Summer Term

Year 7

#MathsEveryoneCan

2019-20

White Rose Maths

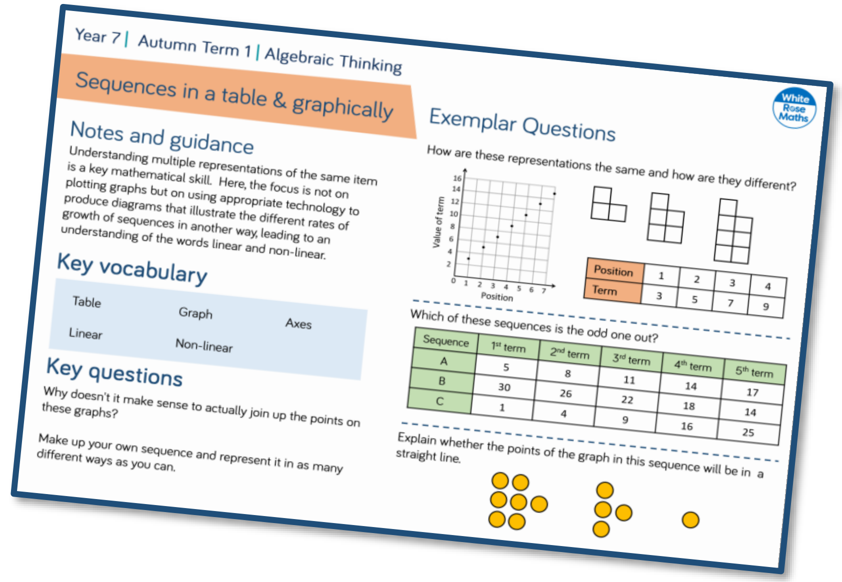
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

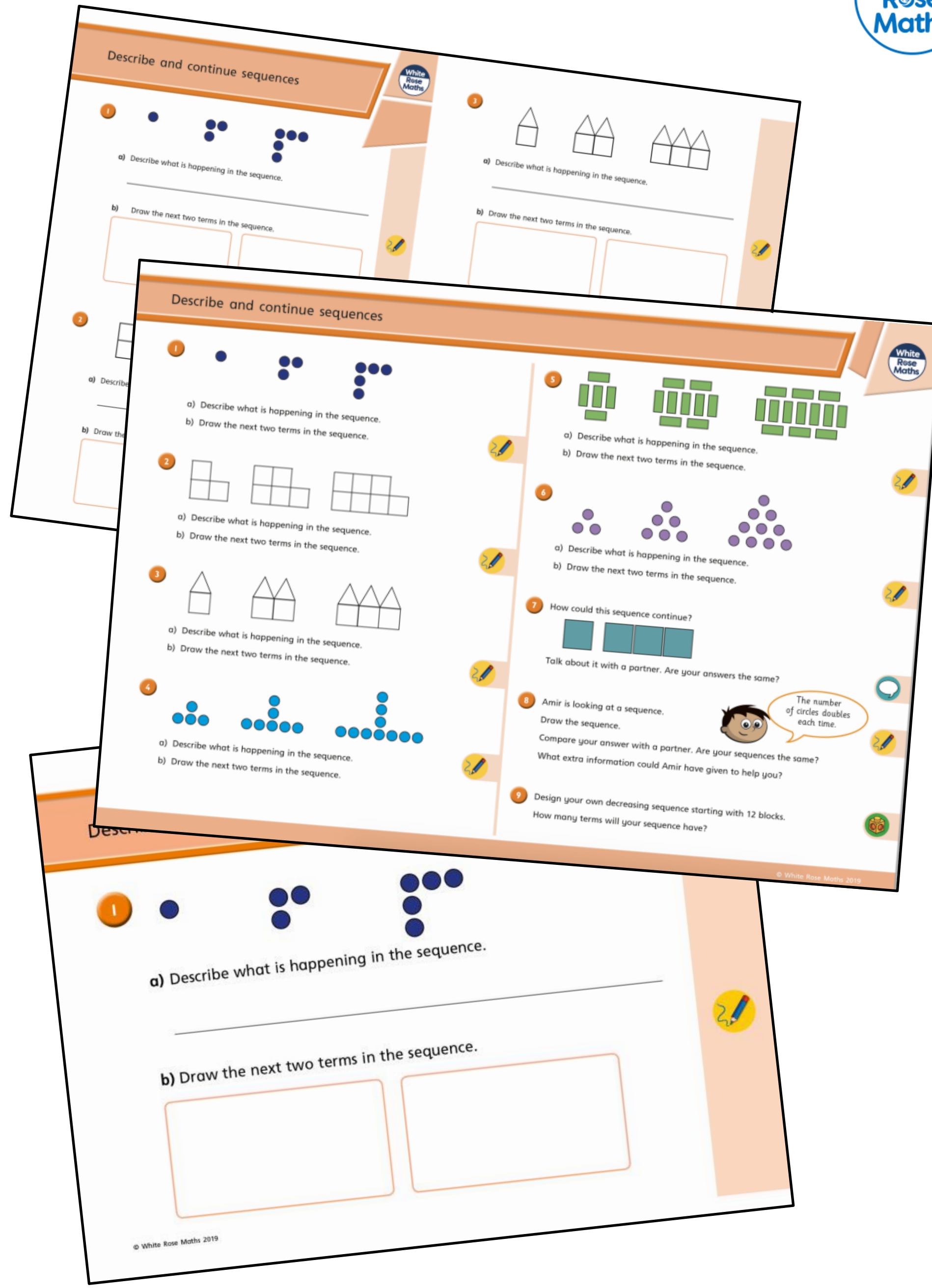
Supporting resources

NEW for 2019-20!

We have produced supporting resources for every small step from Year 1 to Year 8.

The worksheets are provided in three different formats:

- Write on worksheet – ideal for children to use the ready made models, images and stem sentences.
- Display version – great for schools who want to cut down on photocopying.
- PowerPoint version – one question per slide. Perfect for whole class teaching or mixing questions to make your own bespoke lesson.



For more information visit our online training and resources centre www.resources.whiterosemaths.com or email us directly at support@whiterosemaths.com

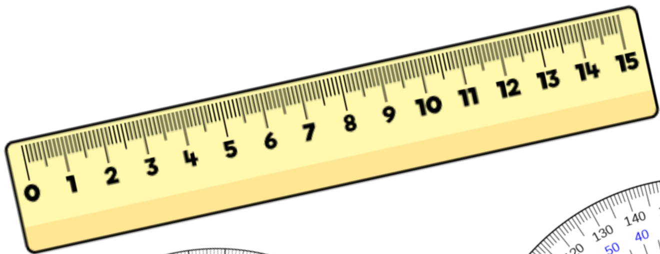
White Rose Maths

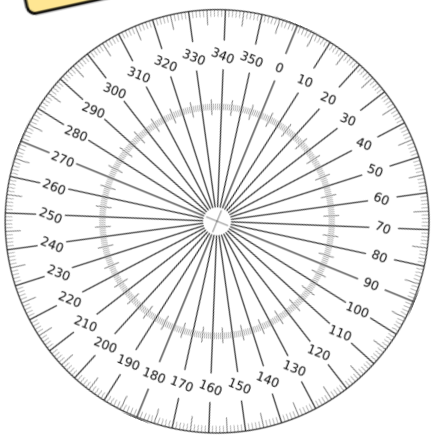
	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Algebraic Thinking**						**Place Value and Proportion**					
	Sequences		Understand and use algebraic notation		Equality and equivalence		Place value and ordering integers and decimals			Fraction, decimal and percentage equivalence		
Spring	**Applications of Number**						**Directed Number**			**Fractional Thinking**		
	Solving problems with addition & subtraction		Solving problems with multiplication and division			Fractions & percentages of amounts	Operations and equations with directed number			Addition and subtraction of fractions		
Summer	**Lines and Angles**						**Reasoning with Number**					
	Constructing, measuring and using geometric notation			Developing geometric reasoning			Developing number sense		Sets and probability		Prime numbers and proof	

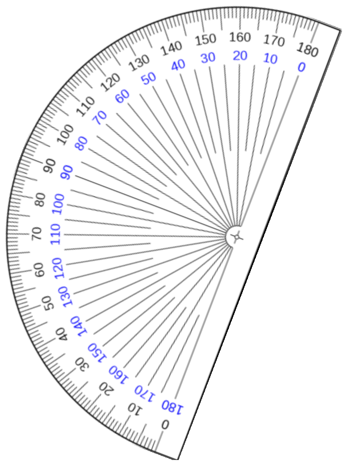
Summer 1: Lines and Angles

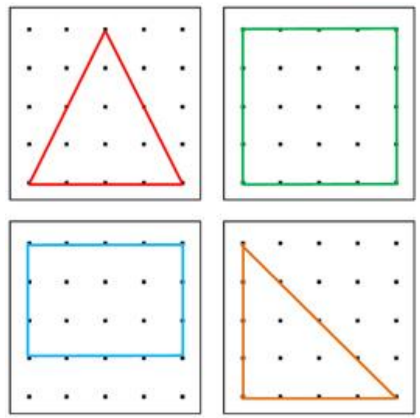
Weeks 1 to 3: Construction, measurement and notation	Weeks 4 to 6: Geometric reasoning
Students will build on their KS2 skills using rulers, protractors and other measuring equipment to construct and measure increasingly complex diagrams using correct mathematical notation. This will include three letter notation for angles, the use of hatch marks to indicate equality and the use of arrows to indicate parallel lines. Pie charts will be studied here to gain further practice at drawing and measuring angles. National curriculum content covered: • use language and properties precisely to analyse 2-D shapes • begin to reason deductively in geometry including using geometrical constructions • draw and measure line segments and angles in geometric figures, including interpreting scale drawings • describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right-angles, regular polygons, and other polygons that are reflectively and rotationally symmetric • use the standard conventions for labelling sides and angles • construct and interpret pie charts for categorical, ungrouped and grouped numerical data • Identify and construct triangles Interleaving/Extension of previous work • revisit four operations	This block covers basic geometric language, names and properties of types of triangles and quadrilaterals, and the names of other polygons. Angles rules will be introduced and used to form short chains of reasoning . The higher strand will take this further, investigating and using parallel line rules. National curriculum content covered: • use language and properties precisely to analyse 2-D shapes, • begin to reason deductively in geometry including using geometrical constructions • describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right-angles, regular polygons, and other polygons that are reflectively and rotationally symmetric • use the standard conventions for labelling sides and angles • derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language and technologies • apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles • apply angle facts, triangle similarity and properties of quadrilaterals to derive results about angles and sides, and use known results to obtain simple proofs • understand and use the relationship between parallel lines and alternate and corresponding angles **(H)** • derive and use the sum of angles in a triangle and use it to deduce the angle sum in any polygon, and to derive properties of regular polygons **(H)** Interleaving/Extension of previous work • forming and solving linear equations • revisiting addition and subtraction, including decimals

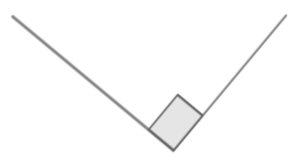
Key Representations

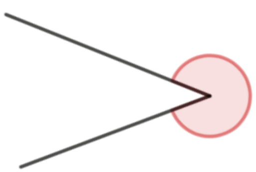


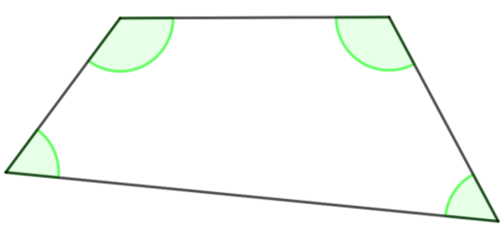


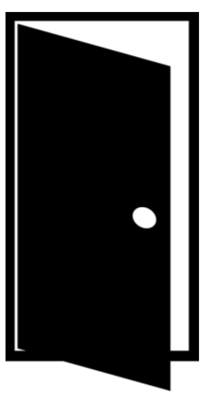


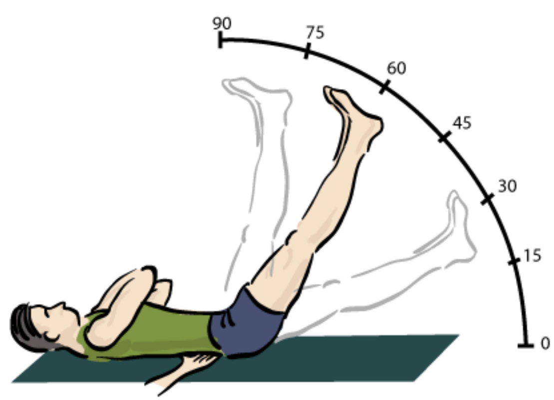












Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Here are a few ideas of equipment and representations that you might use during Construction and Measuring.

Opening and closing a door or a book allows students to visualise angles as a turn. They can also physically turn themselves and use their arms and legs to demonstrate angles and turns as in the lower right diagram.

Pictorial representations should be shown in a variety of orientations to avoid misconceptions such as angles always being measured from the horizontal.

Construction and Measuring

Small Steps

- Understand and use letter and labelling conventions including those for geometric figures
- Draw and measure line segments including geometric figures
- Understand angles as a measure of turn
- Classify angles
- Measure angles up to 180°
- Draw angles up to 180°
- Draw and measure angles between 180° and 360°
- Identify perpendicular and parallel lines
- Recognise types of triangle
- Recognise types of quadrilateral

Construction and Measuring

Small Steps

- Identify polygons up to a decagon
- Construct triangles using SSS
- Construct triangles using SSS, SAS and ASA
- Construct more complex polygons
- Interpret simple pie charts using proportion
- Interpret pie charts using a protractor
- Draw pie charts

Letter and labelling conventions

Notes and guidance

Students need to be able to describe a line segment and geometric figures using letter notation. They should always use a capital letter to define a vertex and know that two letters are required to define a line segment and three letters for an angle. Polygons should be described by naming the vertices cyclically and often but not always in alphabetical order.

Key vocabulary

Line	Line segment	Geometric figure
Notation	Polygon	

Key questions

How many points do you need to define a straight line?

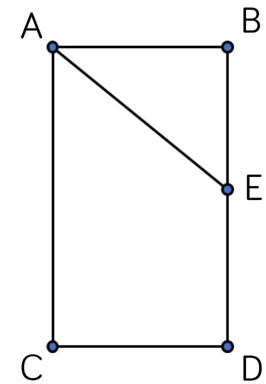
How many points do you need to define a polygon?

Exemplar Questions

A geometric figure is shown to the right.

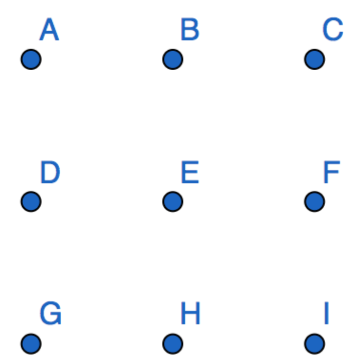
Use letter notation to fill in the blanks in the statements below.

Angle ______ is a right-angle.
Line segments ____ and ____ are vertical.
Shape ________ is a triangle.
Shape __________ is a quadrilateral.



Draw a trapezium and label the vertices KLMN.
Draw a line segment from point L to point N.
Use three letter notation to identify the triangles formed within the trapezium.

How many squares can you construct using the points labelled?
Use letter notation to name each one.



Draw and measure line segments

Notes and guidance

During this small step students will measure line segments within geometric figures to an accuracy of up to 1 mm. Students should be expected to convert freely between metric units. They should measure objects within the classroom and the wider environment and be able to justify the unit used.

Key vocabulary

Line	Segment	Length
Height	Width	Figure

Key questions

What is the difference between a line and a line segment?

