Spring Term

Year 11

#MathsEveryoneCan

WRM – Year 11 Scheme of Learning

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Graphs**						**Algebra**					
	Gradients & lines		Non-linear graphs		Using graphs		Expanding & Factorising		Changing the subject		Functions	
Spring	**Reasoning**						**Revision and Communication**					
	Multiplicative		Geometric		Algebraic		Transforming & Constructing		Listing & describing		Show that...	
Summer	**Revision**						**Examinations**					

Spring 1 : Reasoning

Weeks 1 and 2: Multiplicative Reasoning

Students develop their multiplicative reasoning in a variety of contexts, from simple scale factors through to complex equations involving direct and inverse proportion. They link inverse proportion with the formulae for pressure and density. There is also the opportunity to review ratio problems.

National Curriculum content covered includes:

- compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity
- understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$
- **{construct and}** interpret equations that describe direct and inverse proportion
- extend and formalise their knowledge of ratio and proportion, including trigonometric ratios, in working with measures and geometry, and in working with proportional relations algebraically and graphically

Weeks 3 and 4: Geometric Reasoning

Students consolidate their knowledge of angles facts and develop increasingly complex chains of reasoning to solve geometric problems. Higher tier students revise the first four circle theorems studied in Year 10 and learn the remaining theorems. Students also revisit vectors and the key topics of Pythagoras' theorem and trigonometry.

National Curriculum content covered includes

- reason deductively in geometry, number and algebra, including using geometrical constructions
- **{apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results}**
- interpret and use bearings
- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; **{use vectors to construct geometric arguments and proofs}**

Weeks 5 and 6: Algebraic Reasoning

Students develop their algebraic reasoning by looking at more complex situations. They use their knowledge of sequences and rules to made inferences, and Higher tier students move towards formal algebraic proof. Forming and solving complex equations, including simultaneous equations, is revisited. Higher tier students also look at solving inequalities in more than one variable.

National Curriculum content covered includes:

- make and test conjectures about the generalisations that underlie patterns and relationships; look for proofs or counter-examples; begin to use algebra to support and construct arguments **{and proofs}**
- deduce expressions to calculate the n^{th} term of linear **{and quadratic}** sequences
- solve two simultaneous equations in two variables (linear/linear **{or linear/quadratic}**) algebraically; find approximate solutions using a graph
- solve linear inequalities in one **{or two}** variable**{s}**, **{and quadratic inequalities in one variable}**; represent the solution set on a number line, **{using set notation and on a graph}**

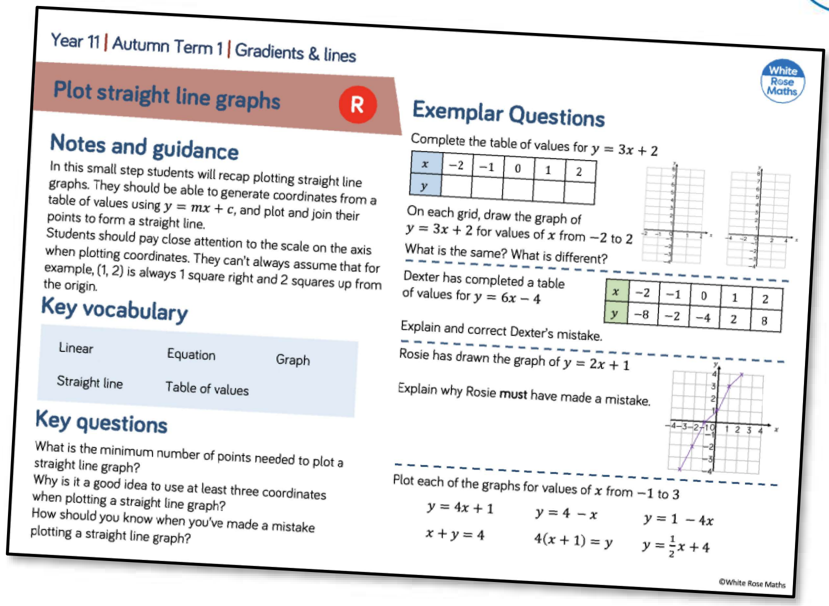
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 11 | Autumn Term 1 | Gradients & lines

Plot straight line graphs R

Notes and guidance

In this small step students will recap plotting straight line graphs. They should be able to generate coordinates from a table of values using $y = mx + c$, and plot and join their points to form a straight line.
Students should pay close attention to the scale on the axis when plotting coordinates. They can't always assume that for example, (1, 2) is always 1 square right and 2 squares up from the origin.

Key vocabulary

Linear, Equation, Graph, Straight line, Table of values

Key questions

What is the minimum number of points needed to plot a straight line graph?
Why is it a good idea to use at least three coordinates when plotting a straight line graph?
How should you know when you've made a mistake plotting a straight line graph?

Exemplar Questions

Complete the table of values for $y = 3x + 2$

x	-2	-1	0	1	2
y					

On each grid, draw the graph of $y = 3x + 2$ for values of x from -2 to 2
What is the same? What is different?

Dexter has completed a table of values for $y = 6x - 4$

x	-2	-1	0	1	2
y	-8	-2	-4	2	8

Explain and correct Dexter's mistake.

Rosie has drawn the graph of $y = 2x + 1$

Explain why Rosie **must** have made a mistake.

Plot each of the graphs for values of x from -1 to 3

$y = 4x + 1$ $\quad y = 4 - x$ $\quad y = 1 - 4x$

$x + y = 4$ $\quad 4(x + 1) = y$ $\quad y = \frac{1}{2}x + 4$

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- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

Multiplicative Reasoning

Small Steps

- Use scale factors R
- Understand direct proportion
- **Construct complex direct proportion equations** H
- Calculate with pressure and density
- Understand inverse proportion
- **Construct inverse proportion equations** H
- Ratio problems R

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Use scale factors

R

Notes and guidance

This step reviews the concept of a scale factor. This is a good opportunity to use scale factors between 0 and 1 (reminding students that this is still an enlargement) as well as those above 1. In this step, students should practise finding scale factors as well as using them. Revisiting the definition of a similar shape is also emphasised in this step. To extend this review step, Higher tier students could revisit area and volume scale factors.

Key vocabulary

Enlargement	Scale factor
Multiplier	Similar

Key questions

If an object is enlarged, does it always get bigger?

How can you work out the scale factor? Is there more than one method?

When are two shapes similar?

Exemplar Questions

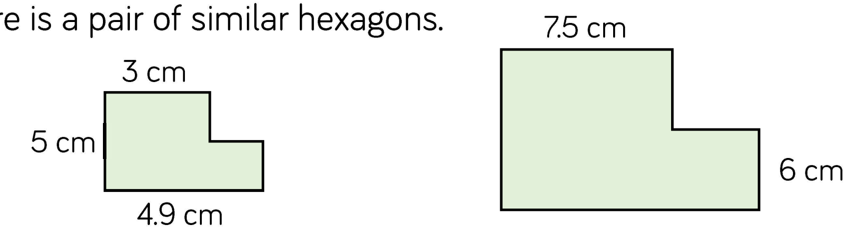
Shape B is an enlargement of shape A.

Mo has incorrectly worked out the value of x. Here is his working.

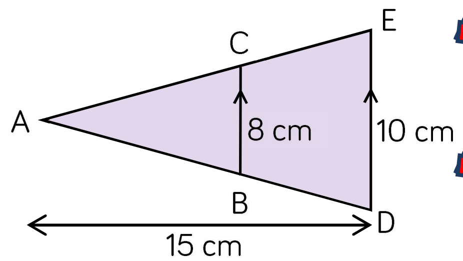
6 cm − 2 cm = 4 cm and 15 cm − 2 cm = 13 cm so x = 13 cm

Explain Mo's mistake and work out the value of x.

Here is a pair of similar hexagons.



Work out all of the missing lengths on both shapes.



Prove that triangles ABC and ADE are similar.

Work out the perpendicular height of ABC.

Is the length scale factor the same as the area scale factor? Use calculations to justify your answer.

Understand direct proportion

Notes and guidance

This aim of this step is to understand direct proportion before introducing $y = kx$. Direct proportion relationships such as diameter and circumference, converting units, currency conversions etc. can all be revisited. Students should be exposed to different representations such as word problems, graphs and equations. Students studying for Foundation GCSE should also form simple direct proportion equations in this step ($y = kx$).

Key vocabulary

Direct proportion	Equation	Origin
Constant ratio	Straight line	Linear

Key questions

List as many examples as you can of relationships that are in direct proportion to one another.

Why aren't (e.g. speed and time) directly proportional to one another?

Describe the key features of a graph representing direct proportion.

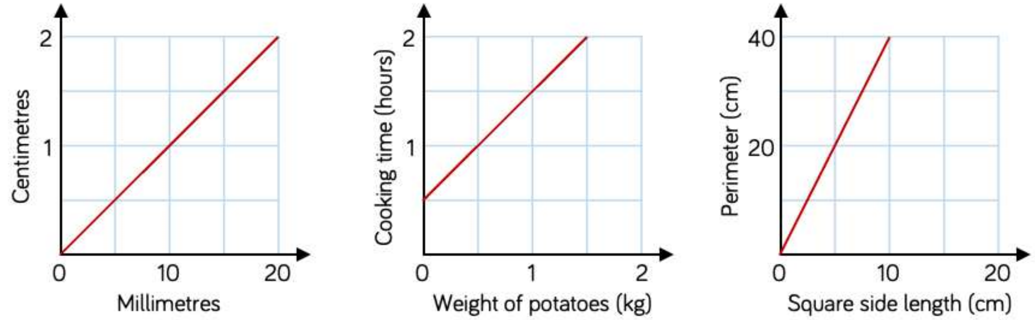
Exemplar Questions

A train is travelling at a constant speed of 140 km/h.
Work out how far the train will it travel in

- 6 hours
- 2.5 hours
- 30 minutes
- 20 minutes

Eva says "distance travelled is directly proportional to the time travelled".
Explain why Eva is correct.

Which of the graphs does not represent a directly proportion relationship? Explain how you know.



Write down a formula to find

- the perimeter of a square if given its side length
- a length in centimetres if given the length in millimetres.

Which of these equations represent direct proportion relationships?

Euros = 1.1 × British Pounds	Mass = Density × Volume
Cost of stapler and staples = 2 + (Number of packs of staples × 0.5)	

Direct proportion equations H

Notes and guidance

Students are introduced to the proportionality symbol, α, and the constant of proportionality, k. Knowledge of constant ratios in a direct proportion relationship leads to the general equation $y = kx$. A common mistake is to find k but then forget to substitute it into the equation. Encourage students to write out the full equation as soon as k has been found can. In more complex problems, students often confuse the use of e.g. 'square/square root' and this needs highlighting.

Key vocabulary

Direct proportion

Constant of proportionality

Equation

Varies directly

Key questions

When is it appropriate to use the equation $y = kx$?
How can we work out k? What is the resulting equation?
How can this equation be used to find x when given y?
How can this equation be used to find y when given x?
What do you notice about the value of y when $x = 0$?
Will this always be true? Why?

Exemplar Questions

a and b are directly proportional to each other.

a	b
1	8
5	
	56
	800

- Calculate the missing values in the table
- Dexter says, "$b = ka$"
  Dexter is correct. Work out the value of k.
- Use your equation to find b when $a = 0.35$

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c is directly proportional to the square of d.
Which of the cards are true?

$c = d^2$	$c \alpha d$	$c \alpha d^2$	$c \alpha kd$
$c = k \times d$	$c = k \times d^2$	$c = \sqrt{d^2}$	$c = kd^2$

When $c = 25$, $d = 2.5$
Work out the value of the constant of proportionality, k.

---

y varies directly with the cube root of x.
When $y = 27$, $x = 216$

- Show that when $x = 1000$, $y = 45$
- Work out y when $x = 1\,000\,000$
- Work out the value of x when $y = 90$

Calculate with pressure & density

Notes and guidance

You might want to revisit rearranging simple equations in the form $4.1 = \frac{3.6}{x}$ before starting this step. Speed, distance and time can also be reviewed, making links to direct proportion. Students then consider the similar formulae for pressure and density. Students should have a good understanding of what these concepts are before progressing onto use of equations. Understanding of the units used is important.

Key vocabulary

Density	Mass	Volume
Pressure	Force	Area

Key questions

If we compare two solids on a table which apply the same force, how can we tell which one will apply a greater pressure?

How do we know which number to substitute?

What is the relationship with direct proportion?

Exemplar Questions

Sort the cards into those that represent density, mass or volume.

8 g/cm^3	1 cm^3	36.2 g	2 m^3	0.01 g/cm^3

Lead has a density of 11.3 g/cm^3.
Complete the table.

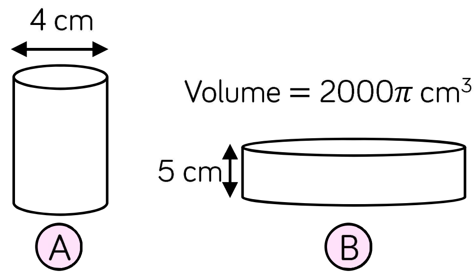
Volume	2 cm^3	3 cm^3	4 cm^3	10 cm^3
Mass				

Teddy says "mass is directly proportional to volume".
Explain why Teddy is correct.

Work out the constant of proportionality for lead.

A piece of tin and a piece of aluminium each have a mass of 1 kg.
The volume of the piece of tin is 137.74 cm^3.
The density of the piece of aluminium is 2.7 g/cm^3.
Compare the density, mass and volume of the two pieces of metal.

Complete the table.



	Cylinder A	Cylinder B
Pressure		10 N/cm^2
Force	300 N	
Area of base		

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Understand inverse proportion

Notes and guidance

Students can now consider the three variables in the speed, distance, time or mass, density, volume relationships to distinguish between direct and inverse proportion. Inverse proportion relationships should be explored in different representations such as word problems, graphs and equations. Students studying for Foundation GCSE should also form simple inverse proportion equations in this step ($y = \frac{k}{x}$).

Key vocabulary

Inverse proportion	Equation
Smooth curve	Constant of proportionality

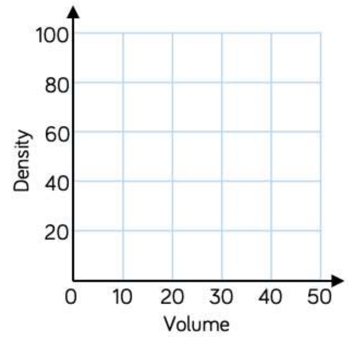
Key questions

What is the same and what is different about direct and inverse proportion?
Will it take less time for more people to complete a task? What assumption have you made?
How can we write an equation to represent inverse proportion?

Exemplar Questions

Complete the table.

Mass	Volume	Density
100 kg	1 cm^3	
100 kg	10 cm^3	
100 kg	20 cm^3	
100 kg	50 cm^3	



- What happens to the density of an object as the volume increases?
- Sketch a graph of volume verses density on a copy of the axis shown.
  What are they key features of the graph?
  How does this differ from direct proportion?

A builder takes 1 day to build a wall.

Alex says "if there were 2 builders it would have taken them 2 days."

Explain why is Alex incorrect.
How long will it take 2 builders if they both work at the same pace?

Decide whether each of these relationships are directly or inversely proportional.

Density of an object and its volume	Number of days it takes to build a house and number of builders	Speed of a car and distance it travels

Inverse proportion equations H

Notes and guidance

By now students should be familiar with both simple direct and inverse proportional relationships. This leads to more complex inverse proportion equations, working in the abstract.

You may wish to explore both the forms $y = \frac{k}{x}$ and $xy = k$ as ways of solving inverse proportion problems. Students again need to be careful when reading questions to identify the correct power of the variable in a relationship.

Key vocabulary

Inverse proportion	Varies inversely
Constant of proportionality	Substitute

Key questions

How can we work out the constant of proportionality, k?

How can we use this equation to calculate x given y and vice versa?

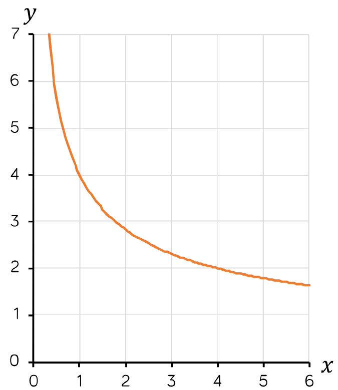
How do we know what to substitute?

