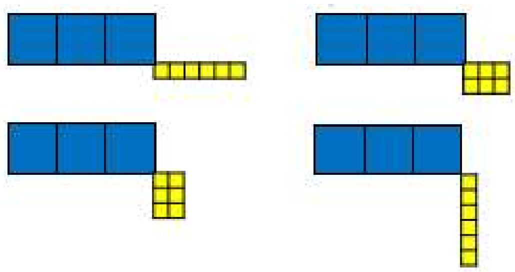
How can trial and improvement be used efficiently to factorise? Are there any pairs that obviously won't work?

Exemplar Questions

Eva is factorising $3x^2 + 11x + 6$

Using algebra tiles, she tries different factors of 6

Which arrangement works?



Eva says, "To factorise $3x^2 + 11x + 6$ I need to think about factors of 6 **and** factors of 3". Explain why Eva is right.

Factorise $3x^2 + 11x + 6$

Use algebra tiles to factorise.

- $2x^2 + 6x + 4$
- $4x^2 + 10x + 4$

Whitney is factorising $3x^2 + 11x - 20$

List the factors of −20

Try different pairs of factors in the brackets and expand.

$$(3x \; \square)(x \; \square) \equiv 3x^2 \; \square \; x - 20$$

Which pair of factors in the expression give $11x$?

Factorise.

- $3x^2 - 28x - 20$
- $3x^2 - 32x - 20$
- $3x^2 - 17x - 20$
- $3x^2 + 59x - 20$

Solve equations equal to 0

Notes and guidance

The purpose of this small step is to prepare students for solving quadratics by factorisation. Firstly students practise solving linear equations equal to zero. They then need to understand that is the product of two numbers or terms is zero then at least one of the two numbers/terms must be zero. This supports understanding of why there are 2 solutions to a quadratic equation.

Key vocabulary

Expression	Term	Quadratic
Solve	Solutions	Product

Key questions

If two numbers/terms multiply to give 0, what do we know about one of the numbers/terms?
Why are there two solutions for x in equations such as $x(x+1)=0$?
How can we find each solution? How can we check the solutions are correct?

Exemplar Questions

Teddy is solving the equation $2 - x = 0$

Teddy says, "The answer must be 2 because $2 - 2 = 0$ so $x = 2$"

Solve these equations.

$x + 3 = 0$ $\quad$ $x - 3 = 0$ $\quad$ $3 - x = 0$ $\quad$ $3 + x = 0$

Dexter and Amir are solving $1 - 3x = 0$

Dexter's method

$1 - 1 = 0$

So $3x = 1$

$x = \frac{1}{3}$

Amir's method

$1 - 3x = 0$

$1 = 3x$

$\frac{1}{3} = x$

$+ 3x$ to both sides

$\div$ both sides by 3

Which method do you prefer? Use your chosen method to solve

$3x - 1 = 0$ $\quad$ $3x + 1 = 0$ $\quad$ $2 - 3x = 0$

---

a	b	ab
2		0
	2	0
$(x + 2)$		0
	$(x + 2)$	0

Complete the table.

What do you notice?

What do you know about a and b if $ab = 0$?

Solve these equations.

$x(x+1) = 0$ $\quad$ $(x+1)(x-1) = 0$ $\quad$ $(2x+1)(x-1) = 0$

Solve quadratics by factorisation

Notes and guidance

It's important to emphasise the difference between factorising and solving. Some students try to solve when they are asked to factorise. Students should make links between the solutions of a quadratic equation and the roots of a quadratic. They should also form quadratic expressions and equations using given information. They should solve quadratic equations in a context and choose the most sensible solution given the context e.g. avoiding negative lengths.

Key vocabulary

Expression	Equation	Factorise
Solve	Solutions	Roots

Key questions

What's the difference between factorising and solving?
What's the difference between the roots of a quadratic equation and the solutions of the same quadratic equation? Explain your answer.
How is an expression different to an equation?
What do we mean by "the difference of two squares"?

Exemplar Questions

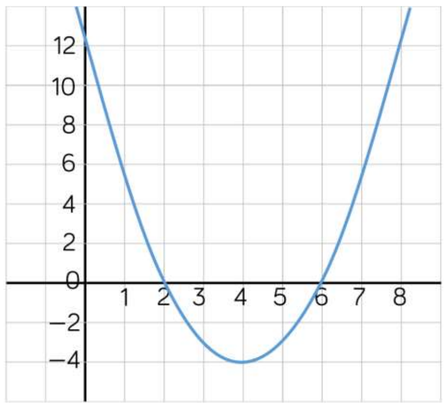
Dani solves $x^2 + 17x + 70 = 0$ and finds $x = 7$ or $x = -10$
Check her answers by substituting them into $x^2 + 17x + 70$
Which answer is incorrect? How do you know?
Solve $x^2 + 17x + 70 = 0$ correctly.

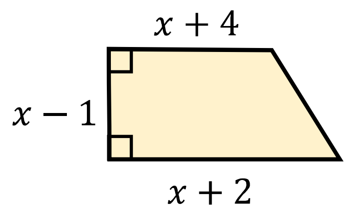
Show that one solution of $x^2 - 8x + 12 = 0$ is $x = 6$

Factorise $x^2 - 8x + 12$

Work out another possible solution for x.

The graph shows $y = x^2 - 8x + 12$
What are the roots of this equation?
What do you notice?





Show that the area of the trapezium is
$x^2 + 2x - 3$
The area of the trapezium is 45 cm^2
What equation can you form?
How can you find the value of x?
Are there one or two solutions?

