Autumn Term

Year 7

#MathsEveryoneCan

2019-20

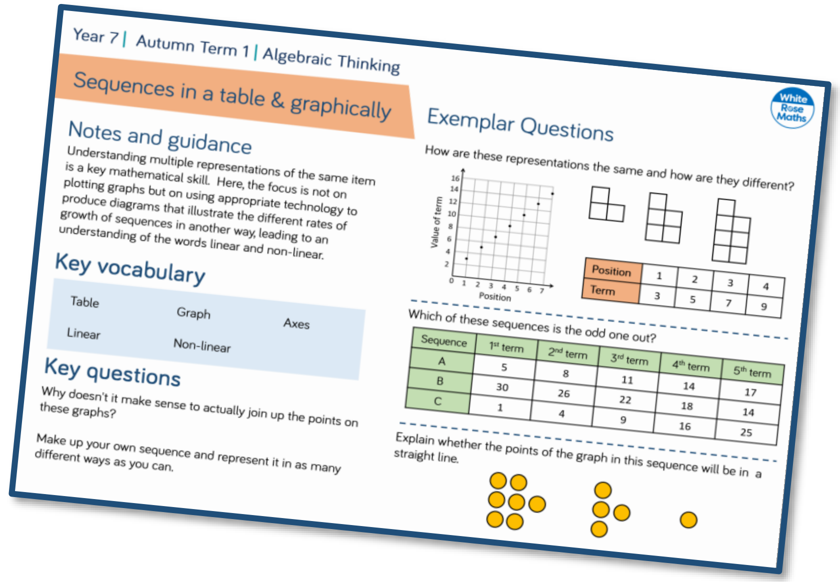
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Algebraic Thinking						Place Value and Proportion					
	Sequences		Understand and use algebraic notation		Equality and equivalence		Place value and ordering integers and decimals			Fraction, decimal and percentage equivalence		
Spring	Applications of Number						Directed Number			Fractional Thinking		
	Solving problems with addition & subtraction		Solving problems with multiplication and division			Fractions & percentages of amounts	Operations and equations with directed number			Addition and subtraction of fractions		
Summer	Lines and Angles						Reasoning with Number					
	Constructing, measuring and using geometric notation			Developing geometric reasoning			Developing number sense		Sets and probability		Prime numbers and proof	

Autumn 1: Algebraic thinking

Week 1: Exploring Sequences

Rather than rushing to find rules for n^{th} term, this week is spent exploring sequences in detail, using both diagrams and lists of numbers. Technology is used to produce graphs so students can appreciate and use the words "linear" and "non-linear" linking to the patterns they have spotted. Calculators are used throughout so number skills are not a barrier to finding the changes between terms or subsequent terms. Sequences are treated more formally later this unit.
National curriculum content covered:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- make and test conjectures about patterns and relationships
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately
- generate terms of a sequence from a term-to-term rule
- recognise arithmetic sequences
- recognise geometric sequences and appreciate other sequences that arise

Weeks 2 to 4: Understanding and using algebraic notation

The focus of these three weeks is developing a deep understanding of the basic algebraic forms, with more complex expressions being dealt with later. Function machines are used alongside bar models and letter notation, with time invested in single function machines and the links to inverse operations before moving on to series of two machines and substitution into short abstract expressions.
National curriculum content covered:

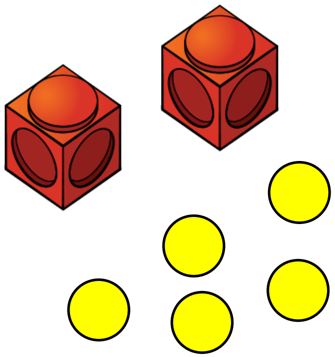
- move freely between different numerical, algebraic, graphical and diagrammatic representations
- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- recognise and use relationships between operations including inverse operations
- model situations or procedures by translating them into algebraic expressions
- substitute values in expressions, rearrange and simplify expressions
- use and interpret algebraic notation, including:
  - ab in place of $a \times b$
  - $3y$ in place of $y + y + y$ and $3 \times y$
  - a^2 in place of $a \times a$
  - ab in place of $a \times b$
  - $\frac{a}{b}$ in place of $a \div b$
- generate terms of a sequence from a term-to-term rule
- produce graphs of linear functions of one variable

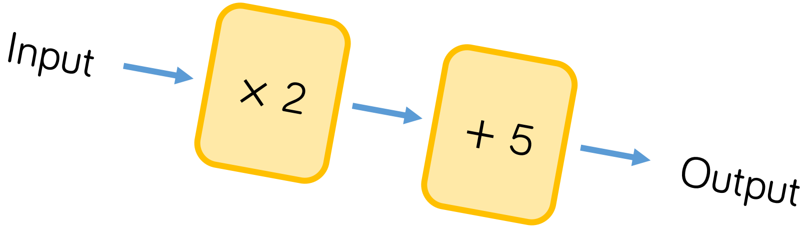
Weeks 5 and 6: Equality and equivalence

In this section students are introduced to forming and solving one-step linear equations, building on their study of inverse operations. The equations met will mainly require the use of a calculator, both to develop their skills and to ensure understanding of how to solve equations rather than spotting solutions. This work will be developed when two-step equations are met in the next place value unit and throughout the course. The unit finishes within consideration of equivalence and the difference between this and equality, illustrated through collecting like terms.
National curriculum content covered:

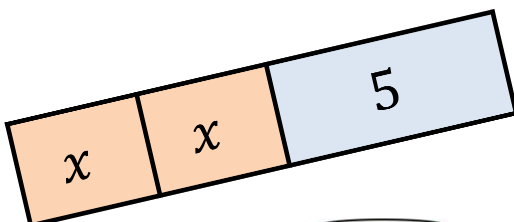
- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- simplify and manipulate algebraic expressions to maintain equivalence by collecting like terms
- use approximation through rounding to estimate answers
- use algebraic methods to solve linear equations in one variable

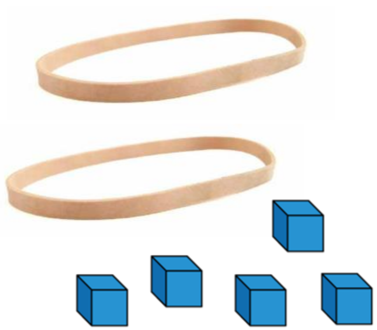
Key Representations

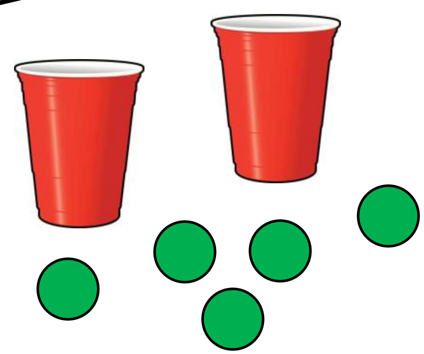




$$2x + 5$$







Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Here are a few ideas for how you might represent algebra. Cups, cubes and elastic bands lend themselves well to representing an unknown, whereas ones (from Base 10) and counters work well to represent a known number.

Be careful to ensure that when representing an unknown students use equipment that does not have an assigned value – such as a Base 10 equipment and dice.

Sequences

Small Steps

- Describe and continue a sequence given diagrammatically
- Predict and check the next term(s) of a sequence
- Represent sequences in tabular and graphical forms
- Recognise the difference between linear and non-linear sequences
- Continue numerical linear sequences
- Continue numerical non-linear sequences
- Explain the term-to-term rule of numerical sequences in words
- **Find missing numbers within sequences** H

H denotes higher strand and not necessarily content for Higher Tier GCSE

Describe and continue sequences

Notes and guidance

Given a sequence of diagrams, students recognise and describe the change(s) from one term to the next. They use their findings to draw the next term, or terms, in the sequence. Sequences chosen should be linear, non-linear, oscillating etc. but this language will be later. Students should have access to counters and other manipulatives to support them.

Key vocabulary

Sequence	Term	Position
Rule	Term-to-term	

Key questions

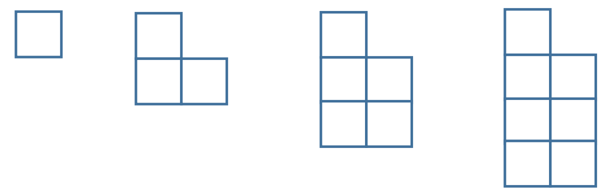
How is each term in the sequence different from the previous term?

Do the terms change in the same way every time?

How could you change the sequence?

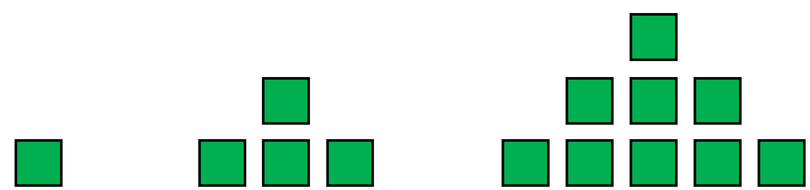
Exemplar Questions

Draw the next two terms in this sequence.



Describe the sequence.

What would the fifth term in this sequence look like?.



How might this sequence continue?

Describe the ways in which your sequences are similar and how they are different.

Predict and check next term(s)

Notes and guidance

Students predict the structure of the next term in a sequence of diagrams e.g. the number of squares/lines in the pattern, and then draw the term to check their prediction. Both linear and non-linear sequences should be used.

Key vocabulary

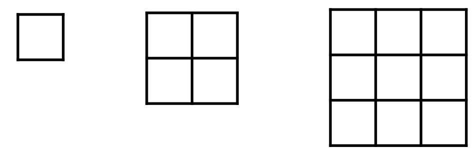
Sequence	Term	Position
Rule	Term-to-term	

Key questions

Is there a quick way of counting the number of squares/circles/lines in each diagram?
Does this help you predict how many squares/circles/lines there are in the 10th term? The 100th term?

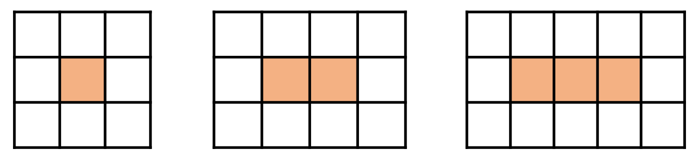
Exemplar Questions

How many squares are there in each diagram in this sequence?

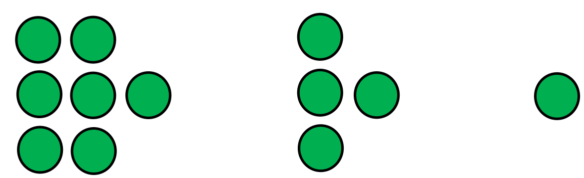


How many squares would there be in the next diagram?

How many white squares would there be in the fifth term of this sequence?



Zena says it's impossible to draw the next term in this sequence.
Is she right? Why?



Sequences in a table & graphically

Notes and guidance

Understanding multiple representations of the same item is a key mathematical skill. Here, the focus is not on plotting graphs but on using appropriate technology to produce diagrams that illustrate the different rates of growth of sequences in another way, leading to an understanding of the words linear and non-linear.

Key vocabulary

Table	Graph	Axes
Linear	Non-linear	

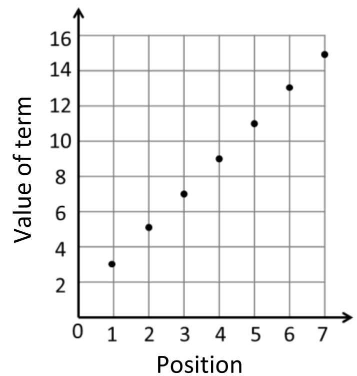
Key questions

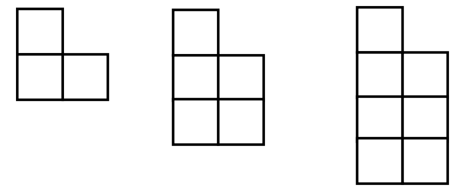
Why doesn't it make sense to actually join up the points on these graphs?

Make up your own sequence and represent it in as many different ways as you can.

Exemplar Questions

How are these representations the same and how are they different?



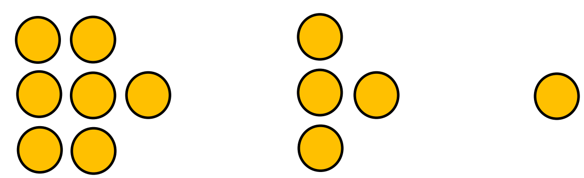


Position	1	2	3	4
Term	3	5	7	9

Which of these sequences is the odd one out?

Sequence	1st term	2nd term	3rd term	4th term	5th term
A	5	8	11	14	17
B	30	26	22	18	14
C	1	4	9	16	25

Explain whether the points of the graph in this sequence will be in a straight line.



Linear & non-linear sequences

Notes and guidance

Building on the previous step, students are now asked to recognise from a list of numbers, rather than from a graph, or a table, whether the sequence is linear or not; you may then wish to demonstrate this graphically. The idea of a constant difference between the terms should be emphasised. If appropriate, discussion of second differences could follow.

Key vocabulary

Linear	Non-linear	Difference
Constant difference	Ascending	Descending

Key questions

How is a linear sequence different from a non-linear sequence?

What do you look for in a sequence to decide if it is linear?

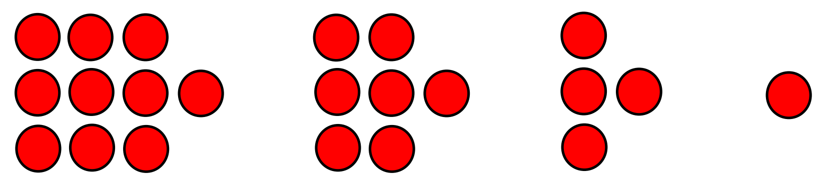
Can a linear sequence be decreasing?

Exemplar Questions

Which of these sequences are linear, and which are not?

- 10 , 20 , 30 , 40 , 50…
- 10 , 100 , 1000 , 10 000 , 100 000…
- 90 , 85 , 80 , 75 , 70…
- 2 , 3 , 5 , 8 , 12…
- 1 , 2 , 3 , 5 , 8 , 13….

Do these pictures show a linear sequence or not? Explain why.



Mo says this sequence is linear because there is a constant difference of one between the terms.
Is he right?

1 , 2 , 1 , 2 , 1 , 2…

Continue linear sequences

Notes and guidance

Students should be taught to work out the next term in a sequence of numbers through finding and using the constant difference. The sequences chosen should include both ascending and descending sequences, including decimals if appropriate. Calculators should be used so atypical sequences are encountered and students can develop their calculator skills.

Key vocabulary

Linear	Non-linear	Difference	Arithmetic
Constant difference	Ascending	Descending	

Key questions

Why does the common difference help us to work out the next term in a linear sequence?

How many terms do you need to be able to write a linear sequence?