Exemplar Questions

x and y are inversely proportional to each other.

x	y
1	24
2	12
3	
	6

- Calculate the missing values in the table
- Annie says "$x \propto \frac{1}{y}$ and so $x = \frac{k}{y}$"
  Annie is correct. Calculate the value of k.
- Use your equation to find x when $y = 1.5$



- Find the value of y when $x = 1$
- y is inversely proportional to the square root of x
  Dexter writes $y = \frac{k}{x^2}$
  Correct Dexter's mistake and calculate k.
- Calculate y when $x = 100$

r is inversely proportional to the cube of s.
When $r = 8, s = 5$

- Show that when $r = 64, s = 2.5$

Give as exact answers in simplest form.

- Work out r when $s = 8$
- Work out the value of s when $r = 10$

Ratio problems

R

Notes and guidance

This step is an opportunity for students to revisit ratio problems and strategies for solving these. Students should be encouraged to use bar models and two-way tables where appropriate. When combining ratios, teachers may want to revisit LCM first. For students studying the higher tier GCSE, problems involving algebra and ratio will be appropriate.

Key vocabulary

Proportion	Fraction	Percentage
Bar model	Two-way table	LCM

Key questions

How can we use a bar model to represent the problem?

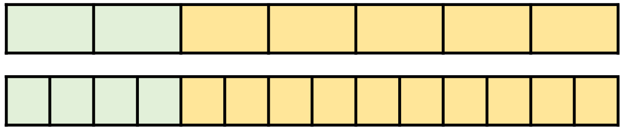
What does the bar model(s) tell us about our problem?

Do we need to divide the bar model up into more equal parts? How can we do this?

How can we find a percentage/fraction of a quantity using a calculator? Without a calculator?

Exemplar Questions

The ratio of boys to girls at a party is 2 : 5
Three-quarters of the boys wore jeans.



Use the bar models to work out the fraction of children at the party who were boys wearing jeans.

Mrs Rose has a salary £25 000 per year.

- 18% of this is spent on taxes and national insurance.
- $\frac{3}{8}$ of her salary is spent on rent.
- She gives £10 a month to charity.

She spends the rest on leisure and living costs in the ratio 2 : 3
What percentage of her overall salary did she spend on leisure?

One morning, a vet treated cats and dogs in the ratio 4 : 5
The same morning, the vet treated cats and rabbits in the ratio 6 : 1
Write down the ratio of cats to dogs to rabbits treated by the vet that morning.

The ratio of $x : y = 1 : 4$ and $3y = 2z$

- Work out the value of z when $y = 4$
- Show that the ratio $y : z = 2 : 3$
- Explain why the ratio $x : y : z$ is $1 : 4 : 6$

Geometric Reasoning

Small Steps

- Angles at points R
- Angles in parallel lines and shapes R
- Exterior and interior angles of polygons R
- Proving geometric facts
- Solve problems involving vectors R
- **Review of circle theorems** H
- **Circle theorem: Angle between radius and chord** H
- **Circle theorem: Angle between radius and tangent** H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Geometric Reasoning

Small Steps

- **Circle theorem: Two tangents from a point** H
- **Circle theorem: Alternate segment theorem** H
- Review Pythagoras' theorem and using trig ratios R

H denotes Higher Tier GCSE content
R denotes 'review step' – content should have been covered at KS3

Angles at points

R

Notes and guidance

This small step provides students with opportunity to revise rules of angles at points. They will revisit angles in a full turn, adjacent angles on a straight line and vertically opposite angles. As students have already seen these rules several times, interleaving other topics such as ratio and equations can be used to maintain the level of challenge whilst still securing this essential knowledge.

Key vocabulary

Angle	Adjacent	Vertically opposite
Point	Full turn	Straight line

Key questions

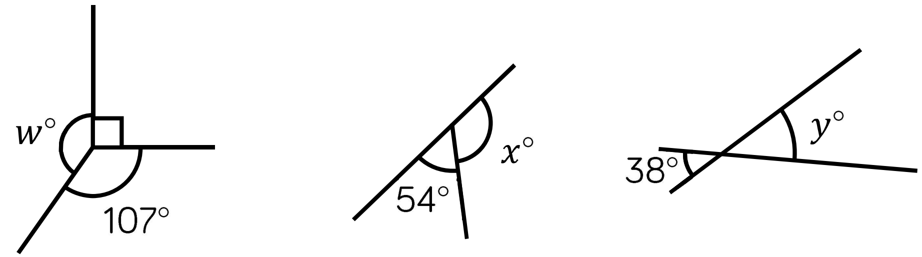
What is the sum of angles around a point?

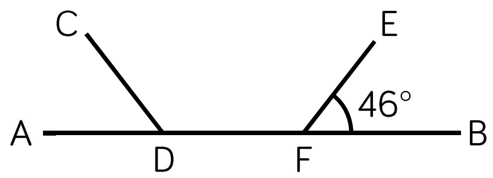
What is important about the word 'adjacent' in 'adjacent angles on a straight line sum to 180 degrees'?

What does it mean for two angles to be vertically opposite?

Exemplar Questions

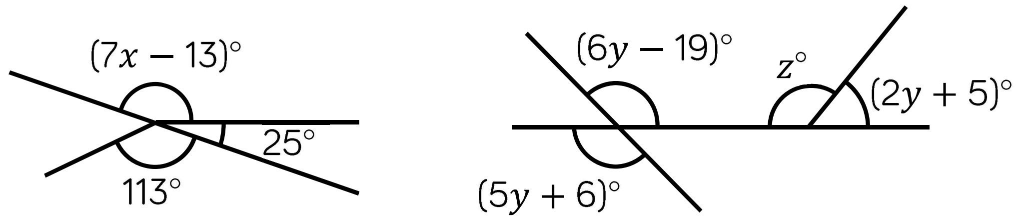
Work out the size of the angles marked with letters.
Give reasons for your answers.





Tommy says "Angle ADC is 134 degrees because angles on a straight line sum to 180 degrees".
Do you agree with Tommy? Explain your answer.

Work out the value of the letters.



Angles in parallel lines

R

Notes and guidance

Here students are reminded of rules of angles in parallel lines and shapes. Students will focus on triangles and quadrilaterals as shapes in this step, as polygons will be recapped more formally in the next small step. Students should be confident what is meant by alternate, corresponding and co-interior angles. This small step provides opportunity to revisit other content such as bearings.

Key vocabulary

Angle	Parallel	Corresponding
Alternate	Bearing	Co-interior

Key questions

What do angles in a triangle/quadrilateral sum to?

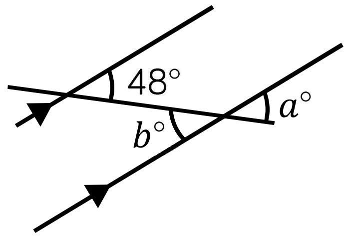
What is meant by alternate/corresponding?

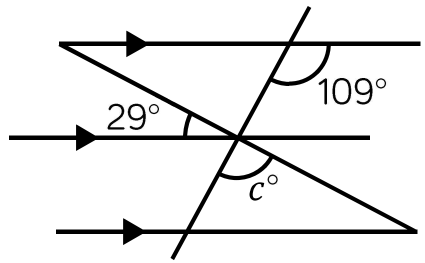
Could you have worked it out another way?

Are the line segments parallel? How do you know?

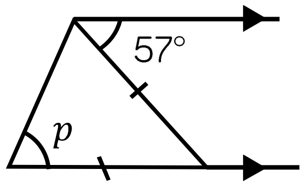
Exemplar Questions

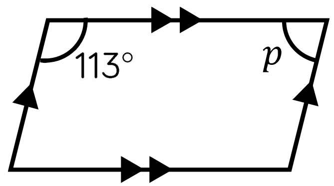
Work out the size of the angles marked with letters.
Give reasons for your answers.





Find the size of the angle marked p in each diagram.





The angles in a triangle are in the ratio 3 : 8 : 4
Work out the size of the smallest angle in the triangle.

The bearing of B from A is 102°
What is the bearing of A from B?

× A

× B

Angles in polygons

R

Notes and guidance

This small step provides opportunity for students to recap rules of angles in polygons. It's important that students understand what is meant by a regular polygon and are able to work out the size of both interior and exterior angles in a regular polygon. Students should be familiar with rules of both interior and exterior angles and should be able to work fluently with both in regular and irregular shapes.

Key vocabulary

Angle	Polygon	Regular
Interior	Exterior	

Key questions

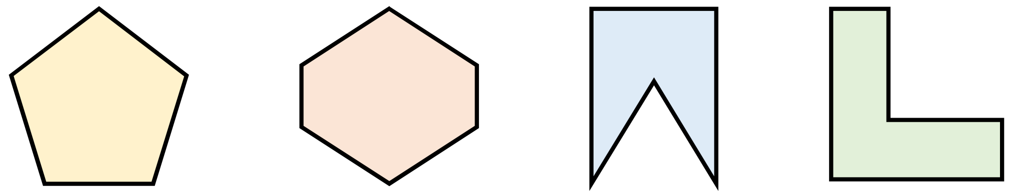
What does it mean for a polygon to be regular?

What is the sum of the angles in this polygon? How do you know?

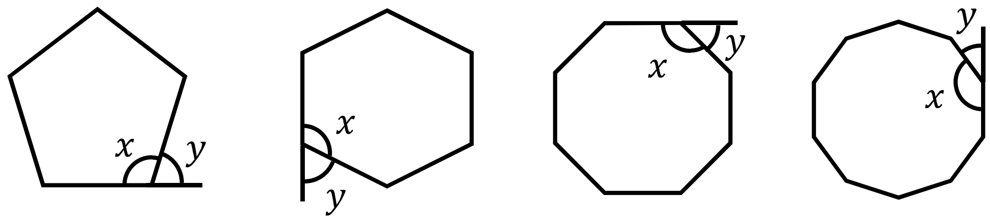
Could you have worked it out a different way?

Exemplar Questions

Find the sum of the interior angles of each shape.



Each of the polygons is regular.
Work out the size of the angles marked x and y.



A regular polygon has 15 sides.
- Work out the size of one interior angle of the polygon.
- Work out the size of one exterior angle of the polygon.

A regular polygon has n sides.
- Write an expression for the size of one interior angle of the polygon.
- Write an expression for the size of one exterior angle of the polygon.

The exterior angle of a regular polygon is 7.2°
How many sides does the polygon have?

Proving geometric facts

Notes and guidance

This small step provides opportunity for students to use all the angle facts they have covered to prove simple geometric facts. This is also a good opportunity to revisit properties of shape covered earlier in the curriculum. There should be a focus placed on the explanations used throughout each proof. Students should know that etc 'angles in a triangle' is not sufficient but 'angle in a triangle sum to 180 degrees' is.

Key vocabulary

Angle	Parallel	Equilateral
Isosceles	Right-angle	Trapezium

Key questions

What do you know? What can you find out?

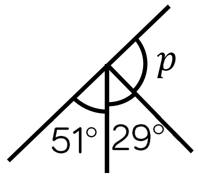
How do you know if a triangle is isosceles?

If the lines are parallel, what must be true?

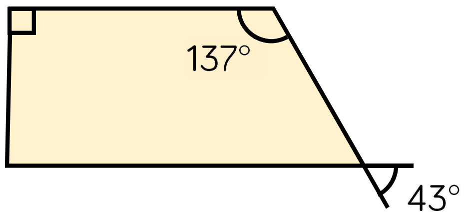
What are the properties of the shape? How does that help?

Exemplar Questions

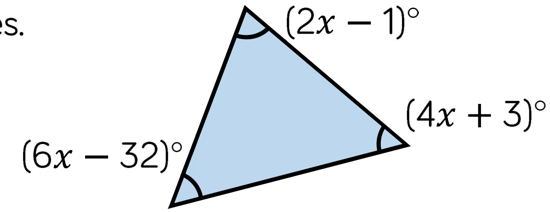
Show that the angle marked p is a right angle.



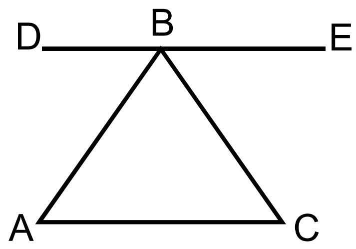
Prove that the quadrilateral is a trapezium.



Prove that this triangle is isosceles.



Line segments AC and DE are parallel.
ABC is a triangle such that point B lies on the line segment DE.
Using rules of angles in parallel lines, prove that the sum of the angles in a triangle is 180 degrees.



Solve problems involving vectors

Notes and guidance

Students should be able to find a column vector given a diagram and vice versa. They need to be able to calculate with vectors using addition and subtraction, and multiply a vector by a scalar. Students should know the conditions that make two or more vectors parallel and be able to prove this. They need to be particularly careful with directions of arrows and calculations involving negative numbers.

Key vocabulary

Vector	Column	Horizontal
Vertical	Position	Parallel

Key questions

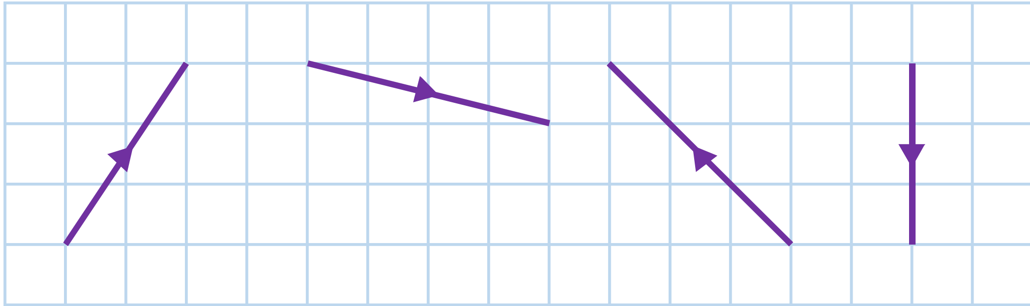
In the column vector $\binom{a}{b}$ what do a and b represent?

How does this link to vectors when performing a translation?

What does it mean for two or more vectors to be parallel?

Exemplar Questions

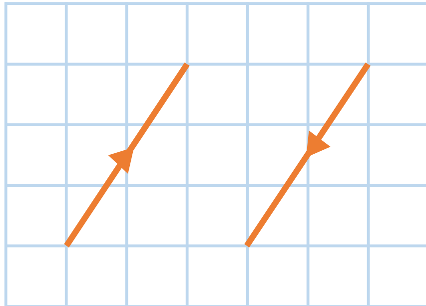
Write each vector in column form.



Two vectors are shown on the grid.

Mo says the vectors are the same.
Rosie says they're different.

Who do you agree with?
Explain your answer.



$\boldsymbol{a} = \binom{5}{-1}$ $\boldsymbol{b} = \binom{-3}{2}$

- Draw a diagram to show that $\boldsymbol{a} + \boldsymbol{b} = \binom{2}{1}$
- Work out $\boldsymbol{a} + 2\boldsymbol{b}$

$\boldsymbol{c} = \binom{4}{0}$ $\boldsymbol{d} = \binom{-2}{5}$ $\boldsymbol{e} = \binom{-6}{0}$

Show that $\boldsymbol{c} + \boldsymbol{d}$ is parallel to $3\boldsymbol{c} + 6\boldsymbol{d} - 2\boldsymbol{e}$

Review circle theorems H

Notes and guidance

Students will review the circle theorems covered in year 10:

- the angle subtended at the circumference is half the angle subtended at the centre
- the angle in a semi-circle is a right-angle
- angles in the same segment are equal
- opposite angles in a cyclic quadrilateral sum to 180°

Students should be able to use and prove each theorem.
Looking for isosceles triangles when solving problems.

Key vocabulary

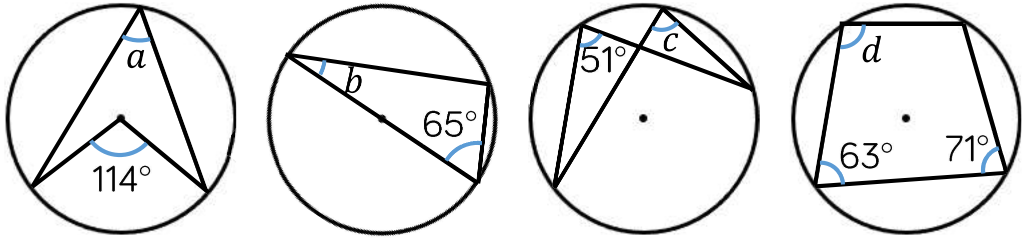
Circle	Segment	Circumference
Centre	Right-angle	Cyclic quadrilateral

Key questions

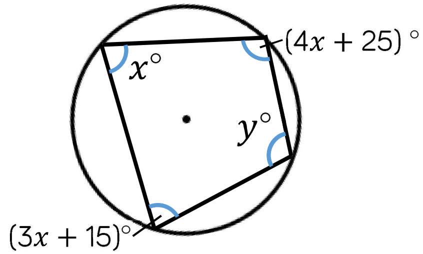
Which is the angle at the circumference?
Is O the centre of the circle? How do you know?
How can you use the fact that the angle at the centre is twice the angle at the circumference to prove that the angle in a semi-circle is 90 degrees?