Brett thinks you can't solve $x^2 - 36 = 0$ because it's impossible to factorise $x^2 - 36$ as it only has two terms.
Brett is wrong.
Find two ways to solve the equation $x^2 - 36 = 0$

Solve complex quadratics H

Notes and guidance

Higher tier students need to solve quadratics where the coefficient of x^2 is greater than 1 by factorisation. Encourage students to make the link between the solutions of a quadratic and the roots illustrated on a graph. Explicitly discuss the possibility of simplifying some quadratic equations by dividing both sides by a common factor, but highlight that it is not possible to simplify an expression in the same way.

Key vocabulary

Quadratic equation	Factories	Solve
Simplify	Solutions	Roots

Key questions

Why do we often use fractions rather than decimals when writing solutions?
How can we check whether the solutions are correct?
How does this link to the graph of a quadratic equation?
Why can we sometimes simplify a quadratic equation, but not an expression?

Exemplar Questions

Eva is solving $2x^2 + 9x - 35 = 0$
Complete her workings.

$$(2x - 5)(x \square) = 0$$

$2x - 5 = 0$ or $x \square = 0$

$2x = \square$ $\quad$ $x = \square$

$x = \frac{\square}{2}$

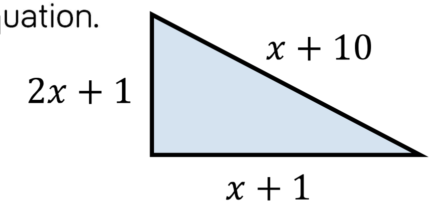
Solve the following quadratic equations.

- $3x^2 + 37x + 44 = 0$
- $3x^2 - x - 44 = 0$
- $3x^2 - 37x + 44 = 0$
- $3x^2 + x - 44 = 0$

Draw each graph using a dynamic geometry package.
What do you notice about the solutions and roots of each equation?

Ron is solving $4x^2 - 14x - 98 = 0$
He says that he can simplify the equation by dividing both sides by 2
Is he right? Factorise and solve this equation.

Here is a right-angled triangle.



Show that $4x^2 - 14x - 98 = 0$

Work out the perimeter of the triangle.

Complete the square

H

Notes and guidance

Students use algebra tiles to understand the structure of completing the square. They understand why halving the coefficient of x is necessary when completing the square. Students should then be encouraged to work abstractly, perhaps with scaffolded activities. They then consider how to solve quadratic equations, where a common mistake is to take only the positive square root, therefore missing a solution.

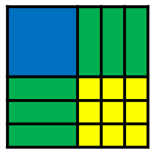
Key vocabulary

Complete the square	Quadratic	In the form
Coefficient	Factorise	Solve

Key questions

Why is this method called completing the square?
How does p in $(x+p)^2$ relate to the original expression/equation?
When solving, what's important to remember about square rooting both sides of the equation? How does this compare to the graph?

Exemplar Questions



Alex factorises $x^2 + 6x + 9$ using algebra tiles.
Write down the factorised expression.
How does the shape connect to the factorised expression?

Explain how the diagrams show that
$x^2 + 6x + 8 \equiv (x+3)^2 - 1$ and $x^2 + 6x + 10 \equiv (x+3)^2 + 1$

Use algebra tiles, to write the expressions in the form $(x+a)^2 + b$.

$x^2 + 4x + 5$ $\quad$ $x^2 + 6x + 7$ $\quad$ $x^2 - 8x + 19$

What connections can you see between the expressions and the "completed square" form?

Complete the workings.

$$x^2 + 3x - 9 \equiv (x - \square)^2 - \frac{9}{4} - 9$$
$$x^2 + 3x - 9 \equiv (x - \square)^2 - \frac{9}{4} - \frac{\square}{4}$$
$$x^2 + 3x - 9 \equiv (x - \square)^2 - \frac{\square}{4}$$

Spot the error.

$$x^2 + 8x + 6 = 0$$
$$(x+4)^2 - 10 = 0$$
$$(x+4)^2 = 10$$
$$(x+4) = \sqrt{10} \therefore x = \sqrt{10} - 4$$

Using the quadratic formula (H)

Notes and guidance

Teachers could introduce this step by exploring the derivation of the formula. Students use the quadratic formula to solve equations both with and without a calculator. Errors in substitution often occur when b is negative; this needs highlighting. Students should be encouraged to breakdown the calculation, even when using a calculator, as this minimises error. They may need to practise simplifying surds before using the quadratic formula without a calculator.

Key vocabulary

Formula	Substitute	Surd
Simplify	Significant figures	

Key questions

How do we know which number to substitute into the formula?

Why do we need to be careful, particularly if b is negative?

Why should we always calculate in more than one step?

If we're not using a calculator, how do we simplify our answer?

Exemplar Questions

Complete the method to solve $3x^2 + 8x - 5 = 0$ using the quadratic formula. Give your answer to 3 significant figures.

$$x = \frac{-8 \pm \sqrt{(64 - 4 \times \square \times \square)}}{2 \times \square}$$

$$x = \frac{-8 \pm \sqrt{\square}}{\square}$$

$x = 0.523$ or $x = \square$

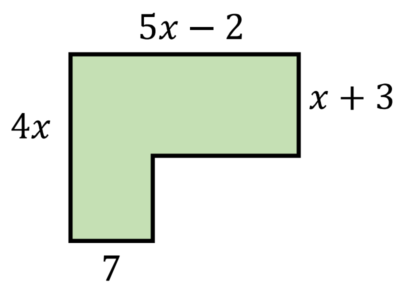
Huan is solving $2x^2 - 6x + 3 = 0$ using the quadratic formula. Find all of his errors.

$$x = \frac{-6 \pm \sqrt{(36 - 4 \times 2 \times 3)}}{2}$$

$$x = \frac{-6 \pm \sqrt{12}}{2}$$

$$x = -6 \pm \sqrt{6}$$

Correct his method and show that $x = \frac{3 \pm \sqrt{3}}{2}$



On the diagram, all measurements are in cm and the area of the hexagon is 100 cm^2.