Exemplar Questions

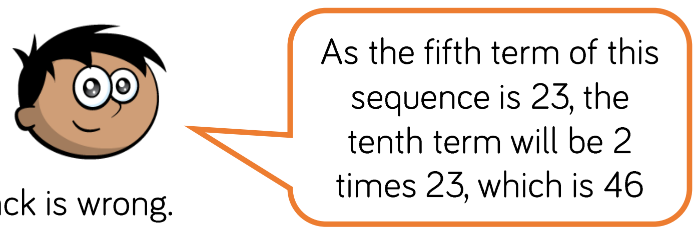
Find the next three terms in each of the following linear sequences.

- 60 , 74 , 88 , ____ , ____ , ____
- 8000 , 11 000, 14 000 , ____ , ____ , ____
- 90 , 85 , 80 , ____ , ____ , ____
- 0.9 , 1.2 , 1.5 , ____ , ____ , ____
- 7.42, 6.81 , ____ , ____ , ____

Here is a linear sequence.

7 , 11 , 15 , 19 , 23…

Jack says,



Explain why Jack is wrong.

How many linear sequences can you create starting with 90 , 88…?

An ascending linear sequence starts with 4.7 and has common difference 2.5. Find the first six terms of the sequence. What do you notice about all the numbers in the sequence?

Create an integer linear sequence whose last digits are always a repeating series.

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Continue non-linear sequences

Notes and guidance

Students should be taught to decide whether a sequence is linear or not by checking to see whether the differences are constant. In the case where they are not, students should be encouraged to find the most efficient way of getting from one term to the next e.g. focusing on the multiplier in a geometric sequences rather than the change in differences.

Key vocabulary

Linear	Non-linear	Difference
Second difference	Ascending	Descending
Geometric	Fibonacci	

Key questions

Why does the common difference help us to work out the next term in a linear sequence?

Do geometric sequences always grow faster than arithmetic?

Exemplar Questions

Find the next two terms in each of the following sequences.

- 1 , 2 , 4 , 8 , ____ , ____
- 64 000 , 32 000, 16 000 , ____ , ____
- 1 , 3 , 6 , 10, , ____ , ____
- 100 , 150 , 225 , ____ , ____ , ____
- 1 , 1 , 2 , 3, 5 , 8 , ____ , ____

Sequence A: 1 , 11 , 21 , 31 , 41…

Sequence B: 1 , 2 , 4 , 8 , 16…

Who is right?

How many sequences, linear or non-linear, can you create starting with 1 , 2 , …?

A sequence starts with the number 17. The next number is found by doubling the previous number and adding 3. Find the first five terms of the sequence. What do you notice?

Create a geometric sequence whose last digits are always 6

Explain the term-to-term rule

Notes and guidance

This step will probably be covered alongside the previous two steps. Students should be encouraged to use full sentences and the key words rather than vague statements like “it doubles” or “you times it be three”.

Key vocabulary

Linear	Non-linear	Arithmetic
Geometric	Fibonacci	

Key questions

How would you explain the difference between an arithmetic and a geometric sequence?

How could you get from the first to …th term in this sequence?

Exemplar Questions

Describe in words how these sequences change from one term to the next:

- 1 , 5 , 9 , 13 , ____ , ____
- 64 000 , 32 000, 16 000 , ____ , ____
- 8 , 24 , 72 , ____ , ____
- 100 , 150 , 225 , ____ , ____ , ____
- 1 , 1 , 2 , 3, 5 , 8 , ____ , ____

The term-to-term rule of a sequence is:

The next term is found by tripling the previous term.

Why can’t we write out this sequence?

How many sequences, linear or non-linear, can you create starting with 1 , 2 , …?
Write the term-to-term rule for each in words.

Find missing terms H

Notes and guidance

Students should start by considering finding a term further away than the next term in a given sequence (see example one). Students should then discover strategies to find missing terms in sequences where the rule cannot be determined from adjacent terms. For most students it might be best to do this with linear sequences only.

Key vocabulary

Difference	Position

Key questions

How many **terms** are there between the first and third term?

How many **differences** are there between the first and third term?

Exemplar Questions

A sequence starts 5, 12 , 19

Ian says,

The fifth term of this sequence is 33, as you add need to add 7 on 19 twice

Check that Ian is right.
Use Ian's strategy to work out the seventh and tenth term of the sequence without working out the terms in between.

Find the missing terms in each of these sequences:

- 2 , 8 , ____
- 2 , ____ , 8
- 2 , ____ , ____, 8
- 6 , ____ , 14 , ____ , 22, ____
- 8000 , ______ , _______ , 6500, ______

The first term of a sequence is 4 and the third term is 16.
If the sequence is arithmetic, what are the second and fourth terms?
If the sequence is geometric, what are the second and fourth terms?
Can you find rules for other sequences that start 4 , ____ , 16?

Understand and use notation

Small Steps

- Given a numerical input, find the output of a single function machine
- Use inverse operations to find the input given the output
- Use diagrams and letters to generalise number operations
- Use diagrams and letters with single function machines
- Find the function machine given a simple expression
- Substitute values into single operation expressions
- Find numerical inputs and outputs for a series of two function machines
- Use diagrams and letters with a series of two function machines
- Find the function machines given a two-step expression
- Substitute values into two-step expressions
- Generate sequences given an algebraic rule
- Represent one- and two-step functions graphically

Single function machines (number)

Notes and guidance

The aim of this small step is for students to become fluent in the use of single function machines with numbers, working from left to right. Students also need to learn the associated vocabulary of “input” and “output”. Wherever appropriate, calculator use should be encouraged.

Key vocabulary

Function	Input	Output
Estimate	Operation	Square

Key questions

How can we check if the answer from our calculator is reasonable?

What happens to the size of the outputs if we change the size of the inputs?

Exemplar Questions

Find the outputs when you input 0, 1, 2, 3, 4 and 5 into these machines. What’s the same and what’s different?

→ $+4$ →   → $\div 4$ →

Find the output for these function machines if 17 is input into each of them.

→ $+86$ →   → -37 →

→ $\div 5$ →   → Square →

Before doing the calculations, can you estimate which of these machines will have the biggest output for the given inputs?

1876 → $+ 12\,000$ →

418 → $\times 96.12$ →

How many functions can you think of where the output is always the same as the input?

Find the input given the output

Notes and guidance

Using students' knowledge of inverse operations, we will now consider using a function machine from right to left to find the input for a given output. Again, calculator skills should be developed including how to use the square and square root functions.

Key vocabulary

Function	Input	Output
Estimate	Operation	Inverse

Key questions

What calculation can we do to check that our answer for the input is correct?

What happens to the size of the outputs if we change the size of the inputs?

Exemplar Questions

Find the input for these function machines if the output for each of them is 100

→ $+ 86$ →   → $\times 1.25$ →

→ $\div 5$ →   → -37 →   → Square →

Find two possible machines that give the output 10 for an input of 5

5 → ? → 10

What could the machines have been if the input had been 10 and the output had been 5?

Find the outputs for this function machine.

3.1 →, 5 →, 8.2 → Subtract from 10 →

Put the outputs back in to the function machine. What do you notice?

Investigate inverse functions on your calculator.

Use letters to generalise number

Notes and guidance

This small step is where students are explicitly taught algebraic notation and may well need a few lessons. At each stage it is important to keep reminding students that each representation stands for a number. Multiple representations including concrete materials and bar models should be used alongside each other to encourage flexible thinking, but emphasis needs to be placed on correct algebraic notation.

Key vocabulary & notation

Bar model	Variable	Coefficient
$3a$ for $a \times 3$	$\frac{a}{3}$ for $a \div 3$	a^2 for $a \times a$
ab for $a \times b$	Commutative	Expression

Key questions

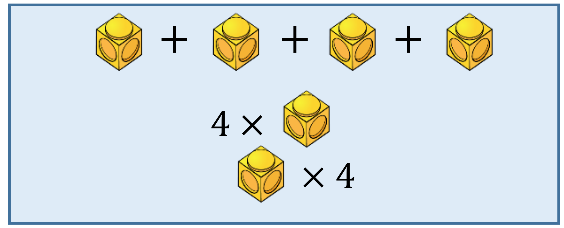
What's different about using a letter to represent a number compared to using a bar?

Exemplar Questions

How are these sets of calculations the same and how are they different?

$10 + 10 + 10 + 10$
4×10
10×4

$6 + 6 + 6 + 6$
4×6
6×4



$a + a + a + a$
$4 \times a$
$a \times 4$

Write these expressions without mathematical operation signs.

$f + f + f + f + f + f$	$7 \times g$
$t \div 5$	$5 \div t$
$m \times m$	$d \times c$

Zeb says p^2, $p2$ and $2p$ are all exactly the same.

Explain why Zeb is **wrong**.

Use diagrams to help.

Single function machines (algebra)

Notes and guidance

Here we are linking the last few steps to reinforce students understanding of algebraic notation and linking it to bar model representations. Use of concrete resources such as multi-link cubes to represent unknowns alongside should also be encouraged, but take care not to use objects like ten-sticks that have a pre-defined value to stand for variables as this can lead to confusion.

Key vocabulary & notation

Bar model	Variable	Coefficient
$3a$ for $a \times 3$	$\frac{a}{3}$ for $a \div 3$	a^2 for $a \times a$
ab for $a \times b$	Commutative	Expression

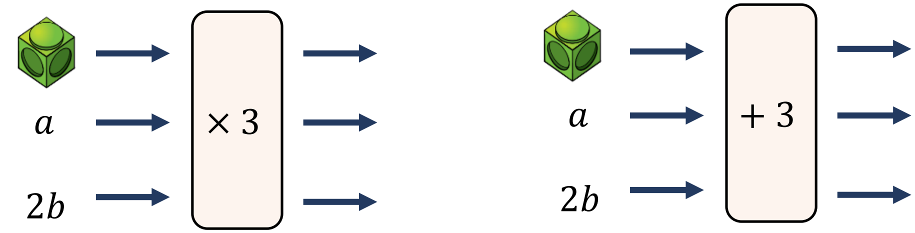
Key questions

What's different about using a letter to represent a number compared to using a bar?

Will outputs like $a + 3$ and $3a$ always, sometimes or never be the same?

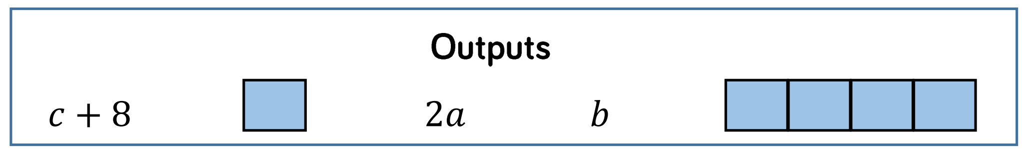
Exemplar Questions

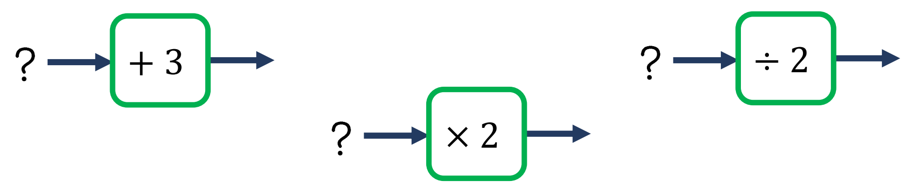
Find the **output** for each of the function machines with these inputs.



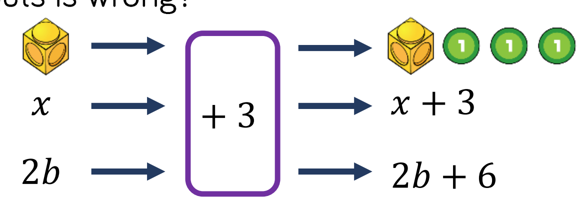
Investigate other function machines e.g. "÷ 2"

Find the **input** for each of the function machines with each of these outputs.





Which of these outputs is wrong?



Find functions from expressions

Notes and guidance

In this small step students are developing their fluency and understanding by reversing the process of the previous step.

Given an expression involving a single operation applied to a variable, they identify the function that has taken place and so find the function machine.

Key vocabulary & notation

Bar model	Variable	Coefficient
$3a$ for $a \times 3$	$\frac{a}{3}$ for $a \div 3$	a^2 for $a \times a$
ab for $a \times b$	Commutative	Expression

Key questions

What does the expression $6a$ mean?

Why are the expressions $\frac{a}{2}$ and $\frac{2}{a}$ different?

Exemplar Questions

For each of these function machines, find the function that gives the outputs shown for the given inputs

a → ? → $5a$

b → ? → $b - 3$

$10c$ → ? → $2c$

y → ? → xy

x → ? → x^2

d → ? → $d - g$

Do any of the machines have more than one possible answer?

Complete the missing information for this function machine.

Input		Output
4	?	8
10		20
?		16
$10c$		?
?		$6b$

Fred says the machine is "$\times$ 2", Bertha says it's "$+ a$".
Who do you agree with?

a → ? → $2a$

Substitute into single expressions

Notes and guidance

In this small step, students are practising their calculator skills and using the expressions they have learnt in the more abstract context of stand-alone expressions. These can be used in conjunction with function machine diagrams if needed. Comparing answers of different expressions will link to sequences studied earlier, and inform later work on equivalence.

Key vocabulary & notation

Expression	Evaluate	Substitute
$3a$ for $a \times 3$	$\frac{a}{3}$ for $a \div 3$	a^2 for $a \times a$
ab for $a \times b$		

Key questions

Are $t + 5$ and $5 + t$ always, sometimes or never equal?

Are $2p$ and p^2 always, sometimes or never equal?

What is different about the expressions $p - 4$ and $4 - p$?

Exemplar Questions

Substitute $a = 5$ into each of these expressions.

$7a$	$\frac{7}{a}$	$19.8 - a$	a^2
$2a$	$a - 3.6$	$a + 3.6$	

Which of these expressions will be equal when $x = 2$?

$2x$	$\frac{x}{2}$	$\frac{2}{x}$	$x + 2$
$2 + x$	$x - 2$	$2 - x$	x^2

Put the expressions in order from smallest to largest for different values of x (Try $x = 1,\ x = 0.4,\ x = 100,\ x = 0$ ...)
Which expressions will always be equal, whatever the value of x?

Substitute $n = 1, n = 2, n = 3,\ n = 4$ and $n = 5$ into all of these expressions.

$n + 7$	$3n$	n^2
$20 - n$	$\frac{n}{2}$	$\frac{2}{n}$

What do you notice about each set of answers?

2-step function machines (number)

Notes and guidance

Students now move on to using two function machines in a row, so that the output of the first machine is the input of the second machine. Students need to become fluent in this process with numbers, both forward and backward, before moving on to the next step where they use concrete objects, diagrams and letters.