Exemplar Questions

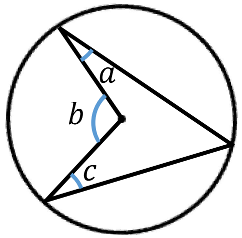
Work out the size of the angles marked with letters.



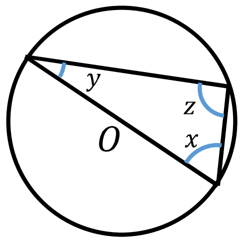
Work out the value of y.



$a : c = 5 : 7$
$a + c = 60$
Work out the size of the angle marked b.



$x : y : z = 3 : 1 : 5$
Is O the centre of the circle?



Angle between radius & chord H

Notes and guidance

Here students are introduced to the circle theorem that the angle between a radius and the midpoint of a chord is a right-angle. Students should be confident in what a chord is and what it isn't. Students should know that a line drawn from the centre of a circle to the midpoint of a chord is a perpendicular bisector of the chord. They then need to be able to apply this in questions.

Key vocabulary

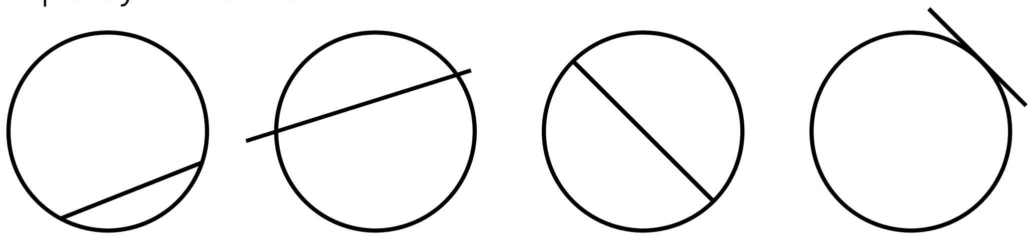
Circle	Angle	Radius
Chord	Bisects	Right-angle

Key questions

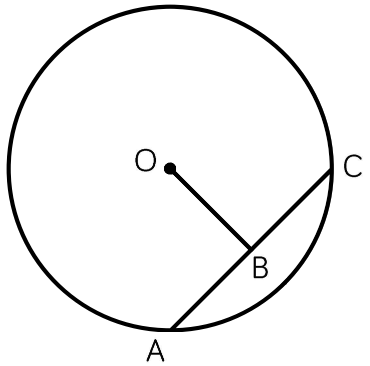
What is a chord?
What is a perpendicular bisector?
Is the diameter of a circle a chord? Why or why not?
If you know the length of the chord and the radius, how can you work out the length of the line from the centre of the circle to the chord?

Exemplar Questions

Which of the diagrams show a chord?
Explain your answer.

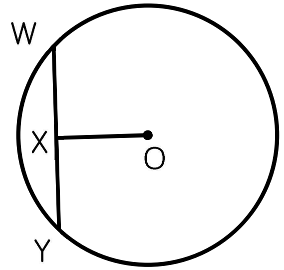


The diagram shows a chord AB drawn through a circle centre O.



- What is the size of $\angle OBC$?
- If $\angle OAB = 29°$, work out $\angle AOB$
- If $AC = 18$ cm, what is the length of AB?
- If $AB = 18$ cm, what is the length of AC?

WY is a chord through a circle centre O.
A straight line is drawn from O to point X on WY.
$WY = 15$ mm, $OY = 10$ mm.
Work out the length of OX.



Angle between radius & tangent H

Notes and guidance

Students are likely to already be confident in identifying a radius of a circle, and it's now important that they fully understand what is meant by tangent as this will be used in the next few theorems. Understanding of a radius meeting a tangent at 90 degrees is essential before going on to look at the alternate segment theorem. As with all the circle theorems, illustrating using dynamic geometry software is useful.

Key vocabulary

Circle	Circumference	Radius
Tangent	Right-angle	Angle

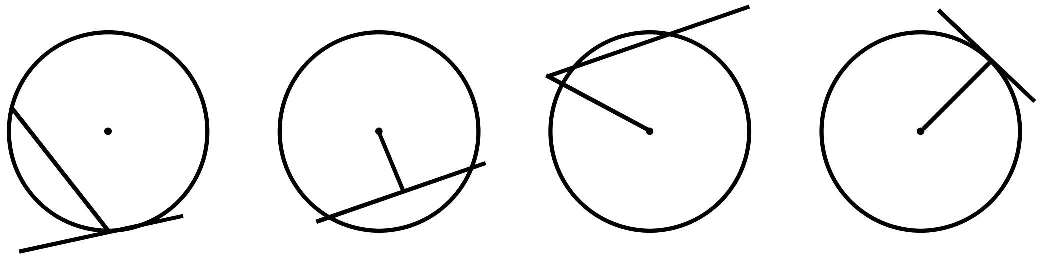
Key questions

What is a tangent to a circle? Where else do we use the word tangent in mathematics?

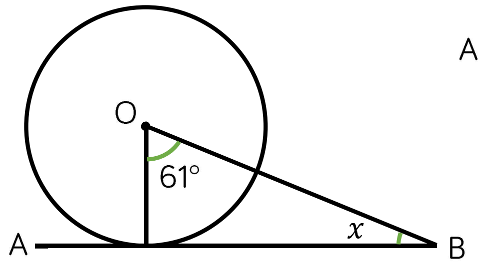
Is this line a tangent? How do you know?

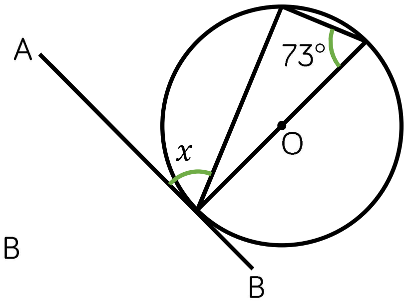
Exemplar Questions

Which diagram shows a radius meeting a tangent?

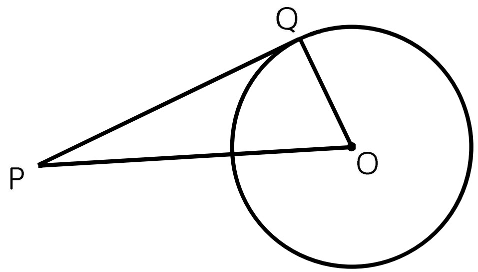


In each diagram, AB is a tangent to the circle centre O.
Work out the size of the angle marked x in each diagram.





O is the centre of the circle
$\angle POQ = 79°$
$\angle OQP = 15°$
Is PQ a tangent to the circle?
Explain your answer.



Two tangents from a point

H

Notes and guidance

Here students are introduced to the circle theorem that states that two tangents from the same point to the circumference of a circle are equal in length. They should relate this fact to other areas of mathematics, including isosceles triangles, pairs of congruent triangles or kites. Students can combine this theorem with their other knowledge of circle theorems to tackle more challenging problems. In particular, combining with the last step produces Pythagorean problems.

Key vocabulary

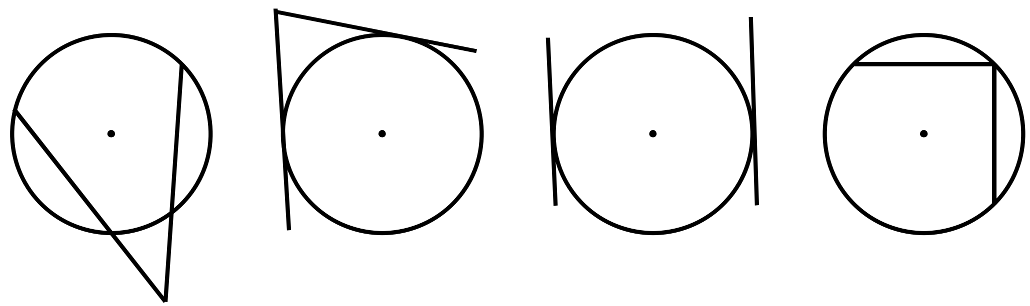
Circle	Circumference	Radius
Tangent	Right-angle	Length

Key questions

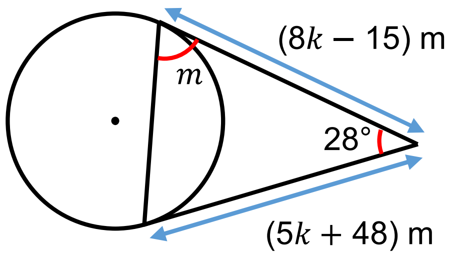
If two tangents drawn from point A meet the circumference of a circle at points B and C, what do you know about triangle ABC?
If the length of AB is 25 cm, what is the length of AC?
What types of triangle/quadrilateral can you see in the diagram?

Exemplar Questions

Which diagram shows two tangents from a common point?



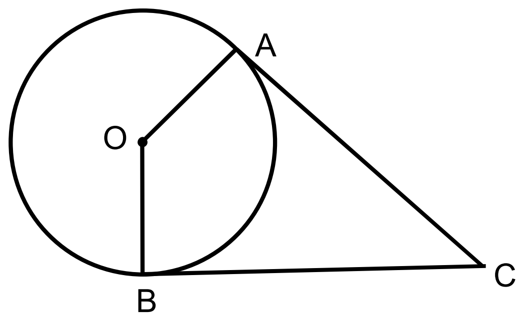
The diagram shows two tangents from a common point to a circle.



Work out the size of the angle marked m.

Work out the value of k.

AC and BC are tangents to a circle centre O.
Prove that triangles OAC and OBC are congruent.



Alternate segment theorem H

Notes and guidance

Here students are introduced to the alternate segment theorem. Students should know and understand that the angle between chord and tangent is equal to the angle in the alternate segment. Students could be introduced to this as a special case of the theorem about the angle between a radius and a tangent. It's important that students understand what is meant by the alternate segment so that they can correctly identify any pairs of equal angles.

Key vocabulary

Circle	Angle	Alternate
Segment	Equal	Tangent

Key questions

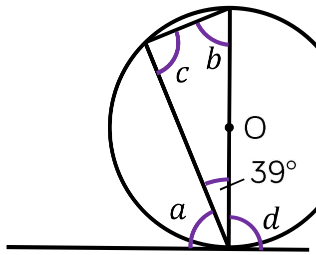
What is a segment?

What is meant by the alternate segment?

What do you know about the angle between a tangent and a radius? How can you use this here?

Exemplar Questions

The diagram shows a tangent to a circle centre O.

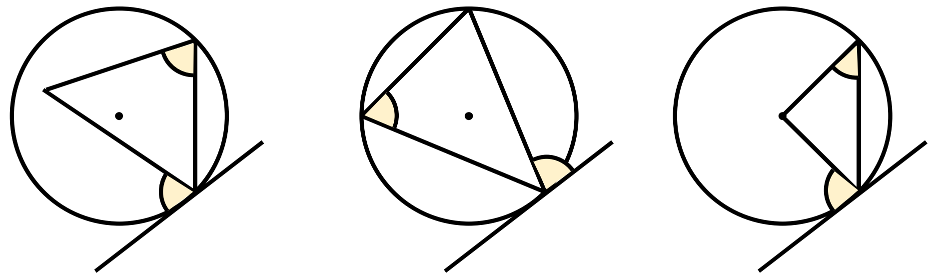


- Write down the size of angles c and d.

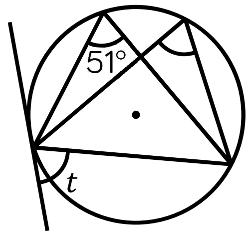
  What do you notice?
- Work out the size of angles a and b.

  What do you notice?
- Why does this happen?

In which diagram are the pair of marked angles equal?



Work out the size of the angle marked t.
Give a reason for your answer.



Prove that the angle between a tangent and a chord is equal to the angle in the alternate segment.

Review Pythagoras' & Trigonometry

Notes and guidance

This small steps provides opportunity to revisit Pythagoras' Theorem and trigonometry. Students studying for foundation tier GCSE could go through key points from the Pythagoras block in year 10 during this block. Links can be made to different areas of the National Curriculum including coordinates and vectors. Students should be able to recognise when to use which rules to answer the questions.

Key vocabulary

Triangle	Opposite	Adjacent
Hypotenuse	Ratio	Inverse

Key questions

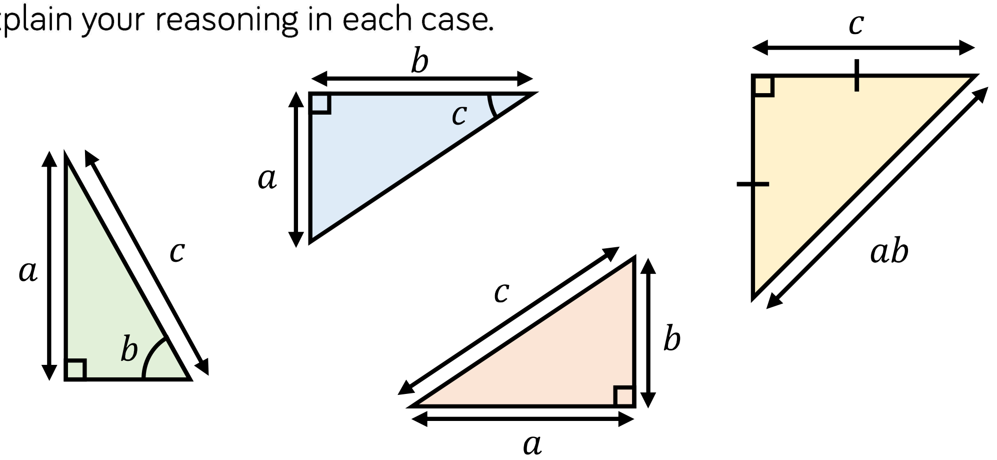
Does this question require Pythagoras' Theorem or trigonometry to solve? How do you know?

Where can you split the shape so that there is a right-angled triangle?

How can drawing a diagram support you?

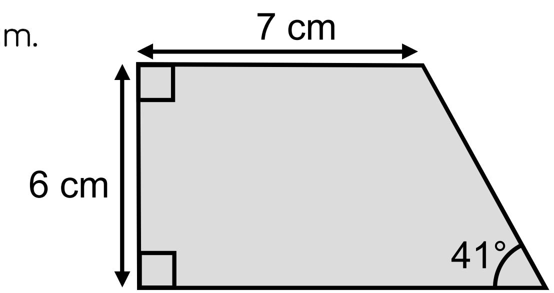
Exemplar Questions

If you knew the value of a and b for each diagram, would you use Pythagoras' Theorem, Trigonometry or either to work out the value of c? Explain your reasoning in each case.



Given that $a = 41$ and $b = 37$, work out the value of c in each diagram.

Work out the area of the trapezium.



Point A has coordinates $(-5, 3)$
Point B has coordinates $(2, -8)$
Work out the length of the line segment AB.

Algebraic Reasoning

Small Steps

- Simplify complex expressions
- Find the rule for the n^{th} term of a linear sequence R
- **Find the rule for the n^{th} term of a quadratic sequence** R H
- Use rules for sequences
- Solve linear simultaneous equations R
- **Solve simultaneous equations with one quadratic** R H
- **Formal algebraic proof** H
- **Inequalities in two variables** H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Simplify complex expressions

Notes and guidance

Here students have an opportunity to revise algebraic notation and the rules for collecting like terms and indices. Answers could be checked by substitution as well if desired. Students should be aware of the associated language but may need a reminder of the word coefficient. Using the precise language makes it much easier to explain the simplifying process, particularly with division.

Key vocabulary

Term	Expression	Simplify
Power	Index	Coefficient

Key questions

How do you know when an expression is in its simplest form?

How does simplifying expressions relate to the laws of indices?