Show that $5x^2 + 34x - 127 = 0$

Find x, giving your answer to 3 significant figures.

Changing the subject

Small Steps

Solve linear equations	R
Solve inequalities	R
Form and solve equations and inequalities in the context of shape	
Change the subject of a simple formula	R
Change the subject of a known formula	
Change the subject of a complex formula	
Change the subject where the subject appears more than once	H
Solve equations by iteration	H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Solve linear equations

R

Notes and guidance

Students are familiar with solving equations from previous years' content. This step provides an opportunity to check the basics are secure. In particular, students should be familiar with equations presented in many forms with different letters and unknowns on either/both sides of the equals sign. Positive, negative, fractional and decimal solutions should all be included. Bar models could still be used if necessary.

Key vocabulary

Equation	Solve	Solution
Unknown	Coefficient	Expand

Key questions

What's the first step you take when solving an equation?
Do you need to expand the brackets when solving an equation?
How do you start solving an equation with unknowns on both sides?
How do you know whether an equation is linear? How many solutions does a linear equation have?

Exemplar Questions

Which of these equations is $x = 5$ a solution of? Which are linear?

$2x = 25$ $\quad$ $x^2 = 10$ $\quad$ $\frac{10}{x} = 2$ $\quad$ $\frac{x}{2} = 2.5$ $\quad$ $2x = 10$ $\quad$ $x^2 = 25$

$8 = 3x - 7$ $\quad$ $3 + 4x = 7x - 12$ $\quad$ $12 - 2x = 2$

Is $x = -5$ a solution of any of the equations?

What's the same and what's different about these equations/problems?

$4a + 3 = 12$ $\quad$ $12 = 4a + 3$ $\quad$ $3 + 4a = 12$ $\quad$ $12 = 3 + 4a$

$4b + 3 = 12$ $\quad$ $(4 \times \square) + 3 = 12$ $\quad$? $\times 4$ $+3$ 12

$3 + a + a + a + a = 12$ $\quad$ $(\square + \square + \square + \square) + 3 = 12$

Explain the steps you would take to solve the equations.

- $\frac{3x+1}{5} = 7$
- $\frac{3x}{5} + 1 = 7$
- $\frac{3(x+1)}{5} = 7$
- $3x + 2 = 5x - 7$
- $3x + 2 = 7 - 5x$
- $2 - 3x = 5x - 7$
- $2 - 3x = 7 + 5$

What's the same and what's different?

Solve inequalities

R

Notes and guidance

Students need to be aware of the similarities and differences when solving inequalities rather than equations, taking care that the appropriate sign is not 'lost'. They also need to be aware that e.g. $x \geq 3$ and $3 \leq x$ are equivalent. Expressing solution sets on number lines as well as algebraically is to be encouraged to ensure familiarity. Higher tier students should also revise giving solutions using set notation.

Key vocabulary

Equation	Inequality	Solution set
Greater/less than	Greater/less than or equal to	

Key questions

What's the difference between an equation and an inequality?

What are the four possible symbols you might see in an inequality? What does each one mean?

Explain how you represent an inequality on a number line?

Do the solutions of inequalities have to be integers?

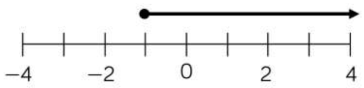
Exemplar Questions

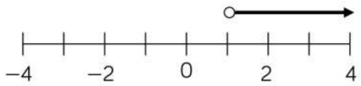
What's the same and what's different about solving these?

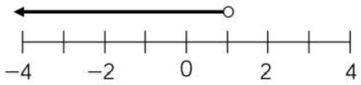
- $3a + 11 = 83$
- $3a + 11 > 83$
- $83 \leq 3a + 11$

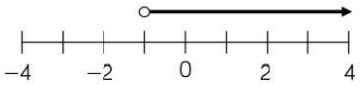
Match the inequalities with the solutions on the number lines.

- $4x + 3 > 7$
- $3x - 1 \leq 4x$
- $4x - 2 > x - 5$
- $2x + 3 < 5$

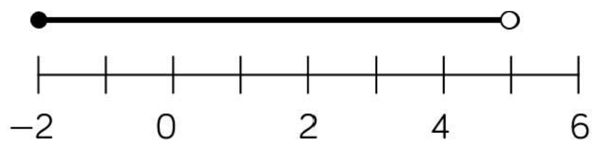








Write the solutions using set notation.



Which inequality is shown by the number line?

$-2 \leq x \leq 5$ $\quad -2 < x < 5$ $\quad -2 \leq x < 5$ $\quad -2 < x < 5$

Given that x is a prime integer, what are the possible values of x?

Ron is solving the inequality $-6 < -3x \leq 12$

He writes $2 < x < -4$

Explain Ron's mistakes and find the correct solution.

Equations/inequalities from shapes

Notes and guidance

Students should be confident in forming as well as solving equations, and this step uses shape as a context to support this. Teachers may well choose other topics which their classes need to revise here if appropriate. Students should be encouraged to check answers by substituting solutions back in to the original problem as well as in the equation or inequality.

Key vocabulary

Form	Solve	Perimeter	Area
Volume	Opposite angles		Check

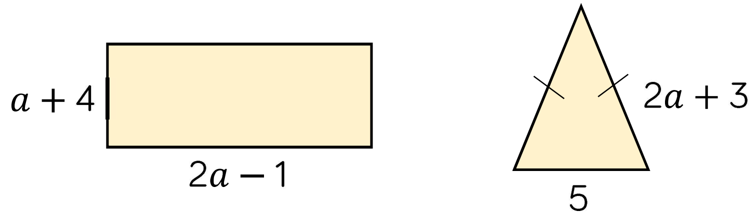
Key questions

How can we form an equation/inequality in this situation?
Which values are equal?