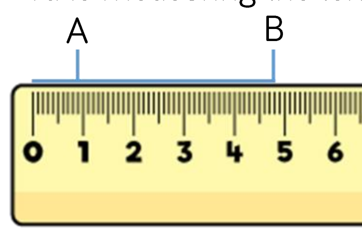
What would you measure in millimetres/centimetres etc.?

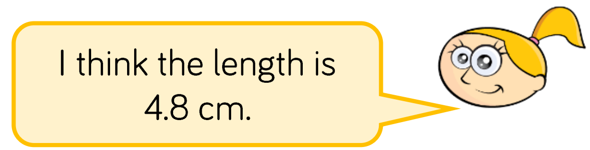
Exemplar Questions

WXYZ is a square. Side XY is 5cm long.

Draw the square and find the lengths of its diagonals WY and XZ.
Is there more than one way to draw this square?
Investigate if WXYZ were a rhombus instead.

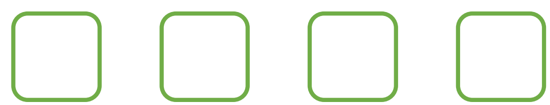
Eva is measuring the length from point A to point B.





What mistake has Eva made? What advice would you give her?

Line A is $4\frac{1}{4}$ cm long. Line B is 70 mm shorter than line A.
Lines C and D are drawn accurately below.
Put these lines in ascending length order.



Angles as a measure of turn

Notes and guidance

This step ensures that students understand that angles are a measure of turn. They will understand that an angle is formed by two lines meeting at a point. A variety of language should be used to describe the size and direction of a turn. Demonstrations of angles as a turn such as opening or closing a door, and the angle formed at the elbow should be shown to students.

Key vocabulary

Quarter/Half/Three Quarter/Full Turn

Degrees | Angles | Rotation

Key questions

How can we measure the size of a turn?

How can we describe the direction of a turn?

Does direction matter for a turn of 180°?

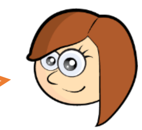
Exemplar Questions

A diver performs a dive with two and a half-turns.
How many degrees do they rotate through?

A ship is sailing North. It turns to face east.
Mo and Rosie are discussing how far the ship has turned.

I think the ship has turned one quarter turn clockwise.

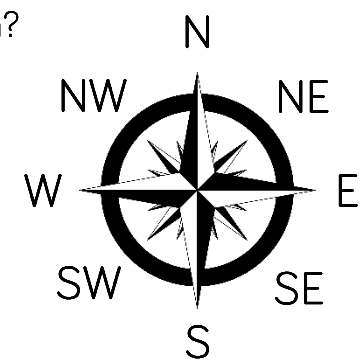
I think the ship has turned 270°



Who do you agree with? Why?

Write down three things in your classroom which turn through an angle.

Sam starts by facing North.
She turns clockwise to face West.
How many degrees has she turned through?



Asif is facing East.
He turns anticlockwise through 540°
Which direction is he now facing?

Alicia starts by facing South East.
She turns clockwise through 270°
Which direction is she now facing?

Classify angles

Notes and guidance

After completing this step students should be able to classify angles by sight, including within geometric figures. They should be familiar with and able to use conventional markings for right-angles. Students should also be introduced to the vocabulary of interior and exterior angles.

Key vocabulary

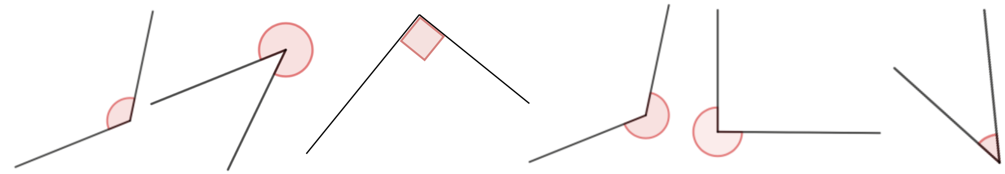
Angle	Acute	Obtuse	Right-angle
Reflex	Interior	Exterior	

Key questions

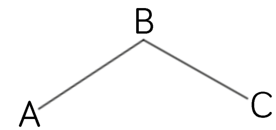
How do we illustrate that an angle is 90°?
How do we know which angle we are measuring?
Will turning through two acute angles result in turning through an obtuse angle?
Will turning through two obtuse angles result in turning through a reflex angle?

Exemplar Questions

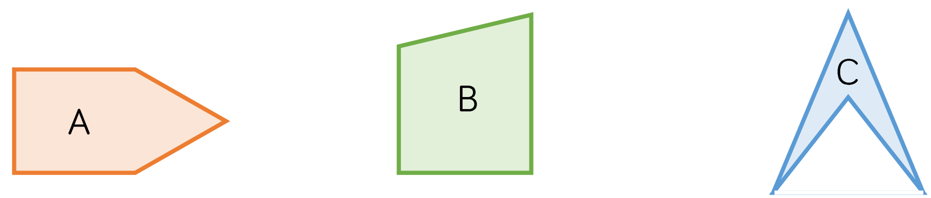
Classify the angles as acute, obtuse, reflex or right-angles.



Amir says angle ABC is obtuse.
Whitney says angle ABC is a reflex angle.
Who is correct and why?



Complete the table about the interior angles for each of the shapes.



Shape	Acute angles	Obtuse angles	Reflex angles	Right-angles
A				
B				
C				

Repeat for the exterior angles of each shape.

Measure angles up to 180°

Notes and guidance

Students use a protractor graduated in degrees to measure angles up to 180° including within geometric figures. Accuracy of measurement should be within a degree. Students should estimate the size of angles before measuring by comparing them to 90° and 180°. Students could estimate and measure angles in the real world such as the angle between the hands of a clock.

Key vocabulary

Protractor	Degrees	Right-angle
Half-turn	Sum	Measure

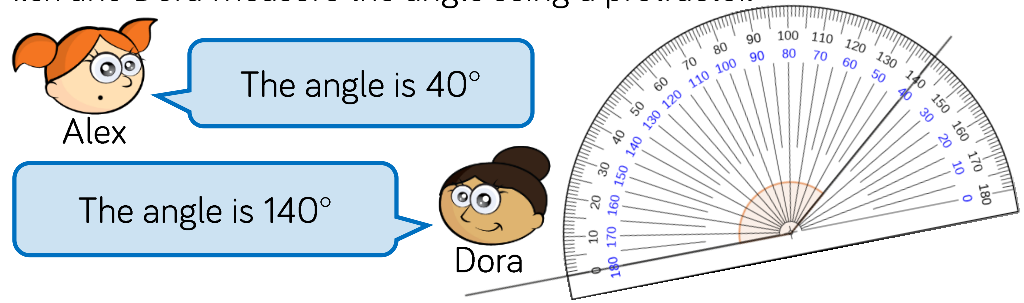
Key questions

How do we know which scale should be used to measure the angle?

How do we know where to put the protractor when measuring an angle?

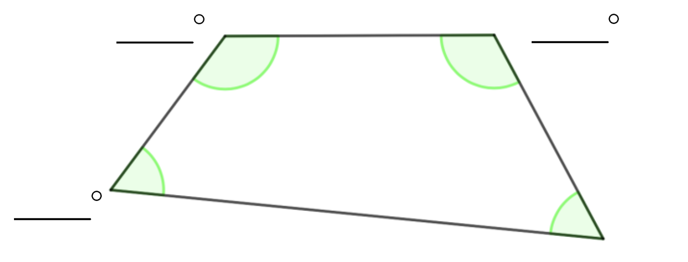
Exemplar Questions

Alex and Dora measure the angle using a protractor.



Who do you agree with? Why?

Measure the size of each of the interior angles in the shape.



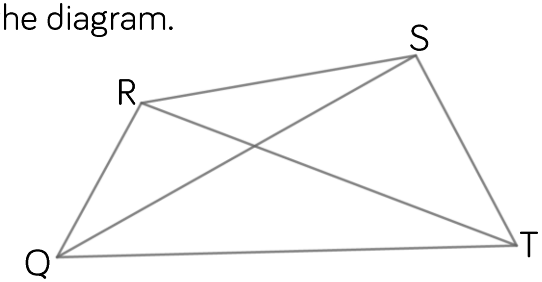
Complete the statements about the diagram.

∠RTS is _____°

∠QRS is _____°

The sum of angles SRT and QRT is _____°

The difference between angles QST and SQT is _____°



Draw angles up to 180°

Notes and guidance

In this small step, students draw angles up to 180° using a ruler and protractor. They should be able to construct the angle either at a specified point on a line or at the end of a line segment. As with measurement, angles should be drawn to an accuracy of within 1 degree. When checking their drawings, students should make comparisons with 90° and 180°

Key vocabulary

Angle	Protractor	Construct

Key questions

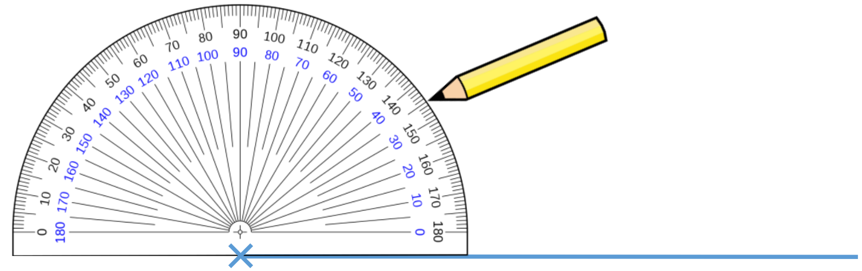
How do you choose which scale to use on a protractor?

Is it possible to draw an angle of 180°?

Why are there two scales on a protractor?

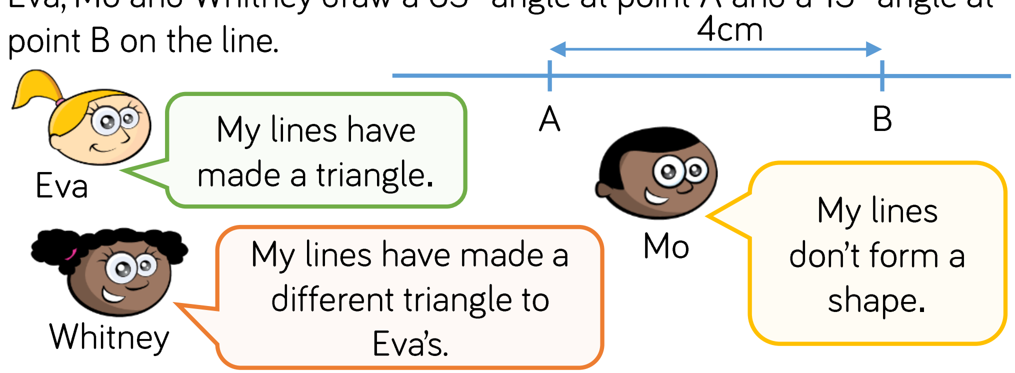
Exemplar Questions

Teddy is drawing an angle of 35°
He marks the angle with his pencil as shown.



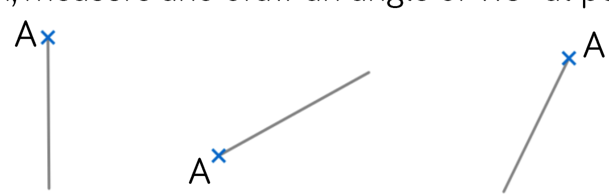
Will Teddy's diagram show the correct angle? How do you know?

Eva, Mo and Whitney draw a 63° angle at point A and a 15° angle at point B on the line.



Who is correct? Demonstrate your reasoning by drawing a diagram.

In each diagram, measure and draw an angle of 115° at point A.



Draw and measure 180° to 360°

Notes and guidance

Students develop their skills acquired during the previous steps by drawing and measuring angles between 180° and 360°

Accuracy should be within one degree when drawing and measuring angles.

Key vocabulary

Angle	Protractor	Construct

Key questions

How many degrees are there in a full turn?

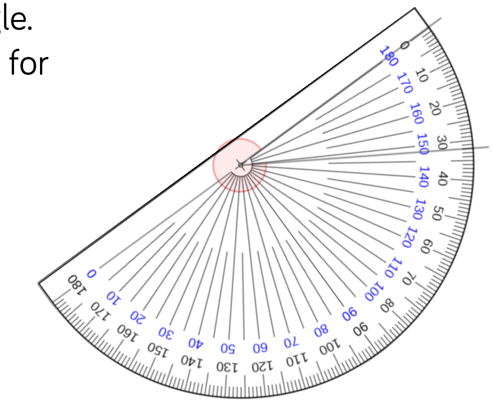
How can we use a protractor that doesn't go up to 360° to draw and measure angles over 180°?

Exemplar Questions

Measure all four angles shown.

Draw an angle of 258° at point A. How many ways can you do this?

Circle the correct size of the angle.
What mistakes have been made for the other answers?



A 32°

B 148°

C 322°

D 212°

Perpendicular and parallel lines

Notes and guidance

Students need to be able to identify parallel and perpendicular lines, including within geometric figures. They should use the correct notation to show where they have been identified. Examples of parallel and perpendicular lines in the real world should be explored.

Key vocabulary

Parallel   Perpendicular   Intersect

Key questions

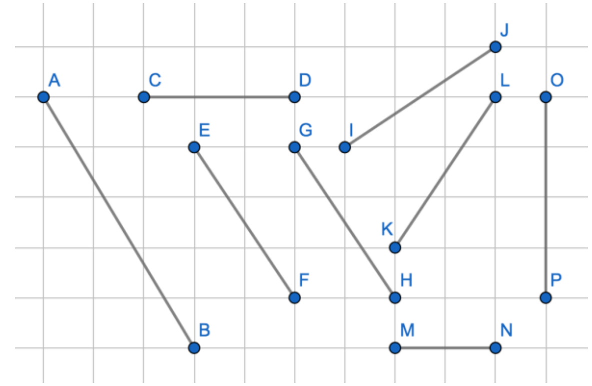
When are two or more lines parallel?

When are two lines perpendicular?

Can curved lines be parallel?

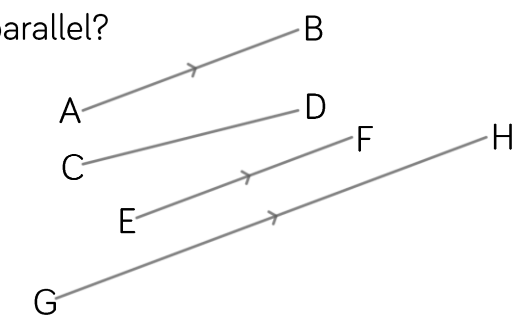
Exemplar Questions

Complete the sentences using two letter notation for each line segment.
CD is parallel to ___
IJ is perpendicular to ____
EF is neither parallel nor perpendicular to ____



Eva thinks that the lines are parallel.
Is she right?

Which of the line segments are parallel?



Tommy says the lines can't be perpendicular because they don't cross.
Do you agree?

Recognise types of triangles

Notes and guidance

Students should be able to recognise different types of triangles. They will be familiar with the properties of triangles from the primary curriculum. Students should be able to measure lengths and angles in triangles in order to classify them.