Key vocabulary

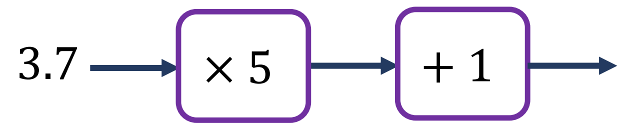
Input	Output	Inverse

Key questions

Why do you do the inverse operations in reverse order when finding the input to a pair of function machines?

Exemplar Questions

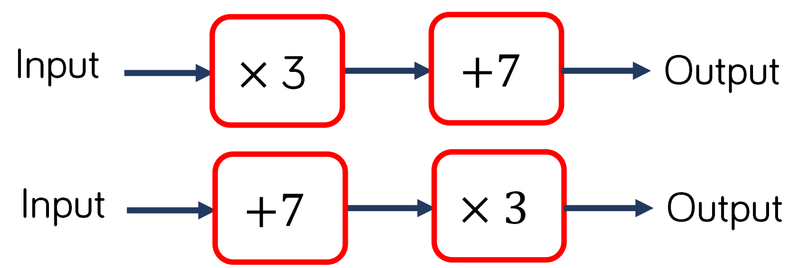
Find the output of this series of two function machines.



I think of a number, double it and then add on 9
The result is 22.4

- Show this using a series of function machines..
- Use inverse operations to work out the number I started with.

Aisha says these pairs of function machines will have the same output as they are the same functions.



Give an example to show that Aisha is wrong.

2-step function machines (algebra)

Notes and guidance

Students now build on their experience of two machines in the previous step by using objects, bar models and letters. They will need to be taught that the order in which the functions are applied is important and will need to be introduced to brackets in algebraic expressions to distinguish between e.g. $2x + 5$ and $2(x + 5)$. Formal expanding of brackets is not expected at this stage.

Key vocabulary

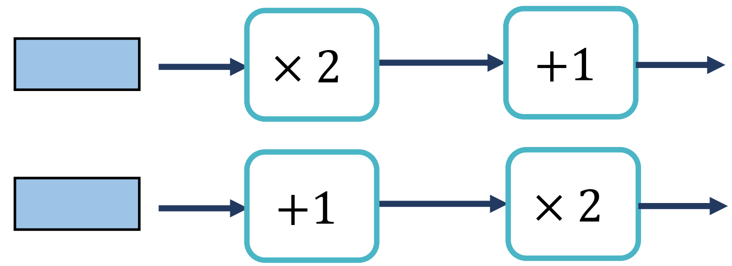
Input	Output	Order
Bracket	Variable	Expression

Key questions

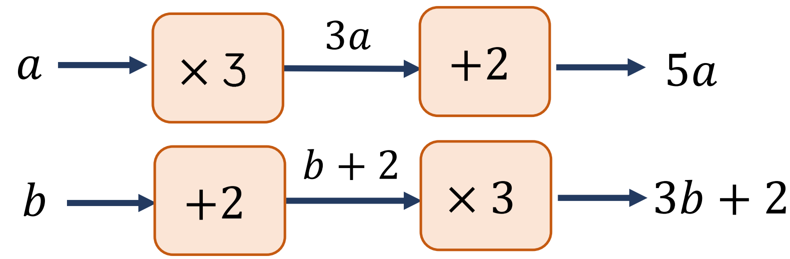
Does it sometimes, always or never make a difference if you change the order of a pair of function machines?

Exemplar Questions

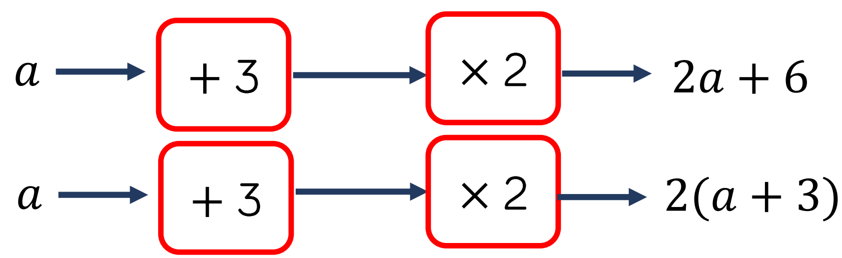
Compare the outputs of these pairs of function machines.



Correct the mistakes in the working below.



Use bars or concrete materials to show that both these answers are correct.



White Rose Maths

Find functions from expressions

Notes and guidance

In this small step, students show their understanding of two-step expressions by reversing the process of the previous step and finding the operations that formed the expressions. It should again be reinforced that the letter represents any number, possibly by teaching the next small step alongside this one.

Key vocabulary

Input	Output	Order
Bracket	Variable	Expression

Key questions

What's the difference between $\frac{a+4}{2}$ and $\frac{a}{2} + 4$?

Is there more than one way of applying two consecutive functions to x and obtaining $2x + 4$?

Exemplar Questions

Fill in the gaps in these function machines.

$x \to \boxed{?} \to \boxed{?} \to 5x - 6$

$y \to \boxed{?} \to \boxed{?} \to \frac{y}{2} - 4$

$w \to \boxed{?} \to \boxed{?} \to 3(w + 1)$

$t \to \boxed{?} \to \boxed{?} \to \frac{t - 2}{4}$

Complete the missing information for this function machine.

$6 \to \boxed{?} \to \boxed{+2} \to 20$

$? \to \boxed{?} \to \boxed{+2} \to 3x + 2$

$? \to \boxed{?} \to \boxed{+2} \to 9y + 2$

Investigate.

$\to \boxed{\times 5} \to \boxed{\div 5} \to$

Substitute into two-step expressions

Notes and guidance

Students are again practising their calculator skills, now using the two-step expressions they have learnt. They can compare the similarities and differences between e.g. $3a + 2$ and $3(a + 2)$ for a wide variety of inputs. Substituting repeatedly into the same expression is a valuable experience with opportunities for discovery.

Key vocabulary

Expression	Evaluate	Substitute
Variable	Constant	

Key questions

How would you use your calculator to wok out the value of the square of a number?

When do you need to use brackets when substituting into expressions using a calculator?

Exemplar Questions

Substitute different values of x into these two expressions – include integers, decimals, negatives and fractions.

$2(x + 4)$ $2x + 8$

What do you notice?
Can you use function machines and diagrams to explain why?

.

Which of these is the largest when $a = 1$ and $b = 0.1$?

ab $\frac{a}{b}$ $\frac{b}{a}$ $a + b$ $a - b$

How would this change if $a = 0.1$ and $b = 0.01$?
Investigate for other values of a and b

Pick values of a and b to substitute into this expression.

$a^2 + 2b$

- How do the values of the expression change if you keep a the same and vary b?
- How do the values of the expression change if you keep b the same and vary a?

Generate sequences from a rule

Notes and guidance

In this small step, students revisit the ideas from week 1 combining their knowledge with that of the substitution they have just learnt. At this stage students do not need to learn a procedure for finding a rule for the n^{th} term of a linear sequence, but they may well make connections between the sequences found and the rules given. The language of sequences can also be reinforced in this step.

Key vocabulary

Sequence	Non-linear	Linear
Rule	Term-to-term	Position-to-term

Key questions

What feature of the difference between terms tells us if a sequence is linear?

Which type of rule is better for finding the 100^{th} term of a sequence?

Exemplar Questions

Substitute $n = 1,\ n = 2,\ n = 3,\ n = 4$ and $n = 5$ into the expression $3n + 5$

- What do you notice about your answers?

Repeat for $3n + 6$ and then $2n + 5$

- What stays the same? What changes?

Use your calculator to find the first ten terms of the sequences given by these rules.

$$n^2 \qquad 2^n \qquad n^2 - 4 \qquad (n-4)^2$$

What are the similarities and differences?

Which of these rules do you think will produce linear sequences?

$$\frac{n}{2} + 4 \qquad 150 - 8n \qquad 6n + 0.2$$

$$3 + n^2 \qquad \frac{n-3}{4}$$

Check by substituting several consecutive values of n.

Represent functions graphically

Notes and guidance

In this small step, students use technology to plot the graphs of some of the functions they have been working with to reinforce the vocabulary of linear and non-linear. There is no need to formally investigate the equations of lines at this stage, but students should be encouraged to spot similarities and differences.

Key vocabulary

Graph	Axis	Axes	Scale
Equation	Linear	Non-linear	Curve

Key questions

How can you tell from an equation whether the graph is going to be linear?

How does this link to linear and non-linear sequences?

Exemplar Questions

Use a graphing program to compare the graph of the sequence given by the rule $2n + 1$ with the graph given by the equation $y = 2x + 1$

What are the similarities and differences?

Compare the graphs of $y = 2x$ and $y = x^2$

What are the similarities and differences?

Without using a graph plotter, decide which of these equations will produce a straight line graph

- $y = 3x + 2$
- $y = 2 + 3x$
- $y = x^2 + 3$
- $y = 6 - \frac{x}{2}$
- $y = \frac{2}{x} + 6$
- $y = 5 - x$

Check your answers with a graph plotter.
Which shapes were most surprising?

Equality and Equivalence

Small Steps

- Understand the meaning of equality
- Understand and use fact families, numerically and algebraically
- Solve one-step linear equations involving +/− using inverse operations
- Solve one-step linear equations involving ×/÷ using inverse operations
- Understand the meaning of like and unlike terms
- Understand the meaning of equivalence
- Simplify algebraic expressions by collecting like terms, using the ≡ symbol

Understand equality

Notes and guidance

Students often misinterpret the equals sign as “makes”. The bidirectional nature of equality needs to be emphasised so that students realise the left hand side and right hand side of an equation are worth the same amount rather than making each other. It is helpful to read the equals sign as “is equal to” to support this.

Key vocabulary

Equality	Equation	Equals
Is equal to		

Key questions

What difference does it make when you swap the right hand side and the left hand side of an equation?

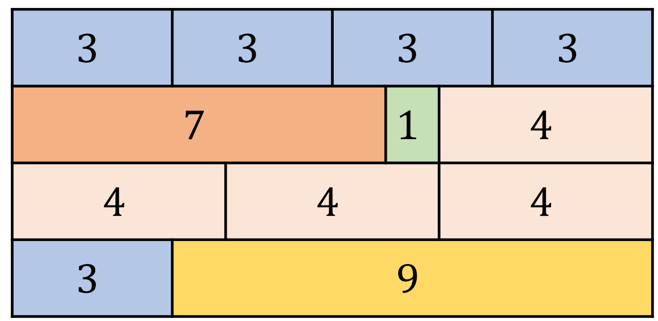
If you change the order of the terms on one side of an equation, will it still be true?

Exemplar Questions

Which of the following are true?

- $6 + 3 = 9$
- $8 = 5 + 3$
- $5 + 6 = 8 + 3$
- $312 + 99 = 312 + 100 - 1$
- $12 + 9 = 3 \times 7$
- $8 \div 0.2 = 80 \div 2$
- $6700 - 67 = 99 \times 67$

Here is a number wall.



How many equations can you find from this number wall?
E.g. $12 = 3 + 9$, $4 \times 3 = 7 + 1 + 4$

Work out the missing numbers in these equations.

$7 + 8 = 10 + ?$

$9 + ? = 20 - 6$

$12 \times 5 = ? + 50$

$? \div 2 = 60 \div 4$

Understand and use fact families

Notes and guidance

Students will be familiar with fact families from their work in previous key stages. This small step extends their knowledge to algebraic fact families in preparation for solving equations and recognising equivalent forms of the same equation.

Key vocabulary

Fact family	Bar Model	Is equal to

Key questions

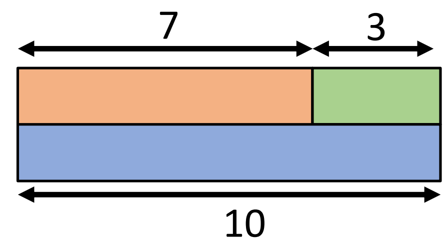
Do bar models need to be drawn to scale?

If you know one addition fact, how many subtraction facts do you also know?

Exemplar Questions

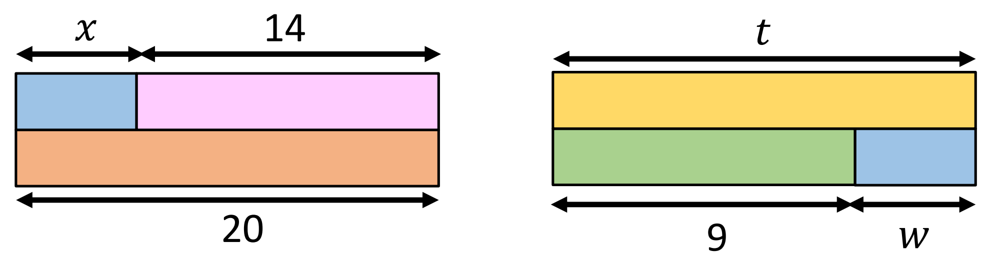
This bar model shows::

$$7 + 3 = 10$$
$$7 = 10 - 3$$

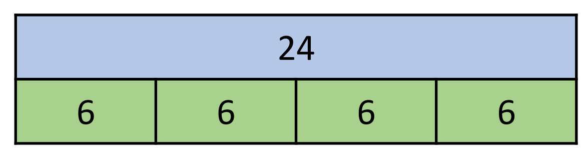


What other facts does it show?

Write the fact families for these bar models.



Write the fact family for this bar model.



$$67.3 - 11.6 = 55.7$$

Using the fact above, write down an addition and another subtraction.

Solve one-step equations (+/−)

Notes and guidance

Calculators should be used to find the solutions to the one-step equations in this step. Avoid equations such as $x + 3 = 8$ as students are likely to "spot" the answer rather than use the inverse operation. Practice questions should include the unknown on either sides and terms in any order. Link also to function machines.

Key vocabulary

Equation	Solve	Solution
Unknown	Inverse	

Key questions

What's the difference between an equation and an expression?

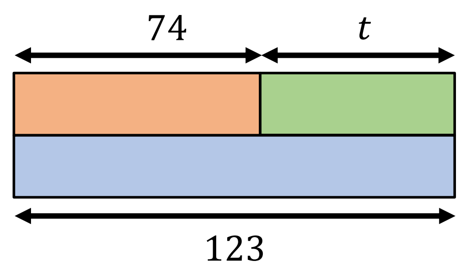
How is an 'unknown' different from a 'variable'?

What is the inverse of 'add on 12'?

Exemplar Questions

Draw a bar model to illustrate $x + 37 = 61$
Write down the fact family for your bar model.
What is the value of x?

What additions does this diagram show?
What subtractions does it show?



What is the value of t?

Solve these equations.

$a + 47 = 93$	$38 = 5.6 + b$	$36 = c - 102$
$4.8 + d = 11$	$91 = 53 + e$	$70 - f = 11.4$

Ken thinks of a number. He subtracts 78 from his number and gets the answer 137. Show this information as an equation and solve the equation to find Ken's number.
How else could you represent the information?