How can we check our solution is correct? Does it make sense?

Exemplar Questions

The perimeter of the rectangle is greater than the perimeter of the triangle. Find the smallest possible integer value of a.

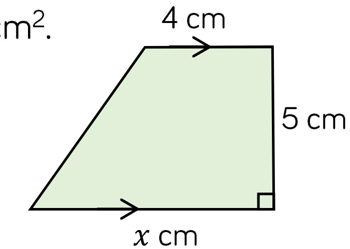


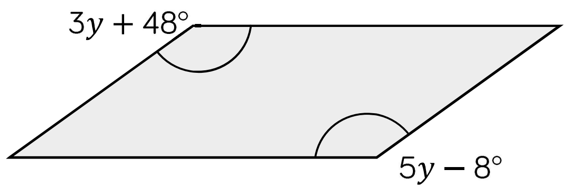
The volume of a pyramid is given by the formula

$$V = \frac{1}{3} \times \text{base area} \times \text{vertical height}$$

The base of a pyramid is a rectangle of length l cm and width 5 cm.
The height of the pyramid is 12 cm.
Given that the volume of the pyramid is 90 cm^3, find the value of l.

The area of the trapezium is 28 cm^2.
Find x.





Find the difference between the largest and smallest angles in the parallelogram.

Change the subject: simple formula R

Notes and guidance

Students have studied changing the subject of the formula in Year 9 and this steps reviews the basic principles. Comparison with solving equations is useful, as is substituting values into the original and final forms to check accuracy. Questions like the second exemplar here are very useful to help students to identify the first step in various situations.

Key vocabulary

Equation	Subject	Rearrange
Change	Inverse	

Key questions

Which letter is the subject of the formula? How do you know?

What is the first step when rearranging this formula?

Why are inverse operations important when rearranging a formula?

Exemplar Questions

The equation of a straight line is $y = 4x$
Complete the coordinates of these points on the line.

(8,) (, 8) (, 60)

Rewrite the equation of the line in the form $x = \ldots$

The equation of another straight line is $y = x + 7$
Complete the coordinates of these points on the line.

(8,) (, 8) (, 60)

Rewrite the equation of the line in the form $x = \ldots$

Make b the subject of the formulae.

$a = b + 3$	$a = b - 3$	$a = c + b$	$a = b - c$
$a = 4b$	$a = \frac{b}{4}$	$a = b^2$	$a = \sqrt{b}$
$a = 4b + 3$	$a = 4b - 3$	$a = 3 + \frac{b}{4}$	$a = \frac{b-3}{4}$

In the formula $v = u + at$, v is final velocity, u is initial velocity, a is acceleration and t is time.
The initial velocity of an object is 3 m/s and its acceleration is 5 m/s^2.
Find the time it takes to reach a final velocity of 20 m/s.
Rearrange $v = u + at$ to make t the subject of the formula.

The relationship between pressure (P), force (F) and area (A) is given by the formula $P = \frac{F}{A}$.

- Rearrange the formula to make F the subject.
- Rearrange your answer to make A the subject.

White Rose Maths

Change the subject: known formula

Notes and guidance

This step could be covered in conjunction with the previous step. Changing the subject can be a rather abstract concept, so it can be useful for students to see it in the context of formulae with which they are familiar. It is particularly useful in checking the accuracy of the rearrangement as they know what the letters represent and make sense of their answers.

Key vocabulary

Equation	Subject	Rearrange
Change	Inverse	

Key questions

Which letter is the subject of the formula? How do you know?

What is the first step when rearranging this formula?

How can we check that the rearrangement is correct?

Exemplar Questions

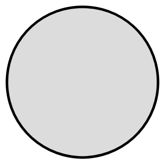
The speed (S) of an object is found by dividing the distance travelled (D) by the time taken (T).

- Write this formula algebraically.
- Rearrange the formula to make D the subject.
- Rearrange your answer to make T the subject.

Check your rearrangements work with e.g. a car travels 30 m.p.h. covering a distance of 120 miles in 4 hours.

The circumference of a circle radius r is given by $C = 2\pi r$.
Which is the correct rearrangement to make r the subject?

$r = \frac{2C}{\pi}$ $\quad r = \frac{C}{2\pi}$ $\quad r = \frac{2\pi}{C}$ $\quad r = \frac{\frac{C}{2}}{\pi}$



A circle of radius r has area A

Show that $r = \sqrt{\frac{A}{\pi}}$

The perimeter (P) of a rectangle of length l and width w can be found using the formula $P = 2(l + w)$.
Explain why both $l = \frac{P}{2} - w$ and $l = \frac{P-2w}{2}$ are both correct rearrangements to make l the subject of the formula.

The volume of a pyramid is given by the formula

$V = \frac{1}{3}Ah$, where A is the base area and h is the vertical height.

Rearrange the formulae to make A the subject.

Change the subject: complex formula

Notes and guidance

When students are comfortable with rearranging one-step and two-step formulae, they can then move on to multi-step formulae such as those in the final exemplar. The order in which steps are taken is paramount, so comparing similar formulae is useful. Students should also be able to identify errors as part of AO2 reasoning, and this topic provides good practice to support developing their communication skills.

Key vocabulary

Subject	Formula	Order
Inverse	Square/square root	

Key questions

If you are multiplying or dividing, why is it important to do this to every term? When should squaring/square rooting take place?

What is the first step when rearranging this formula?

How can we check that the rearrangement is correct?