Key vocabulary

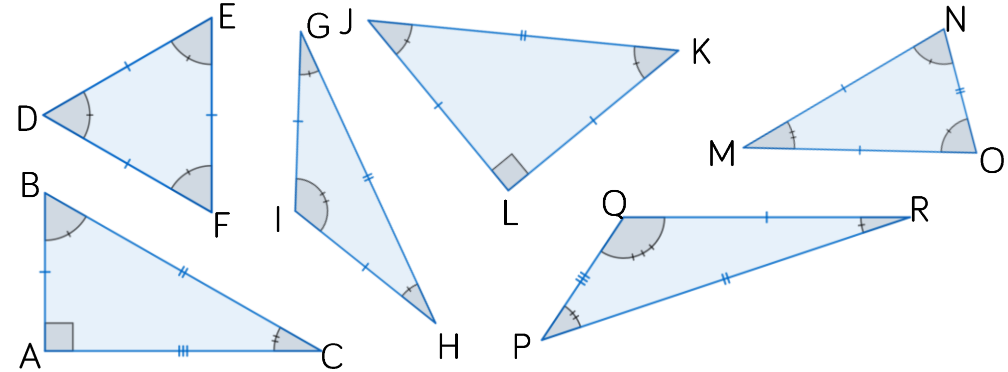
Equilateral	Isosceles	Scalene
Right-angled	Length	Angle

Key questions

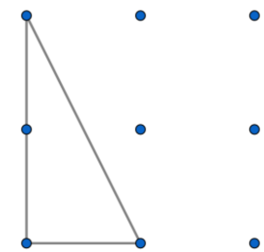
What is the difference between a scalene and an isosceles triangle?
What would you need to know about a triangle to be sure that it was equilateral?
Which types of triangle can also be right-angled?
Is an equilateral triangle also an isosceles triangle?

Exemplar Questions

Classify the following triangles as equilateral, isosceles, scalene or right-angled. Is there more than one category for each triangle?



Make as many different triangles as possible using a 3 by 3 geoboard. Classify each triangle.
One example is shown.
Are there any types of triangle you cannot make?



Use Cuisenaire rods to construct triangles using one piece per side.
What types of triangles can you create?
Are there any types that you can't create?
Can you generalise?

Record your results in a table.

Recognise types of quadrilaterals

Notes and guidance

Students will be familiar with the vocabulary of quadrilaterals from key stage 2. This step revisits and consolidates their understanding. Students should be able to fluently distinguish between types of quadrilateral using appropriate terminology to justify their decisions.

Key vocabulary

Square	Rectangle	Kite	Rhombus
Parallelogram	Trapezium	Parallel	Perpendicular

Key questions

What property does every quadrilateral share?
Is a quadrilateral a polygon?
Which quadrilaterals always have an interior right-angle?
Which quadrilaterals sometimes have an interior right-angle?
Explain why a square is a rectangle and a parallelogram.

Exemplar Questions

Create a 4 by 4 square on your geoboard.
By moving only one vertex, which of the following can you make?

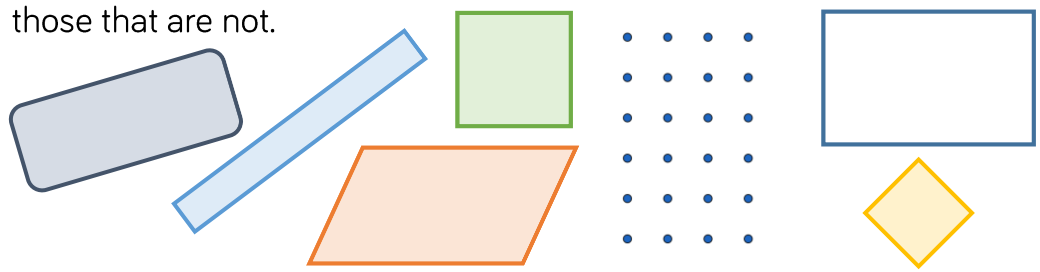
Kite	Trapezium	Parallelogram	Rhombus

Create a 4 by 6 rectangle on your geoboard.
By moving only one vertex, which of the following can you make?

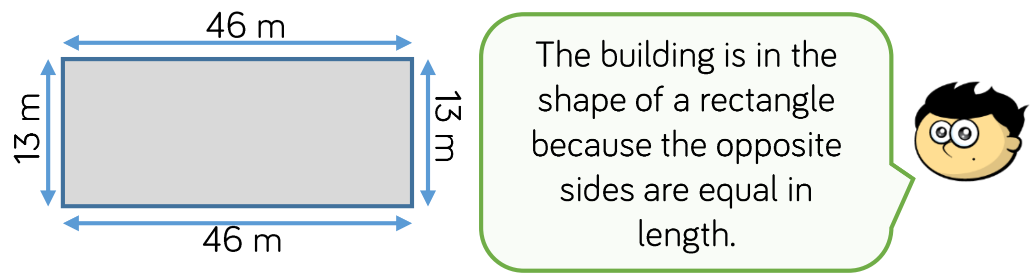
Kite	Trapezium	Parallelogram	Rhombus

How do your answers change if you're allowed to move 2 vertices?

Sort the shapes into two groups, those that are rectangles and those that are not.



A plan of a building is shown.
What else does Jack need to check to show that he is correct?



Identify polygons up to a decagon

Notes and guidance

Students should identify polygons up to a decagon. They need to be able to distinguish between regular and irregular polygons. Students often think shapes are regular when their side lengths are equal without considering interior angles. They should relate vocabulary with other areas of mathematics as well as in the real world. For example, associating 'dec' with decimal and decathlon.

Key vocabulary

Polygon	Edges	Vertices	Angles
Equal	Length	Triangle	Decagon

Key questions

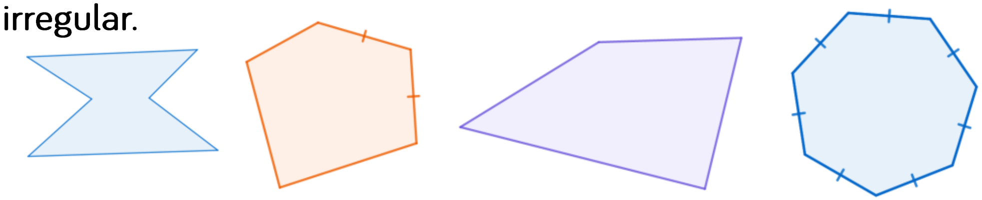
When is a polygon regular?

What name do we give to a regular three sided polygon?

What name do we give to a regular four sided polygon?

Exemplar Questions

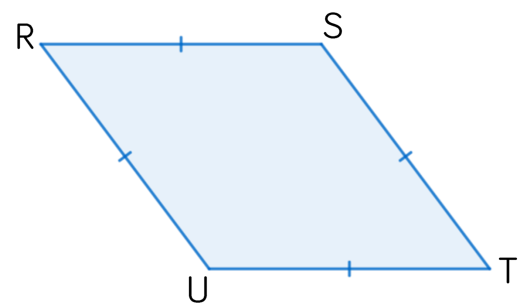
Write the name of each polygon and decide whether it is **regular** or **irregular**.



________ ________ ________ ________

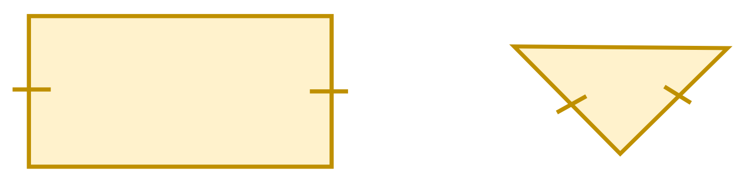
________ ________ ________ ________

Jack says that the shape, RSTU, is a regular polygon.



Explain Jack's mistake.

How many types of polygon can be made by attaching the following two shapes at their edges?



Construct triangles – SSS

Notes and guidance

Students need to understand how to construct a triangle where all 3 sides are given. You may want students to try and do this using just a ruler and pencil at first as this will highlight that this is inaccurate. They should realise that it is more accurate to use a compass. Students should be able to explain why a certain set of side lengths will not make a triangle.

Key vocabulary

Pair of Compasses		Construct	
Side	Edge	Vertex	Point

Key questions

Is it possible to accurately construct a triangle given the side lengths using just a pencil and ruler?

Why is more accurate to use a pair of compasses?

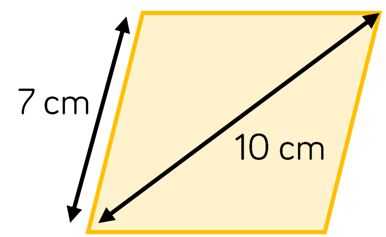
Exemplar Questions

Construct a line segment AB 10 cm long.
Plot point C such that AC and BC have lengths 8 cm and 5.5 cm respectively.
Measure the angle at each vertex and classify the triangle.
What equipment did you use?
Why is it more accurate to use a pair of compasses?

Construct a triangle with side lengths 6 cm, 8 cm and 10 cm.
What do you notice about this triangle?

Explain why you cannot construct a triangle with side lengths 4 cm, 5 cm and 11 cm.
Can you come up with another set of side lengths that will not make a triangle?
How do you know?

Here is a rhombus with side lengths of 7 cm and a longest diagonal of 10 cm.



Use a pair of compasses to construct a rhombus with sides of 5 cm and a longest diagonal of 8 cm.

Construct triangles – SSS, SAS, ASA

Notes and guidance

Students need to be familiar with the phrases side-side-side, side-angle-side and angle-side-angle. They should also understand why they represent the minimum information to draw a distinct triangle. They should be exposed to ambiguous cases when it is possible to construct two distinct triangles from the information given.

Key vocabulary

Isosceles	Equilateral	Scalene	Right-angled
Side	Edge	Vertex	Point

Key questions

Is it possible to construct a unique triangle given all three angles?

Why is it sometimes possible to draw two distinct triangles when given an angle and the length of two sides?

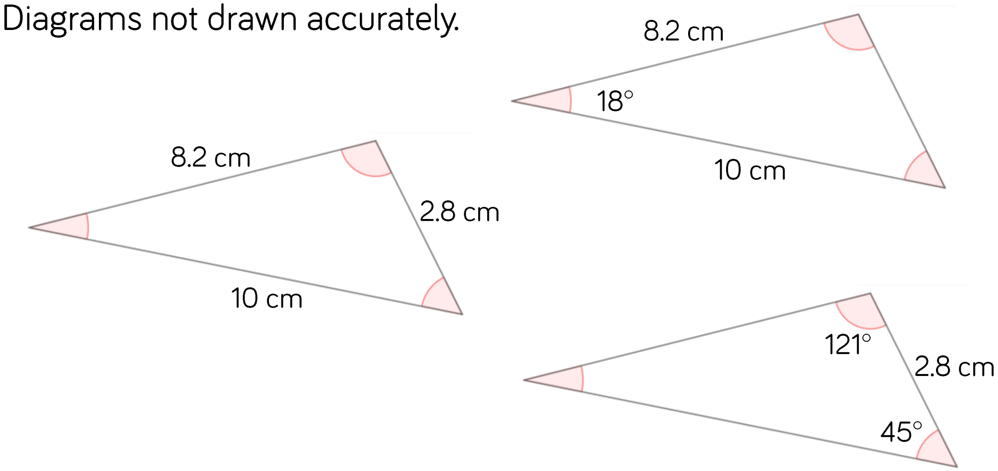
Exemplar Questions

Construct a triangle WXY such that angle WXY is 40° and lengths WX and WY are integers that add to make 13 cm.
How many unique triangles can you construct?
Explain why this is possible.

Which of the triangles are identical?

You must draw each triangle accurately and give reasons for your answer

Diagrams not drawn accurately.



Describe three different ways to construct an equilateral triangle with perimeter 189 mm.

Construct more complex polygons

Notes and guidance

Students should be able to draw more complex polygons and diagrams constructed using multiple polygons. This step is an opportunity to recap perimeter.
Letter notation should continue to be used for line segments, polygons and angles.

Key vocabulary

Polygon	Regular	Side	Vertex
Vertices	Rhombus	Diagonals	Compound

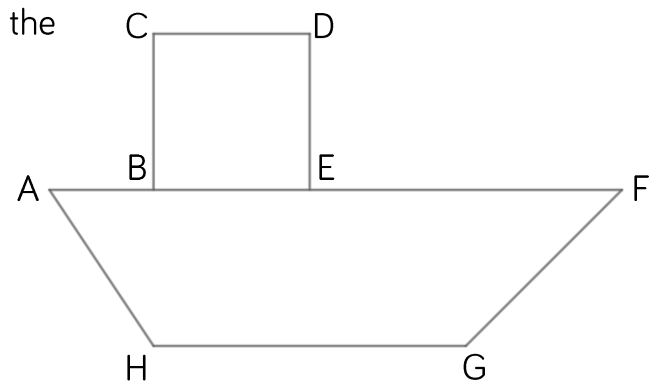
Key questions

Is it possible to construct an irregular polygon with equal angles?

Is it possible to construct an irregular polygon with equal side lengths?

Exemplar Questions

The boat has been constructed using 2-D shapes.
BCDE is a square with side length 3 cm.
Line segment BA is 2.4 cm long.
Point H is 3 cm directly beneath point B.
Line segment BF is 8.8 cm long.
Angle EFG is 45°
Construct an accurate diagram of the boat and complete the information below.



Length AH = ____

∠AHG = ____

Perimeter = ____

The height of the boat is ____.

Construct a rhombus EFGH with side length 5.7 cm and angle FGH measuring 53°
Measure and state the lengths of the diagonals using two-letter notation.

Can you construct a regular hexagon with side lengths 7 cm without using a protractor?

Interpret pie charts using proportion

Notes and guidance

In this small step students will interpret pie charts divided into equal portions, given the whole or part of the total frequency. Students should be able to make comparisons between multiple pie charts. Students should acknowledge that although they can compare proportions, this does not necessarily mean they can compare frequencies.

Key vocabulary

Proportion	Frequency	Fraction
Total	Comparison	Sector

Key questions

What do pie charts show us?

If two parts of the pie chart are the same size, what does that tell us?

If one part of two different pie charts are the same size, do they represent the same frequency?

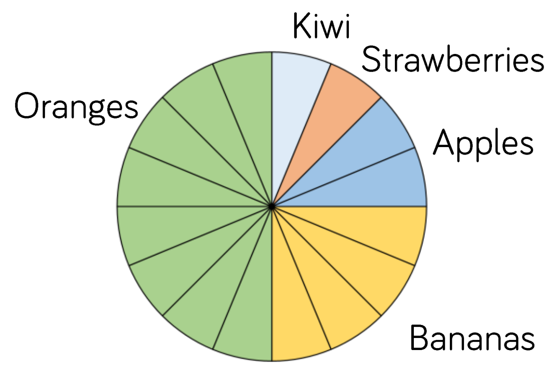
Exemplar Questions

The pie chart below shows the favourite fruits of a class. There are 32 students in the class.

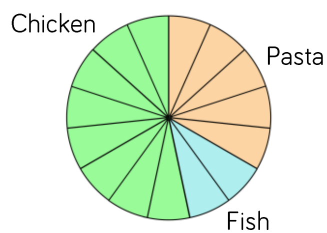
What fraction prefer bananas?

How many more students prefer oranges to apples?

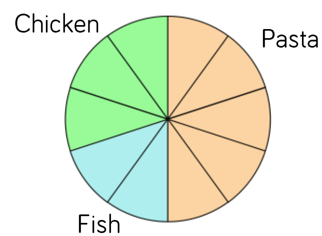
What was the least popular fruit?



The pie charts below show what students at two different schools chose for their lunch one day. Do you agree with the statements?



Fawton High School

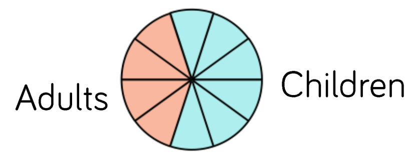


Burton Primary School

Half of the children at Burton Primary chose pasta.

More children chose pasta at Fawton High than at Burton Primary.

The pie chart shows the proportions of adults and children attending a musical. 240 children attended the musical. How many people attended the musical altogether?