Solve one-step equations ($\times / \div$)

Notes and guidance

Calculators should again be used to find the solutions to these equations involving multiplication and division. This will allow for a variety of questions, avoiding misconceptions such as answers always been integers. Emphasis should again be on inverse operations with unknowns in different places, not just $ax = b$.

Key vocabulary

Equation	Solve	Solution
Unknown	Inverse	

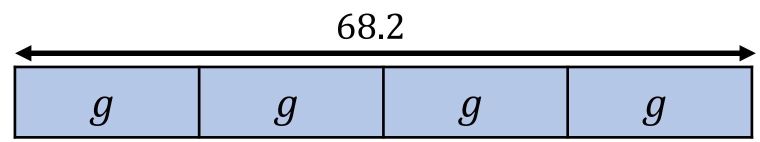
Key questions

Are the equations $3x = 192$ and $192 = 3y$ the same or different?

How can we check the answers to our equations are correct?

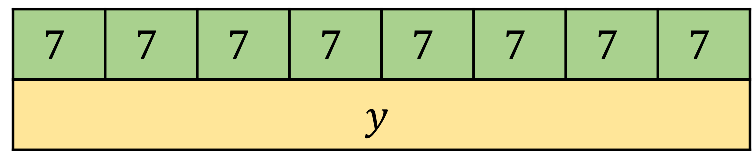
Exemplar Questions

Write the fact family for this bar model.



Work out the value of g.

Write the fact family for this bar model.



Work out the value of y.

Marta thinks of a number.

She divides her number by 7 and gets the answer 42

Write this information as an equation.

Solve your equation to find Marta's number.

Solve these equations.

$\frac{a}{23} = 9$ $\quad 46 = \frac{b}{11}$ $\quad 187 = 5c$ $\quad 12d = 3$

Understand like and unlike terms

Notes and guidance

This small step is vital in supporting simplification of algebraic expressions. Displaying a pair of terms on the board and using a true/false activity with mini-whiteboards is a good way to assess.

Key vocabulary

Term	Like	Unlike
Coefficient	Index	

Key questions

Why are $3x$ and $3x^2$ unlike terms?

What is the coefficient of d in the term $-7d$?

Exemplar Questions

Like terms
$5a, 6a$
$10t, -3t$
$2xy, 4xy$
$10. -7$
$3a^2, 7a^2$

Unlike terms
$5a, 5b$
$-10t, -3$
$2xy, 4xz$
$10, 7a$
$3a^2, 7b^2$

Explain why the terms in the left are called 'like terms' and the terms on the right are called 'unlike terms'.

Sort the expressions below into sets of like terms

3	$3a$	$-3b^2$	$-3a$
a^2	-3	$6a$	-6
$6b^2$	6	$-6a$	$12b$

Jemima says $3xy$ and $6yx$ are like terms because the expressions involve the same powers of the same letters just in a different order. Do you agree?

The meaning of equivalence

Notes and guidance

Students often get confused between equality and equivalence and try to 'solve' when they are asked to simplify. This step is needed to illustrate the difference and to start to appreciate the idea of an identity as opposed to an expression or an equation. It is worth noting that repeated substitution is a demonstration rather than a proof.

Key vocabulary

Expression	Term	Expression
Equivalent		

Key questions

Are the expressions $2x$ and x^2 equivalent? Why or why not?

Write down as many expressions as you can that are equivalent to $5p$

Exemplar Questions

Substitute $x = 7$ into each of these expressions.

$5x$	$2x$	$8x - 3x$	$x + x$
$2 + 4x$	$3x + 2x$	$6x - x$	$4x + 2$

Which expressions give you the same answers? Why?
Repeat with a different value of x.
What do you notice?

Which of the following expressions are equivalent?

$6m$	$2m + 4m$	$\frac{m}{6}$	$10m - 4m$
$2m \times 3$	$m + 6$	$30m \div 5$	$4m - m + 3m$

Check by substituting several values of m.

Work out the expressions below for several values of y.

$2y + 10$ $\qquad$ $2(y + 5)$

What do you notice. Will this always be the case?

Collect like terms using ≡ symbol

Notes and guidance

Building on the last two small steps, we now move to simplifying expressions by collecting like terms. It is important to write e.g. $3x + 2x \equiv 5x$ rather than the commonly seen $3x + 2x = 5x$ and to discuss with the students that the former is true for any value of x, whereas equations are only true for specific values of x.

Key vocabulary

Like/unlike	Equation	Equivalent
≡	Simplify	Collect

Key questions

What's the difference between equality and equivalence?

Can you simplify unlike terms?

Exemplar Questions

Which of the following are true and which are false?

$6x + 2x \equiv 8x$ $\quad$ $6x - 2x \equiv 4x$ $\quad$ $2x \equiv 8x \div 4$

$3x + 2 \equiv 5x$ $\quad$ $3x + 2y \equiv 5xy$ $\quad$ $x + 2 \equiv 2 + x$

$5x - 5 \equiv x$ $\quad$ $10x \equiv 5x \times 2$

Simplify these expressions so they have only one term.

$7a + 2a$	$3a + 4a + 5a$	$10b - 3b + 5b$
$6x^2 + 5x^2$	$2ab + 6ab - 3ab$	$10 + 6 - 3$

Correct the mistakes in the simplifications below.

$5x + 3x \equiv 8x^2$ $\quad$ $10y - 3y \equiv 13y$ $\quad$ $9p + 4p = 94p$

Simplify the expressions below by collecting like terms

$3a + 4 + 5a$ $\quad$ $6b + 2c - 2b + 6c$ $\quad$ $5d + 3e + 2d - 3e$

Find expressions that simplify to $8x + 10y$

Autumn 2: Place Value and Proportion

Weeks 1 to 3: Place Value and Ordering

In this unit, students will explore integers up to one billion and decimals to hundredths, adapting these choices where appropriate for your groups e.g. standard index form could additionally be introduced to student following the Higher strand. Using and understanding number lines is a key strategy explored in depth, and will be useful for later work on scales for axes. When putting numbers in order, this is a suitable point to introduce both the median and the range, separating them from other measures to avoid getting them mixed up. Rounding to the nearest given positive power of ten is developed, alongside rounding to one significant figure. Decimal places will come later, again to avoid too similar concepts being covered at the same time. Topics from last term such as sequences and equations, will be interleaved into this unit.

National curriculum content covered:

- Consolidate their understanding of the number system and place value to include decimals
- understand and use place value for decimals, measures and integers of any size
- order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, , ≤, ≥
- work interchangeably with terminating decimals and their corresponding fractions
- round numbers to an appropriate degree of accuracy
- describe, interpret and compare observed distributions of a single variable through: the median and the range
- interpret and compare numbers in standard form

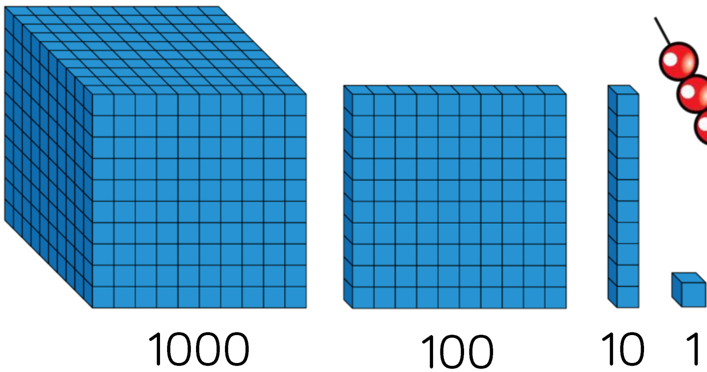
Weeks 4 to 6: Fraction, Decimal and Percentage Equivalence

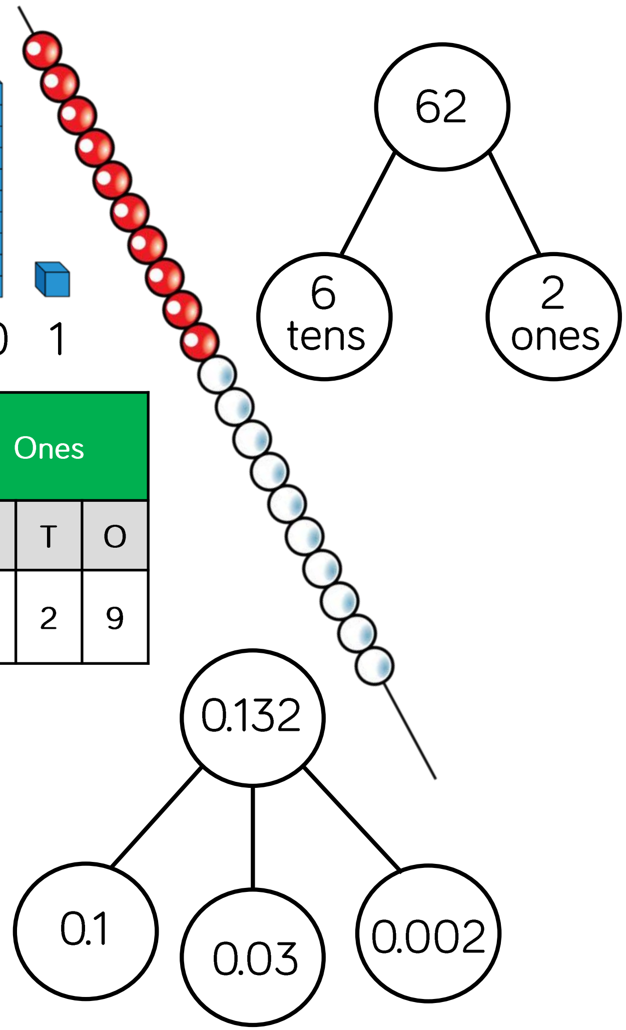
Building on the recent work on decimals, the key focus for this three weeks is for students to gain a deep understanding of the links between fractions, decimals and percentages so that they can convert fluently between those most commonly seen in real-life. The Foundation strand will focus will be on multiples of one tenth and one quarter whilst the Higher strand will look at more complex conversions. Whilst looking at percentage is, pie charts will be introduced. In addition, various forms of representation of any fraction will be studied, focusing on equivalence, in an appropriate depth to the current attainment of students; this will be revisited later in the year. The focus is very much on a secure understanding of the most common fractions under one, but fractions above one will be touched upon, particularly in the Higher strand.

National curriculum content covered:

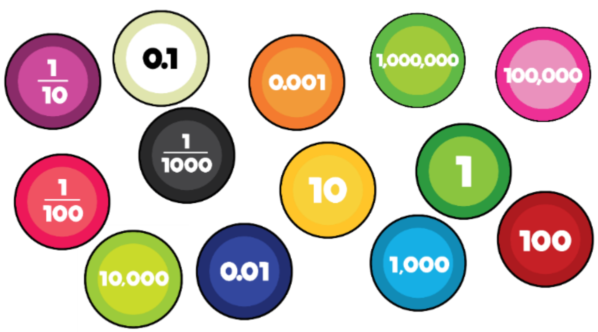
- consolidate their understanding of the number system and place value to include decimals, fractions
- move freely between different numerical representations [for example, equivalent fractions, fractions and decimals]
- extend their understanding of the number system; make connections between number relationships
- express one quantity as a fraction of another, where the fraction is less than 1 and greater than 1
- define percentage as 'number of parts per hundred', interpret percentages as a fraction or a decimal
- compare two quantities using percentages
- work with percentages greater than 100%
- interpret pie charts

Key Representations





Billions			Millions			Thousands			Ones		
H	T	O	H	T	O	H	T	O	H	T	O
		3	1	4	8	0	3	3	0	2	9



Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Here are a few ideas for how you might represent place value.

Base 10 equipment is beneficial to students who need to get a sense of the size of numbers. Reassignment could also be used to support work with decimals, for example a one becomes 0.1, etc. The same could be done with a bead string. What if each bead represents 0.1? 0.01? However, care must be taken with this approach and careful explanation is needed.

Place Value

Small Steps

- Recognise the place value of any number in an integer up to one billion
- Understand and write integers up to one billion in words and figures
- Work out intervals on a number line
- Position integers on a number line
- Round integers to the nearest power of ten
- Compare two numbers using $=, \neq, <, >, \leq, \geq$
- Order a list of integers
- Find the range of a set of numbers
- Find the median of a set of numbers
- Understand place value for decimals
- Position decimals on a number line
- Compare and order any number up to one billion

Place Value

Small Steps

- Round a number to 1 significant figure
- **Write 10, 100, 1000 etc. as powers of ten** (H)
- **Write positive integers in the form A x 10^n** (H)
- **Investigate negative powers of ten** (H)
- **Write decimals in the form A x 10^n** (H)

(H) denotes higher strand and not necessarily content for Higher Tier GCSE

Recognising integer place value

Notes and guidance

Students have met numbers up to ten million at KS2. This small step revises and extends this knowledge. Students should write and represent the numbers in several ways and need to see a mixture of smaller and larger integers. If appropriate, you could discuss the meaning of trillion etc. This step may well be taught alongside the next step.

Key vocabulary

Place value	Digit	Billion
Placeholder	Integer	

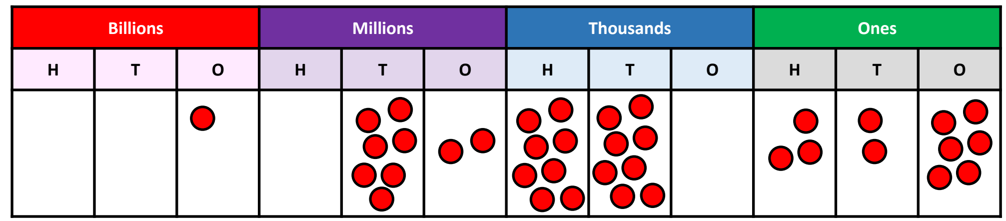
Key questions

Why do we need placeholders?

What strategies can you use to work out the value of a digit in a very long integer?

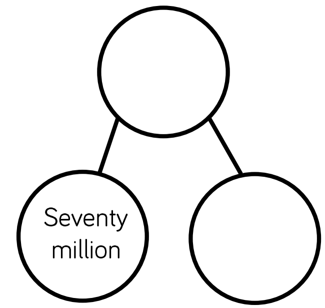
Exemplar Questions

Complete these representations so they all show the same number.



1 073 080 529

One billion, …….



State the value of the 5 in each of these numbers.

650	6500	560 000	60 500
65 000	56	6 005 000	56 000 000
65 000 000	665 066 600		

Write down any number that has:

- Three in the millions place and five in the thousands place
- Three in the ten millions place and five in the hundred thousands place
- Three in the hundreds place and five in the ten millions place

Understand and write integers

Notes and guidance

Following on from the last step, students should become fluent in converting integers from numeral form to words and vice-versa. They should also be comfortable in dealing with other representations. Populations and government finances provide good contexts for real numbers. Comma notation should be taught alongside the more common spacing between every three digits.