How many expressions can you find that simplify to (e.g.) $24x^2y^3$?

Exemplar Questions

Simplify the expressions, giving the answers as a single term.

$6 \times a$	$a \times 6$	$a \times 6a$	$6a \times b$
$6a \times 2b$	$6a \times 2ab$	$6a^2 \times 2ab$	$6a^2b \times ab^2$

Simplify the expressions, giving the answers as a single term.

$\frac{18xy}{3}$	$\frac{18xy}{3x}$	$\frac{18xy}{3y}$	$\frac{18xy}{3xy}$	$\frac{18xy}{8x}$	$\frac{18x^3y}{8x^2y}$	$\frac{18x^3y}{9x^2y^2}$

Explain the difference between these divisions.

- $12a^2 \div 6$
- $12a^2 \div 6a$
- $6a^2 \div 12a$
- $6a \div 12a^2$

Rosie thinks $(3x^2)^3$ can be rewritten as $9x^6$

Explain why Rosie is wrong.

Write these expressions without brackets.

- $(2a^3)^2$
- $(5p^3)^3$
- $(6a^2b^4)^2$
- $(2ab)^3 \times 3a^3b$

Which of the expressions cannot be simplified?

$xz + yz + xy$	$yx + xy$	$3yx + 3xz$
$3yx \times 3xz$	$yz + \frac{1}{2}zy$	$3yx \div 3xz$

n^{th} term of a linear sequence R

Notes and guidance

Students are very familiar with linear sequences and this review step is a reminder of previous learning. Encourage students to check their answers by substituting several values for n. To extend challenge, students could look at patterns and explain how the values of a and b in the rule $an + b$ relate to the pattern. Beware of the misconception that, for example, a sequence with a constant difference of 4 has the rule $n + 4$

Key vocabulary

Linear	Sequence	Non-linear
Term	Expression	Coefficient

Key questions

What's the connection between the coefficients of n in the rule for a linear sequence and the behaviour of the sequence? How would this change if n were negative?

When you know the coefficients of n in the rule for the n^{th} term of a sequence, explain how you find the rest of the rule.

Exemplar Questions

Match each sequence to the rules for the n^{th} term.

3, 6, 9, 12 …	$3n + 1$
1, 4, 7, 10 …	$3n$
4, 7, 10, 13 …	$3n - 2$

A sequence starts 10, 12, 14, 16 …
Dexter says the rule for the n^{th} term of the sequence is $n + 2$
Explain why Dexter is wrong, and find the correct rule.

Draw the next pattern in the sequence.

Find the rule for the number of counters in the n^{th} pattern.

- What is the 100th even number?
- What is the 100th odd number?
- Find rules for the n^{th} even number and the n^{th} odd number.

Here are the first five terms of an arithmetic sequence.

2 8 14 20 26

- Find an expression, in terms of n, for the n^{th} term of the sequence.
- Find an expression, in terms of n, for the $(n + 1)^{th}$ term of the sequence.

n^{th} term - quadratic sequence H

Notes and guidance

Higher tier students should have studied this in Year 10, so this step serves as revision. Students may try to apply the technique for finding the rule of a linear sequence, and should be encouraged to check their rule works by substitution of more than one value of n, and to identify the type of sequence before embarking on a solution.

Key vocabulary

Linear	Non-linear	Quadratic
Constant	Difference	Second difference

Key questions

What's the same and what's different about linear and quadratic sequences?

How do you find first and second differences? Do they relate to the coefficients of n in the rule for the n^{th} term of a sequence?

Exemplar Questions

Classify each sequence as linear, quadratic, or neither, explaining how you know.

1	3	4	7	11	…
18	15	12	9	6	…
8	13	20	29	40	…
6	5	2	-3	-10	…
1	2	4	8	16	…

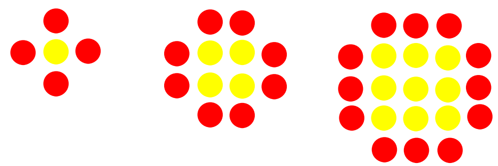
Explain how you can match the sequences to the rules without working out the rules or substitution.

4, 11, 22, 37, 56 …	$n^2 + 4n - 3$
2, 9, 18, 29, 42 …	$2n^2 + n + 1$
4, 7, 10, 13 …	$\frac{1}{2}(n^2 + n + 6)$

Find the rules for the n^{th} term of the sequences.

- 4, 12, 22, 34, 48 …
- 1, 5, 13, 25, 41 …

Draw the next pattern in the sequence.



Find the rule for the number of counters in the n^{th} pattern.
Explain why the rule works.

Use rules for sequences

Notes and guidance

Here students build on their learning and use reasoning to determine, for example, whether a term is a member of a sequence or not. Students may need support to realise that the questions can be approached through forming and solving equations and inequalities rather than trying to list an excessive amount of terms. Here students can be reminded about geometric and Fibonacci sequences as well.

Key vocabulary

Linear	Geometric	Quadratic
Fibonacci	Equation	Inequality

Key questions

How could forming an equation help here? What is the variable? What does it mean if the solution to the equation is not an integer?

What's the difference between a linear sequence and a geometric sequence?

How can you identify a Fibonacci sequence?

Exemplar Questions

Show that 27 is a term in all these sequences.

15, 17, 19, 21 …	-1, 4, 3, 7, 10 …	$5n + 2$	$90 - 7n$
$8n - 5$	864, 432, 216, 108 …	$n^2 + 2$	

Here are the first five terms of an arithmetic sequence.

2 8 14 20 26

Find an expression, in terms of n, for the n^{th} term of the sequence.
Hence determine whether 150 is a term in the sequence.
Which term in the sequence is the first to exceed 300?

The n^{th} term of a sequence is given by $n^2 + 9$
Write down the first three terms of the sequence.
Which term of the sequence is 109?

Whitney says that all terms in the sequence $4n + 2$ are even.
Determine whether Whitney is correct. Explain why.

A Fibonacci sequence starts $a, b, a + b$.
Which of these terms are in the sequence?

$\boxed{a + 2b}$ $\boxed{2a + b}$ $\boxed{3a + 5b}$

Annie thinks the terms of the sequence given by the rule $4n + 6$ will be double the terms of the sequence given by the rule $2n + 3$
Do you agree with Anne?

Linear Simultaneous Equations R

Notes and guidance

Students explored solving a pair of linear simultaneous equations in Autumn Year 10, so this provides a timely reminder. Students could explore different approaches, including substitution where appropriate as opposed to relying only on the elimination method. Students may also need help to see when it is necessary to form simultaneous equations.

Key vocabulary

Coefficient	Linear	Simultaneous
Eliminate	Substitute	

Key questions

What does simultaneous mean?

How can you check your solutions to a simultaneous equations question? Does it matter which equation you use?

What clues in a question suggest you need to form a pair of simultaneous equations?

Exemplar Questions

Write down a pair of simultaneous equations with solutions.

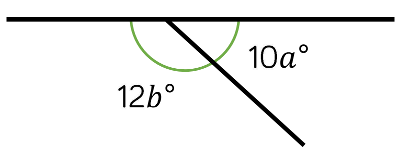
- $x = 5 \qquad y = 3$
- $x = 5 \qquad y = -3$
- $x = -5 \qquad y = -3$

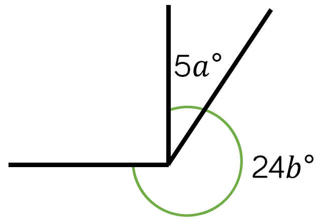
Compare your strategy with a partner's.

What's the same and what's different about these three pairs of simultaneous equations? How would you go about solving each pair?

$x + y = 60$ $x - y = 8$	$x + y = 60$ $2x + y = 73$	$x + y = 60$ $y = 2x$

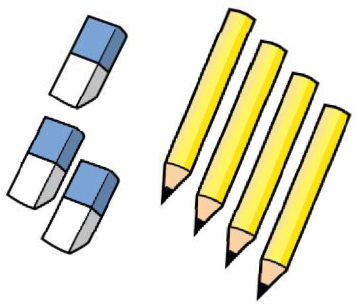
Use an algebraic method to work out the missing angles.





Two pencils and three erasers cost £2.85
Three pencils and two erasers cost £3.30

Eva wants to buy six pencils and an eraser.
She has £3
Does Eva have enough money?



Simultaneous Equations

H

Notes and guidance

Here Higher tier students revisit solving a pair of simultaneous equations where one is quadratic. Since first meeting this in Year 10, students have learnt other methods of solving quadratic equations and studied many other areas of maths. This means they can now practise using the quadratic formula as well as factorisation and work through equations in other contexts e.g. where a line meets a curve.

Key vocabulary

Coefficient	Linear	Simultaneous
Eliminate	Substitute	

Key questions

What's the same and what's different about solving two linear simultaneous equations and a pair where one is linear and the other quadratic?

How can you solve a quadratic equation when you cannot factorise?

Exemplar Questions

Use graphing software to draw the graphs of $x^2 + y^2 = 25$ and $y = x + 1$

- Find the coordinates of the points where the graphs meet.
- Verify algebraically that these points are the solutions to the simultaneous equations.

$$x^2 + y^2 = 25, \qquad y = x + 1$$

- Use the graphing software to draw other straight line graphs with gradient ± 1 that meet $x^2 + y^2 = 25$ at points with integer coordinates.
- Check your results using substitution and by solving pairs of simultaneous equations.

Compare solving these two pairs of simultaneous equations.

$$\boxed{\begin{array}{c} x^2 - 4x - 3y = 0 \\ x + y = 4 \end{array}} \qquad \boxed{\begin{array}{c} x^2 - 4x - 3y = 0 \\ x + y = 5 \end{array}}$$

What's the same and what's different?

- Explain how you can tell, without drawing the graphs or doing any calculations, that the pair of simultaneous equations

$$y = 3x + 5 \qquad y = x^2 + 4x + 5$$

have a solution when $x = 0$?

- Find the other solution.

Will a pair of simultaneous equations where one is linear and the other quadratic always have 2 solutions? Justify your answer.

Formal algebraic proof

H

Notes and guidance

Students need to be comfortable with algebraic manipulation to complete formal algebraic proof, so it is worth starting with a reminder of this. Students can use cubes or bar models to appreciate than $2n$ is even for integer n and that $2n+1$ and $2n-1$ are both odd. Likewise they should know e.g. $5k$ is a multiple of 5 They should also appreciate the meaning of the word counterexample and how to show a conjecture is false.

Key vocabulary

Proof	Demonstration	Counterexample
Justify	Even	Odd

Key questions

Why isn't a list of examples a proof?

If k is an integer, tell me some numbers (e.g.) $12k$ must be a multiple of.

If n is even, what can you say about $n+1$? $n+2$? $2n+3$?

Exemplar Questions

By expanding the brackets, explain why $3(9x+6) \equiv 9(3x+2)$

Find some other expressions that are equivalent to $6(8x+4)$

n is a positive integer. Which of these expressions must represent an even number, which must be odd and which could be either?

$2n$	$3n$	$4n$	n^2
$2n+1$	$2n-1$	$3n+1$	$4n+6$

Would any of your answers change if you knew n were
- even
- odd?

Investigate other expressions of the form $an+b$. Can you generalise?

Here is Dexter's proof that the sum of any two even numbers is even.

$2m = 2 \times m$ is even
$2m + 2m = 4m$
$4m = 2 \times 2m$, so it's even too.

Explain why Dexter's proof does not cover all pairs of even numbers and write a correct proof.
Show also that the sum of two odd numbers is also even.

- Explain why $8k$ must be a multiple of 8
- What is the next odd number after $2n+1$?
- Prove that the difference between the squares of two consecutive odd numbers is always a multiple of 8

Inequalities in two variables H

Notes and guidance

Here students explore inequalities in more than one variable, using a graphical approach. You may wish to start by exploring which side of a single line such as $y = x$ is the region $y > x$ and which is the region $y < x$. You may also need to remind students about the equations of lines parallel to the axis. The "cover-up" method is a quick way of finding where equations like $2x + 3y = 12$ meet the axes.

Key vocabulary

Inequality	Region	Satisfy
Equation	Integer	Test

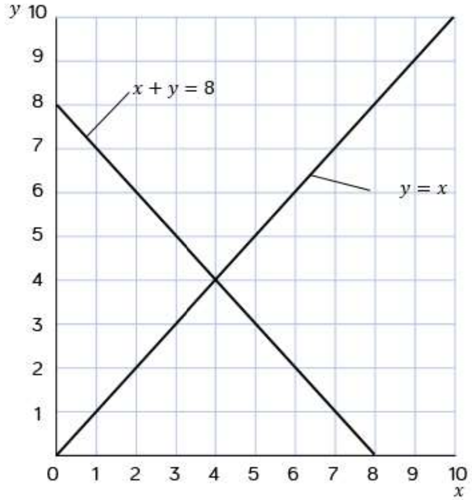
Key questions

How can you use the coordinates of a point in a region to test what inequalities it satisfies?

How do you choose a region when there is more than one condition to be satisfied?

What's the difference between finding a region and finding integer values that satisfy a set of inequalities?

Exemplar Questions



On separate copies of the grid and graphs shown, shade the regions where

- $y > x$
- $x + y < 8$
- $y > x$ and $x + y < 8$
- $y < x$ and $x + y < 8$
- $y < x$ and $x + y > 8$

Use another copy of the grid, draw suitable lines to find the integer values of x and y that satisfy all three of the inequalities $y > 4, x < 3$ and $x + y < 8$

Where does the line $5x + 4y = 20$ meet the axes?
What's the same and what's different about the line $5x - 4y = 20$?

Draw a pair of axes from -6 to 6 in both directions.
By drawing suitable graphs, mark with a cross the integer values of x and y that satisfy all the inequalities.

$3y + 2x < 12 \qquad y < 2x + 1 \qquad 0 < y < 3$

Spring 2 : Revision & Communication

Weeks 1 and 2: Transforming & Constructing

Students revise and extend their learning from Key Stage 3, exploring all the transformations and constructions, relating these to symmetry and properties of shapes when appropriate. There is an emphasis on describing as well as performing transformations as using the language promotes deeper thinking and understanding. Higher tier students extend their learning to explore the idea of invariance and look at trigonometric graphs as a vehicle for exploring graph transformations.

National Curriculum content covered includes:

- describe translations as 2D vectors
- reason deductively in geometry, number and algebra, including using geometrical constructions
- interpret and use fractional **{and negative}** scale factors for enlargements
- **{describe the changes and invariance achieved by combinations of rotations, reflections and translations}**
- recognise, sketch and interpret graphs of **{the trigonometric functions (with arguments in degrees) for angles of any size}**
- **{sketch translations and reflections of the graph of a given function}**

Weeks 3 and 4: Listing & Describing

This block is another vehicle for revision as the examinations draw closer. Students look at organisation information, with Higher tier students extending this to include the product rule for counting. Links are made to probability and other aspects of Data Handling such as describing and comparing distributions and scatter diagrams. Plans and elevations are also revised. You can adapt the exact content to suit the needs of your class.

National Curriculum content covered includes:

- explore what can and cannot be inferred in statistical and probabilistic settings, and express their arguments formally
- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
- **{calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams}**
- apply systematic listing strategies, **{including use of the product rule for counting}**
- construct and interpret plans and elevations of 3D shapes

Weeks 5 and 6: Show that

This is another block designed to be adapted to suit the needs of your class. Examples of communication in various areas of mathematics are provided in order to highlight gaps in knowledge that need addressing in the run up to the examinations. "Show that" is used to encourage students to communicate in a clear mathematical fashion, and this skill should be transferred to their writing of solutions to any type of question.

National Curriculum content covered includes:

- know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**
- apply the concepts of congruence and similarity
- make and use connections between different parts of mathematics to solve problems
- **{change recurring decimals into their corresponding fractions and vice versa}**
- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; **{use vectors to construct geometric arguments and proofs}**

Transforming and Constructing

Small Steps

- Perform and describe line symmetry and reflection R
- Perform and describe rotation/rotational symmetry R
- Perform and describe translations of shapes R
- Perform and describe enlargements of shapes R
- **Perform and describe negative enlargements of shapes** R H
- Identify transformations of shapes R
- Perform and describe a series of transformations of shapes
- **Identify invariant points and lines** H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Transforming and Constructing

Small Steps

- Perform standard constructions using ruler and protractor or ruler and compasses R
- Solve loci problems
- **Understand and use trigonometrical graphs** H
- **Sketch and identify translations of the graph of a given function** H
- **Sketch and identify reflections of the graph of a given function** H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Line symmetry and reflection R

Notes and guidance

Using paper-folding activities or mirrors can help to unpick common mistakes regarding rectangles, parallelograms and reflecting in a diagonal line. A good strategy when reflecting in a diagonal, is to reflect each vertex before joining up the edges. This step provides a good opportunity to revisit equations of a straight line. Students need to check scales on axes before giving the equation of the mirror line.