Exemplar Questions

Teddy rearranges the formula $y = 2x^2 + 5$ to make x the subject.
Here are the first two lines of his working.

$$y = 2x^2 + 5$$
$$\frac{y}{2} = x^2 + 5$$

Explain what Teddy has done wrong.
Make x the subject of $y = 2x^2 + 5$

Dora rearranges the formula $p = q + t^2$ to make t the subject.
Here is her working.

$$p = q + t^2$$
$$p - q = t^2$$
$$\sqrt{p} - \sqrt{q} = t$$

Explain what Dora has done wrong.

What's the same and what's different about rearranging these formulae to make x the subject?

- $A = 3x^2 - b$
- $A = 3(x^2 - b)$
- $A = x^2 - 3b$
- $A = 3\sqrt{x} - b$
- $A = \sqrt{3x} - b$
- $A = \sqrt{3x - b}$
- $A = \frac{x^2 - b}{3}$
- $A = \left(\frac{x - b}{3}\right)^2$
- $A = \left(\frac{\sqrt{x} - b}{3}\right)^2$

Repeated subject

H

Notes and guidance

This Higher tier step requires students to rearrange a formula where the subject appears more than once. Students need to collect together the terms that feature the intended subject, which will often require factorisation and sometimes expansion first. Depending on which side terms are collected, sometimes students will arrive at equivalent but different answers - this provides a good point for discussion.

Key vocabulary

Subject	Expand	Factorise
Collect like terms		

Key questions

How many times does the new subject appear in this formula? What is the first step we need to take?

Do we need to expand any brackets or not?

Is it possible to collect like terms or factorise?

Exemplar Questions

Complete the working to make b the subject of the formula.

$$t = 3b + ab$$
$$t = b(\square + \square)$$
$$\frac{t}{\square + \square} = b$$

Aisha is making y the subject of these formulae.

- $h = 7y + 8g + 4y$
- $v = 7y + 8g + xy$

Her first step is to collect all the terms involving y on one side. Explain why she has to factorise to rearrange one of the formulae but not the other.

Make t the subject of each formula.

- $5(t + a) = 3(b + t)$
- $5(t + a) = xt$
- $5(t + a) = x(b + t)$
- $t(2 + a) = 3(t + 7)$
- $t(2 + a) = x(t + a)$
- $a = \frac{t + 4}{t + 2}$
- $a = \frac{t + x}{t - y}$
- $a = \frac{x + t}{y - t}$

What's the same and what's different?

Esther's answer to a question is $x = \frac{5-a}{11-a}$

The answer is the textbook is $x = \frac{a-5}{a-11}$

Has Esther made a mistake or not?

Solve equations by iteration H

Notes and guidance

Students met the notation u_n to define sequences in Year 10. Here they use the notation when solving equations using iterative processes, and they can either find rearrangements or confirm that a given iterative formula rearranges to the original equation. If there is time, it is interesting to explore which rearrangements of a given formula will converge to a solution and which will not.

Key vocabulary

Iterate	Repeat	Rearrange
Solution	Converge	

Key questions

How can we check the rearrangement is correct?

What do x_1, x_2, x_3 etc. mean?

How can we check that our last iteration is a good estimate for the solution of the equation?

Exemplar Questions

Complete the workings to show that the equation $3x^2 + 5x - 4 = 0$ can be rearranged to give the equation $x = \sqrt{\frac{4-5x}{3}}$

$$3x^2 + 5x - 4 = 0$$
$$3x^2 = 4 - 5x$$
$$x^2 = \ldots$$
$$x = \ldots$$

A sequence is given by the rule $u_{n+1} = 2u_n - 1$
Given that $u_1 = 3$, find the values of u_2, u_3 and u_4

Using $x_{n+1} = \frac{7}{x_n^2+3}$ with $x_0 = 1$, find the values of x_1, x_2, x_3 and x_4

By rearranging $x = \frac{7}{x^2+3}$, show that the iteration formula gives an estimate for the solution of $x^3 + 3x - 7 = 0$

- Show that the equation $x^3 + 2x = 5$ has a root between 1 and 2
- Show that the equation $x^3 + 2x = 5$ can be rearranged to give $x = \sqrt[3]{5 - 2x}$
- Starting with $x_0 = 1$, use the iteration formula $x = \sqrt[3]{5 - 2x}$ three times to find an estimate for the solution of $x^3 + 2x = 5$

Functions

Small Steps

Use function machines	R
Substitution into expressions and formulae	R
Use function notation	
Work with composite functions	H
Work with inverse functions	H
Graphs of quadratic functions	
Solve quadratic inequalities	H
Understand and use trigonometric functions	R

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Use function machines

R

Notes and guidance

Students will recap using function machines in order to aid their understanding when moving onto more abstract functions in later steps. It's important that they can find an output for a given input, and also an input for a given output in both one- and two-step function machines. The link between a numerical input/output and an algebraic input/output of a given function machine is essential knowledge for this block.

Key vocabulary

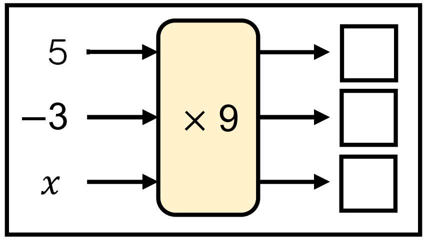
Input	Output	Function
Operation	Inverse	Variable

Key questions

If the input is ___, what is the output? How do you know?
If the output is ___, what is the input? How do you know?
How can you check your answer?
How do you calculate the input given the output?
What is the difference between $2x + 3$ and $2(x + 3)$?