Key vocabulary

Place value	Digit	Billion
Placeholder	Integer	

Key questions

Why do we use spaces or commas in large integers?

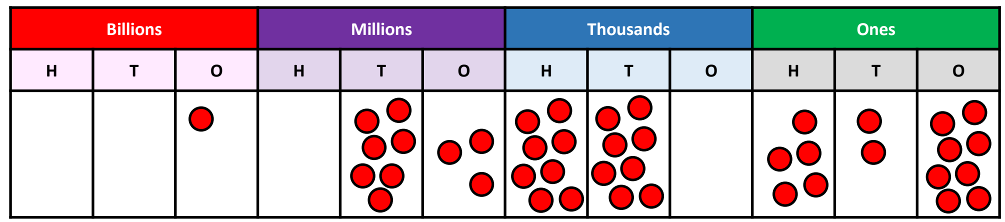
Where do we put them?

Exemplar Questions

Write in figures.

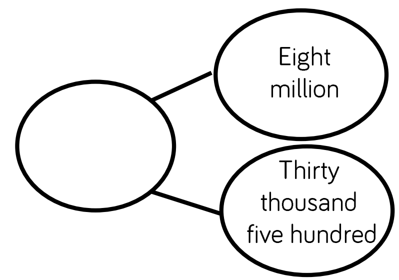
- Thirty-five thousand million
- One and a half billion
- Two hundred and three thousand, five hundred and twelve
- Eighty-eight million, eighty-eight thousand
- Half a million
- One billion, ten thousand and one

Write the numbers represented below in words.



72 007 270

1 402 140 206



Write down the numbers that are:

- Three million more than 917 000 000
- The sum of three hundred million and 700 000 000
- 30 000 000 more than nine hundred and sixty million
- The difference between one billion and seventy-five million

Work out intervals on a number line

Notes and guidance

This key skill will also be useful with later work on fractions and reading/scaling graphs. Students should be taught to work out the intervals given the number of spaces on a line and to fill in missing values. Although the focus should be on the most common values such as 5 and 10, it is worth exploring other values. Using other scales that use number lines will also be useful.

Key vocabulary

Equal division	Interval	Scale
Gap	Spaces	

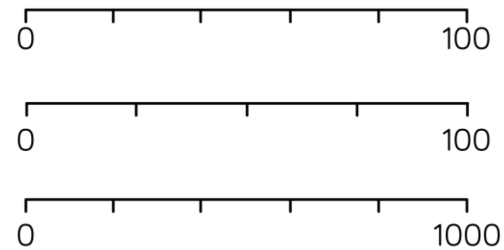
Key questions

Why do we count the number of spaces rather than the number of marks on a number line?

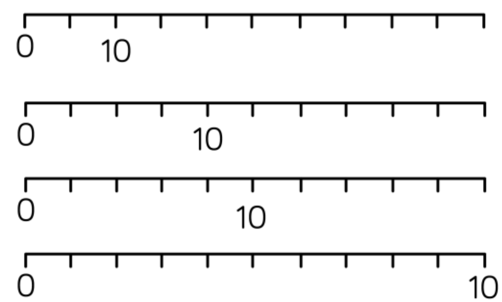
Which are the most important points to label on a number line or other scale? Why?

Exemplar Questions

Work out the value of each of the intervals in these number lines.

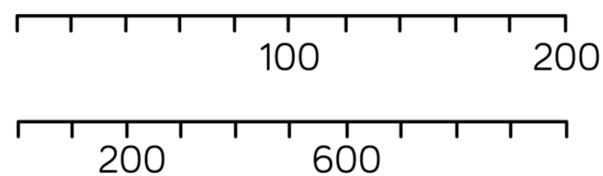


Fully label these number lines.



Repeat for lines where 10 is replaced by 20, 100 and 1000

What is each interval worth on these scales?



Position integers on a number line

Notes and guidance

Once familiar with how intervals on number lines work, students can start to use these to place integers and to read values. Making links to reading from common scales such as weighing scales, measuring jugs and thermometers will be helpful.

Key vocabulary

Equal division	Interval	Scale
Gap	Spaces	Approximate

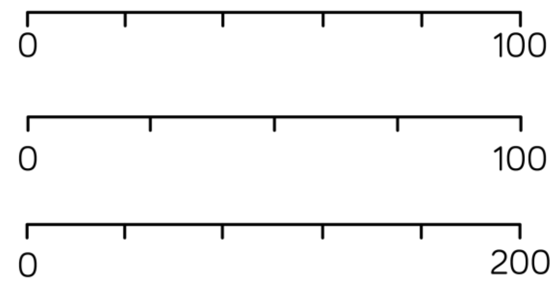
Key questions

Why can we mark some numbers exactly on a number line but others only approximately?

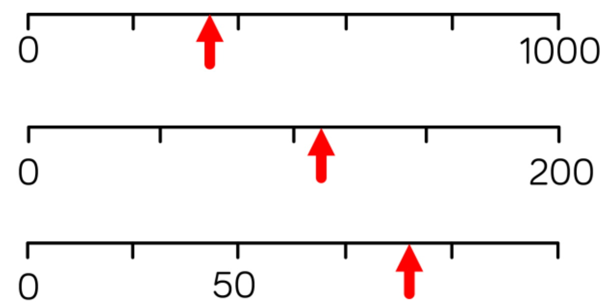
Describe the steps you need to take to read a number off a line of a scale.

Exemplar Questions

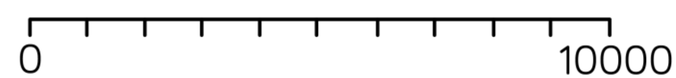
Where would 80 be on each of these number lines?



Estimate where the arrow is pointing to in each of these number lines.



Mark the approximate positions of 3500, 6100 and 1785 on this number line. Explain which ones are easier to do and why.



Round integers to powers of ten

Notes and guidance

Using appropriate number lines to support, and building on previous knowledge, from Key Stage 2, students should now be able to round to the nearest 10, 100, 1000 etc. Emphasis should be placed on "nearest" meaning proximity, encouraging students to think about the size of the number rather than rote-learned rules. "Rounding up" for halfway should be explained as a convention.

Key vocabulary

Round	Approximate	Nearest
Convention	Halfway	

Key questions

Why do we round numbers?

When talking about the population of the UK, would you round to the nearest hundred, thousand or million? What about the population of Leeds?

Exemplar Questions

Use your calculator to find the answers to these calculations. Round your answers to the nearest hundred.

- 47 × 68
- 65 681 ÷ 77
- 97 813 – 88 694

Which of these numbers would be sensible to round to the nearest 10? What would be the most sensible choice for rounding the other numbers?

9761	145	48 312	603 156
287	48	19 201	671
5.9	797	23.5	1542

Jim says "There are 2000 students at my school". Do you think there are exactly 2000 students? How many to you think there might be?

To the nearest thousand, 84 000 people attend a pop concert. What's the greatest possible number of people that were at the concert? What is the least possible number?

White Rose Maths

Compare integers using $=, \neq, <, >$

Notes and guidance

Students need to compare two integers confidently before they can go to order a larger list of numbers. Students will be familiar with the equals sign but may need introducing to $\neq$. Encourage the use of “greater than” and “less than” rather than “bigger than”/”smaller than” etc. and pay attention to reading statements like “$829 < 850$” both from left to right and from right to left.

Key vocabulary

Compare	Digit	Equal
Not equal	Greater than	Less than

Key questions

What do you look at first when comparing the size of two integers? What do you look at next?

Is it true that if $a > b$ and $b > c$ then $a > c$?

Exemplar Questions

Complete the following using $=$ or $\neq$.

Two and a half million	◯	2 500 000
300 000 000	◯	Three billion
Six thousand and eighty	◯	68 000

$86 < 101$ and $101 > 86$ are both **true**.
Decide which statements below are true and which are false.
Rewrite the false statements, using the same numbers, making them true. Can you do this in more than one way?

$902 < 93$	$8106 > 8099$	$3751 < 3699$
$203\,000 < 199\,987$	$32\,150 = 31\,205$	$809 > 820$
$601 \times 1000 > 10\,000 \times 59$		$903\,000 \div 100 > 88\,000$

$$46\,000 < 400\,010$$

Joe says the statement says “46 000 is less than 400 010”.
Jay says the statement says “400 010 is greater than 46 000”.
Who do you agree with?

White Rose Maths

Order a list of integers

Notes and guidance

Students can now use their skills of identifying the values of digits in a number, supported if necessary by number lines, to put a series of integers in order. Introducing the term “leading digit” may be helpful for those who find this challenging; emphasis on the difference between a number and digit is important as students can get confused.

Key vocabulary

Order	Ascending	Descending
Place Value	Leading digit	

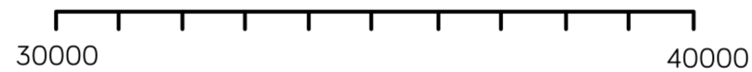
Key questions

Why is the leading digit of a number important when ordering?

For a set of integers, is the longest number always the largest number?

Exemplar Questions

Decide where these numbers belong on the number line and then list them in **ascending** order.



34 601 | 30 641 | 36 104 | 34 016

31 604 | 36 401 | 31 046

Put your answers to the following in **descending** order.

1. 180 000 – 42 781
2. 360 × 25
3. One billion divided by forty-thousand
4. The **sixth** term of the sequence 200 , 800 , 3200 ….
5. The value of x^2 when $x = 305$
6. Two hundred thousand more than 610 408

How many different four digit numbers can you make using these cards? Put your numbers in order, starting with the smallest.

8 | 4 | 9 | 4

Find the range of a list of integers

Notes and guidance

Now students are able to confidently order integers, finding the range is straightforward. Care needs to be taken so that students remember to find the difference between the greatest and least values rather than state "they range from ___ to ___". It is worth revisiting the concept regularly in lesson starters or within other topics

Key vocabulary

Range Greatest Least

Difference

Key questions

How do you calculate the range of a set of numbers?

When given a list of numbers to find the range of, what might it be helpful to do first.

Exemplar Questions

What is the range of the ages:

- In your mathematics class?
- In your household?
- In your school?

The table shows the heights of the highest mountains in some of the countries in Europe.

Work out the range of these heights.

Country	Height (m)
France	4808
Belgium	694
England	978
Sweden	2104
Russia	5642
Croatia	1831

Substitue $x = 30$ into each of these expressions and find the range of your answers.

x^2 $25x - 80$ $10\,000 - 14x$ $\frac{180\,000}{x}$ $2500 + 17x$

Gemma says the range of the numbers below is "from 63 to 111"

68 63 79 111 104

Explain why Gemma is **wrong**.

Find the median of a list of integers

Notes and guidance

Students need to be taught how to find the median from a list with both an even amount of numbers and an odd amount of numbers. As the mean has not yet been formally introduced, it is best to focus on the latter. As a homework, students could explore the use of the median average in real-life e.g. median wage etc.

Key vocabulary

Median	Middle	Order
Average		

Key questions

What do you need to do first when finding the median of a list of numbers?

What is different about the median and the range?

Exemplar Questions

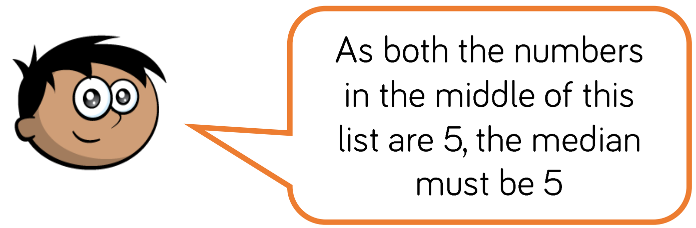
Gertrude has seven chickens. Over a week they lay the following amount of eggs: 5 4 5 4 3 2 3

Work out the median of the number of eggs laid.

---

9 4 5 5 7 9

Eric says



Explain why Eric is **wrong**, and find the actual median of the list.

Which of these sets of data is it possible to find the median of?

- The shoe sizes of students in your class
- The number of pets owned by students in your class
- The eye colour of students in your class
- The number of siblings of students in your class

Explain what is the same and what is different about finding the median of these three lists of numbers.

3000	3500	3500	3500	3600
3000	3500	3500	3600	
3000	3500	3600	3600	

Understand place value - decimals

Notes and guidance

Students following the Foundation strand should focus on proper understanding of tenths and hundredths during this step, and throughout this unit. Only move on to thousandths and beyond if appropriate for the students in your class. Conversion between fractional and decimal forms of tenths and hundredths are covered in depth in the next block.

Key vocabulary

Tenth — Hundredth — Decimal

Decimal point

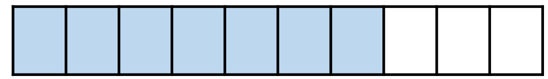
Key questions

Why do we say "0.37" as "nought point three seven" rather than "nought point thirty-seven"?

Why is 0.4 bigger than 0.29, even though twenty-nine is bigger than 4?

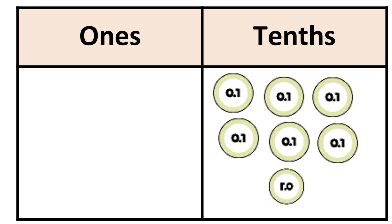
Exemplar Questions

Explain why these representations are all the same.



0.7

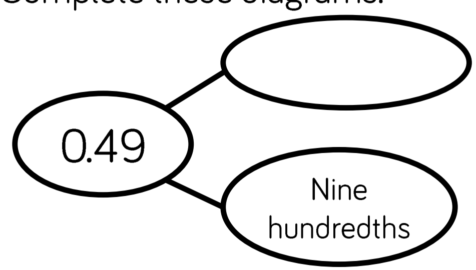
$\frac{7}{10}$

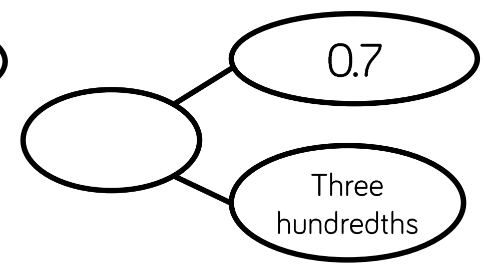


Write the value of the 4 in each of these numbers.

34 601 — 3.46 — 43.6 — 3.64 — 14.63

Complete these diagrams:





Write these numbers in figures.

- Seven thousandths
- Seventeen thousandths
- Seventy thousandths
- Seven hundred thousandths

Position decimals on a number line

Notes and guidance

Students may need help with finding the intervals in decimal number lines, and this key skill will be revisited in the upcoming FDP work. The focus in this step is appreciating the place value of decimal numbers and how this affects their relative positioning. Challenge can be added if appropriate by looking at intervals of 0.2, 0.05 etc,

Key vocabulary

Tenth	Hundredth	Decimal
Decimal point	Interval	

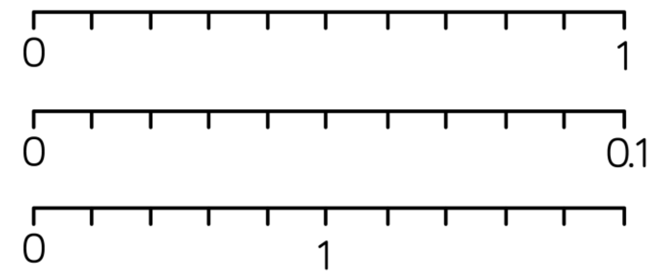
Key questions

How do we work out the size of an interval on a number line?