Interpret pie charts using a protractor

Notes and guidance

Students will extend the skills developed in the previous step to interpret pie charts given the angles for each section.

Students should be familiar with a full turn being 360°

Key vocabulary

Protractor	Proportion	Frequency
Angle	Degrees	Sector

Key questions

If two pie charts are identical, do they represent identical frequencies?

What if the angle measured is between two marks on your protractor?

Exemplar Questions

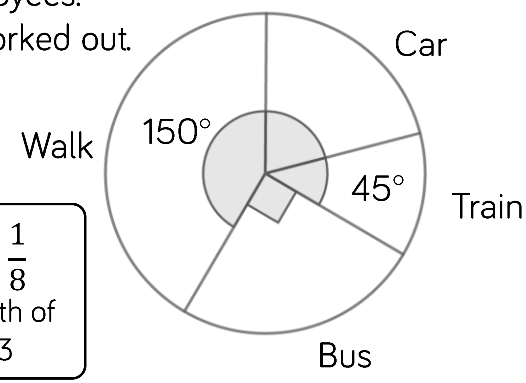
The pie chart shows the proportion of how a group of employees travel to work. There are 24 employees.

Explain how the following were worked out.

"a quarter of the people go by bus. So 6 people go by bus."

$\frac{150}{360} \times 24 = 10$
So 10 people walk.

$\frac{45}{360} = \frac{1}{8}$
One eighth of 24 is 3



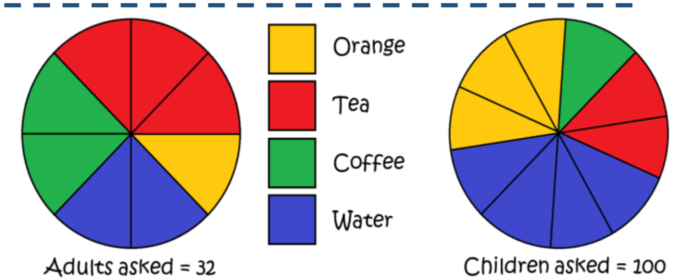
How many people came by car?

How many ways could you work this out?

The pie charts shows favourite drinks.

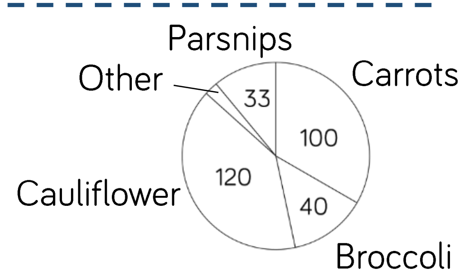
What fraction of adults prefer tea?

Which group has the most tea drinkers?



---

300 students were asked to name their favourite vegetable given the options carrots, broccoli, cauliflower, parsnips and other. How many chose other? Which vegetable was chosen by 40% of the students?



Draw pie charts

Notes and guidance

Students should be able to draw a pie chart given a complete or incomplete frequency table.

Students should be encouraged to consider whether a pie chart is the most appropriate representation for given data.

Key vocabulary

Protractor	Proportion	Frequency
Angle	Degrees	Sector

Key questions

What do you do when the total frequency is neither a multiple nor a factor of 360?

How do you calculate the angle of a sector of a pie chart?

Exemplar Questions

The table contains information about the cars in a car park.
Complete the table and draw a pie chart to represent the data.

Manufacturer	Frequency	Angle of sector
Volvo	10	
BMW	8	
Ford	16	
Kia	11	

The table contains information about the colour of students eyes in class 8a. There are 30 students in class 8a. Complete the table and draw a pie chart to represent the data.

Manufacturer	Frequency	Angle of sector
Brown	6	
Hazel	8	
Blue		60°
Green		48°
Silver	2	
Amber		

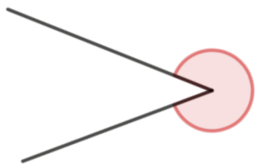
Tahir is drawing a pie chart using the frequency table.
Tahir's calculations are in red. Explain the mistake Tahir has made and then construct an accurate pie chart.

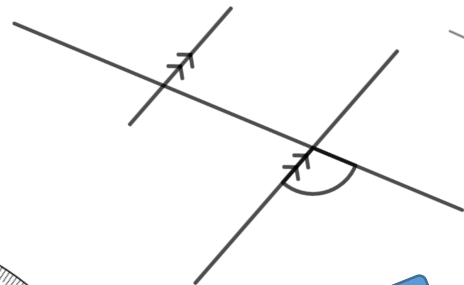
Sport	Frequency
Hockey	20
Rugby	28
Football	16
Basketball	26

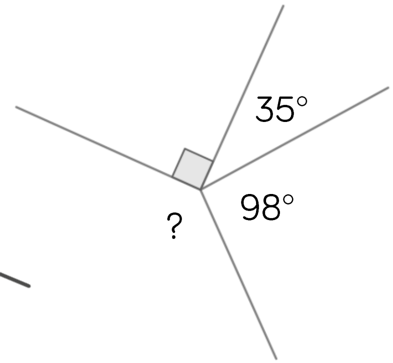
$0.25 \times 20 = 5°$
$0.25 \times 28 = 7°$
$0.25 \times 16 = 4°$
$0.25 \times 26 = 6.5°$

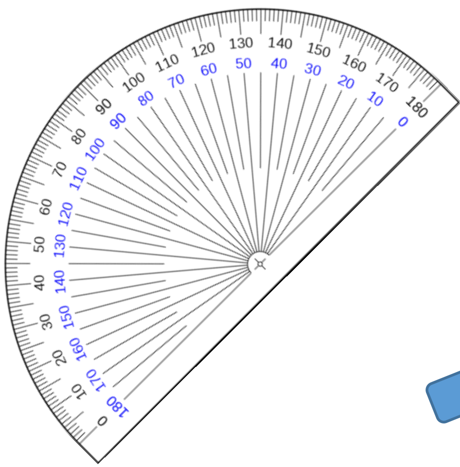
$20 + 28 + 16 + 26 = 90$ $90 \div 360 = 0.25$

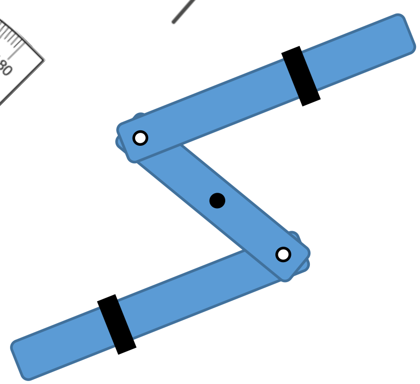
Key Representations

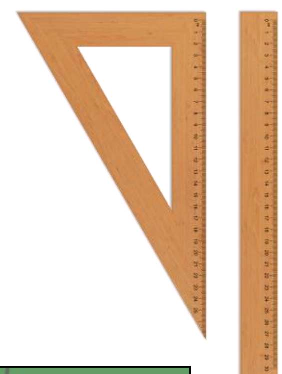


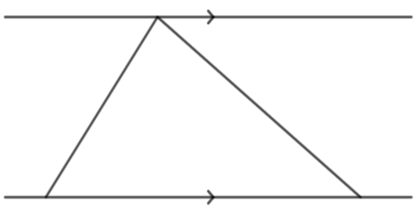












Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Here are a few ideas of equipment and representations that you might use during Geometric reasoning.

A variety of pictorial models should be shown in a range of orientations.

The blue diagram shows a reverse motion linkage that students should be familiar with from primary school. The upper and lower parts will always remain parallel. Parallel line constructions can also be demonstrated using a set square and ruler. Real life uses of parallel could be discussed such as "the roads run parallel."

Geometric Reasoning

Small Steps

- Understand and use the sum of angles at a point
- Understand and use the sum of angles on a straight line
- Understand and use the equality of vertically opposite angles
- Know and apply the sum of angles in a triangle
- Know and apply the sum of angles in a quadrilateral
- Solve angle problems using properties of triangles and quadrilaterals
- Solve complex angle problems

Geometric Reasoning

Small Steps

- Find and use the angle sum of any polygon H
- Investigate angles in parallel lines H
- Understand and use parallel line angle rules H
- Use known facts to obtain simple proofs. H

H denotes higher strand and not necessarily content for Higher Tier GCSE

Sum of angles at a point

Notes and guidance

Students should know that angles at a point sum to 360°. They should understand that this is a definition and that it is not possible to prove that angles at a point sum to 360°. Interactive geometry software should be used by teachers and students to demonstrate and explore this step.

Key vocabulary

Sum	Angle	Degrees
Line Segment	Notation	

Key questions

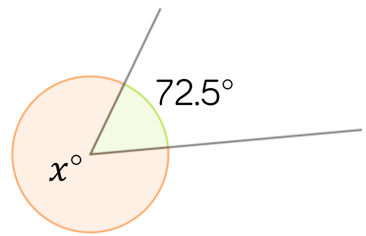
What is the sum of angles at a point?

How many right-angles fit around a point?

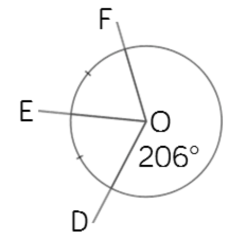
How does 180° compare to the sum of angles at a point?

Exemplar Questions

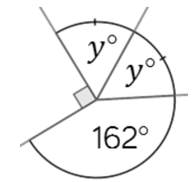
Work out the size of each angle. Diagrams are not drawn accurately.



$x =$ ______



FOE = ______

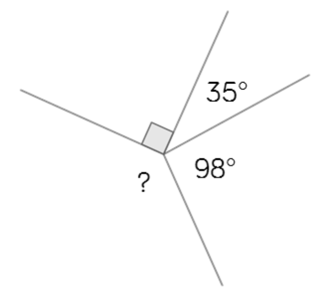


Form and solve an equation and find the value of y.

Mo and Rosie are discussing how to find the missing angle in the diagram below. Who do you agree with and why?

Mo: I think you just need to subtract 35 and 98 from 360

Rosie: I think that you need to subtract another 90



Four line segments are drawn from a point O. They are OA, OB, OC and OD. A, B, C and D are points drawn clockwise in order around O. Angle AOB is 54°. Angle BOC is 106°. Angle BOD is 187°. What is the size of angle AOD? Circle the correct answer.

13°   81°   119°   347°

Sum of angles on a straight line

Notes and guidance

Students should know that adjacent angles at a point on a straight line sum to 180°. They should recognise when this fact can or cannot be applied. Non-examples should be shown where two angles on a straight line do not share a common point. Interactive geometry software can be used to demonstrate this step and allow students to explore when this rule can and cannot be applied.

Key vocabulary

Sum	Angle	Degrees
Line Segment	Notation	Adjacent

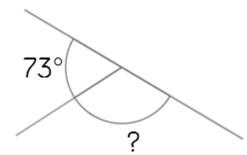
Key questions

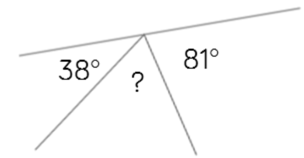
What is the sum of angles at a point on a straight line?
How many right-angles would fit on a straight line?

John measures three angles on a straight line. They are 81°, 47° and 51°. Has John measured the angles correctly? Explain your answer

Exemplar Questions

Calculate the size of the missing angle in each diagram.





Amir and Whitney are discussing a diagram.
Angle WXY is 19°. Angle YXZ is 161°.

Y, X and Z all lie on the same line.

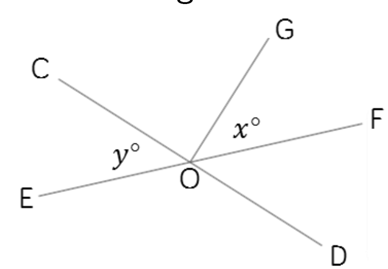
Amir

XY and XZ are two different lines intersecting at X.

Whitney

Who do you agree with? You must justify your answer with a sketch.

CD and EF are straight lines.
Write expressions for the size of any missing angles.
Use the correct three letter and geometric notation where appropriate.



Vertically opposite angles

Notes and guidance

Students should know that vertically opposite angles are equal. They should understand that vertically opposite angles are formed when two or more lines intersect at a point. They should be able to show that vertically opposite angles are equal by considering angles at a point on a straight line.

Key vocabulary

Angle	Vertically Opposite	Line
Intersect		

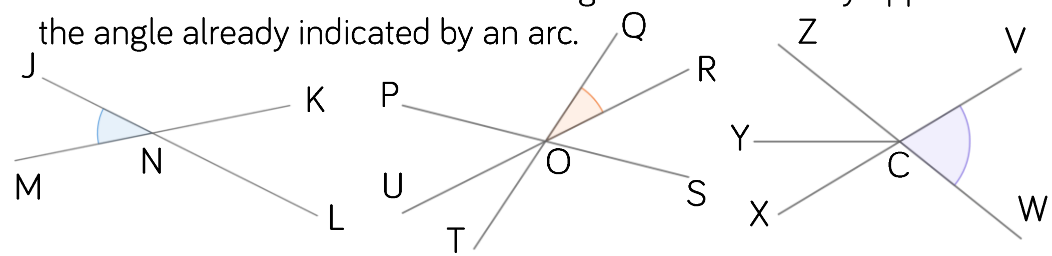
Key questions

When are vertically opposite angles formed?

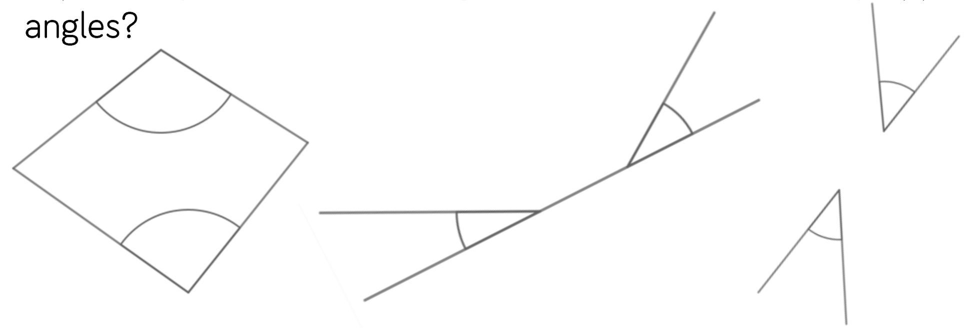
Given an angle formed at the intersection of two straight lines, is it always possible to find all angles at that point?

Exemplar Questions

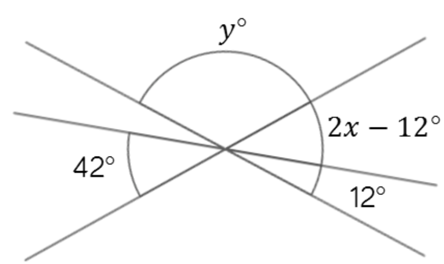
Use letter notation to describe the angle that is vertically opposite to the angle already indicated by an arc.



Explain why each of these diagrams do not show vertically opposite angles?



The following diagram shows three straight lines that intersect at a single point. Work out the value of x and y.



Sum of angles in a triangle

Notes and guidance

Students should know that the interior angles in a triangle sum to 180°. Students may be familiar with and could investigate tearing the corners from a triangle and using them to form 180°.

Demonstrations using interactive geometry software may also aid understanding.