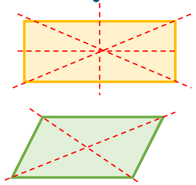
Key vocabulary

Line symmetry	Reflection	Diagonal
Vertex	Side	Mirror Line

Key questions

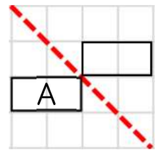
How many lines of symmetry do regular polygons have?
How can we work out where a vertex is positioned when reflecting in a diagonal line?
Are reflected images congruent to the original object?
How do we know the equation of the straight line is (e.g. $x = 5$, $y = x$ etc.)?

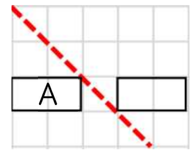
Exemplar Questions

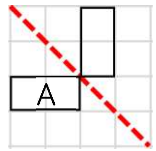


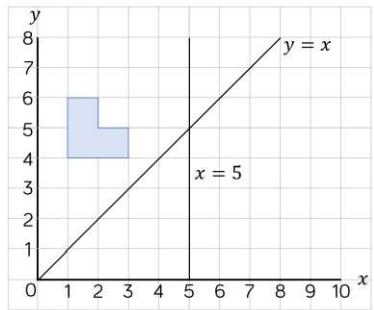
Mo says, "A rectangle has 4 lines of symmetry but a parallelogram only has 2 lines of symmetry."
Show that Mo is incorrect.

Rectangle A is reflected in the dotted line. Which of these is correct?





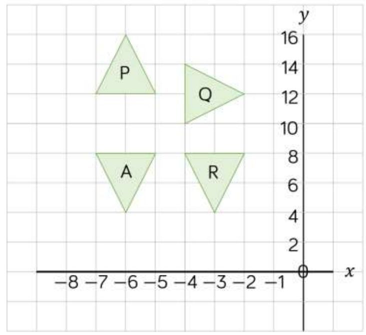




Reflect the shape in the lines:

- $x = 5$
- $y = x$
- $y = 3$
- $x + y = 8$

What's the same and what's different?



Object A has been reflected to form each of the images P, Q and R.

For each image, write down the equation of the line of reflection.

Rotation/Rotational symmetry R

Notes and guidance

This is an opportunity to revisit names of shapes and revise and build on the study of rotation in Year 9. It's important students can rotate a shape no matter where the centre of rotation is, so include centres inside the shape, on a side of a shape and outside of the shape. Teachers will need to model how to perform a rotation, and then how to describe a rotation. Tracing paper and dynamic geometry software packages will help students to visualise rotations.

Key vocabulary

Rotate	Clockwise	Anticlockwise
Centre	Order of rotational symmetry	

Key questions

What is the order of rotational symmetry for different regular polygons? What do you notice?
Are rotations of an object congruent to the object?
What do you notice about a rotation 90° clockwise compared to one that is 270° anticlockwise?
How is tracing paper used to help with rotation?

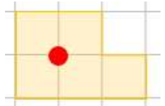
Exemplar Questions

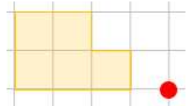
Use a pencil and ruler to draw a sketch of a
- Scalene triangle
- Rhombus
- Parallelogram
- Trapezium
- Kite

For each of your shapes, write down the order of rotational symmetry.

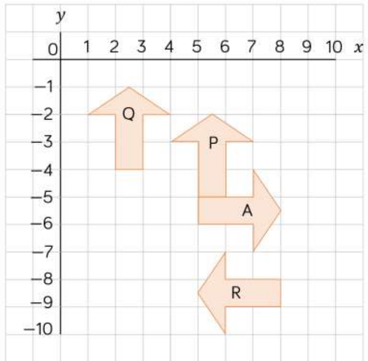
The red circle represents the centre of rotation for each question.
Rotate each object by the angle and direction of turn given.


180°


90° clockwise


90° anti-clockwise

A trapezium ABCD has vertices A (3, 3), B (3, 5), C (6, 3) and D (5, 5)
Draw trapezium ABCD on x- and y-axes from 0 to 10
Rotate trapezium ABCD:
- 90° clockwise, centre (3, 3)
- 180°, centre (4, 4)
- 90° anticlockwise, centre (4, 6)



Object A has been rotated to form each of images P, Q and R.

For each image, describe the rotation fully.

Translation of shapes

R

Notes and guidance

When translating shapes, a common misconception is that the vector of translation is found by counting the squares/measuring the distance between shapes rather then the number of squares/distance between corresponding vertices. Students can also mix up which direction the numbers in the vector represent. Students could also revisit adding vectors here in the context of repeated translation.

Key vocabulary

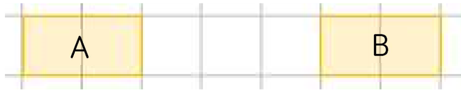
Translation	Vector	Axes
Scale	Congruent	Vertex

Key questions

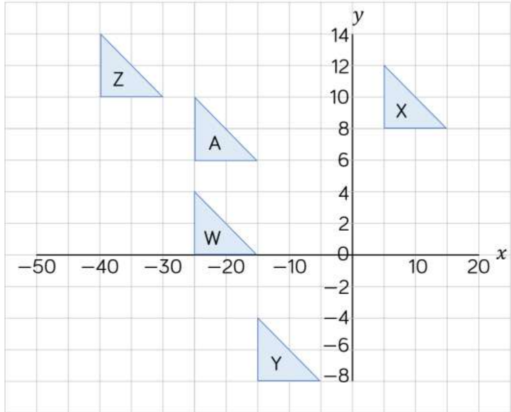
After a translation, is the image congruent to the object?
Why do we measure from one vertex on the object to the corresponding vertex on the image?
How do we know which direction to translate the object in?
Why is it important to consider the scales of axes when giving a vector of translation?

Exemplar Questions

Which of the statements are correct? What mistakes have been made?



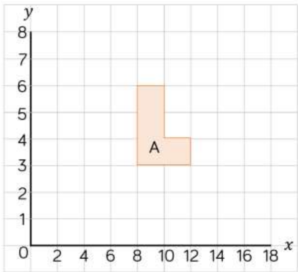
- Shape A is translated by vector $\begin{pmatrix}3\\0\end{pmatrix}$ to image B.
- Shape A is translated by vector $\begin{pmatrix}5\\0\end{pmatrix}$ to image B.
- Shape A is translated by vector $\begin{pmatrix}0\\5\end{pmatrix}$ to image B.
- Shape B is translated by vector $\begin{pmatrix}-5\\0\end{pmatrix}$ to image A.



What do you notice about the axes?

Triangle A has been translated to images W, X, Y and Z.

Describe each translation using a column vector.



Draw a translation of shape A by each vector.

- $\begin{pmatrix}-4\\0\end{pmatrix}$
- $\begin{pmatrix}6\\2\end{pmatrix}$
- $\begin{pmatrix}-8\\-3\end{pmatrix}$
- $\begin{pmatrix}5\\-1\end{pmatrix}$

Enlargements of shapes

R

Notes and guidance

Students start by identifying an enlargement and its scale factor and revise the term 'similar'. Students then consider using centres of enlargements inside, on and outside of a given shape, before moving onto using axes. The 'counting squares from the centre method' is usually more reliable than drawing rays, although both methods can be explored. Students then revise how to fully describe an enlargement and how to perform enlargements.

Key vocabulary

Enlargement	Scale Factor	Multiplier
Similar	Centre of enlargement	Ray

Key questions

If an object is enlarged, does it always get bigger?

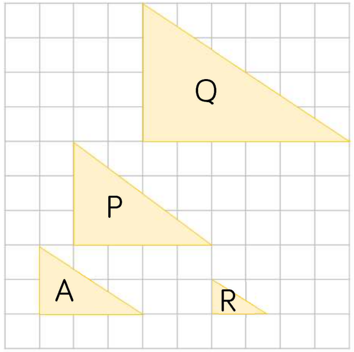
How can you work out the scale factor?

What makes two shapes mathematically similar?

How do we know where each vertex of the image should go?

How do we identify the centre of enlargement?

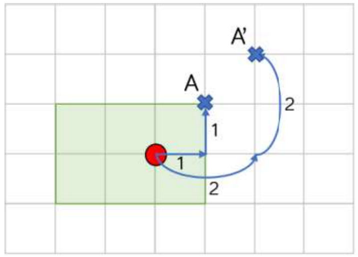
Exemplar Questions



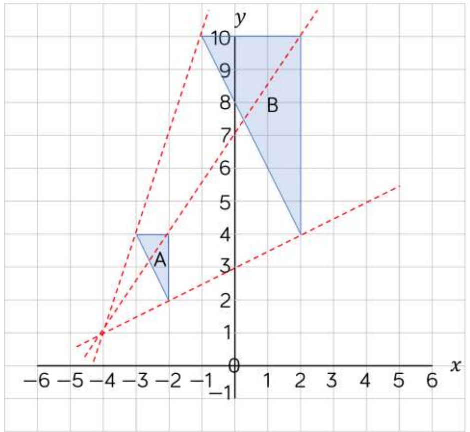
Which triangles are an enlargement of triangle A? How do you know?

What is the scale factor or enlargement for each?

Which triangles are similar to triangle A? Explain how you know.



Huda enlarges the rectangle by scale factor 2, using the marked centre of enlargement.
The first step is shown.
Explain Huda's method and complete the enlargement.



Mo says, "Triangle B is an enlargement of triangle A, scale factor 3"

- Which other information has Mo forgotten to give?
- Enlarge triangle A by scale factor 2, centre of enlargement (0, 1)
- Enlarge triangle A by scale factor 1.5, centre of enlargement (−2, 2)

Negative enlargements

R H

Notes and guidance

Reinforce the impact of a negative scale factor, by giving lots of visual examples starting with −1, so that students make the link with rotation by 180°. Some students have the misconception that a negative scale factor reduces the dimensions of the shape. To perform an enlargement using a negative scale factor, the ‘counting squares from the centre method’ again is useful. Students may need to be reminded to count from the centre to each vertex.

Key vocabulary

Negative	Direction	Rotation
Centre	Scale Factor	Multiplier

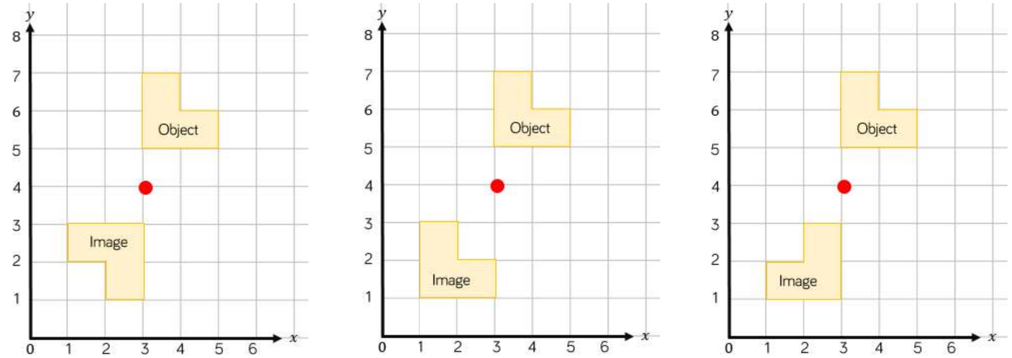
Key questions

Is enlarging a shape by scale factor −1 the same as rotating it by 180°? Explain your answer.

Does a negative scale factor always reduce the dimensions of a shape?

What’s the same and what’s different about enlarging a shape by the scale factors 2 and −2?

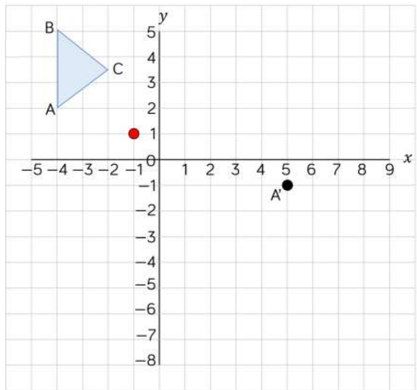
Exemplar Questions



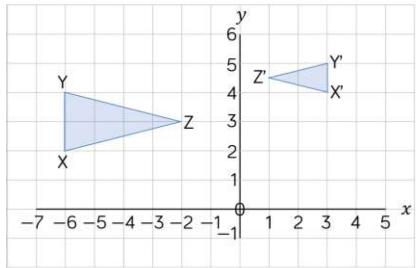
Which diagram shows an enlargement of the object by scale factor −1 using the centre of enlargement marked? Explain your answer.

Teddy says, “Enlarging a shape by scale factor −2 will make it half the size.”
Is Teddy correct? Explain why or why not.

Complete the enlargement of triangle ABC, scale factor −2, centre (−1, 1). Label each vertex of the image A’, B’ and C’.



Describe the transformation of triangle XYZ onto triangle X’Y’Z’.



Identify transformations

R

Notes and guidance

Students sometimes describe a series of transformations, rather than giving a single transformations, and sometimes use language that is non-mathematical (e.g. flip, mirror). Therefore, it's important to promote preciseness in use of language as well as identifying the single transformation. Students also need to describe the transformation fully e.g. including the centre for rotation and enlargement. Tracing paper is very useful in this small step.

Key vocabulary

Single	Equation of a line	Translation
Enlargement	Reflection	Rotation

Key questions

Give the names of the four types of transformation.

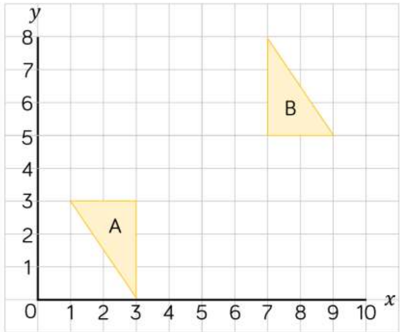
How much information is needed to describe each one?

Can any/all rotations be described as a series of reflections?

What is meant by 'single transformation'?

Exemplar Questions

Describe fully the single transformation that maps triangle A onto triangle B.



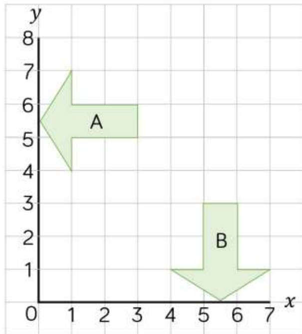
Annie says, "Reflect triangle A in the line $y = 4$ and then reflect this triangle in the line $x = 5$ to get triangle B".

Does Annie's description map triangle A onto triangle B?

Why is Annie's answer incorrect?

Give the correct answer.

Here are some different descriptions students gave for the single transformation that maps shape A onto shape B.



- Flip arrow A to get arrow B
- Turn arrow A by 90° clockwise
- Move arrow A up by 3 and then do a quarter of a turn clockwise and then move it along by 3

Why is each answer incorrect?

Find a single reflection that maps shape A onto shape B.

Can you find a single rotation that maps shape A onto shape B?

Series of transformations

Notes and guidance

Here students perform a series of transformations on an object. They should be encouraged to draw each stage of the series of transformations, rather than trying to do it all in one step. Students will not be asked to describe a series of transformations, and should be reminded about the importance of identifying single transformations. This provides the opportunity to practise the earlier steps.

Key vocabulary

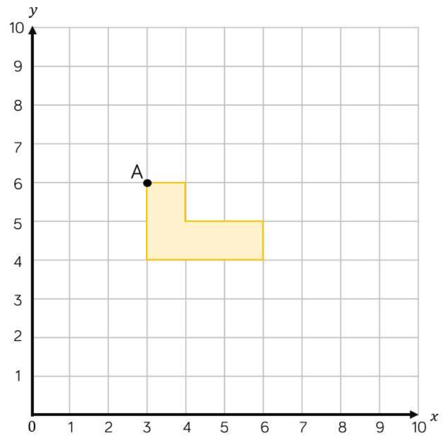
Single	Series	Translation
Enlargement	Reflection	Rotation

Key questions

Does the order a series of transformations are performed in always, sometimes of never make a difference?