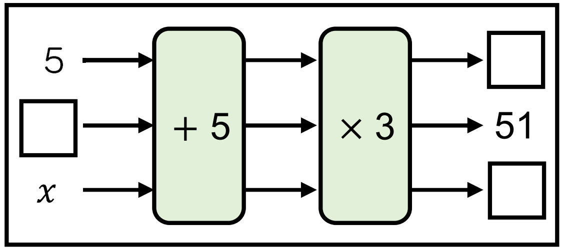
Exemplar Questions

Work out the missing outputs for the function machine.



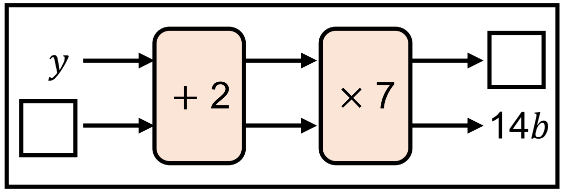
- Work out the value of $9x$ when $x = 5$
- Work out the value of $9x$ when $x = -3$
- What do you notice?

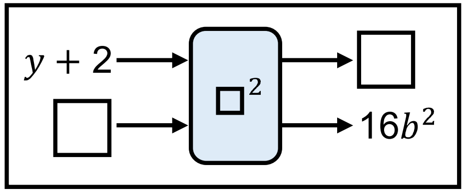
Work out the missing inputs and outputs for the function machine.

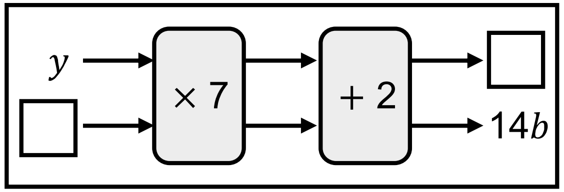


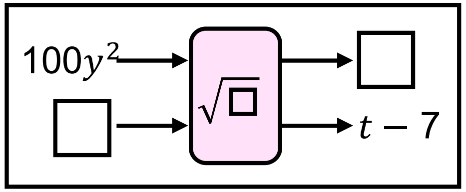
- Evaluate $3(x + 5)$ when $x = 5$
- Solve $3(x + 5) = 51$
- What do you notice?

Work out the missing inputs and outputs for the function machines.









Substitution

R

Notes and guidance

This small step provides opportunity for students to revise substituting into expressions and formulae. There is also plenty of opportunity to recap other areas of the curriculum such as fractions, decimals, directed numbers, area and volume etc. as meets your students' needs. It is useful to explore misconceptions such as $2x^2 = (2x)^2$. Students should be exposed to examples involving two or more variables.

Key vocabulary

Evaluate	Substitute	Expression
Formulae	Variable	

Key questions

What does it mean to substitute a value?

What is the difference between the expressions $x - 7$ and $7 - x$?

How can you use substitution to show that $x = 5$ is the solution to the equation $3x - 9 = 6$?

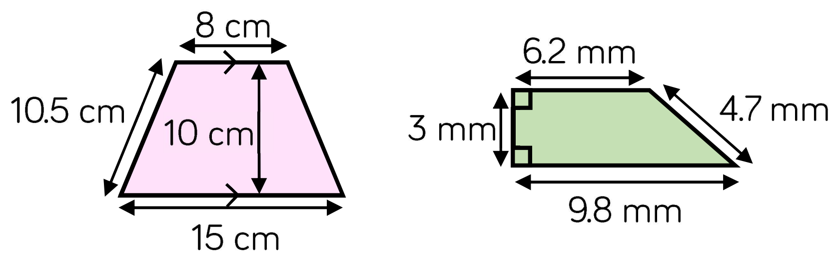
Choose values for x to show that $5x^2 \neq (5x)^2$

Exemplar Questions

Evaluate each expression when $x = 6$

- $5x + 11$
- $30 - 2x$
- $\frac{x+7}{4}$
- $6(x - 1)$
- x^2
- $2x^2$
- $(2x)^2$
- $x^2 + 5x - 3$

The area A of a trapezium is given by the formula $A = \frac{1}{2}(a + b)h$ where a and b are the lengths of the parallel sides, and h is the perpendicular distance between them.
Work out the area of each trapezium.



Speed (S), distance (D) and time (T) are connected by the formula $S = \frac{D}{T}$.
A car travels 420 miles in 8 hours. Work out its speed in mph.

Use substitution to show that $y = 5$ and $y = -2$ are solutions to the equation $y^2 - 3y - 10 = 0$

Use function notation

Notes and guidance

In this small step, students are introduced to formal function notation for the first time. The most common function notation is $f(x)$ which reads 'f of x'. $f(x)$ is a function applied to x, and $f(5)$ for example would be worked out by substituting $x = 5$ into the function. Students should also be aware that other letters can be used, with different letters used to distinguish between different functions within the same question.

Key vocabulary

Function   Variable   Evaluate   Solve

Key questions

What's the difference between $f(x)$ and $f(2)$?
What's the difference between $f(x)$ and $f(a)$?
What's the difference between $f(x)$ and $g(x)$?
If you know that $h(x) = 3x + 7$, how can you work out $h(2x)$?
If you know that $f(x) = 12$, what else do you know?

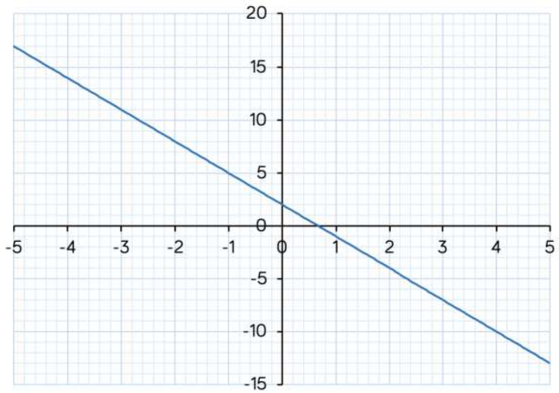
Exemplar Questions

The function $f(x)$ is defined by $f(x) = 5x + 2$
Work out

- $f(3)$
- $f(-4)$
- $f(0.5)$
- $f(\frac{1}{5})$
- $f(2a)$

How does this relate to finding points on the line $y = 5x + 2$?