Key vocabulary

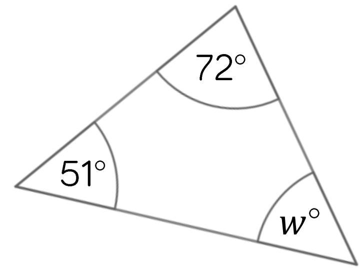
Angle	Isosceles	Equilateral
Scalene	Right-angled	Sum

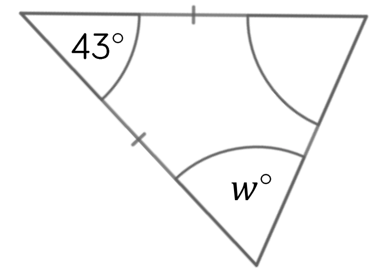
Key questions

What is the sum of the interior angles of a triangle?
How many angles do you need to know to be able to find all of the interior angles of a triangle?
If one angle in an isosceles triangle is 60°, is it an equilateral triangle?
Can a triangle have two right-angles?

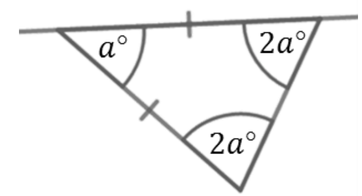
Exemplar Questions

Calculate the size of the angle marked in each diagram.

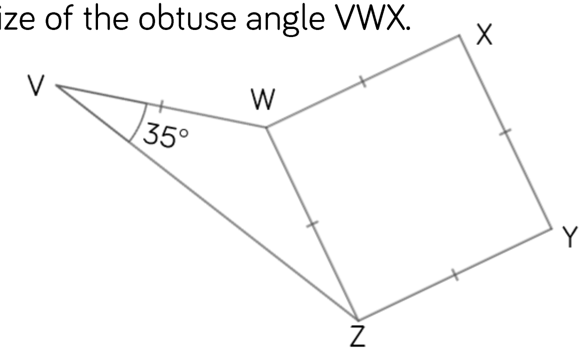




Bella says, "Angle a can be any angle and $2a$ is double that."
Milo says, "Five lots of angle a must be equal to 180°."
Who do you agree with? Why?



Two shapes are connected at their edges as in the diagram below.
Calculate the size of the obtuse angle VWX.



One angle in an isosceles triangle is 30°. What are the other two angles? Give two possible solutions.

Sum of angles in a quadrilateral

Notes and guidance

Students should know and be able to derive that the sum of angles in a quadrilateral is 360°. Both convex and concave quadrilaterals should be considered. Students should derive the angle sum by considering quadrilaterals as two triangles. Interactive geometry software can be used to demonstrate this. Students should revisit the properties of quadrilaterals.

Key vocabulary

Quadrilateral	Convex	Concave
Sum	Parallelogram	Rhombus

Key questions

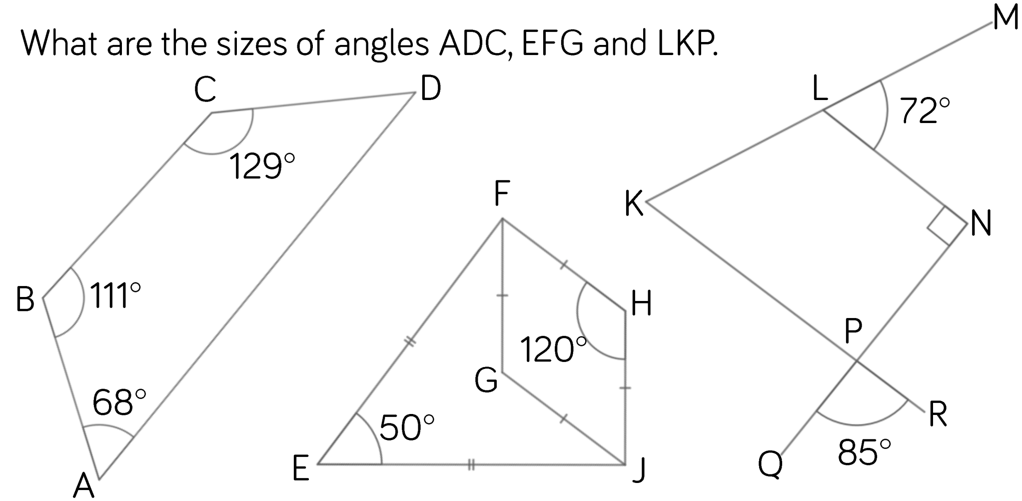
What is the sum of interior angles in a quadrilateral?

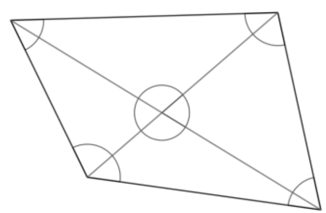
How can you demonstrate that the sum of the interior angles of a quadrilateral is 360°?

If a quadrilateral has four right-angles, is it a square?

Exemplar Questions

What are the sizes of angles ADC, EFG and LKP.



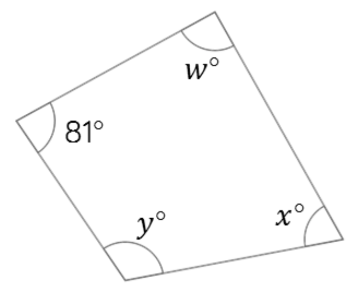


David draws the diagram to the left. He says the interior angles in a quadrilateral sum to 720° because he has split the shape into four triangles and four lots of 180° is 720°.

Explain why David's diagram does not show this.

Which of the following equations are correct for the diagram?

$$81 = w + x + y$$
$$w + x + y + 81 = 360$$
$$279 = w + x + y$$
$$x = 81$$
$$180 = x + 81$$



Angle problems

Notes and guidance

Within this small step, students should use one known angle fact to find a missing angle.

The focus should be on reasoning which angle fact should be applied to each scenario. Justifications using the correct vocabulary and notation should be used throughout.

Key vocabulary

Angle	Sum	Vertically opposite
Point	Straight Line	Polygon

Key questions

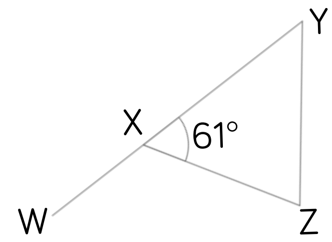
How did you decide which angle fact to use and apply?

Could you have applied a different angle fact?

Which angle facts do you know?

Which angle facts do you think you will need to apply to this question?

Exemplar Questions



Complete the sentences.
Angles on a straight line sum to ______.
Therefore angle WXZ is ________.
The interior angles in a triangle sum to ______ so the other two angles in the triangle must sum to ______.

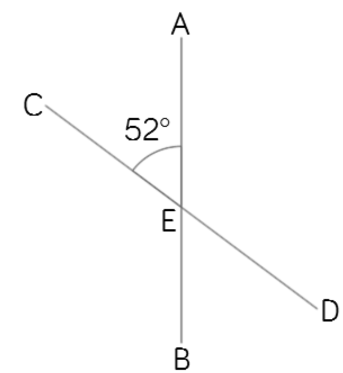
Complete the sentences.

Angle CEB is _____ because ___________

________________________________.

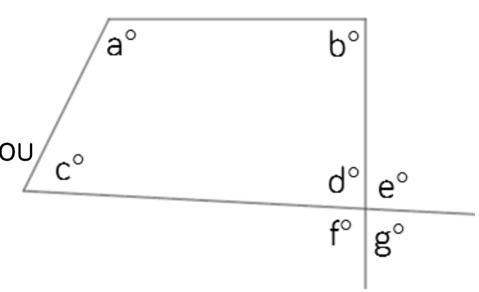
∠BED is _____ because ___________

________________________________.



Write as many equations as possible for the diagram to the right.

You must state the angle facts that you have considered to write each equation.



Complex angle problems

Notes and guidance

This step considers angle problems where two or more known angle facts need to be applied to a problem. Students should always give reasons for their solutions, ensuring that they use the correct vocabulary. Different chains of reasoning should be explored alongside discussion of which are the most efficient methods.

Key vocabulary

Angle	Sum	Vertically opposite
Point	Straight Line	Polygon

Key questions

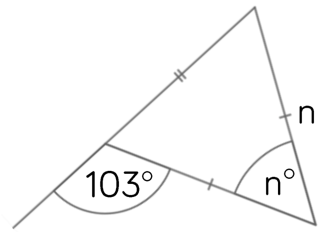
How did you decide which angle facts to apply?

Could you have considered the same angle facts in a different order?

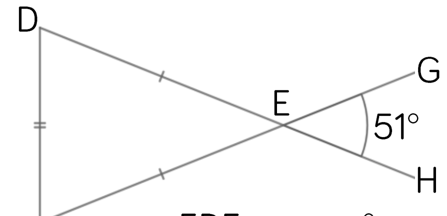
Could you have applied a different angle fact?

Exemplar Questions

Find the size of the angle specified for each diagram.



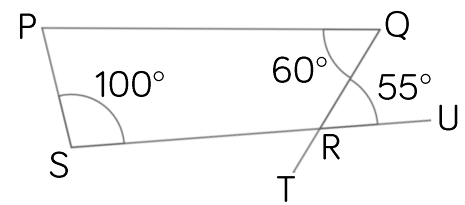
n = ____°



EDF = ____°

Below is a diagram and a student explanation of finding the size of angle QPS. Is each stage of reasoning correct? Could the solution have been more efficient?

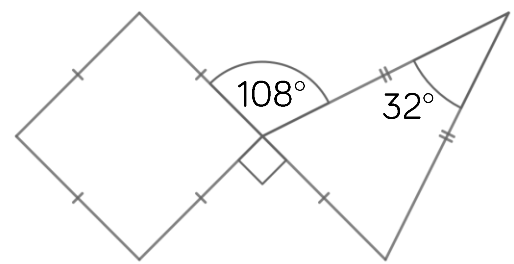
TRU = 125° because angles on a straight line sum to 180°.
QRS = TRU = 125° because vertically opposite angles are equal.
QPS = 75° because angles in a quadrilateral sum to 360°.



The diagram below shows two polygons connected by one of their vertices at a point. Determine if the quadrilateral is a square or not.

How do you know?

The diagram is not drawn accurately.



Angle sum of polygons

H

Notes and guidance

Students should know how to find and use the angle sum of any polygon.

They should investigate interior and exterior angles at vertices.

Students can investigate sums by partitioning polygons into triangles from a single vertex.

Key vocabulary

Polygon	Interior	Sum
Regular	Convex	Concave

Key questions

Explain why the interior angle of any polygon is a multiple of 180°.

How can you calculate the angle sum of any polygon?

Does your method work for concave polygons?

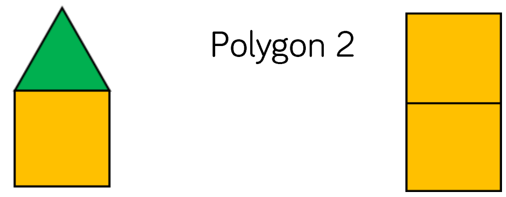
Exemplar Questions

Aliya and Theo are forming polygons by combining pattern blocks.

- Aliyah puts a square and a triangle together to form polygon 1.
- Theo puts two squares together to form polygon 2.

Polygon 1

Polygon 2

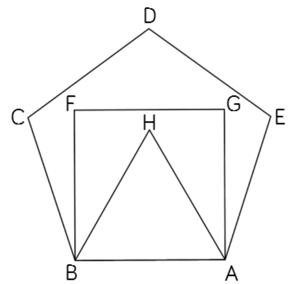


Aliya: The interior angles of a square and triangle sum to 180° and 360° respectively.
The interior angles of the pentagon must sum to 540°.

Theo: The interior angles of polygon 2 must be 720° because the sum of the interior angles in each square is 360°.

Explain whether you agree or disagree with each person.

The diagram to the right shows an equilateral triangle, a square and a regular pentagon all sharing the edge AB.
Find the size of angles:



- CDE
- EAH
- FBH

A regular polygon has an interior angle of 172°.
How many edges does the polygon have?

Angles in parallel lines H

Notes and guidance

Students investigate angles in parallel lines by measuring. They should not formally consider parallel line angle rules during this step. Students should make and test conjectures. They should also be encouraged to use known angle facts to justify any of their conjectures. Both computer software and physical mechanical apparatus should be used to demonstrate to students.

Key vocabulary

Parallel	Perpendicular	Line segment	
Conjecture	Equal	Opposite	Transversal

Key questions

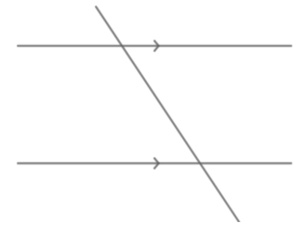
How do you denote that two or more lines are parallel?

What do you notice about the sum of angles ___ and ___ ?

What do you notice about angles ___ and ___ ?

Exemplar Questions

Measure all of the angles below. What do you notice?

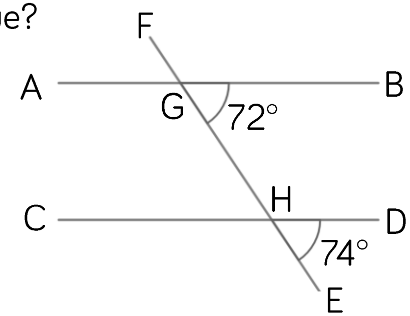


Use dynamic geometry software to see if this is always the case.

All of the odd numbered streets are parallel. All of the even numbered avenues are parallel. Mark on all of the parallel lines with appropriate notation.

Mark any angles that you think will be the same. Does it matter whether the transversal lines are perpendicular to the streets?

Which of the following statements are true?



- AB is parallel to CD.
- Lines through AB and CD will meet exactly once.
- ∠CHE = ∠FHD

White Rose Maths

Parallel line angle rules

H

Notes and guidance

In this step, students should build on the previous step by looking formally at alternate, corresponding and co-interior angles. They should be able to identify these types of angles and use them to find other angles in parallel lines. Students should also be aware of the converses e.g. if a pair of corresponding angles are equal, then the lines are parallel.

Key vocabulary

Parallel	Intersect	Transversal
Co-interior	Corresponding	Alternate

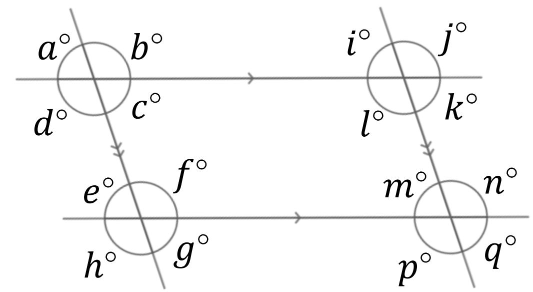
Key questions

How do you identify corresponding/alternate/co-interior angles?

Why are co-interior angles different to corresponding and alternate angles?

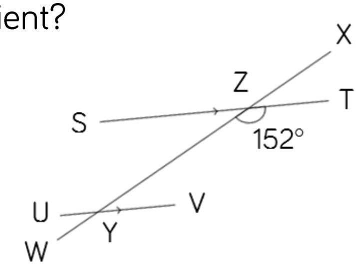
Exemplar Questions

Find as many pairs of angles as possible that are the same.
Find as many pairs of angles as possible that sum to 180°.
State whether each pair is corresponding, alternate or co-interior.



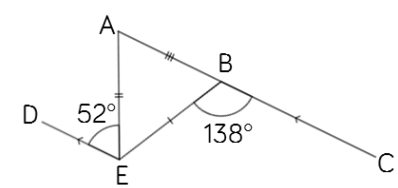
Find the size of the following angles. You must give reasons for your solution. How many different ways can you justify your solution? Which justification is the most efficient?

- UYZ = _____°
- WYV = _____°
- SZX = _____°
- VYZ = _____°
- XZT = _____°



Points A, B and C all lie on a straight line. Find the size of angle BAE in the diagram below.

Give reasons for your solution.



Simple proofs

H

Notes and guidance

Students need to be able to obtain simple proof using known facts from previous steps. They should explore the difference between a demonstration and a proof. The teacher should demonstrate the proof that angles in triangle add up to 180° Writing of proofs should be encouraged following discussions about efficiency and generalisation.