Can a series of transformations ever be reduced to just a single transformation?

Exemplar Questions



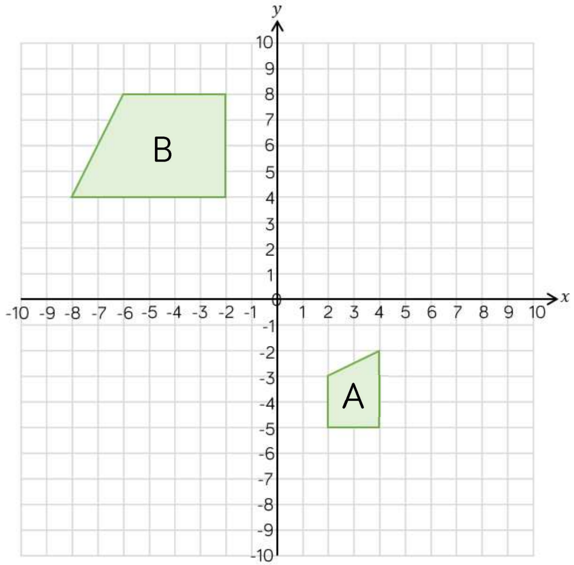
The shape undergoes a series of transformations

- a rotation 90° anticlockwise, centre (6, 7)

followed by

- a translation by the vector $\binom{-5}{-4}$

Show that vertex A is mapped to (2, 0)



Shape A undergoes a series of transformations.

- a reflection in the line $x + y = 4$
- an enlargement by scale factor 2 about centre (10, −4)

Describe the final transformation in the series needed to map shape A onto shape B.

Invariant points and lines

H

Notes and guidance

This is an opportunity for students to continue to practise transformations, and could be extended to exploring a series of transformations. Students both identify invariant points given a transformation, and also give a transformation so that a specific point or points are invariant. Finding the equation of a straight line is revisited here. Students could explore shapes and transformations where all points on a shape are invariant.

Key vocabulary

Invariant	Transformation	Reflection
Rotation	Translation	Enlargement

Key questions

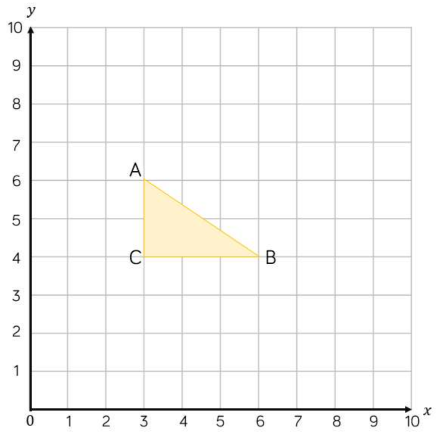
Is it possible for any point to be invariant after a translation?

Is it possible for all points on a shape to be invariant after a rotation?

What do you notice about the centre of enlargement and invariant points?

How do you find the equation of a line, given two points on the line?

Exemplar Questions



Reflect triangle ABC in the line $x = 6$

What do you notice when vertex B is mapped to the image?

Complete the sentence:

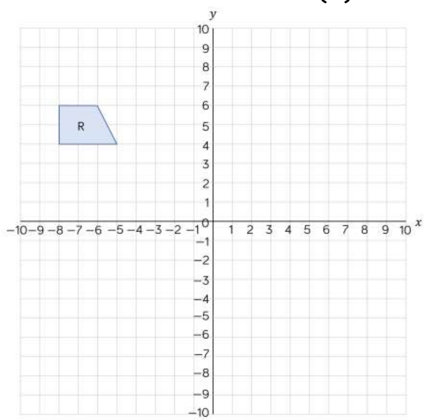
C is an invariant point when triangle ABC is rotated 90° clockwise, centre (___, ___)

Write down a transformation so that both

- B and C are invariant points.
- A and B are invariant points.

Shape R is transformed so that exactly one point is invariant.

Which transformation(s) could have been used?



- Reflection in the line $y = 4$
- Translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$
- Enlargement, SF −1, centre (−5, 4)
- Enlargement SF 2, centre (−10, 8)
- Rotation, 180°, centre (−7, 5)

Find some other transformations so that exactly one point on R is invariant.

Find the coordinates of the invariant points when shape R is rotated 180°, centre (−2, 1) and then reflected in the line $y = 2$ and then translated by the vector $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$.

Constructions

R

Notes and guidance

Practising general skills with a pair of compasses might be a necessary starting point for some students before moving on to revising the standard constructions. This is a good opportunity to interleave the properties of quadrilaterals and their diagonals. Students can also sketch triangles, construct perpendicular heights, from the opposite vertex to the base, to revise calculating the area of the triangle. Midpoints of line segments is also another topic that can be revised here.

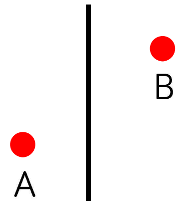
Key vocabulary

Construct	Angle Bisector	Locus
Equidistant	Perpendicular Bisector	

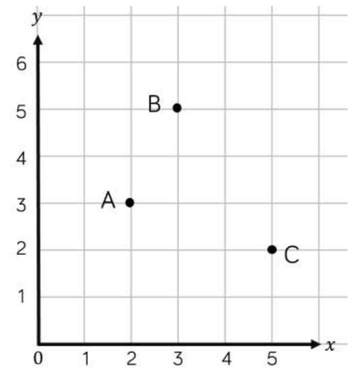
Key questions

What do we know about all points on the perpendicular bisector in relation to A and B?
How can I construct a 60° angle? 30° angle? 45° angle?
What's the same and what's different about drawing a perpendicular to a point and a perpendicular from a point?
What is the point is at the end of the line?

Exemplar Questions



Annie has sketched the perpendicular bisector of line segment AB. Why is this incorrect? Construct and accurate perpendicular bisector of AB.



Dora works out that the midpoint of line segment AB is $\left(\frac{2+5}{2}, \frac{3+2}{2}\right)$ which is (3.5, 2.5)

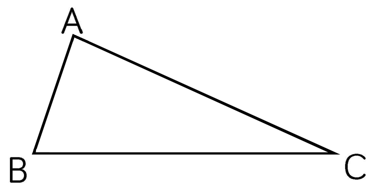
Give the coordinates of a point which is on the perpendicular bisectors of line segment AB and BC.

Check your answers by constructing the perpendicular bisectors.

Draw a triangle ABC like the one shown.

Construct the perpendicular bisector of BC from point A.

How is this different from constructing the perpendicular bisector of line segment BC?



Construct a triangle with side lengths 7 cm, 8 cm and 9 cm.
Construct the angle bisector of each side.
What do you notice about where these bisectors intersect?

Solve loci problems

Notes and guidance

You may wish to review the locus of points equidistant from a single point, a straight line, two intersecting lines, two points and between two parallel lines, linking to the last step, before considering loci problems. You could also interleave revision of bearings and scale drawings. Students should then be encouraged to perform constructions, leaving in all construction marks. They can then reflect on the question posed, for example, shading in the relevant area.

Key vocabulary

Locus/Loci	Equidistant	Circle
Perpendicular Bisector	Angle Bisector	

Key questions

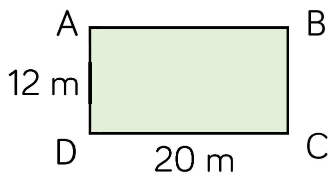
What does equidistant mean?

How can I show the area that is at least a certain distance away from a point, two points, two intersecting lines, two parallel lines, a straight line? How do I ensure accuracy when using a pair of compasses?

How can we use scale to work out actual distances?

Exemplar Questions

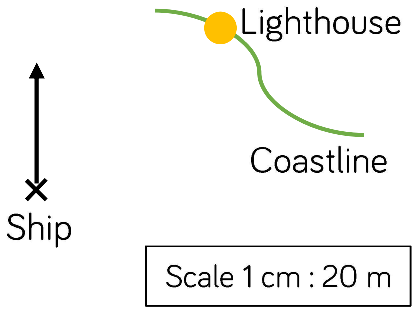
A rectangular garden is 20 m long and 12 m wide.
Use a scale of 1 cm : 2 m to make a scale drawing of the garden.



Dani wants to put in a water feature. It needs to be

- at least 8 m away from A
- closer to DC than BC
- closer to DC than AB

On your diagram, shade the region where Dani can add a water feature.



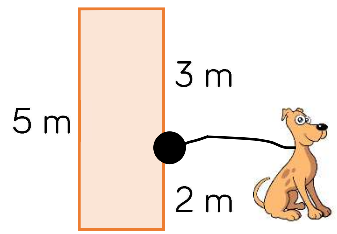
A ship is sailing on a bearing of 075° towards the coastline.

It must not travel within 60 m of the lighthouse otherwise it will be in danger.

Will the ship be in danger?

Foggy the dog is tied to a post outside a shed.
His lead is 4 m long.

- Make a scale drawing of the shed and lead (1 cm : 0.5 m)
- Shade in the area of grass Foggy can reach.



Trigonometrical graphs

H

Notes and guidance

Dynamic software showing the unit circle and the construction of trigonometric graphs might be a useful starting point. Giving time to explore the graphs, considering the key features of each and then differences and similarities helps understanding. Realising that the graphs are periodic and repeat an infinite number of times is also key. The main focus here is using the symmetry of the graphs to work out different values of x which give the same value or y.

Key vocabulary

Sin/Cos/Tan	Asymptote	Period
Symmetry	Repeating	

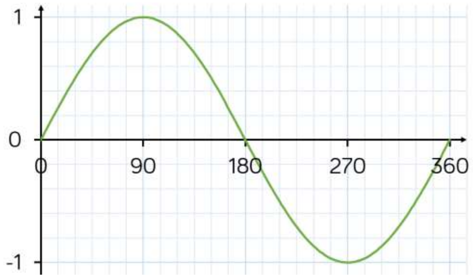
Key questions

Why can angles be greater than 360°? Is there a limit for the size of a measure of turn?

Why is the maximum and minimum value of y on the sine and cosine graph 1?

Why is there a repeating pattern? How regularly does this pattern repeat itself?

Exemplar Questions



This is the graph of $y = \sin x$

Calculate y when x is equal to

- 30° and 150°
- 225° and 315°

Show your solutions on the graph.

What do you notice?

Use this graph to complete the following.

- $\sin 60 = \sin\square$
- $\sin 0 = \sin\square$
- $\sin\square = \sin\square$

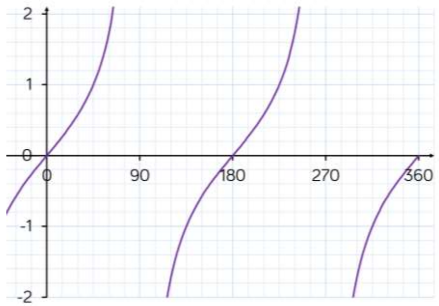
Complete the table for $y = \cos x$

x	0	30	60	90	120	150	180	210	240	270	300	330	360
y													

Use your table to draw the graph of $y = \cos x$ for $0° \le x \le 360°$

Explore what happens when $x \le 0$ and $x \ge 360°$

What's the same and what's different about the graphs of $y = \sin x$ and $y = \cos x$?



This is the graph of $y = \tan x$.

What happens when $x = 90°$? Why?

Draw a right-angled triangle to help you explain your answer.

Find three values of x that satisfy the conditions $x > 360°$ and $\tan x = 0$

White Rose Maths

Translations of the graph

H

Notes and guidance

Dynamic geometry software is very useful here, allowing students to explore translations and to make generalisations. Translations horizontally and vertically should be considered separately. A common misconception is that $y = f(x)$ is translated $\binom{a}{0}$ onto $y = f(x + a)$. To avoid this, ensure that students can explain why this is incorrect by comparing coordinates on each graph. Trigonometric graphs should be included in this step.

Key vocabulary

Mapping	Translation	Horizontal
Vertical	Vector	

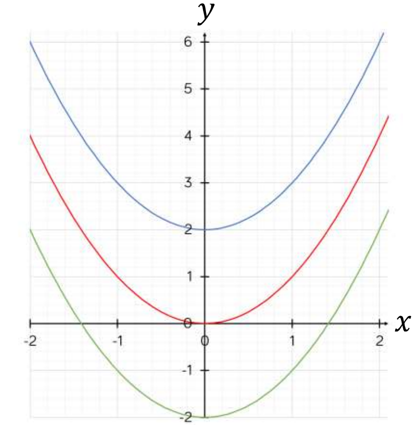
Key questions

How do we know, by considering the equation, which direction the translation is in?

How can we tell from a vector if a translation is horizontal or vertical?

Why is $y = f(x)$ mapped onto $y = f(x + a)$ by $\binom{-a}{0}$ and not $\binom{a}{0}$?

Exemplar Questions



Match the label to each graph.

$y = x^2$	$y = x^2 + 2$	$y = x^2 - 2$

Explain how you worked each out.

Sketch the graph of

- $y = x^2 + 3$
- $y = x^2 - 3$

Describe the transformation that maps $y = x^2$ to each of these graphs.

Mo says, "$y = f(x)$ maps to $y = f(x) + a$ by translation $\binom{0}{a}$"

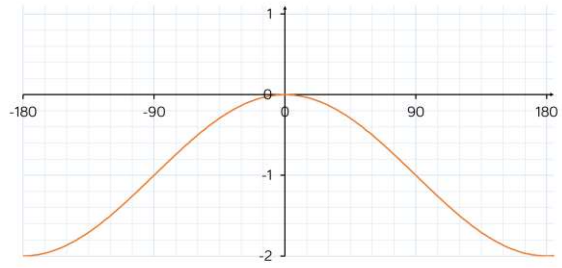
Do you agree with Mo? Explain your answer.

Eva and Huda are describing the translation that maps $y = x^2$ onto $y = (x + 1)^2$.

Eva says, "Use vector $\binom{1}{0}$"	Huda says, "Use vector $\binom{-1}{0}$"

Who is correct? Explain why.

Can you generalise this for a mapping of $y = f(x)$ to $y = f(x + a)$?



This is the graph of $y = \sin(x + a) + b$ where a and b are integers.

Eva says, "a could be 90°".

Eva is correct. Show why.

Now work out the value of b.

Reflections of the graph

H

Notes and guidance

It's important that students understand why the graph reflects in either the x- or the y-axis rather then trying to remember a series of rules. Dynamic geometry software helps students to explore relationships between graphs. Students should be able to both sketch a graph mapped by the transformation, as well as describe the transformation. Exploring functions which are symmetrical in the x-axis or y-axis is also useful. You could also revisit completing the square.

Key vocabulary

Mapping	Reflection	y-axis
x-axis	Invariant	

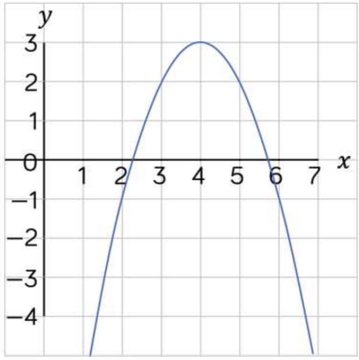
Key questions

Why is the relationship between $y = f(x)$ and $y = -f(x)$ a reflection in the x-axis?

Why is the relationship between $y = f(x)$ and $y = f(-x)$ a reflection in the y-axis?

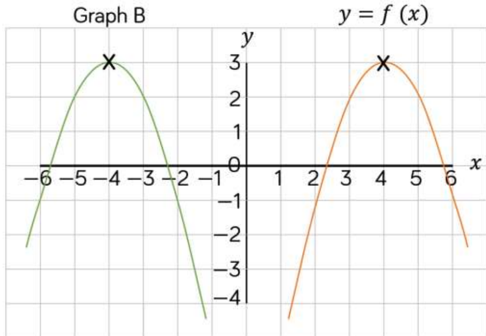
Give an equation for $y = f(x)$ that will remain the same when mapped to $y = f(-x)$.

Exemplar Questions



The graph has equation $y = f(x)$

- What happens to the point (4, 3) if we change the equation to $y = -f(x)$?
- What happens to the other points on the graph?
- Sketch the graph $y = -f(x)$
- Describe the transformation that maps $y = f(x)$ onto $y = -f(x)$.



What's the relationship between $y = f(x)$ and the Graph B? What's the same and what's different about the coordinate of the marked point on each graph?

Sketch the graphs of $y = \cos x$ and $y = \sin x$, on separate axes, between $-180° \leq x \leq 180°$

Nijah thinks that the graphs of $y = \cos(-x)$ will look the same as the graph of $y = \cos x$.