A function, $g(x)$, is shown on the graph.



- Find $g(4)$
- Find $g(0)$
- Find $g(-2)$
- Solve $g(x) = 11$

The function $h(x)$ is defined by $h(x) = \frac{x+3}{7}$

- Solve $h(x) = \frac{5}{7}$
- Solve $h(x) = -9$

The function $f(x)$ is defined by $f(x) = x^2 - 9$

- Evaluate $f(2x)$
- Evaluate $f(x + 5)$
- Solve $f(x) = 0$

Composite functions

H

Notes and guidance

A composite function is a function made of other functions, where the output of one is the input of the other. For example, $fg(x)$ reads as 'f of g of x' and students will first need to evaluate $g(x)$ before substituting the result into $f(x)$. The order is important, and students should explore the difference between $fg(x)$ and $gf(x)$, knowing they should "work form the middle" when evaluating composite functions.

Key vocabulary

Function	Variable	Evaluate
Solve	Composite	Substitute

Key questions

What does $fg(x)$ mean?
What does $gf(x)$ mean?
What does $ff(x)$ mean?
Calculate $fg(5)$ and $gf(5)$. What's the same? What's different?
If $f(2) = 6$ and $g(6) = 17$, what is $gf(2)$?

Exemplar Questions

Given that $f(x) = 7x + 11$ and $g(x) = 10 - x$, work out

- $f(2)$
- $g(2)$
- $fg(2)$
- $gf(2)$
- $ff(2)$
- $gg(2)$

$g(b) = 5 + 3b$ $\qquad$ $h(b) = 2b^2$

Tommy and Eva are working out $hg(b)$ but they have each made a mistake. Spot the mistakes and work out $hg(b)$.

Tommy

$$hg(b) = 5 + 3(2b^2) = 5 + 6b^2$$

Eva

$$hg(b) = 2(5 + 3b)^2 = 2(25 + 9b^2) = 50 + 18b^2$$

$f(x) = 4x - 13$ $\qquad$ $g(x) = 15 - 8x$ $\qquad$ $h(x) = x^2 - 36$

Find expressions for:

- $fg(x)$
- $gf(x)$
- $fh(x)$
- $hf(x)$
- $gh(x)$
- $hg(x)$

Solve the equations:

- $fg(x) = 30$
- $fg(x) = gf(x)$

Inverse functions

H

Notes and guidance

In this small step students will be introduced to inverse functions, making the link to inverse operations. The inverse of $f(x)$ is denoted $f^{-1}(x)$ and can be confused with the reciprocal x^{-1}. Students need to be secure in rearranging formula before looking at inverse functions. Working backwards through function machines is suitable for simple cases, but not for more complex cases.

Key vocabulary

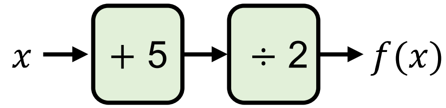
Function	Variable	Evaluate
Solve	Inverse	Rearrange

Key questions

What is an inverse operation?
If $y = x + 9$, how would you work out x given y?
What is an inverse function?
If $f(7) = 19$, what is $f^{-1}(19)$? How do you know?
Work out $ff^{-1}(x)$ and $f^{-1}f(x)$. What do you notice? Will this always happen?

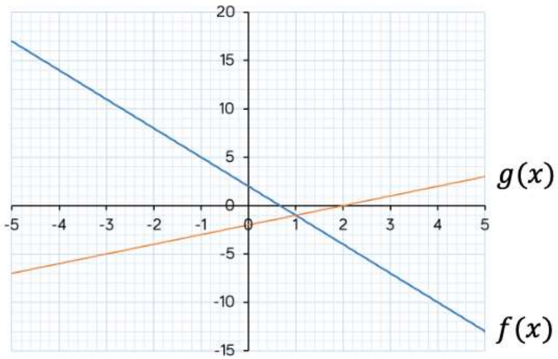
Exemplar Questions

Here is a function machine.



- Write an expression for $f(x)$
- If the output is 19, what was the input?
- Work out $f^{-1}(x)$

Two functions, $f(x)$ and $g(x)$ are shown on the graph.



- Solve $f(x) = g(x)$
- Find an expression for $f^{-1}(x)$
- Find an expression for $g^{-1}(x)$
- Solve $f^{-1}(x) = g^{-1}(x)$

$$g(x) = \frac{7x-1}{2}$$

- Find $g^{-1}(x)$
- Find $gg^{-1}(x)$
- Find $g^{-1}g(x)$

Given that $h(x) = \frac{5x+2}{x+4}$, find an expression for $h^{-1}(x)$

Graphs of quadratic functions

Notes and guidance

This small step consolidates quadratic graphs. All students should be able to recognise and plot the graph of a quadratic function. They need to be able to estimate solutions and identify the coordinates of the turning point. Students sitting Higher tier GCSE should also be able to identify the turning point by completing the square, recognising the turning point of $y = (x + a)^2 + b$ has coordinates $(-a, b)$.

Key vocabulary

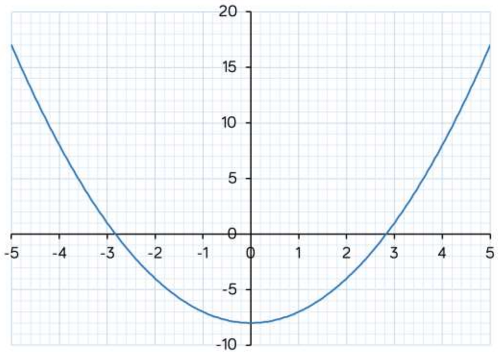
Quadratic	Function	Graph
Intercept	Turning point	Roots

Key questions

How do you recognise the graph of a quadratic function?
How many turning points will it have?
At what point will the graph of $y = x^2 + 5x - 1$ intercept the y-axis?
How do you identify the turning point from the completed square form?