Key vocabulary

Proof	Demonstration	Opposite
Interior	Exterior	Parallel

Key questions

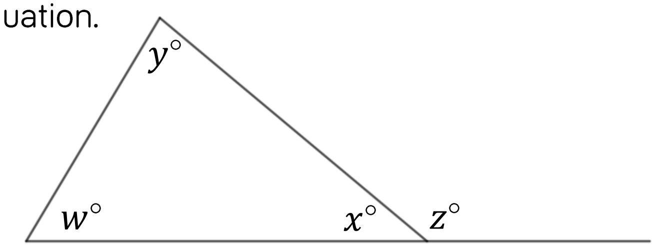
What is the difference between a proof and a demonstration?

Is it possible to prove something in more than one way?

Can you prove that there are 360° in a full turn?

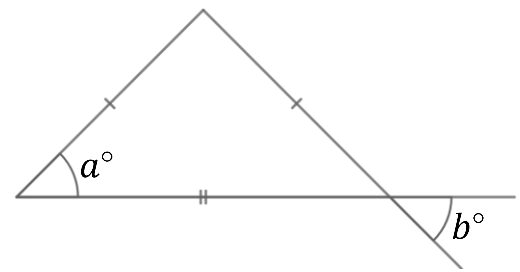
Exemplar Questions

Write down as many equations as possible that you know to be true using the diagram below. You must provide reasons for each equation.



Use your equations to prove that the exterior angle of a triangle is equal to the sum of the two opposite interior angles.

Prove that angle a is equal in size to angle b in the diagram below. Explain your reasoning in full.



By considering a convex kite as two different isosceles triangles. Prove that a convex kite has a pair of opposite angles that are equal.

Does your proof work for a concave kite? If not, can you find a proof that will work for all kites?

Summer 2: Reasoning with Number

Weeks 7 to 8: Developing Number Sense

Students will review and extend their mental strategies with a focus on using a known fact to find other facts. Strategies for simplifying complex calculations will also be explored. The skills gained in working with number facts will be extended to known algebraic facts.

National curriculum content covered:

- consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals, fractions, powers and roots
- select and use appropriate calculation strategies to solve increasingly complex problems
- begin to reason deductively in number and algebra

Interleaving/Extension of previous work

- Generating and describing sequences
- Substitution into expressions
- Order of operations

Weeks 9 to 10: Sets and Probability

FDP equivalence will be revisited in the study of probability, where students will also learn about sets, set notation and systematic listing strategies.

National curriculum content covered:

- record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness, fairness, equally and unequally likely outcomes, using appropriate language and the 0-1 probability scale
- understand that the probabilities of all possible outcomes sum to 1
- enumerate sets and unions/intersections of sets systematically, using tables, grids and Venn diagrams
- generate theoretical sample spaces for single and combined events with equally likely and mutually exclusive outcomes and use these to calculate theoretical probabilities
- appreciate the infinite nature of the sets of integers, real and rational numbers

Interleaving/Extension of previous work

- FDP equivalence
- Forming and solving equations
- Adding and subtracting fractions

Weeks 11 to 12: Prime Numbers and Proof

Factors and multiples will be revisited to introduce the concept of prime numbers, and the Higher strand will include using Venn diagrams from the previous block to solve more complex HCF and LCM problems. Odd, even, prime, square and triangular numbers will be used as the basis of forming and testing conjectures. The use of counterexamples will also be addressed.

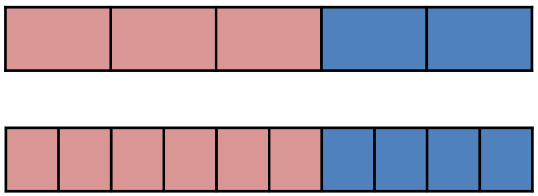
National curriculum content covered:

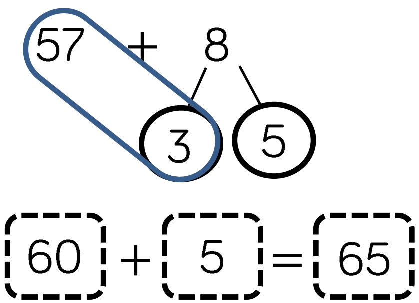
- use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation property
- use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5
- make and test conjectures about patterns and relationships; look for proofs or counterexamples
- begin to reason deductively in number and algebra

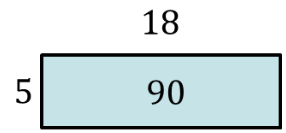
Interleaving/Extension of previous work

- Generating and describing sequences
- Factors and multiples

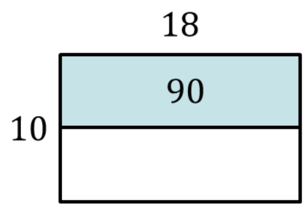
Key Representations



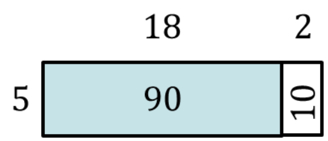




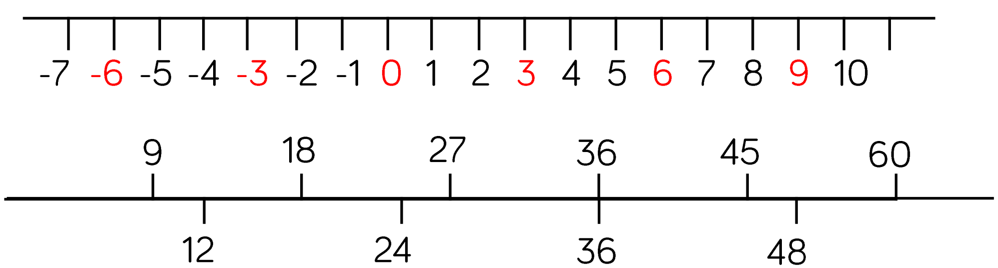
$$10 \times 5 = 50$$
$$8 \times 5 = 40$$
$$50 + 40 = 90$$



$$18 \times 10 = 180$$
$$180 \div 2 = 90$$



$$20 \times 5 = 100$$
$$2 \times 5 = 10$$
$$100 - 10 = 90$$



Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Here are a few ideas for how you might use representations of number to develop reasoning skills.

Many of the small steps are linked to area of a rectangle to support understanding.

For example:
18×5 can be calculated in many different ways. It could be partitioned into 10×5 and 8×5 or 18 could be halved and 5 could be doubled to change the calculation to 9×10

Developing Number Sense

Small Steps

- Know and use mental addition and subtraction strategies for integers
- Know and use mental multiplication and division strategies for integers
- Know and use mental arithmetic strategies for decimals
- Know and use mental arithmetic strategies for fractions
- Use factors to simplify calculations
- Use estimation as a method for checking mental calculations
- Use known number facts to derive other facts
- Use known algebraic facts to derive other facts
- Know when to use a mental strategy, formal written method or a calculator

Mental addition and subtraction

Notes and guidance

This small step is for students to understand the properties of addition and subtraction, and how these can be used to simplify mental strategies in calculations. The explicit use of the vocabulary commutative and associative is important in ensuring that students approach calculations appropriately and with flexibility. Techniques such as the 'make ten' strategy highlight useful shortcuts to simplify calculations.

Key vocabulary

Compensation	Number Line	Addition
Subtraction	Associative	Commutative

Key questions

How can you check answers to subtraction problems using addition?

Can you explain why addition is commutative using concrete manipulatives? Does the same apply to subtraction?

Exemplar Questions

Which of these are true? Explain why.

$37 + 45 = 45 + 37$

$180 - 64 = 180 - 60 - 4$

$24 - 8 = 24 - 10 + 2$

$32 - 9 = 32 - 10 - 1$

$48 + 14 + 16 = 48 + 30$

$24 - 8 = 8 - 24$

$360 - 147 = 360 - 100 - 40 - 7$

$180 - 64 = 124$

Use the 'make 10' strategy to calculate

- $27 + 19 + 33$
- $76 + 34 - 99$
- $28 + 662$
- $342 + 15 + 58$

Tommy suggests three different ways of calculating $68 + 99$

$67 + 100$ | $68 + 100 - 1$ | $60 + 90 + 8 + 9$

All three ways are correct. Explain why. Which one do you prefer?

Using addition and subtraction, how many different ways can you make 37 using the following digit cards:

11 | 18 | 19 | 45 | 8 | 26

Mental multiplication and division

Notes and guidance

Teacher modelling of different strategies to simplify calculations, using concrete and pictorial representations alongside the abstract helps students to develop a flexible approach to problem solving as well as giving them the confidence to choose an appropriate strategy. Partitioning of numbers and using factors to simplify calculations, including spotting multiples such as 5 and 10 are important skills to develop.

Key vocabulary

Partition	Multiply	Divide
Commutative	Associative	Factors

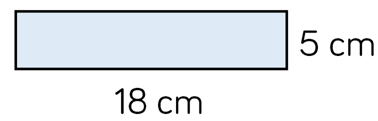
Key questions

What does partitioning mean?

Why do we do some multiplications by portioning and adding, but others by partitioning and subtracting?

Exemplar Questions

To calculate the area of the rectangle, Tommy knows he need to work out 18×5. Below are four different methods of doing this.



A: $10 \times 5 + 8 \times 5$
B: 9×10
C: $20 \times 5 - 2 \times 5$
D: $(18 \times 10) \div 2$

Show that each calculation represents an area of 90 cm^2 by drawing diagrams. Explain why each method works.

The easiest way to calculate $3000 \div 25$ is to start by thinking about how many lots of 25 there are in 100

Whitney

What would Whitney's next steps be to complete the calculation?
Write down a different way of doing this calculation.
Compare the two methods. Which is the easiest one to use? Why?

Which strategy would you use for the following calculations?
Work out the answers using your strategy.

150×4	43×9	17×11	53×98
$2800 \div 14$	$42 \div 3$	$800 \div 20$	$900 \div 15$

Mental strategies for decimals

Notes and guidance

In this small step, students should recognise that previous strategies used to calculate with integers can be extended to decimals.

It's key that students have a sound grasp of place value so that they can use the language of thousandths, hundredths and tenths confidently.

Key vocabulary

Place Value	Estimate	Tenths
Hundredths	Thousandths	

Key questions

How does estimation help us check if answers are reasonable?
Does multiplication always make a number bigger?
Why is multiplying by 0.1 the same as dividing by 10?
Can you just "add a zero" to multiply by 10?

Exemplar Questions

Which strategy would you use to find 2.3 + 2.4? Why?

$2 + 2 = 4$ $0.3 + 0.4 = 0.7$ $4 + 0.7 = 4.7$	Double 2.4 is 4.8 2.3 + 2.4 is 0.1 less than this so the answer is 4.7	Double 2.3 is 4.6 2.3 + 2.4 is 0.1 more than this so the answer is 4.7

Here are two methods for calculating 1.2×0.03

$12 \times 3 = 36$
$1.2 \times 3 = 3.6$
$1.2 \times 0.3 = 0.36$
$1.2 \times 0.03 = 0.036$

$$\begin{array}{ccccc} 12 & \times & 3 & = & 36 \\ \downarrow \div 10 & & \downarrow \div 100 & & \downarrow \div 1000 \\ 1.2 & \times & 0.03 & = & 0.036 \end{array}$$

Compare the methods to calculate:

- 300×0.01
- 0.08×1.1
- 0.4×0.5

Alfie's lunch costs £4.57. He pays with a £10. How much change does he get? Do the following methods work? Explain your answer.

Start with £9.99 and subtract £4.57 Now add one back on to get the answer.

Start with £9.99 and subtract £4.56 This gives the answer.

Now use a number line to work out Alfie's change by counting up from £4.57 to £10. Which of the methods do you prefer?

Mental strategies for fractions

Notes and guidance

This step ensures that students understand the role of the denominator as the divisor in finding a fraction of a quantity. Students understand the idea of sharing a whole to find the value of each equal part and then multiplying to find the required fraction. Use concrete and pictorial representations alongside jottings or mental calculations to support understanding.

Key vocabulary

Whole	Equal parts	Multiply
Numerator	Denominator	

Key questions

Is $\frac{1}{2}$ of an amount always bigger than $\frac{1}{4}$ of an amount?

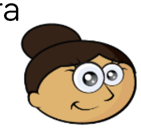
Is it possible to find $\frac{5}{3}$ of a number?

What is the relationship between the denominator, numerator and finding a fraction of an amount?

Exemplar Questions

How much money did Dora have to start with?

Dora

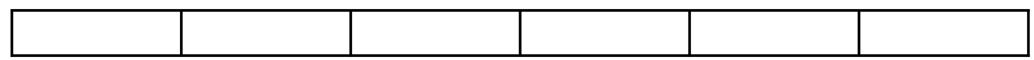


I've spent $\frac{3}{5}$ of my money. I've got £3 left.

A bar model might help you:

Use the bar model to help you find

$\frac{1}{6}$ of 36	$\frac{5}{6}$ of 36	$\frac{1}{3}$ of 36
$\frac{4}{3}$ of 36	$\frac{1}{12}$ of 36	$\frac{12}{12}$ of 36



If the whole were 18 instead, how would this change your answers?
What if the whole were 12? What if the whole were 48?

Special offer: Buy two items and save $\frac{1}{4}$ of the total price.

Ron buys a TV for £200 and a pair of trainers for £55. He only pays $\frac{1}{4}$ of the price. He says the total cost is about £65. Explain whether Ron's answer is:

Too Big | Too Small | Correct

Use factors to simplify calculations

Notes and guidance

Through this step, students will develop flexibility in representing numbers using their factors.

They will be able to choose the most efficient representation in terms of allowing a calculation to be simplified. In particular, looking for combinations of 25 and 4, 125 and 8 etc. is useful.

Key vocabulary

Factor	Equivalent	Calculation
Commutative	Associative	Multiple

Key questions

What numbers are easiest to multiply by?

What factors should you look for to make a calculation easier?

Why does using a different form of the number still give you the same answer?

Exemplar Questions

Rosie thinks 16×20 can be written in lots of different ways.
She writes down the following ways

A: $16 \times 2 \times 10$

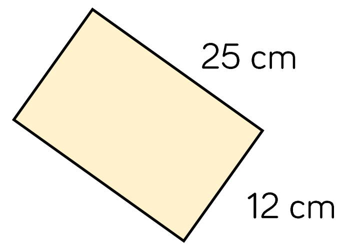
B: $8 \times 2 \times 2 \times 10$

C: $2 \times 2 \times 2 \times 2 \times 2 \times 10$

D: $4 \times 4 \times 2 \times 10$

Explain why you agree or disagree with each factorisation.
Which calculation would you choose to work out 16×20

Find the area of the rectangle.



By using factors of 12 and 25, how many different ways can you find to calculate 12×25?

How many different ways did your whole class find?

36×15 | $18 \times 2 \times 15$ | $18 \times 3 \times 10$

$6 \times 6 \times 10 \times 5$ | $2 \times 9 \times 3 \times 10$ | $36 \times 3 \times 5$

Which calculation is the odd one out? Why?
Which calculation is equivalent to 36×15?

$16 \times 75 = 4 \times 4 \times 25 \times 3 = 4 \times 100 \times 3 = 1200$

Use a similar strategy to work out:

- 125×12
- 28×75
- 125×80
- $36 \times 25 \times 20$

Estimation

Notes and guidance

Students need to be challenged to find the most appropriate estimate in different contexts, it is not always suitable to round to 1 sf. Students should consider whether their rounding will lead to overestimates or underestimates. Rounding to one significant figure should however be revised, including working with numbers less than 1

Key vocabulary

Rounding	Place Value	Significant Figures
Estimate	Overestimate	Underestimate

Key questions

Why is estimation useful?