She also thinks this means that the graph of $y = \sin(-x)$ will look the same as the graph of $y = \sin x$.

Explore Nijah's claims. Is she correct?

Explain your answer using your sketches.

Listing and Describing

Small Steps

- Work with organised lists
- Sample spaces and probability R
- **Use the product rule for counting** H
- Complete and use Venn diagrams R
- Construct and interpret plans and elevations R
- Use data to compare distributions R
- Interpreting scatter diagrams R

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

White Rose Maths

Work with organised lists

Notes and guidance

Students revise how to generate a list systematically. They should be encouraged to explain how they know they have all possibilities. Thinking about what they have changed and what they have kept the same each time can help students to talk about a systematic method. Teachers may want to interleave stem-and-leaf diagrams in this small step, reminding students why an organised list is useful and revisiting averages.

Key vocabulary

Systematic	Exhaustive	Arrangement
Stem-and-Leaf	Median	Range

Key questions

Explain what the word systematic means.

Describe a good systematic method for listing items. What did you keep the same and what did you change each time?

Why is it useful to put the data in order when designing a stem-and-leaf diagram?

Exemplar Questions

Ron is listing all possible 3-digit numbers he can make from the digits 1, 2 and 3

123, 321, 213, 312

- Explain why it is difficult to tell if Ron has listed all possibilities.

Alex also starts a list.

123, 132, 213, 231, …

- Describe Alex's strategy and complete her list.
- Find all 24 arrangements of the letters A, B, C and D

To get the meal deal, you have to choose one of each option.

Sandwiches		Crisps		Drink
Cheese	+	Plain	+	Orange
Ham		Beef		Water

Use a systematic method to show that there are 8 different options. Explain why your method was systematic.

Here are the times taken by some runners in a 100 m race.

Key: 9 | 9 means 9.9 secs

9	9 8 2 1 5 8 7 5 9
10	5 6 1 2 4 3 3 8 6 5
11	1 7 3 1

The range is 11.1 – 9.9 = 1.2 secs

The median 10.1 secs

Why are the range and the median both incorrect?
Work out the correct range and median of the times.

Sample spaces and probability R

Notes and guidance

When students don't use a sample space for multiple trials, they often give an incorrect answer based on flawed intuition (e.g. P(1 Head and 1 Tail) = $\frac{1}{3}$). Remind students that probabilities can only be written as fractions, decimals or percentages. For some students, it might be appropriate to revise constructing and interpreting tree diagrams, moving on to problems without replacement for Higher tier students.

Key vocabulary

Sample space	Two-way table	Event
Tree diagram	Replacement	Outcome

Key questions

How can we list all possible outcomes so that none are missed out?
How can we work out a probability from a sample space?
How do we know which cells to consider when finding a probability from a two-way table?
How can we tell if events are equally likely or not?

Exemplar Questions

- Ron flips a coin twice.
  He says, "the probability of getting a head and a tail is 50%".
  Is Ron correct? Justify your answer.
- Annie rolls a dice twice.
  She says, "the probability of getting two sixes is $\frac{1}{12}$"
  Is Annie correct? Justify your answer.

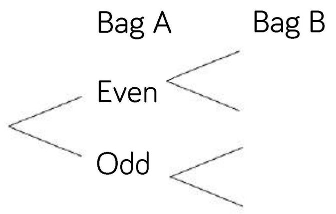
In a school, there are 24 teachers and 720 students.
20% of teachers eat a hot lunch and the rest eat a cold lunch.
The ratio of students who eat a hot lunch to a cold lunch is 3 : 5
Complete the two-way table.

	Hot lunch	Cold lunch	Total
Teachers			
Students			
Total			

A person is selected at random. Work out the probability that they eat a hot lunch.

A student is selected at random. Work out the probability that they eat a cold lunch.

Bag A contains cards numbered 1, 2, 3, 4 and 5
Bag B contains cards numbered 5, 6, 7, 8 and 9
A card is taken from Bag A and then from Bag B.
Copy and complete the tree diagram.



Work out the probability of getting

- 2 odd numbers
- An even and an odd number
- At least one even number

White Rose Maths

Product rule for counting

H

Notes and guidance

Systematic listing and structures like tree diagrams will support students to see the general rule that the total number of arrangements is the product of the number of possible choices for each 'part'. Students need to understand the difference between working out the total number of possible options with and without replacement and when "repeats" occur e.g. the total number of handshakes in a group if each pair shakes hands once.

Key vocabulary

Systematic	Generalise	Product
Product rule	Repeats	Replacement

Key questions

How can you represent the different options?

Can you generalise my findings?

What happens to the total number of options if I can't re-use an item? What happens if I can re-use an item?

Why do I divide by 2 when working out the number of options for pairs of students from a group?

Exemplar Questions

Starter	Main course
Samosa (S)	Meat Curry
Bhaji (B)	Vegetable Curry
	Paneer Cheese Curry

A meal is a starter and main course.

Show that there are 6 possible meal options.

Whitney notices that there are 2 starters and 3 main courses. $2 \times 3 = 6$, and 6 is the number of meal options.

Explore Whitney's idea further by changing the number of starters and number of main courses. What can you conclude?

---

0 1 2 3 4 5 6 7 8 9

Ron is working out how many 4-digit numbers he can make if he uses each card once. Complete his workings.

No. of options: 10 9 ___ ___

Total no. of possible options $= 10 \times 9 \times$ ___ $\times$ ___ $=$ ___

How many 4-digit numbers can be made if each number can be reused?

There are 4 boys in a group. A teacher selects one pair of boys. Eva thinks that the teacher has 12 options ($4 \times 3 = 12$).

Joe thinks that the teacher has 6 options ($\frac{4 \times 3}{2} = 6$)

Joe is correct. Explain why.

How many options does the teacher have if there are 5 boys in the group? Can you make a generalisation?

Venn diagrams

R

Notes and guidance

Teachers may want to use a 'shading activity' to review what the regions on a Venn diagram represent before using these to work out probabilities. A common error is to read a sentence such as '15 people had mustard' in the second exemplar question as 'having mustard only', when in fact, they may also have had tomato sauce. Encourage students to start with the intersection of two events when completing a Venn diagram to overcome this misconception.

Key vocabulary

Venn diagram	Probability	Union
Intersect	Complement	

Key questions

Do the circles in a Venn diagram always overlap? Why or why not?
Can one set lie completely within another set?
Where do we start when completing a Venn diagram? Why?
What do we mean by the words union, intersect and complement?

Exemplar Questions

Draw a Venn diagram to represent the following information.

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$
$A = \{\text{multiples of } 3\}$ $B = \{\text{factors of } 48\}$

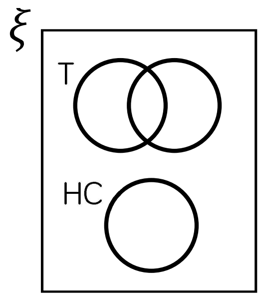
One of the numbers is selected at random.
Work out:

- $P(A \cap B)$
- $P(A \cup B)$
- $P(A)$
- $P(A')$
- $P(A' \cap B)$
- $P(A' \cup B)$
- $P(A \cap B)'$
- $P(A \cup B)'$

---

50 people bought hot dogs.
33 people had tomato sauce on their hot dog.
15 people had mustard on their hot dog.
12 people had both tomato sauce and mustard on their hot dog.
Draw a Venn diagram to represent this information.
A person is selected at random. Work out the probability that they had

- Only tomato sauce on their hot dog.
- Tomato sauce given that they had mustard.



33 teenagers were asked about drinks.
20 drink tea (T) and 10 drink coffee (C).
8 people drink hot chocolate.
All 33 teenagers drink tea, coffee or hot chocolate.
How can you tell from the diagram that no-one who drinks tea or coffee drinks hot chocolate (HC)?
Complete the Venn diagram.

Plans and elevations

R

Notes and guidance

To support this step, students might use multi-link cubes so that they can move around the object to appreciate the different views. Dynamic geometry software can also be very helpful here. Include sketching elevations and plans as well as accurately drawing them and labelling dimensions. Students may need reminding on how to use isometric paper. When reviewing the language, check and compare the language that is used in other areas of the curriculum such as Technology.

Key vocabulary

Plan view	Side elevation	Front elevation
Isometric	Face	Edge

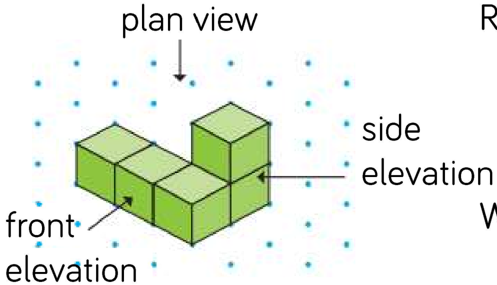
Key questions

What's the difference between the net of a 3-D shape and the plan view?

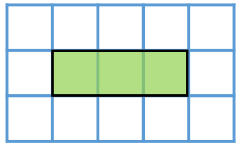
What can you see looking at the shape from the front/ side/ above? Are there any parts you cannot see?

How do you know the dimensions of the elevations and plan?

Exemplar Questions



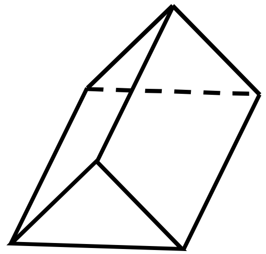
Ron has drawn the front elevation.

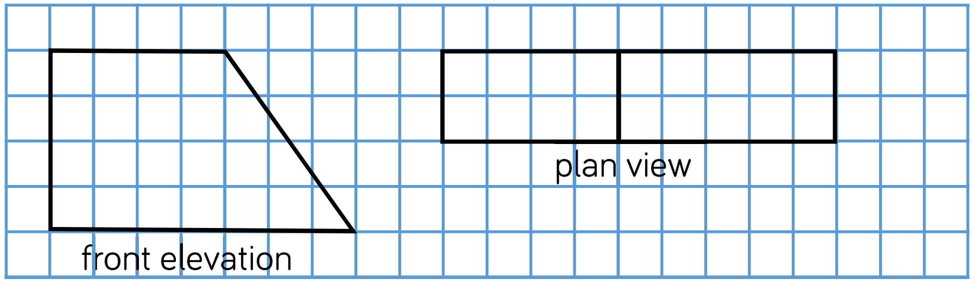


What mistake has he made?

- Build other 3D shapes using 5 multi-link cubes.
- Each time draw the front elevation, plan view and side elevation.

Sketch the front and side elevations and the plan view of the triangular prism.





The front elevation and the plan view of a trapezium-faced prism are shown. On squared paper, draw the 2 different side elevations.
Make a drawing of the 3-D shape on isometric paper.

Compare distributions

R

Notes and guidance

Students should appreciate that a good comparison involves both an average and a measure of spread. Students can find it difficult to construct these, so sentence stems may help. There may be other aspects that they can also comment on, such as outliers. Be aware students may need to compare their findings with a given hypothesis. Higher tier may need to revise cumulative frequency diagrams and box plots here.

Key vocabulary

Hypothesis	Average	Spread
Interquartile Range	Range	Outlier

Key questions

Are there any outliers in the data? How will an outlier affect the mean, median, mode, range, interquartile range?
Which are the most appropriate calculations to do?
What graphs can be drawn? How will they help to compare the data?
What can you conclude? Have you written in full sentences? Is your conclusion fully justified?

Exemplar Questions

Whitney says, "Year 7 students will be slower than Year 11 students at solving a puzzle."
Compare the data. Is Whitney correct? Justify your answer.

	Time taken to complete the puzzle (to the nearest minute)									
Year 7	11	4	6	8	10	7	13	5	12	6
Year 11	4	5	16	20	3	5	5	7	8	9

The number of texts 30 students received on two different days of the week was recorded. Compare the data for the two days.

Number of texts on a Saturday (x)	Frequency
$0 < x \le 20$	0
$20 < x \le 40$	2
$40 < x \le 60$	5
$60 < x \le 80$	17
$80 < x \le 100$	6

Number of texts on a Monday(x)	Frequency
$0 < x \le 20$	5
$20 < x \le 40$	8
$40 < x \le 60$	8
$60 < x \le 80$	9
$80 < x \le 100$	0

The following data was obtained from a register at a national gym.

Eva says, "The youngest person on the register for the gym is male."
Is she right?
Explain your answer.
Compare the data.

	%	Mean age (years)	Interquartile range of ages (years)
Male	56	28.1	7.3
Female	44	34.0	11.7

Interpreting scatter diagrams R

Notes and guidance

Students may need reminding of the vocabulary around correlation. They should also know that a line of best fit has to be straight, but it does not have to go through the origin and nor does it have to join the first and last marked points. The issues with extrapolating from a line of best fit should be discussed. Students should also be aware the correlation does not imply causation.

Key vocabulary

Negative/Positive	Correlation	Causation
Estimate	Outlier	Extrapolate

Key questions

What steps do I take when drawing a line of best fit?

Why do I need a line of best fit when estimating values?

What are the problems caused by extending a line of best fit outside the range of given data?

Why does correlation not imply causation?

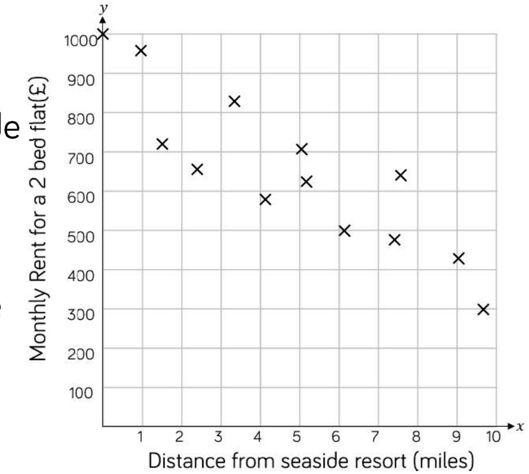
Can two data sets have a relationship which isn't linear?

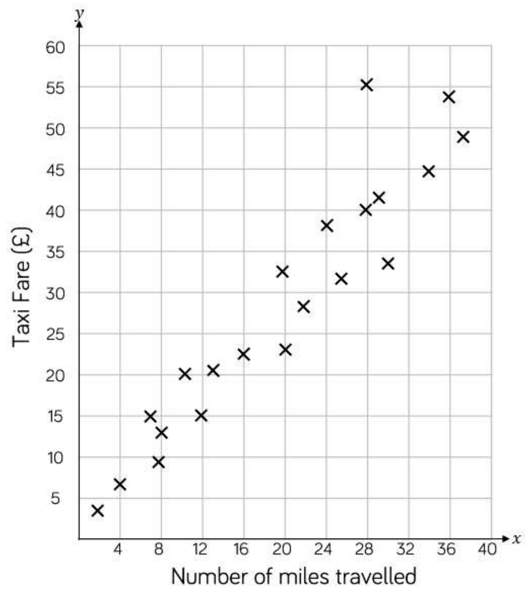
Exemplar Questions

What type of correlation is shown?

Mo says, "Being closer to the seaside resort causes monthly rent to rise". Why might Mo be incorrect?

Use a line of best fit to estimate the rent of a flat which is 7 miles away from the seaside resort.





What type of correlation is shown?

Describe the relationship shown.

Are there any outliers?

Use your graph to estimate the

- price of journey that is 32 miles long
- length of a journey that costs £50

Explain why it may not be appropriate to use your line of best fit to estimate the cost of a journey that is 100 miles long.

Show that

Small Steps

- "Show that" with number
- "Show that" with algebra
- "Show that" with shape
- "Show that" with angles
- "Show that" with data
- **"Show that" with vectors** H
- "Show that" with congruent triangles
- **Formal proof with congruent triangles** H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

"Show that" with number

Notes and guidance

As well as developing students' reasoning skills, this step provides an opportunity for students to revise arithmetical techniques. The suggestions provided can be adapted as suits your class to practice any aspect of number work, including fractions and directed number, the equivalence of fractions, decimals and percentages and, for Higher tier students, fluency with converting recurring decimals and surds.

Key vocabulary

Equivalent	Sum	Product
Difference	Simplest form	Surd

Key questions

How can you determine whether two fractions are equivalent?

How do you find the sum/difference/product/quotient of a pair of fractions? What models could you use to help you?

How can you find a percentage of a number without a calculator?

How can you show that a number isn't prime?