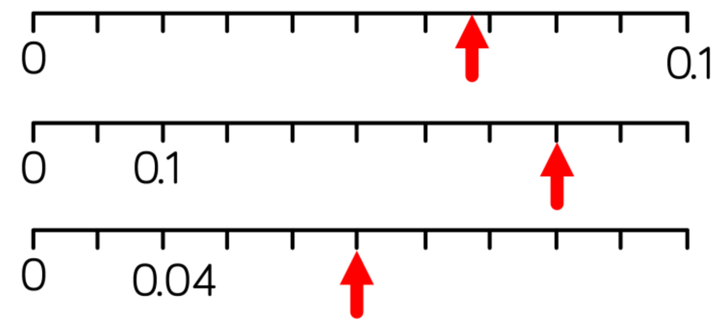
What is different when thinking about the position of 0.3 and 0.03?

Exemplar Questions

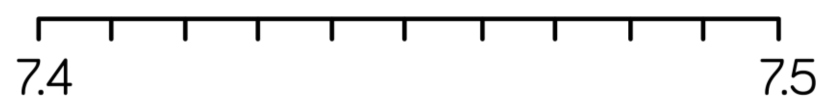
Fully label these number lines.



Estimate where the arrow is pointing to in each of these number lines.



Mark the approximate positions of 7.45, 7.48 and 7.425 on this number line.



Compare and order any numbers

Notes and guidance

Students should now be able to compare decimal numbers as well as integers. It is important that students read numbers correctly e.g. "nought point three five" as opposed to "nought point thirty-five" as this leads to misconceptions such as $0.35 > 0.4$. Students following the Foundation strand should focus on numbers with up to two decimal places at this stage,

Key vocabulary

Decimal	Integer	Leading digit
Ascending	Descending	

Key questions

When you see a list of decimal numbers, is the longest number always the largest number?

When ordering numbers, why are the leading digits important?

Exemplar Questions

Put these numbers in **ascending** order.

346.01 306.41 361.04 340.16
316.04 364.01 310.46

Write these numbers in decimal form and then put them in order, starting with the smallest.

Zero point three five
Seventy-two hundredths
Fifty hundredths
One tenth
Two hundredths
Nought point nought seven
Nought point nought three

Work out these calculations and put the answers in descending order.

1. $47 \div 100$
2. $400 \div 1000$
3. $5 - 0.93$
4. The **sixth** term of the sequence 0.59 , 0.62 , 0.65…
5. The solution to the equation $x + 0.16 = 0.9$
6. $0.168 + 0.232$

Round to 1 significant figure

Notes and guidance

In this step, students learn the key skill of rounding to 1 significant figure both with integers and decimals, as this key skill in estimation is much more useful than rounding to decimal places which is covered later in the scheme. This should be revisited regularly whenever appropriate. You may wish to explore two or three significant figures with some students, but this is not essential at this stage.

Key vocabulary

Round	Approximate
Leading digit	Significant figure

Key questions

If when two numbers are rounded to one significant figure you get the same answer, does it mean the two numbers were the same?

Explain how you estimate the answer to seventeen million multiplied by two point nine six.

Exemplar Questions

Round these numbers to one significant figure:

37
Thirty-seven million
0.37
0.000037
4.37
4.0037
Four million and thirty seven

Work out the value of these expressions if $a = 72, b = 0.6$ and $c = 125$. Give your answers correct to one significant figure.

- $a + b + c$
- $\frac{a}{b}$
- ab
- $\frac{b}{c}$
- c^2

To one significant figure, the population of Scotland is given as five million. What is the greatest possible population of Scotland? What is the least possible population?

Investigate positive powers of 10 H

Notes and guidance

As a precursor to writing numbers in standard index form., this small step looks at writing numbers like 10 000 in the form 10^n. A calculator could be used to introduce this, and it will also provide good practice for using terms like billion. Students following the Foundation strand could access this if time allows but it will be covered in the future if more time is needed to gain fluency with earlier steps.

Key vocabulary

Power	Index	Million
Billion		

Key questions

What does "ten to the power five" mean? How does this link to place value columns?

Why do we write very large numbers as powers of 10 rather than out in full?

Exemplar Questions

Enter the number 10 on your calculator.
Multiply by 10 and note the answer (you know it anyway!)
Multiply by 10 again and note the answer.
Keep going!
What happens after several multiplications?
Why do you think this is the case?

Put these numbers in order of size, starting with the smallest:

10^5 10^7 Ten billion

One hundred million 1000^2

10^{12} 1000^3

A googol is the number formed by writing 1 followed by one hundred zeros.

- Write a googol as a power of ten
- How many times bigger than a billion is a googol?

Work out $10^6 \times 10^6$.
How many similar calculations can you find with the same answer?

+ve integers in the form $A \times 10^n$ H

Notes and guidance

As standard index form is studied in depth in Year 8, this step focuses on writing and interpreting numbers like 8×10^9 rather than numbers that need decimals such as 7.4×10^6. The intention is to get a good understanding of the basics rather than rushing to learning procedural rules.

Key vocabulary

Power	Index	Standard Form
Scientific notation		

Key questions

Why do we use standard index form?

How can you convert numbers like millions and billions easily to standard index form?

Exemplar Questions

Put all these numbers into standard form and then write them in ascending order.

700 000 × 100

Thirty billion

Twenty million

10^7

200^3

1 000 000 × 50 000

Half a billion

These numbers are **not** in standard form. Rewrite them so they are.

30×10^7

$10 \times 3 \times 10^5$

300×10^8

3000×10^4

$300\,000 \times 10^0$

Whitney says:

Even though 9 is greater than 7, 7×10^6 is greater than 9×10^4.

Whitney is **right.** Explain why.

Investigate –ve powers of 10

H

Notes and guidance

Similarly to the earlier step on positive powers of 10. students here explore powers of 10 for numbers between zero and one. Negative numbers have been introduced during KS2 and so students should be aware that e.g. -2 is greater than -4, although this may need reinforcement.

Key vocabulary

Power	Index	Standard Form
Scientific notation	Negative	

Key questions

What's the difference between positive and negative powers of 10?

Why is 10 to the power zero not equal to zero?

Exemplar Questions

Enter the number 10 on your calculator.
Divide by 10 and note the answer (you know it anyway!)
Divide by 10 again and note the answer.
Keep going!
What happens after several divisions?
Why do you think this is the case?

Put these numbers in order of size, starting with the largest:

10^3 | 10^{-4} | One tenth

One thousandth | 10^0 | 10

100 ÷ 10 000 | 10^{-7}

Christina says:

5 is greater than 3, so 10^{-5} must be greater than 10^{-3}

Explain why Christina is **wrong.**

White Rose Maths

Decimals in the form $A \times 10^n$ H

Notes and guidance

Again the focus is on writing and interpreting numbers like 2×10^{-3} rather than numbers that need decimals such as 2.4×10^{-3}. Although this might come up in discuss ion and be addressed briefly, bear in mind that this knowledge and understanding will be considered in depth during Year 8.

Key vocabulary

Power	Index	Standard Form
Scientific notation	Negative	

Key questions

What's different about writing large numbers and small numbers in standard index form?

Where might you see and use standard index form?

Exemplar Questions

Put all these numbers into standard form and then write them in ascending order.

$7 \div 100\,000$ Four thousandths

Three millionths 10^{-7} 0.0004

$50 \div 1\,000\,000$ Two billionths

Put these numbers in ascending order.

5×10^6 6×10^5 6×10^6

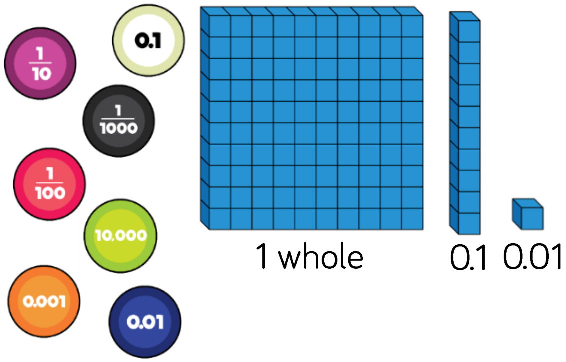
5×10^{-6} 6×10^{-5}

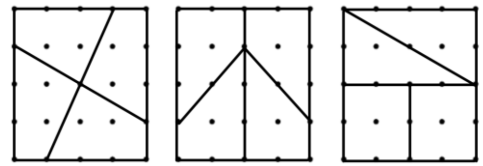
5×10^5 6×10^{-6} 5×10^{-5}

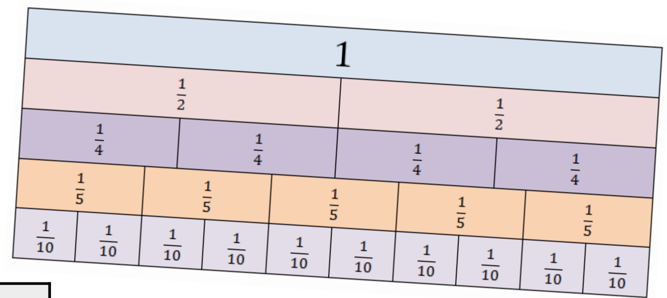
Explain why 3×10^{-4} is less than 4×10^{-3}.

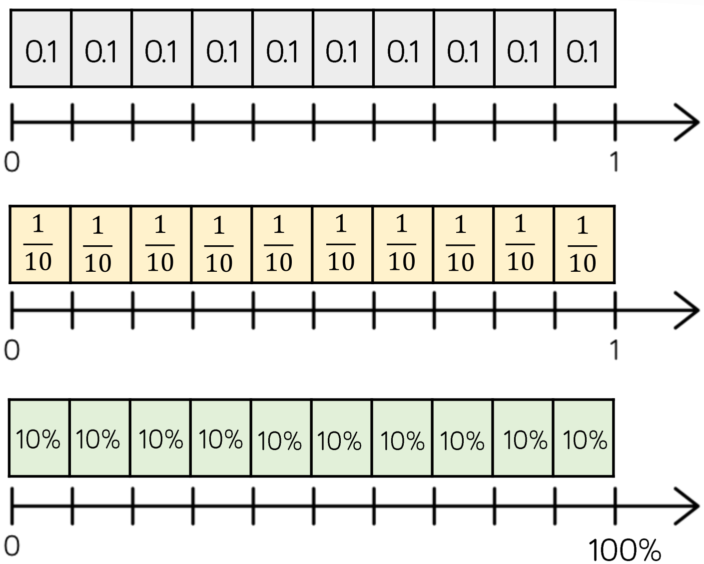
$0.0006 = 6 \times 10^{-4}$ and $0.0007 = 7 \times 10^{-4}$.
How do you think we might write 0.00065 in standard index form?

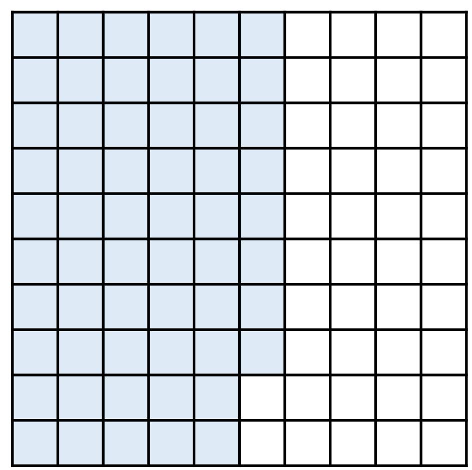
Key Representations











Here are a few ideas for how you might represent fractions, decimals and percentages.

- Number lines are a useful way of assessing whether children understand the size of a F, D or P. Extending the number line above 1 is an option for some students.
- Paper strips can be folded to represent different F, D and P.
- Tiles such as equilateral triangles can be placed together to build patterns, allowing students to consider what fraction is represented by a specific coloured tile. This can be extended to include questions such as “what happens if I add another blue triangle”.
- Different types of paper (isometric, square) to draw non-standard representations of fractions, decimals and percentages
- Bar models are particularly useful when comparing F, D, P.

White Rose Maths

FDP Equivalence

Small Steps

- Represent tenths and hundredths as diagrams
- Represent tenths and hundredths on number lines
- Interchange between fractional and decimal number lines
- Convert between fractions and decimals – tenths and hundredths
- Convert between fractions and decimals – fifths and quarters
- **Convert between fractions and decimals – eighths and thousandths** H
- Understand the meaning of percentage using a hundred square
- Convert fluently between simple fractions, decimals and percentages
- Use and interpret pie charts

H denotes higher strand and not necessarily content for Higher Tier GCSE

FDP Equivalence

Small Steps

- Represent any fraction as a diagram
- Represent fractions on number lines
- Identify and use simple equivalent fractions
- Understand fractions as division
- Convert fluently being fractions, decimals and percentages
- **Explore fractions above one, decimals and percentages** H

H denotes higher strand and not necessarily content for Higher Tier GCSE

Tenths & hundredths as diagrams

Notes and guidance

Students will recognise tenths and hundredths when represented diagrammatically. They should be exposed to various representations and be able to make a number using different diagrams. It is important that students can also work the opposite way, and be able to write a given representation in numerals and words.

Key vocabulary

Place value	Digit	Placeholder
Tenths	Hundredths	

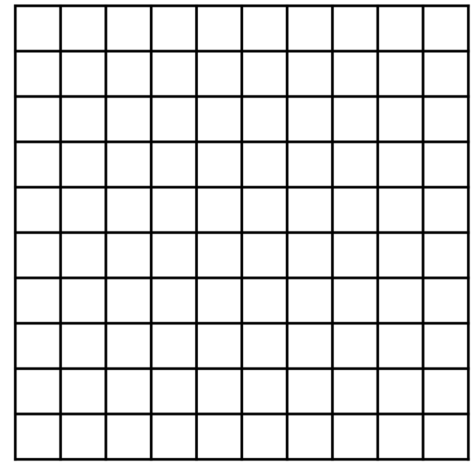
Key questions

Is it possible to represent 120 hundredths on one hundred square? What could you do?
What's the same and what's different between the counters showing decimals and the counters showing fractions?
How can you work out the value of each piece of Base 10?