Is estimating the same as rounding?

Is estimating the same as approximating?

Exemplar Questions

Sam said the following is true because the numbers are too small.

$325 + 773 < 1200$

Help Sam explain his reasoning by using estimation to argue why the statement must be true.

By rounding each number to one significant figure, find estimates to these multiplications.

58×19 $\quad$ 64×23 $\quad$ 57×22

Is it possible to tell whether your estimates are too large or too small? How do you know?

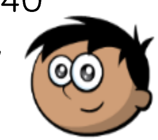
Use estimation to decide whether these statements are true or false.

862×0.9 $\quad$ $296 \times 0.78 < 250$ $\quad$ $8716 + 12\,423 < 15\,000$

$\frac{2}{5}$ of $19\,800 > \frac{1}{3}$ of $17\,900$ $\quad$ $37 \div 3.68 < 4.86 + 6.71$

Amir receives £80 for his birthday. He wants to buy as many of his favourite author's books as possible. Each book costs £7.40

Amir uses £8 as an estimate for each book instead of £7
Do you agree with Amir's strategy? Why?



Number facts to derive other facts

Notes and guidance

Students need a firm understanding of the structure of the an operation (e.g. addition) in order to manipulate this to find other facts. It is important to involve all students in discussing their approaches to a question (possibly through 'talking trios'). Setting one question and asking students to share their approaches, allows the class teacher to model these, thus sharing ideas and encouraging flexibility when approaching calculations.

Key vocabulary

Equivalent	Addend	Compensate
Product	Quotient	

Key questions

What's remains the same about the question, what's different?
How does multiplying one number in a calculation affect the answer? What about both numbers?
How can I change both numbers in a division but keep the answer the same?

Exemplar Questions

How could you change the calculation but keep the **total** the same?

$231 + 428 = 659$

What happens to the **total** if:
428 is changed to 438?
Both addends are reduced by 1?
Both addends are multiplied by 1000?

$23 \times 42 = 996$

Use this number fact to derive the answers to:

2.3×4.2	996×42
23×21	$996 \div 230$

How many other number facts can you derive from this fact?

Use the fact that $1798 \div 29 = 62$ to find $1798 \div 2.9$

Rosie

The answer must be 6.2. If 29 is divided by 10, then we have to *divide* 62 by 10.

But I think 2.9 will fit into 1798 more times than 29. I think we need to *multiply* 62 by 10 to get an answer of 620.

Mo

Who's right, Rosie or Mo? Explain why.

Algebraic facts to derive other facts

Notes and guidance

Students demonstrate an understanding of the difference between an equation and an expression by being able to identify equivalent facts. This small step also allows manipulation of number facts to be extended to rearranging equations without the need of a formal introduction to this.

Key vocabulary

Equation	Expression	Equal
Equality		

Key questions

Explain the difference between an equation and an expression.

If I double both sides in an equation, is the value of the unknown the same?

What does the $=$ sign mean?

Exemplar Questions

Use the fact $a + b = 6$ to find the values or expressions

$a + b = 6$

$\frac{1}{2}a + \frac{1}{2}b =$

$a + b + 1 =$

$= 600$

$a + b - 1 =$

$= 8$

$0.1a - 0.1b =$

$10a + 10b =$

$2a + b =$

Use the fact $x = 2y + 8$ to find the values or expressions of

$10x =$	$= y + 4$	$= 4y + 16$

If $\frac{n}{4} = 2$ explain whether the following are true or false.

$\frac{n}{4} + 1 = 3$	$n = 8$	$\frac{n}{2} = 1$	$\frac{3n}{4} = 6$

White Rose Maths

Choosing the best strategy

Notes and guidance

In this step, the choice of method and strategy should be the focus rather than final answers. Students should become able to quickly identify whether an efficient mental method should be used, or whether a formal written method is more appropriate. They should also know when to use their calculator and to interpret the calculator display in the units referred to in the problem (e.g. money).

Key vocabulary

Estimate	Mental	Calculator
Formal	Efficient	Interpret

Key questions

Is your mental method more efficient than a written method? Is it quicker or slower than using a written method?

Can you interpret your calculator display in terms of the context of the question?

Can time calculations be done on a calculator e.g. how long is it from 1835 to 1920?

Exemplar Questions

Sort each question into the table depending on whether you would use a formal written method or a mental strategy.

$28.72 + 3.41$	41×15	$6007 - 11$
$3.2 \div 5$	$897 + 398$	$\frac{1}{2}$ of 77.8

Mental Strategy	Formal Written Method

Now calculate the answers using your method.

Did you use the same method?

Which ones are easier to do using a formal written method? Why?

Decide on the most efficient method to solve each problem.

A – Formal written method B – Mental strategy C - Calculator

8 979 people watch a netball match. 5 602 are male. How many are female?

Can Mr Hussein buy 40 bags of crisps if he has £10 and each pack costs 27p?

It is 45 miles from Leeds to Manchester. Can you travel from Leeds to Manchester in 40 minutes if you travel at an average speed of 70 miles per hour?

Sets and Probability

Small Steps

- Identify and represent sets
- Interpret and create Venn diagrams
- Understand and use the intersection of sets
- Understand and use the union of sets
- **Understand and use the complement of a set** H
- Know and use the vocabulary of probability
- Generate sample spaces for single events
- Calculate the probability of a single event
- Know that the sum of probabilities of all possible outcomes is 1

H denotes higher strand and not necessarily content for Higher Tier GCSE

Identify and represent sets

Notes and guidance

In this small step, students begin to systematically organise information into sets using set notation. They can identify members of sets given a description, and describe simple sets given the elements. All should find the idea of a set familiar, much of the language will be unfamiliar and will need revisiting regularly to aid retention.

Key vocabulary

Universal Set	Inclusive	Element
Member	Set	

Key questions

What makes a group of objects a set?

Do sets just have to be numerical?

Can you have a set with an infinite number of elements?

Exemplar Questions

In each case, are the sets A and B the same or different?

- A { 2, 4, 6, 8 } — B { 8, 4, 6, 2 }
- A { -2, -4, -6, -8, -10 } — B { 2, 4, 6, 8, 10 }
- A { names of girls in your class }
- B { names of girls in your school }

List the element of the sets.

Set A: Types of triangle

Set B: Quadrilaterals with at least one pair of parallel sides

Set C: Factors of 30

Given that ξ = { Integers between 1 and 50 inclusive }

List the following sets.

Set A: Multiples of 9 Set B: Factors of 9 Set C: Factors of 100

How would sets A, B and C change if:

ξ = { Integers between 1 and 100 inclusive } ?

ξ = { Integers between 1 and 10 inclusive } ?

Describe these sets in words. Compare your answers to a partner's. Which ones can be described in more than one way?

{ 1, 3, 5, 7, 9 } { 3, 6, 9, 12 } { 1, 2, 5, 10 }

{ a, b, c, d, e, f } $\{-2, -1, 0, 1, 2, 3, 4, 5\}$

Interpret and create Venn diagrams

Notes and guidance

By understanding the structure of a Venn Diagram, students will be able to sort information efficiently, seeing whether sets intersect, or whether they are mutually exclusive. Linking this to probability can help students to understand how Venn diagrams can be used as a strategy in working out answers to other problems.

Key vocabulary

Venn diagram	Intersection
Mutually Exclusive	Union

Key questions

How many circles or ellipses are needed in a Venn diagram?

Do we always need a box around the circles/ellipses? Why or why not?

Do the circles/ellipses always need to overlap? Why or why not?

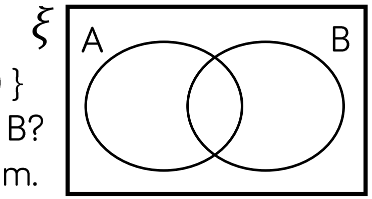
Exemplar Questions

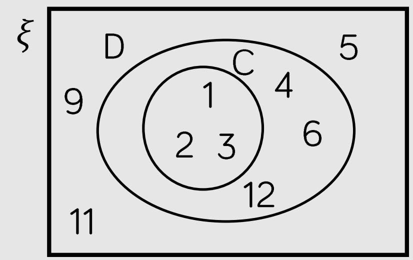
ξ = { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }

A = { even numbers } and B = { factors of 10 }

Which numbers are elements of both A and B?

Represent this information on a Venn diagram.





The Venn diagram shows set C is a subset of set D. All its elements are members of D as well as C.

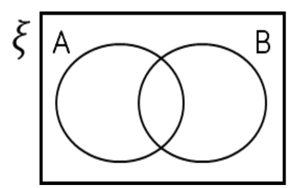
List the members of sets C, D and ξ.

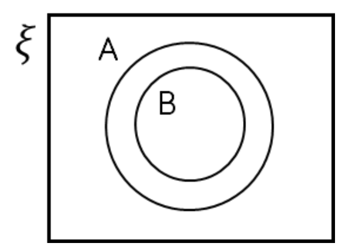
Suggest possible descriptions of each set.

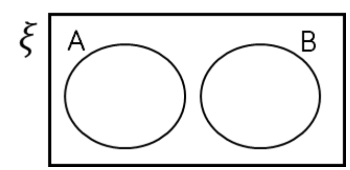
Given that ξ = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }

Choose and complete a Venn diagram to show:

- A = { even numbers } and B = { odd numbers less than 8 }
- A = { even numbers } and B = { even numbers less than 7 }
- A = { even numbers } and B = { square numbers }







Intersection of sets

Notes and guidance

Having already explored the structure of a Venn diagram, students should be able to identify and interpret the part that represents the intersection of two or more sets. Using colour to highlight these areas is an effective way of finding this. Students need to be explicitly taught to associate the word 'and' with intersecting sets. Exploring the intersection of areas where both sets are not true (e.g. doesn't like singing or guitar) extends thinking on this topic.

Key vocabulary

Venn Diagram	Intersection	Element
Intersect	Complement	And

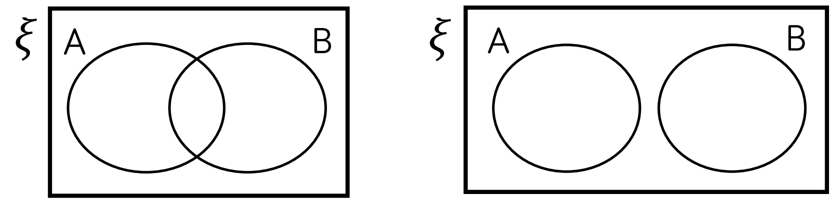
Key questions

What's the same and what's different about the following Venn diagrams?
Why do you think we use different Venn diagrams for different problems? Do all sets intersect? What does the overlapping region represent?

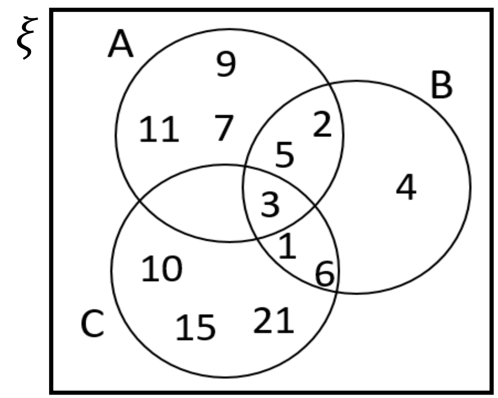
Exemplar Questions

Which Venn diagram would you use to represent sets A and B?

- A = { 5, 10, 15, 20, 25, 30 }
- B = { 1, 2, 3, 5, 6, 10, 15, 30 }



Write down the set A ∩ B.



Write down the elements in the sets:

- A ∩ B
- A ∩ B ∩ C
- B ∩ C
- A ∩ C

Martin says that A ∩ C is exactly the same set as A ∩ B ∩ C. Is he right?
Explain your answer.
Describe in words the sets A, B and C.

In a group of 100 children, 30 like singing, 40 play the guitar, and 20 like singing and play the guitar.
Draw a Venn diagram to represent this information.
How many children don't like singing and don't play the guitar?

Union of sets

Notes and guidance

The step is to support students to distinguish between the union of sets – members belong to A or B or both – from the intersection covered in the previous step.

Labelled Venn diagrams and the use of colour are useful representations to develop understanding. Students need to be explicitly taught to use the words 'and' and 'or'.

Key vocabulary

Union	Or	Element
Intersection	And	Both

Key questions

What's the same and what's different between the union of sets and the intersection of sets?

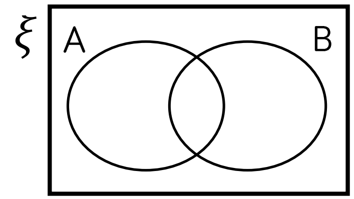
What does the union of two sets look like if they have no intersection?

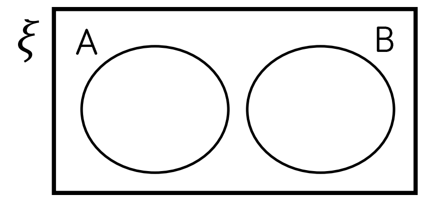
What's the same and what's different about A ∪ B in these situations?

Exemplar Questions

Which Venn diagram would you use to represent sets A and B?

- A = { -10, -8, -6, -5, -4, -2, 0 }
- B = { -2, -1, 0, 1, 2 }





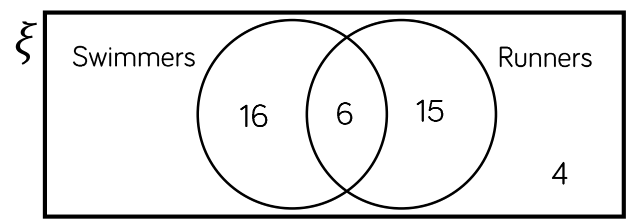
Write down the sets A ∪ B and A ∩ B. Why are they different?

If A = { factors of 36 } , B = { multiples of 3 between 1 and 20 }

Draw a Venn diagram to represent this information.

List:

A ∪ B A ∩ B



The Venn Diagram shows how many students in a class to some sports.

- How many swimmers are there?
- How many students swim or run?
- Using S and R to represent the sets, write the above using set notation.

The complement of a set H

Notes and guidance

Students need to be taught that the complement of a set is the members of the universal set that are not members of the set. Matching activities where children have to match complements of sets to the pre-shaded Venn diagrams help to embed this understanding. Students could be introduced to the notation A' to represent the complement of set A.

Key vocabulary

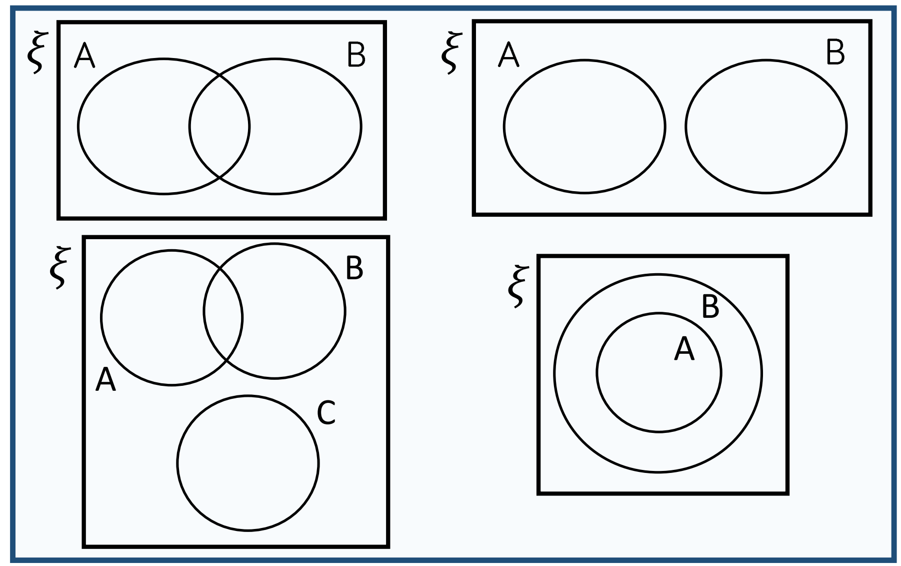
Complement	Member	Element
Universal Set	Not	

Key questions

Do all sets have a complement?
What is the relationship between the complement of a set, the set itself and the universal set?
Can a set whose elements are not numbers have a complement?