Exemplar Questions

Amir says, "$\frac{5}{6}$ is greater than $\frac{3}{4}$"

Show that Amir is correct

- by drawing a diagram.
- by converting both numbers to decimals.
- by converting both numbers to fractions.

Can you find any other ways to show that Amir is correct?

Show that

- $\frac{330}{440}$ is equivalent to $\frac{450}{600}$
- $\frac{2}{5}$ of $60 = \frac{4}{5}$ of 30
- 60% of $80 = \frac{4}{7}$ of 84
- $\frac{7}{12}$ of $600 > \frac{9}{20}$ of 600

Can you show any of these results without performing calculations?

Show that the product of 0.8 and 5 is less than the sum of 0.8 and 5

Show that the product of $5\frac{3}{4}$ and $\frac{7}{8}$ is greater than the difference between $5\frac{3}{4}$ and $\frac{7}{8}$

Determine which of the statements on the cards are true.

$0.\dot{4}\dot{5} = \frac{5}{11}$	$0.\dot{7} - 0.2 = \frac{26}{45}$	$3\sqrt{20} = 2\sqrt{45}$

"Show that" with algebra

Notes and guidance

This step can be used to revise solving linear equations and inequalities, sequences, substitution, expanding brackets and factorisation as appropriate. You could also link to the equation of a straight line or identifying lines and curves. Higher tier students can build on the fraction calculations in the last step to manipulate algebraic fractions. You could also revisit completing the square and identifying turning points.

Key vocabulary

Term	Expression	Equation
Identity	Solve/solution	Sequence

Key questions

What does the symbol ≡ mean?
How can you show that two straight lines are parallel?
What's the same and what's different about solving an equation and an inequality?
How do you use a rule to find a term in a sequence?
How do you factorise a quadratic expression? Is this the same as or different from completing the square?

Exemplar Questions

Show that $x = 4$ is a solution to all of the equations on the cards.

$8x - 3 = 29$	$2 = 10 - 2x$	$\frac{3x}{2} = 6$	$\frac{x + 10}{2} = 7$

Here are the equations of three straight lines.

$y = 2x + 5$	$y + 2x = 5$	$10 = 4x - 2y$

Show that
- Two of the lines have the same gradient.
- Two of the lines meet the y-axis at the same point.

Show that
- $4(3x + 5) \equiv 3(2x + 8) + 2(3x - 2)$
- $(y + 5)^2 \equiv y^2 + 10y + 25$
- $(p + 3)(p - 3) \equiv p^2 - 9$
- $(2k^2)^3 \equiv 8k^6$

The nth term of sequence A is given by $7n - 2$
The nth term of sequence B is given by $2n^2 + 3n - 1$
Show that the fourth term of sequence A is equal to the third term of sequence B.

Show that
- $\frac{2}{x+1} - \frac{3}{(x+2)} \equiv \frac{1-x}{x^2+3+2}$
- $\frac{5x+10}{3x^2+7x+3} \equiv \frac{5}{x+2}$

"Show that" with shape

Notes and guidance

Here students have the opportunity to revise finding areas and perimeters of rectilinear and other shapes. Revisiting Pythagoras' theorem and similarity are also included. Showing shapes are similar using angles rules is also incorporated in the next step and congruence is dealt with separately. You could also build in forming and solving equations, including quadratic equations if these need more practice.

Key vocabulary

Trapezium	Parallelogram	Similar
Corresponding	Circumference	Area

Key questions

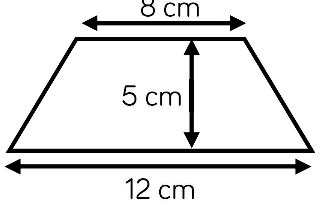
What area formulae to you know? What does each letter represent?

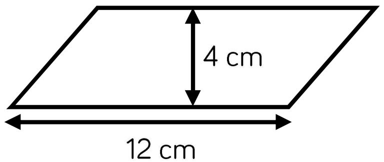
How do you work backwards to find the height if given an area? Is it the same or different if there is a fraction in the area formula?

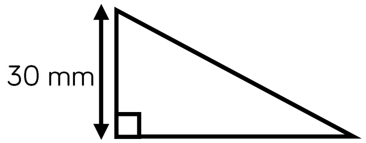
What do you know about the corresponding sides of a pair of similar shapes? What about three similar shapes?

Exemplar Questions

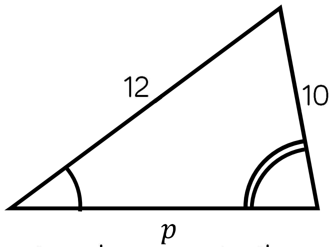
Show that the area of the trapezium is greater than the area of the parallelogram.

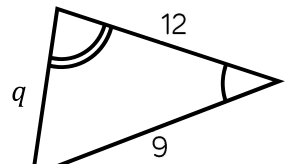






The area of the triangle is 600 mm^2.
Show that the perimeter of the triangle is 120 mm.





The triangles are similar.
Show that $p = 16$ and work out the value of q.

Show that the area of a circle of radius x cm is smaller than the area of a square of side $2x$ cm.

Show that the formula for the area of a trapezium can be used to find the area of a parallelogram, a rectangle and a square.

"Show that" with angles

Notes and guidance

Students should be familiar with basic angles rules (parallel lines, isosceles triangles etc.) from earlier years, but may need reminding of the precise wording and how to 'give reasons for your answer.' Model and encourage clear detailed solutions. You could also use this step to revisit trigonometrical ratios, including the exact values that students need to know. Higher students can also revisit circle theorems here.

Key vocabulary

Alternate	Corresponding	Co-interior
Adjacent	Circle Theorems	Trigonometry

Key questions

What's the difference between corresponding and corresponding-interior angles?

What sides in a triangles do you need to know to find an angle's cosine? What do you know about the sides if the cosine of one of the angles is $\frac{1}{2}$?

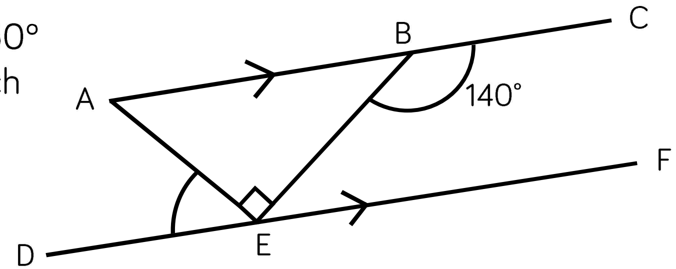
How many circle theorems do you know?

Why are isosceles triangles important?

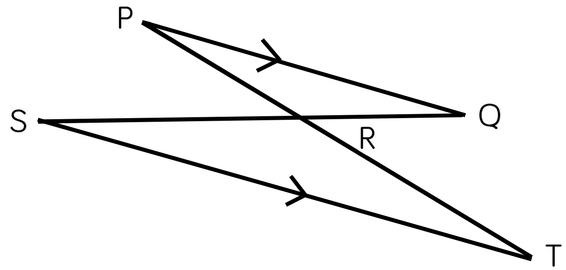
Exemplar Questions

Show that $\angle AED = 50°$

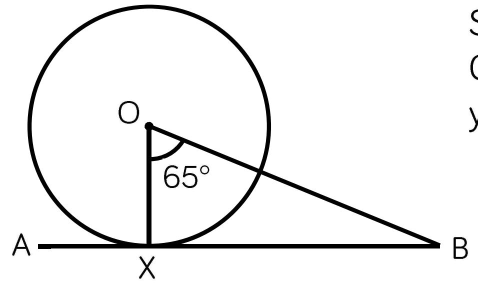
Give a reason for each step of your working.



Show that triangles PQR and RST are similar.



- The angles in a triangle are in the ratio 3 : 4 : 5
  Show that all the angles in the triangle are acute.
- The angles in a triangle are in the ratio 1 : 2 : 3
  Show that the triangle is right-angled.



Show that $\angle OBX = 25°$

Give a reason for each step of your working.

"Show that" with data

Notes and guidance

As with the previous steps, feel free to adapt this step to suit the needs of your class. Students need to be comfortable with the vocabulary surrounding data and in interpreting as well as constructing charts and calculating measures. Probability is also included within this step and you can extend by asking "show that" questions involving more than one event through tree and/or Venn diagrams.

Key vocabulary

Mean	Median	Mode
Range	Quartile	Interquartile range

Key questions

What is the purpose of averages? How do they help us to compare data sets? What other measures can you use?
Which averages are useful in different situations? Why?
How do you construct a frequency polygon? How is this different from a cumulative frequency polygon?
Why is a probability of −0.2/1.03 impossible?
How do you find the probability of combined events?

Exemplar Questions

Here are two sets of data.

Set A	6	12	12	9	11	
Set B	8	14	7	18	9	4

Rosie says the median of the numbers in Set A is 12

- Show that Rosie is wrong.
- Show that the means of both sets of data are equal.
- Show that the range of Set B is greater than that of set A.

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12 adults and 15 children took a spelling test.
The mean mark scored by the adults was 80%
The mean mark scored by the children was 90%
Show that the mean mark scored by everyone who took the test is not 85%

The table shows the probabilities that a die will land on each number.

Score	1	2	3	4	5	6
Probability	0.3	0.2	0.1	0.2	x	$3x$

Show that the probability that the die lands on a prime number is 0.35

A set of numbers have lower quartile 47, upper quartile 79
The difference between the median and the upper quartile is three times the difference between the median and the lower quartile.
Show that the median of the set of numbers is 55

"Show that" with vectors H

Notes and guidance

Foundation tier students need to be able to use vectors to perform and describe translations. You could include other transformations here. Students should also find sums and differences of vectors, and the product of a scalar and a vector, understanding when two vectors are parallel. In addition, Higher tier students need to able to construct geometric arguments e.g. proving a set of points are collinear.

Key vocabulary

Vector	Component	Parallel
Translate	Transform	Collinear

Key questions

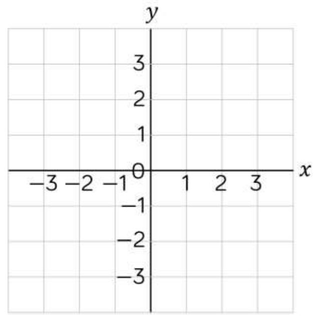
What is a vector?

Can a translation always, sometimes or never be described as a vector? Can a translation always, sometimes or never be described using other transformations?

How do you know when two vectors are parallel when they are written in component form? Can you tell when two vectors are parallel when they are written using letters?

Exemplar Questions

Show that the result of translating the point (1, 1) by the vector $\begin{pmatrix}1\\2\end{pmatrix}$ is the same as the result of translating the point (−1, −1) by the vector $\begin{pmatrix}3\\4\end{pmatrix}$

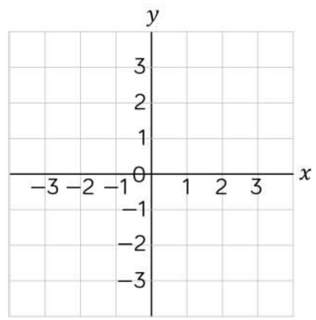


$\boldsymbol{a} = \begin{pmatrix}4\\3\end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix}2\\-1\end{pmatrix}$

- Show that $\boldsymbol{a} + \boldsymbol{b}$ is parallel to $\begin{pmatrix}3\\1\end{pmatrix}$
- Show by calculation that $3\boldsymbol{a} - 2\boldsymbol{b} = \begin{pmatrix}8\\11\end{pmatrix}$

Show that the transformation of shape A to shape B can be described as

- a single translation.
- a single reflection.
- Can the transformation of shape A to shape B be described as a single reflection?



In triangle ABC, X is the midpoint of AC and Y is the point on BC such that BY : YC = 3 : 2
Show that XY is not parallel to AB.

"Show that" with congruent triangles

Notes and guidance

This is the first of two steps on congruent triangles. Concentrate on examples where numerical values are given in this step. Students may need reminding of the four sets of conditions for congruency and the last exemplar illustrates that sometimes more than one option is available. Students needs to take particular care with the difference between AAS and SAS and what is meant by "corresponding side" in the former.

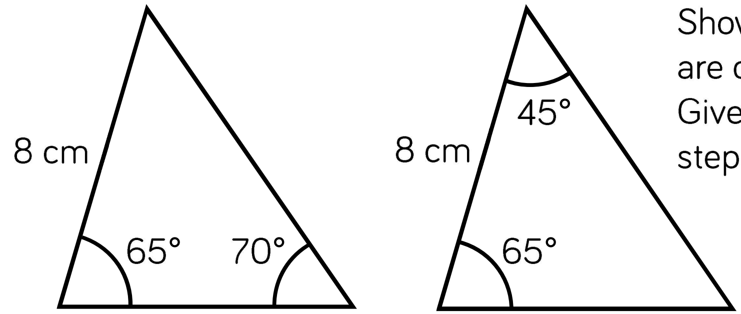
Key vocabulary

Congruent	Condition	Similar
Corresponding	Prove	

Key questions

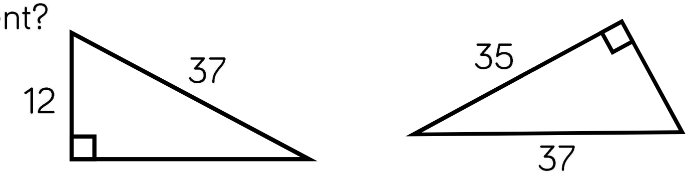
What are the four sets of conditions that show a pair of triangles are congruent? What does each letter stand for in the abbreviated forms?
What is meant by an 'included' angle?
What is meant by a 'corresponding side'? How is this different from corresponding angles?

Exemplar Questions



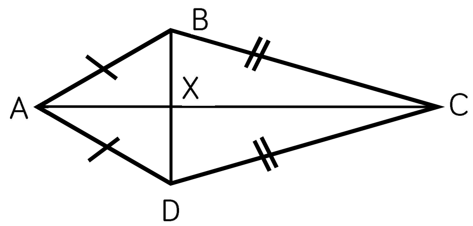
Show that the triangles are congruent.
Give a reason for each step in your working.

What calculation(s) do you need to perform to show that the triangles are congruent?



Which conditions for congruency could you use?

The diagonals of a kite ABCD meet at point X.



Show that three pairs of congruent triangles are formed by joining the diagonals.
Tommy thinks the same result will be true for a parallelogram.
Show that Tommy is wrong.

Proof with congruent triangles

H

Notes and guidance

This extends the previous step to look at more formal proof, particularly in cases when no side lengths or angles are given. Properties of special quadrilaterals and circle theorems may again be interleaved here. Model the setting out of step-by-step proofs for students, and encourage them to write down what sides/angles they can identify as equal if they are struggling to see a starting point.

Key vocabulary

Congruent	Condition	Similar
Corresponding	Prove	Common

Key questions

What's the same and what's different about showing that a pair of triangles are similar triangles and showing that a pair of triangles are congruent?

How does (e.g.) AB being common to both triangles help?

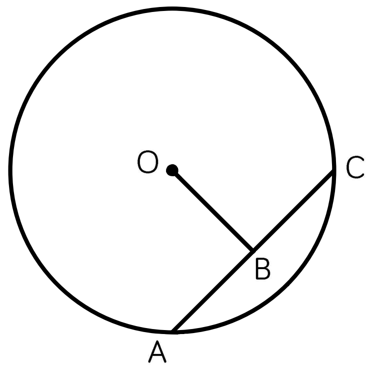
Identify a pair of equal sides. How do you know they are equal? Identify a pair of equal angles. How do you know they are equal?

Exemplar Questions

The diagram shows a chord AB drawn through a circle centre O.

Show that tringles OAB and OCB are congruent

- using the condition RHS.
- using the condition SSS.



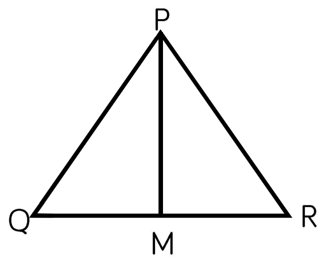
Show that a diagonal of a rhombus splits the rhombus into a pair of congruent triangles.

What other quadrilaterals is this true for?

PQR is an equilateral triangle.

M is the midpoint of QR.

How many ways can you find to show that triangles PQM and PRM are congruent?



Triangle ABC is an isosceles triangle with ∠ABC = ∠ACB.

X and Y are points on AB and BC respectively such that AX = AY.

Show that triangle ABY is congruent to triangle ACX.