Exemplar Questions

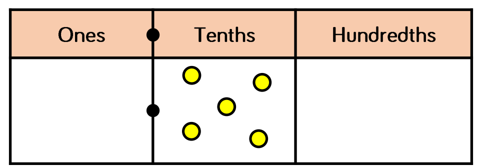
If the hundred square is worth 1 whole, represent:

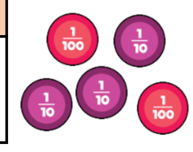
- 1 tenth
- 1 hundredth
- 4 tenths
- 40 hundredths
- 10 tenths
- 120 hundredths

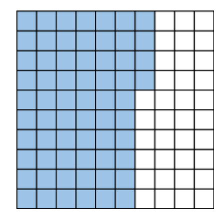


What do you notice?

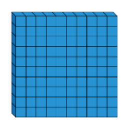
Write the numbers represented below in figures and words.

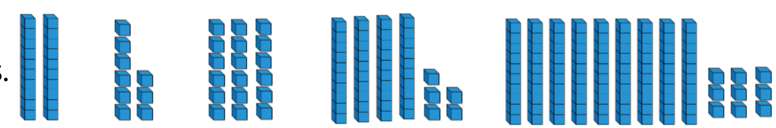






Is there more than one way?

If  is equal to one, write the given representations in both words and figures.



Tenths & hundredths on number lines

Notes and guidance

Students will recognise tenths and hundredths when represented on different number lines. They should be exposed to number lines split into different intervals and be able to estimate the value of a number highlighted. It is important that students can also work the opposite way, and be able to write a highlighted number in both figures and words.

Key vocabulary

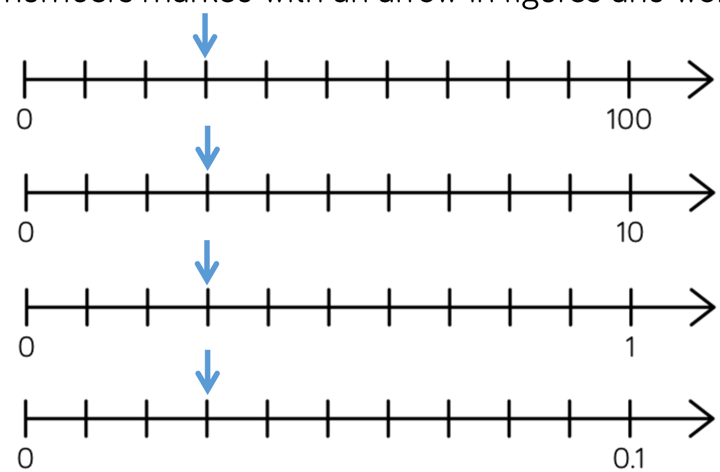
Place value	Digit	Placeholder
Tenths	Hundredths	Interval

Key questions

Do we need to split a number line into a hundred parts in order to show hundredths?
If a number line is split into tenths, how can use it to show hundredths?

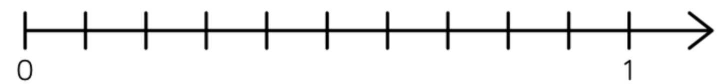
Exemplar Questions

Write the numbers marked with an arrow in figures and words.

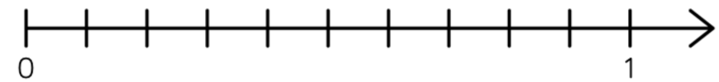


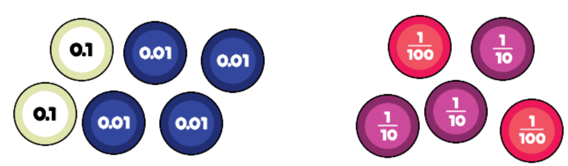
Draw an arrow to show $\frac{9}{10}$ on the number line.

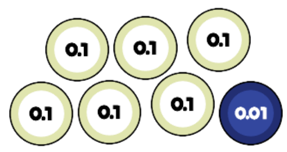
How can you show $\frac{11}{10}$?



Draw arrows to show where the numbers would lie on the number line.







Fractional & decimal number lines

Notes and guidance

Students will be able to use both fractional and decimal number lines and be able to move freely between the two. They should understand the equivalence of 0.1 and one tenth etc., and use this to show both decimals and fractions on the same number line.

Key vocabulary

Tenth	Hundred	Fraction
Decimal	Number line	Interval

Key questions

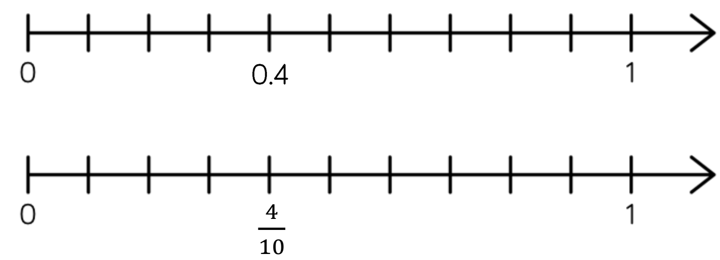
What interval is the number line going up in? How do you know?

Are we counting in tenths or hundredths? How do you know?

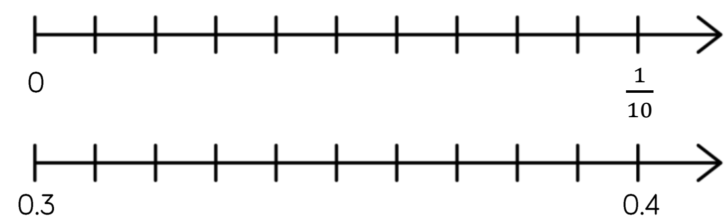
How can we show fractions and decimals on the same number line?

Exemplar Questions

What's the same? What's different?



Complete the number lines.



On a number line, draw arrows to show the approximate position of:

- 0.65
- $\frac{9}{10}$
- Three tenths
- $\frac{9}{100}$
- Three hundredths
- $\frac{99}{100}$

Explain why we can represent 8 tenths on a number line in the same position as 80 hundredths.

Convert tenths and hundredths

Notes and guidance

In this small step, students will understand and explore fractional and decimal representation of tenths and hundredths and be able to convert between them. They should make connections between the place value of the decimal notation and the fraction. Diagrams or concrete resources can be used to aid understanding.

Key vocabulary

Place value	Tenths	Hundredths
Placeholder	Fraction	Decimal

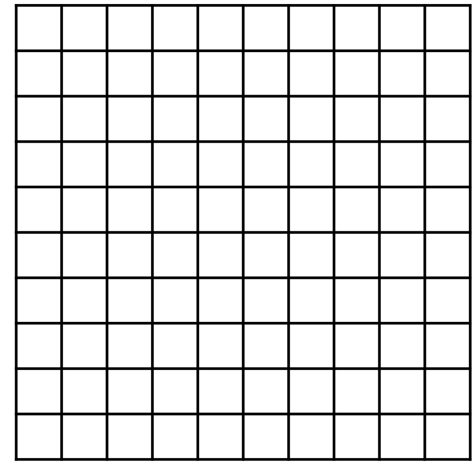
Key questions

How can we easily convert from tenths to hundredths?
Why might it not be so easy to convert from hundredths to tenths?
Is there more than one solution? Compare yours with a friend.

Exemplar Questions

Use the 100 square to show that:

- 0.3 is equivalent to $\frac{3}{10}$ and $\frac{30}{100}$
- $\frac{27}{100}$ is equivalent to 0.27
- $\frac{2}{10}$ is greater than 0.19



Continue the linear sequences.

$\frac{1}{10}, \frac{25}{100}, \frac{4}{10}, \ldots$

$0.57, \frac{6}{10}, 0.63, \ldots$

Work out

$\frac{3}{10} + 0.6$

$\frac{21}{100} - 0.1$

$1 - \frac{9}{10}$

Complete the boxes to make the statements correct.

$\frac{\square}{10} > 0.5$

$\frac{\square}{100} < 0.\square7$

Convert fifths and quarters

Notes and guidance

Continuing the theme of previous small steps, students now focus on fifths and quarters and their relationships to tenths and hundredths. Use of pictorial and concrete resources will be necessary to cement understanding of equivalence in value.

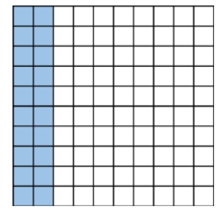
Key vocabulary

Fifth	Quarter	Tenth
Hundredth	Equivalent	

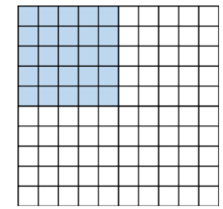
Key questions

Can we write fifths as tenths and hundredths?
Can we write quarters as tenths and hundredths?
Why? Why not?
How is the hundred square representation and the bar model representation the same? How is it different?

Exemplar Questions

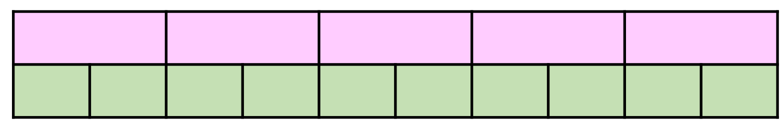


$\frac{1}{5}$ of the grid is shaded.
How many tenths is this?
What is this as a decimal?



$\frac{1}{4}$ of the grid is shaded.
How many hundredths is this?
What is this as a decimal?

Use a hundred square to represent $\frac{3}{4}$, $\frac{2}{5}$, $\frac{3}{5}$ and $\frac{4}{5}$ then write each one as a decimal.



Use the bar model to complete the following.

$\frac{1}{5} = \frac{\square}{10} = 0.\square$ $\qquad$ $\frac{\square}{5} = \frac{8}{10} = 0.\square$ $\qquad$ $\frac{\square}{5} = \frac{\square}{10} = 0.6$

Find three ways to show that $\frac{4}{5}$ is greater than $\frac{3}{4}$

Circle the expressions that are equivalent to three-quarters of the number x?

$\frac{3x}{4}$ $\qquad$ $\frac{3}{4}x$ $\qquad$ $0.34x$ $\qquad$ $0.75x$ $\qquad$ $\frac{x}{4} \times 3$

Convert eighths & thousandths H

Notes and guidance

Students who are comfortable with all the conversions met so far should advance to this step, but if more time is needed to consolidate earlier concepts then it may be omitted. The move from hundredths to thousandths should echo the move from tenths to hundredths. Then, starting from recognising one eighth as half of a quarter, students should be able work with multiples of one eighth.

Key vocabulary

Thousandths	Eighths	Tenth
Hundredth	Equivalent	Quarter

Key questions

Why are we unable to write $\frac{1}{8}$ in tenths?

Can we write $\frac{1}{8}$ in hundredths? Explain your answer.

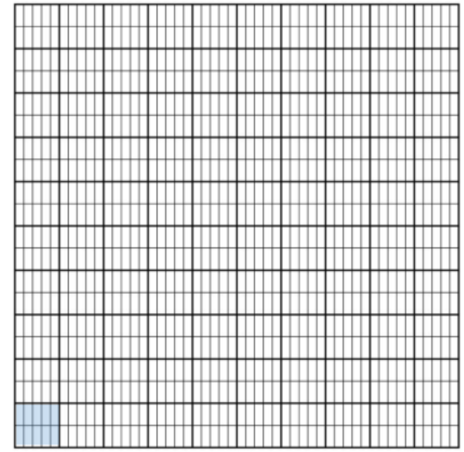
Can we write $\frac{1}{8}$ in thousandths? Explain your answer.

Exemplar Questions

What has this hundred square been divided up into?

How many thousandths are in one hundredth?

How many thousandths are in one tenth?



Complete the table.

Fraction	Tenths	Hundredths	Thousandths
$\frac{1}{2}$			$\frac{500}{1000}$
$\frac{1}{4}$		$\frac{25}{100}$	
$\frac{1}{8}$			
$\frac{5}{8}$			

Put these numbers in order of size, starting with the smallest:

$\frac{5}{8}$ $\frac{607}{1000}$ $\frac{4}{5}$ $\frac{3}{4}$ $\frac{63}{100}$

Percentages on a hundred square

Notes and guidance

In this small step, familiarisation with representing percentages on a hundred square enables students to quickly identify the percentage not shaded using the fact that one whole is 100%. They need to recognise that to represent percentages above 100% more than one hundred square is required.

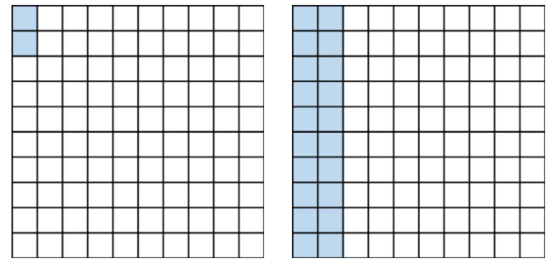
Key vocabulary

Hundredth	Percentage	Shaded
Percent	Out of one hundred	

Key questions

Is it possible to give "110%" effort?
Is it possible to find 110% of an amount?
What does 100% mean? What does 110% mean?
What's the same and what's different about 30% and 3%?

Exemplar Questions



Express the amount shaded in each of these hundred squares as a percentage.

What percentage is not shaded?

I subtract the shaded percentages, what percentage is shaded now? Is it possible to have a negative percentage? Explain your answer.

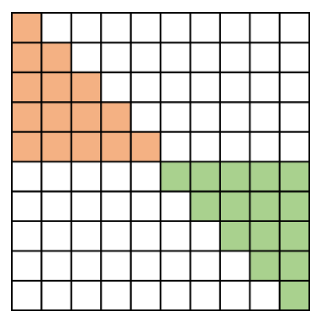
Represent the following percentages on a hundred square:

10%   30%   5%   1%   36%

If you shade $\frac{3}{10}$ of a hundred square, what percentage is shaded?

If x% of a hundred square is shaded, what percentage is not shaded?

How could you use a percentage square to show fiftieths?



Sam thinks 25% of the grid is shaded in total. Is he right?

What percentage of the shape is not shaded?

Sam wants to represent 330% using hundred squares. How many hundred squares does he need?

Convert simple FDP

Notes and guidance

In this small step, students draw together their knowledge of the previous steps to gain fluency in converting simple fractions, decimals and percentages. The focus remains of familiarity with commonly seen FDP rather than rote memorisation of a technique for any value – in particular students should be confident in converting multiples of 10% and 25% given in any form.

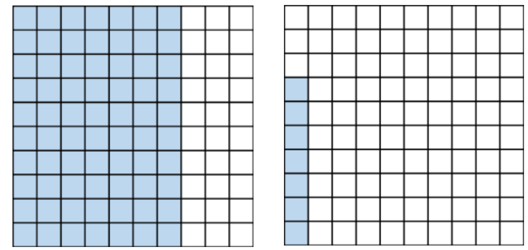
Key vocabulary

Convert	Equivalent	Half
Three-quarters	Tenth	

Key questions

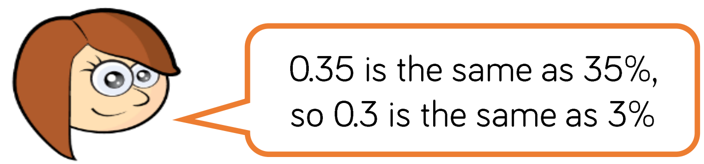
How is a fraction related to a decimal?
How is a percentage related to a fraction?
How is 100% represented as a fraction? Is there more than one way?
How is 100% represented as a decimal?