Exemplar Questions

The function $f(x) = x^2 - 8$ is shown on the graph.



- Is the function linear, quadratic, cubic or other? How do you know?
- What are the coordinates of the turning point of the graph?

Complete the table of values for $y = x^2 + 3x - 1$

x	-3	-2	-1	0	1	2	3
y							

- Plot the graph of $y = x^2 + 3x - 1$
- Identify the turning point of the graph of $y = x^2 + 3x - 1$
- Estimate the solutions to $x^2 + 3x - 1 = 0$

By writing each equation in the form $y = (x + a)^2 + b$, identify the coordinates of the turning point of each quadratic function.

$y = x^2 + 8x - 7$ $\qquad$ $y = x^2 - 12x + 1$ $\qquad$ $y = x^2 - 7x$

Solve quadratic inequalities H

Notes and guidance

This topic was previously covered in the Autumn term of year 10. Here it provides opportunities for students to consolidate factorising, and then link their factorisation to the solution set. They need to be able to represent their solutions on a graph, a number line and using set notation. Look out for erroneous statements such as "$x < -3$ and $x > 3$", as x cannot satisfy both conditions at once.

Key vocabulary

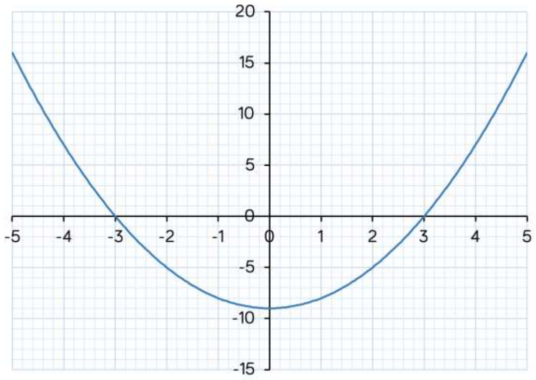
Quadratic	Inequality	Solve
Represent	Set	Solution

Key questions

How do you identify the region on a graph which shows where $y < 0$? So how can you find the values of x for which $y < 0$?
What is the difference between $>$ and $\geq$?
How do you represent your solutions on a number line?
Why will $x^2 + 23$ be positive for all values of x?

Exemplar Questions

The function $f(x) = x^2 - 9$ is shown on the graph.



- Shade the region on the graph where $f(x) < 0$
- Shade the region on the graph where $f(x) > 0$
- Solve $x^2 - 9 \leq 0$

A function, g, is given by $g(x) = 2x^2 - 7x - 30$

- Sketch $g(x)$ highlighting any roots and intercepts.
- Solve $g(x) > 0$ giving your answer in set notation.

Amir and Mo are solving $12 - x - x^2 > 0$
Mo says "The solution is $-4 < x < 3$"
Amir says "The solution is $x < -4$ and $x > 3$"
They are both incorrect.
Explain any mistakes and work out the correct solution.
Show your solution on a number line.

Solve $3x^2 + 32x + 72 \leq 27$

Trigonometric functions

R

Notes and guidance

This step provides a timely opportunity to remind students how to find missing sides and angles in right-angles triangles, relating the trigonometric ratios to the corresponding functions, last studied in Year 10. Depending on your class' needs, you might also remind them of exact trig values. Higher tier students could also revise using the sine and cosine rule. You could use Year 10 worksheets to support this step.

Key vocabulary

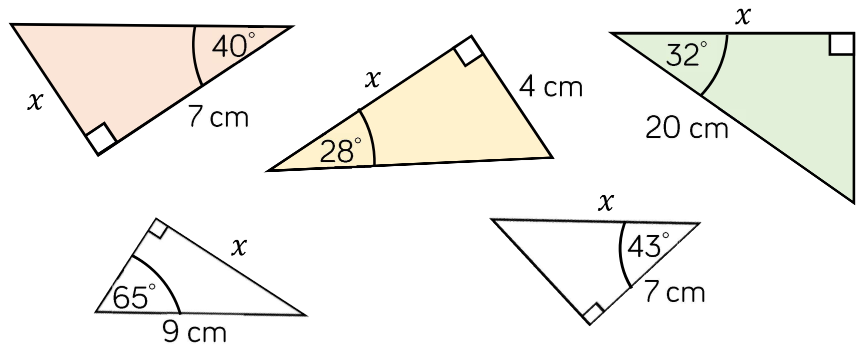
Ratio	Sine	Cosine	Tangent
Opposite	Adjacent	Hypotenuse	

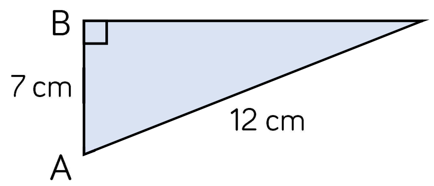
Key questions

How do you decide which ratio to use to find a missing side or angle in a right-angled triangle?
How do you know which side is which?
How can I use trigonometry in a rectangle or a non-right-angled isosceles triangle?
What's the difference between $\sin x$ and $\sin^{-1} x$?

Exemplar Questions

Which trigonometric ratio would you use to find the sides labelled x in each right-angled triangle?





- Calculate the size of angle BAC
- How can you calculate the length BC without using trigonometry?

Work out the size of:

- angle BAD
- angle ABC

Find the area of the trapezium.