Exemplar Questions



Express the amount shaded as
- a fraction
- a decimal
- a percentage
- Repeat for the amount not shaded.

Christina says:



Explain why Christina is **wrong.**

Which of the following are equal in value to two-fifths?

$\frac{4}{10}$ 25% 2.5 $\frac{40}{100}$ 0.25 0.40 0.4

Explain why both of these statements are true:
- 0.08 is smaller than 10%
- $\frac{1}{4} < 0.4 < 45\%$

Fill the blanks below with a suitable percentage:
- $0.7 <$ _____ $< \frac{3}{4}$
- $\frac{1}{10} <$ _____ $< \frac{1}{5} <$ _____ $< 0.23 < \frac{1}{4} <$ _____

Use and interpret simple pie charts

Notes and guidance

The focus here will be on pie charts where the fractions are clearly visible rather than on measuring and constructing, which will be covered later. It is also an opportunity to discuss estimation and assumptions e.g. unless labelled how can we tell whether exactly a half (etc.) is shaded? This is a good opportunity to revisit the conversions covered in the previous step.

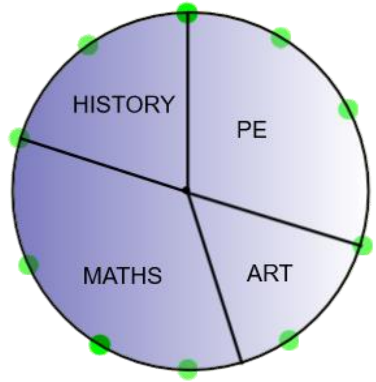
Key vocabulary

Pie chart	Fraction	Decimal
Percentage	Equal parts	Sector

Key questions

Why is it impossible to compare quantities by looking at two pie charts? What can we compare?
How do fractions and percentages help us to do this?
How accurate can we be estimating proportions from a pie chart?

Exemplar Questions

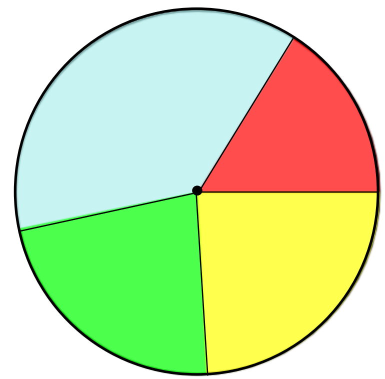


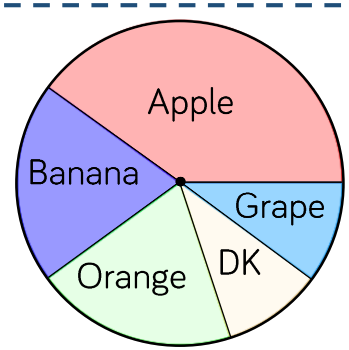
This pie chart has markings at each 10%. It shows some students' favourite subjects.

- What percentage chose PE?
- Estimate the percentage that chose maths?
- What fraction chose History?

Estimate the percentage of each colour in this pie chart.

Suppose $\frac{2}{5}$ is blue, 0.2 is green and $\frac{1}{4}$ is yellow, what percentage is red?





This pie chart shows the results of a survey of people's favourite fruits. 10% said "don't know" (DK).
What fraction chose a fruit?
Estimate the total percentage that chose either apple or grape.
What other questions could you ask?

Represent any fraction as a diagram

Notes and guidance

Now students are confident with simple fractions, they can extend their experience to include less commonly seen fractions, with the emphasis still on the need for equal parts. Non-standard examples of representations of fractions helps to reinforce the importance of equal parts rather than same shaped parts.

Key vocabulary

Fraction	Equal parts

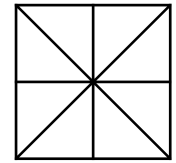
Key questions

Can you use the same diagram to represent both one-third and two-thirds?

Does a diagram have to be cut into equal parts in order to identify the fraction shaded or not shaded?

Exemplar Questions

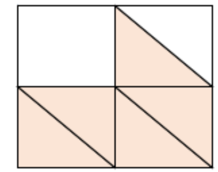
How many ways can you show $\frac{5}{8}$ on this square?



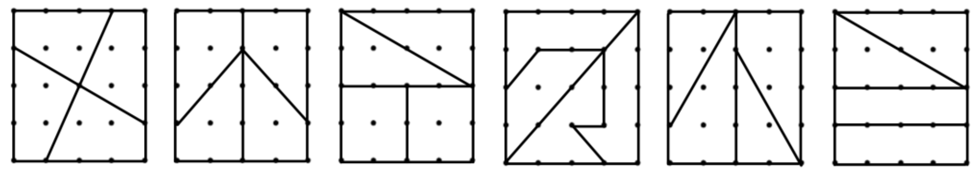
In the rectangle, Jack thinks $\frac{5}{8}$ is shaded.

Teddy thinks $\frac{5}{7}$ is shaded.

Who's right? Explain why.



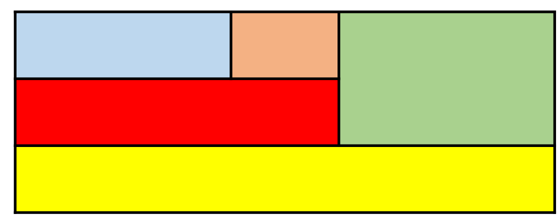
Which of these shapes are split into quarters and which are not?



How many more ways can you find to split a 4 by 4 dotty square into quarters?

Why can't we say what fraction of the shape is shaded red without further measuring?

What reasonable assumptions could you make?



Represent fractions on number lines

Notes and guidance

Students should be able to identify, or where appropriate estimate, fractions represented on different types of number lines (intervals marked and unmarked). This can be used to compare fraction size. We can also use lines to compare relative sizes of fractions. It is important that students think of fractions as numbers on a number line, not just a fraction of an object.

Key vocabulary

Fraction	Denominator	Numerator
Part	Whole	

Key questions

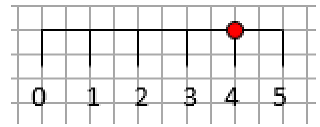
How can we represent the 'whole' on a number line?

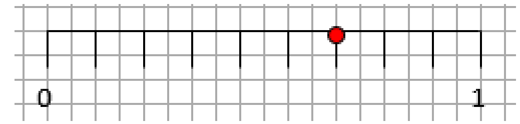
Which fractions are easier to recognise on a number line?

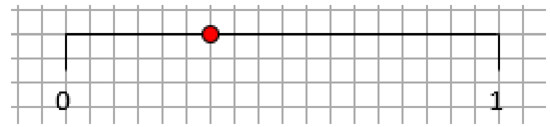
How can we use these to recognise other fractions on a number line?

Exemplar Questions

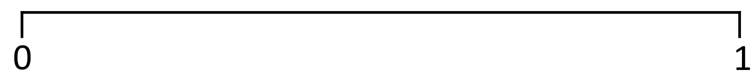
How far along the line is the point marked?
Write your answer as a fraction.







Represent the following on a number line, where the 1 represents an hour.



- Khrish spends half an hour on his homework
- Suki spends a quarter of an hour learning her French vocabulary
- Oli spends a 20 minutes researching for his History homework
- Bella spends $\frac{1}{5}$ of an hour proof-reading her English homework

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Would the representation of one quarter of £100 on a number line look different or the same as the representation of one quarter of £200 on a number line? Explain your answer.

Use number lines to show that $\frac{3}{5} < \frac{7}{10}$

How else could you show this?

Identify and use equivalent fractions

Notes and guidance

Students must understand that a fraction represents a number that can be written in an infinite number of ways. Thinking of individual fractions as part of an "equivalence set" can be useful. This conceptual understanding should be supported with concrete and pictorial representations. These should sit alongside the abstract notation so that the student can make the necessary links.

Key vocabulary

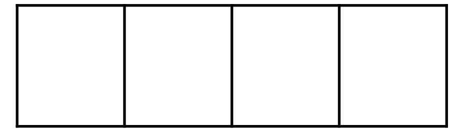
Fraction	Equivalence	Equal
Denominator	Numerator	Whole

Key questions

What makes a fraction equivalent to another fraction?
How many equivalent fractions are there for any one fraction?
Why are equivalent fractions useful in making comparisons? How are they used in day to day life?

Exemplar Questions

Shade $\frac{2}{2}$ of the shape.



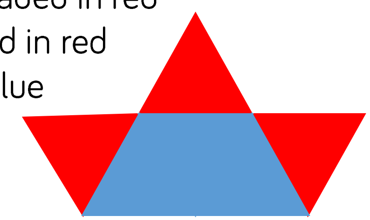
Does this picture also represent $\frac{3}{3}$?
Explain your answer.
Write down 3 other names for the fraction shaded.

Using pictures and words, show which fraction in each pair is larger.

$\frac{2}{5}, \frac{3}{10}$ | $\frac{8}{9}, \frac{4}{3}$ | $\frac{2}{3}, \frac{4}{5}$

The shape is made up of a trapezium and 3 identical triangles.

- Eva says that half of the model is shaded in red
- Mo says that three quarters is shaded in red
- Amir says three sixths is shaded in blue



Who's right? Explain your answer.

- Dora says that if she adds another two red triangles to the shape then Mo will be right

Dora has made a mistake.
How could you make Mo's statement correct?

Copy and complete the following: $\frac{\square}{6} < \frac{5}{8} < \frac{\square}{10}$
What numbers could work in the inequality? What numbers couldn't?

White Rose Maths

Understand fractions as division

Notes and guidance

Students need to understand that a fraction also represents a division, rather than just a comparison with one whole. Therefore, contextualised problem solving questions should be interspersed throughout. A good starting point is to consider tenths, a concept students are already familiar with, and the different representations.

(e.g. $\frac{1}{10}$, 10%, 0.1). This illustrates $\frac{1}{10}$ being the same as 0.1 and $1 \div 10$

Key vocabulary

Division	Quotients	Denominator
Operator		

Key questions

When are fractions used as quotients?

What's the difference between a fraction as a quotient and a fraction as an operator?

Exemplar Questions

Use a calculator to work out

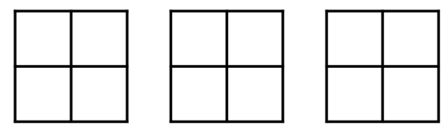
- $1 \div 10$
- $3 \div 100$
- $7 \div 10$

Which fraction corresponds to each of your three calculations?

$\frac{1}{10}$ $\frac{10}{7}$ $\frac{3}{10}$ $\frac{7}{10}$ $\frac{3}{100}$

Three friends share four square pizzas.

Use the diagrams below to show that $3 \div 4$ is the same as $\frac{3}{4}$



Which of these are the same as $7 \div 10$?

$\frac{100}{70}$ $\frac{7}{10}$ $\frac{70}{100}$ 0.7 $\frac{10}{7}$ 0.70

Write a division for each of those that are not the same as $7 \div 10$

Sort these cards into two matching groups.

Sam has 12 dog treats and shares them out between his 5 dogs.

$5 \div 12$

$12 \div 5$

$\frac{5}{12}$

$2\frac{5}{2}$

$\frac{12}{5}$

$2\frac{2}{5}$

Julie has 5 pizzas and she shares them between 12 children.

White Rose Maths

Convert fluently between FDP

Notes and guidance

Students can now extend their knowledge of conversion to include any fraction, decimal and percentage. Calculators should be used where appropriate, but if students are happy to work mentally then of course they may, Mental strategies will be a focus later in the year. At this stage, there is no need to deal with "difficult" numbers and rounding, but thirds are worth exploring.

Key vocabulary

Place value	Equivalence	Fraction
Decimal	Percentage	

Key questions

Why do we use all three representations of fractions, decimals and percentages? Where would you see each type?
What happens if we try to change thirds into a decimal or percentage?
What's the relationship between fifths and tenths? Twentieths and hundredths? Eighths and thousandths?

Exemplar Questions

Use a calculator to find the matching pairs.

$\frac{13}{25}$ $\frac{27}{50}$ $\frac{11}{20}$ $\frac{23}{40}$

0.575 52% 54% 0.55

In each of these lists, **two** of the numbers are not equal to the others. Which two?

- $\frac{3}{10}$ 0.03 0.3 $\frac{1}{3}$ 30%
- $\frac{8}{100}$ 80% $\frac{4}{50}$ 0.08 $\frac{100}{8}$
- 35% $\frac{7}{20}$ 0.14 0.305 $\frac{14}{40}$
- 0.125 13% $\frac{4}{32}$ 0.125 12.5%

Use your calculator the convert $\frac{1}{3}$ to a decimal. What do you notice?
How can you convert two-thirds into a percentage?

Describe the following sequence.

$\frac{1}{8}$, 0.225 , 42.5% , $\frac{5}{8}$

Write down the next term, firstly as a fraction and then as a decimal and a percentage.

Explore fractions above one

H

Notes and guidance

In this Higher strand step, students look at fractions above one and their decimal and percentage equivalents. Being able to convert between improper fractions and mixed numbers as well as linking these representations to percentages and decimals is the aim. To solve problems like this students might use a number line or count up. Formal multiplication of fraction is introduced later.

Key vocabulary

Improper	Fraction	Mixed Number
Rational	Recurring	Convert

Key questions

How can one whole be represented using decimals, different fractions and percentages?

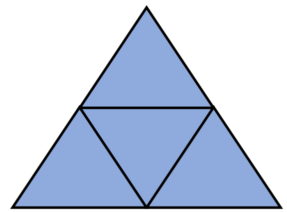
Why do some numbers above one have a direct percentage and decimal equivalence, where others need to be rounded before converting?

Exemplar Questions

Amir is baking pies. Each pie requires $\frac{2}{3}$ cup of flour.

Amir wants to bake 5 pies. He has 3 cups of flour. Is this enough?

A pattern is made out of 4 identical triangles.



Rachel has 15 of these triangles. How many of these whole patterns will she be able to make?

What fraction of the whole pattern is left over?

Write $\frac{19}{4}$ as a mixed number.

Find the next three terms of this sequence.

$1, \quad \frac{8}{5}, \quad \frac{11}{5}, \quad \_\_\_\_, \quad \_\_\_\_, \quad \_\_\_\_$

Write any improper fractions as mixed numbers.

Write these as percentages and then decimals.

What will be the next integer be in the sequence?

The rule for the n^{th} term of a sequence is given by $\frac{3n}{4}$.

Write the first four terms of the sequence as mixed numbers or integers where appropriate.

How often will the terms of the sequence be integers?

Create a linear sequence where every other term is an integer.

Create a linear sequence where every third term is an integer.