Exemplar Questions

Copy each Venn diagram. On each one shade in the area representing the complement of set A.



If ξ = { Integers between 1 and 20 }
Write down the elements in the complements of the following sets.

- A = { factors of 15 }
- B = { square numbers }
- C = { numbers in the sequence where the nth term = 2n + 1 }

ξ = { Integers between 1 and 20 }. If the complement of set X contains the elements { 1, 4, 6, 8, 10,12, 14, 15, 16, 18 }
List the elements in set X. Describe set X

Use the vocabulary of probability

Notes and guidance

Students should be encouraged to think about factors that affect the likelihood of an event happening (such as 'it will rain tomorrow') as this informs their judgements. Sometimes students assume that there is an equal chance of an event happening or not. Exposing these misconceptions by using well chosen examples is crucial (such as scoring a 3 on a die, or not scoring a 3 on a die).

Key vocabulary

Impossible	Likely	Even	Unlikely
Certain	Random	Bias	Event

Key questions

What is the difference between 'almost certain' and 'certain'? Give me examples of events that are 'certain' to happen and those that are 'almost certain'.
Give an example of an experiment with two outcomes that are equally likely. Give an example of an experiment with two outcomes that are not equally likely.

Exemplar Questions

Tommy

Tomorrow I can go to school, or not go to school. There is an even chance of me going to school tomorrow.

Is Tommy right? Explain your answer.

Decide whether these statements are true or false.
Discuss this with your partner. Share your ideas with the class.

If you flip a coin 4 times, you are more likely to get HTHT than HHHH

If you throw a dice and get a 6, you've used up some of your luck so will be less likely to get a 6 next time.

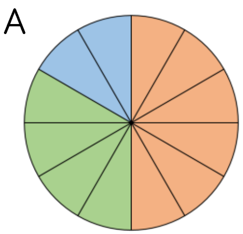
If you flip a coin 8 times and get 8 heads then it is certain that the coin is biased.

It's impossible that I will ever travel into space.

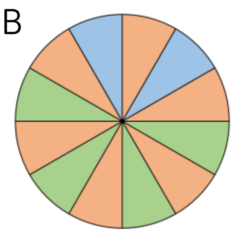
If a test has only true or false questions, it's certain that I'll get at least half of the questions right.

It's certain that I'll watch TV this week.

A



B



Spinner A and Spinner B are the same size. Which spinner is more likely to land on red? Why?

Sample spaces

Notes and guidance

Following on from systematically listing outcomes, students now explore writing exhaustive lists for a single event thus defining a sample space. They also need to recognise whether a list is a sample space or whether elements are missing. This step provides opportunities to link with the concepts of sets and set notation.

Key vocabulary

Sample Space	Possibilities	Event
Outcomes	Element	Set

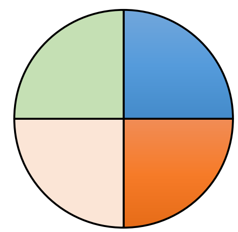
Key questions

How do you know you have a complete sample space?
How does a sample space help you to work out whether something is equally likely to happen or not?
If a sample space has just two possible outcomes, does this mean they are equally likely? If a sample space has 12 outcomes, does this mean they are equally likely?

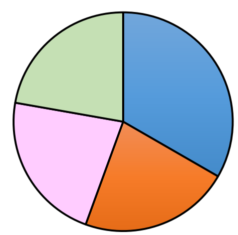
Exemplar Questions

Jack rolls a dice.
The sample space for the possible outcomes is $S = \{ 1, 2, 3, 4, 5, 6 \}$
Write the sample space for the possible outcomes when:

- A coin is flipped
- An eight-sided dice is rolled
- A letter is picked at random from the word MATHEMATICS
- A letter is picked at random from the word PROBABILITY
- The total score when two six-sided dice are rolled



This spinner is divided into four equal sections.
Write down the sample space of the possible outcomes when this spinner is spun.
Do all the outcomes have the same chance of happening?



What's the same and what's different about this spinner?

Design a spinner with six sections so that,

- All the outcomes have the same chance of happening
- All the outcomes have different chances of happening

Probability of a single event

Notes and guidance

In this small step, students need to be taught how to calculate a single probability giving their answer as a fraction, decimal or percentage but not in ratio notation. Vocabulary such as 'random', 'bias' and 'equally likely' needs to be discussed. This is a good opportunity to practise converting fractions decimals and percentages. The fact that probabilities need to be between 0 and 1 needs to stressed, explicitly discussing why a probability of 60% is possible but 60 isn't.

Key vocabulary

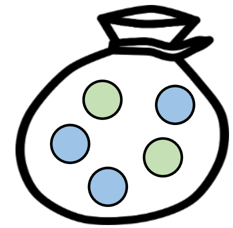
Random	Event	Simplify
Equivalent	Equally Likely	Bias

Key questions

Is $\frac{25}{100}$ a larger probability than $\frac{1}{4}$? Explain your answer.
What does 'random' mean?
Is the probability of rolling a 6 on a dice always $\frac{1}{6}$? Why or why not?

Exemplar Questions

A ball is taken out of this bag at random.
Write down the probability that the ball is blue?
Decide if the probability of selecting a blue
A: stays the same B: increases C: decreases
when,



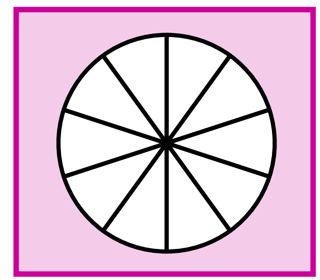
- 5 more blues and 4 more greens are added to the bag
- 6 more blues and 4 more greens are added to the bag
- 1 blue and 1 green are removed from the bag

Justify your answer each time.

Mustafa is making a game using the spinner shown below.

For his game to work, he needs the probability of the spinner landing on:

- an odd number to be $\frac{4}{5}$
- a square number to be 30%
- a number that is 25 or less to be 1



Copy and complete the spinner so that Mustafa's game works.
Make up your own game using an octagon spinner split into 8 sections.

Sum of probabilities

Notes and guidance

This small step requires students to understand that the sum of probabilities for all possible outcomes is 1. They can then calculate unknown probabilities using this fact. Using this same fact, they should also be able to calculate the probability of an event not happening, It is important that students understand that the probability of an event that is certain is 1 Finding unknown probabilities can be linked back to forming and solving equations.

Key vocabulary

Certain	Impossible	Whole
Equivalence	Outcomes	Sum

Key questions

Can a probability be 120%? Why or why not?
Why can't a probability be less than 0?
Why do the sum of probabilities for all possible outcomes add up to 1? Why not 2? or 100?
Why are 100 and 100% different?

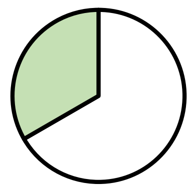
Exemplar Questions

There are some red, blue and green balls in a bag.
The probability of getting a blue ball (when taken at random) is $\frac{3}{10}$

Now think about the probability of getting a red ball and complete the following sentences:

The probability of getting a red ball
- might be……………..
- must be……………..
- cannot be……………..

The probability of a spinner landing on purple is 30%.
The only other two colours on the spinner are yellow and pink.



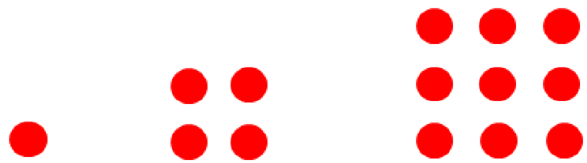
Lami thinks that the probability of a yellow could be 22.7% and the probability of a pink could be 47.3%
Is he right? Explain your answer.
Are there other possible pairs of answers?

Julie randomly selected chocolates from a box containing dark, milk, white and mint chocolates. There is an equal chance of selecting a white or a mint chocolate. Copy and complete the table below to show the probabilities of selecting each type of chocolate:

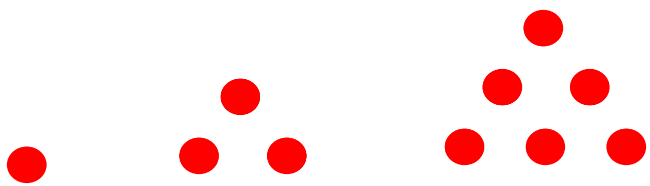
Dark	Milk	White	Mint
0.15	0.35		

What's the probability of Julie selecting a chocolate that isn't mint?

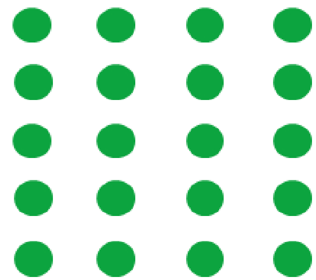
Key Representations



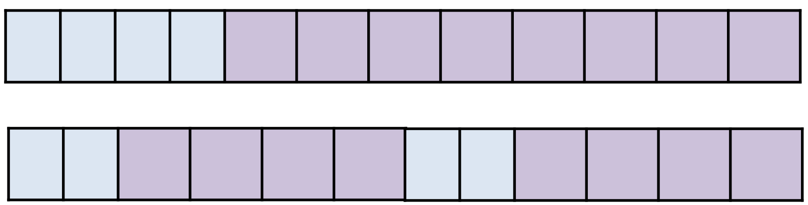
Square numbers

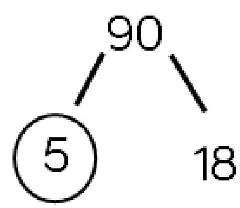


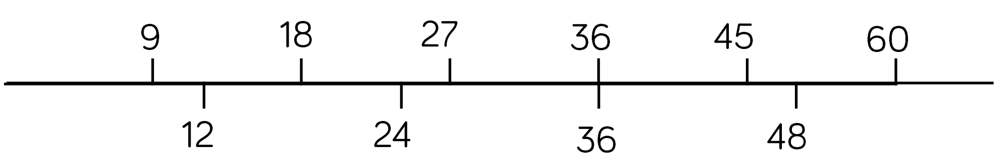
Triangular numbers



Arrays







Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Here are a few ideas for how you might use representations of number to develop reasoning skills.

Many of the small steps are linked to area of a rectangle to support understanding.

For example:
18 × 5 can be calculated in many different ways. It could be partitioned into 10 × 5 and 8 × 5 or 18 could be halved and 5 could be doubled to change the calculation to 9 × 10

Prime Numbers and Proof

Small Steps

- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors of a set of numbers including the HCF
- Find common multiples of a set of numbers including the LCM
- Write a number as a product of its prime factors
- **Use a Venn diagram to calculate the HCF and LCM** H
- Make and test conjectures
- Use counterexamples to disprove a conjecture

H denotes higher strand and not necessarily content for Higher Tier GCSE

Find and use multiples

Notes and guidance

Building on their understanding from KS2, it is important to emphasise that multiples are found by multiplying any number by a positive integer. Skip counting on a number line can support understanding of multiples, as can arrays and Cuisenaire rods. Students often confuse multiples and factors, so this step and the next, need to be planned carefully to overcome this.

Key vocabulary

Multiples	Integer	Positive
Zero		

Key questions

How many multiples of 11 are there?

Can you have multiples of $\frac{1}{3}$?

Does zero have any multiples? Explain your answer.

Exemplar Questions

State whether the statements are true or false. Explain why each time.

4.5 is a multiple of 3

1002 is a multiple of 3

0 is a multiple of 3

Freya's age is a multiple of 7. Next year her age will be a multiple of 5. How old could Freya be?

Freya then says that two years ago her age was a multiple of 4
How old must Freya be?

If x is a positive integer, state whether the following expressions are always, sometimes or never a multiple of 4
Explain your choice of answer each time.

$4x$	$4x + 2$	$4x + 4$	$x + 4$
$x \div 4$	$x - 4$	$8x$	$4x - 4$

Pens come in packs of 8. Pencils come in packs of 6
Mrs Potts buys packs of both and has 120 pens and pencils in total.
How many packs of each did she buy?
Ravi thinks there are 4 different combinations of packs of pencils and pens that Mrs Potts could have bought. Find the four ways.

Factors of numbers & expressions

Notes and guidance

Students will also have met factors at KS2. They should understand that a factor divides exactly into a number with no remainder. Sometimes students do not realise that a number is a factor of itself. Representing numbers as different arrays can help to find the factors of a number.
It is important to distinguish between factors and multiples e.g. 10 is a multiple of 5, but a factor of 20

Key vocabulary

Factor	Divisible	Remainder	Term
Factorise	Divisor	Multiple	

Key questions

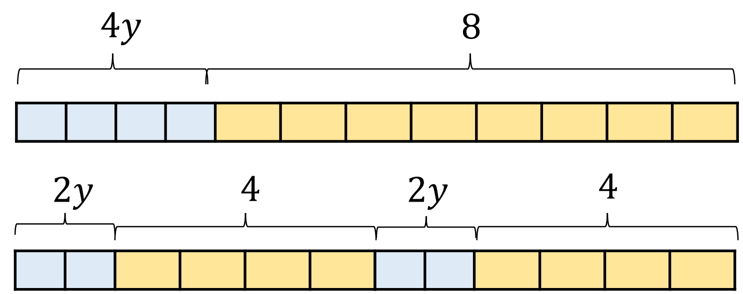
Explain the difference between a factor of a number and a multiple of a number.
Can a number be both a factor and a multiple?
Can zero be a factor of a number?
Can negative numbers be factors of positive numbers?

Exemplar Questions

Use arrays of 24 counters to find all the factors of 24

How many ways can you find to show that 3 is not a factor of 10? What diagrams can you use? What sentences can you write?

Sally uses a bar modelling to represent $4y + 8$



Sally notices that 2 lots of $2y + 4$ is the same as $4y + 8$
She concludes that both 2 and $2y + 4$ are factors of $4y + 8$
Draw another bar model, or an array, to represent different factors of $4y + 8$

In each list, what's the same? What's different?

6	$6x$	$6xy$	$6x^2y$
$2x + 2$	$4x + 4$	$2x - 2$	$8x - 4$

Find the factors of each expression e.g.
$6x = 3 \times 2x$ so 3 and $2x$ are both factors of $6x$
$6x = 3x \times 2$ so ….

Prime numbers

Notes and guidance

Ensure students know that prime numbers are integers greater than 0 that have exactly two factors. Emphasise that the first prime number is 2, as 1 only has one factor.

There is an opportunity to interleave previous topics such as Venn diagrams into this small step e.g. sets that show prime and odd numbers.

Key vocabulary

Prime number	Factor	Odd
Even	Digit	

Key questions

When you add together two prime numbers, do they always give an even number? Explain your answer.

Which large numbers can you tell are not prime just by looking at their digits?

Exemplar Questions

Find the factors of all the integers from 1 to 20
How many of the first 20 integers are prime?
How many have an odd number of factors?
Investigate further.

Explore these statements by substituting in values of x into the expressions. Are Dora and Whitney correct *all* of the time, *some* of the time or *never*? Why?

Dora

That means the expression $2x - 1$ could be a prime number.

Whitney

The expression $2x$ is always even and so is never prime.

Raffle tickets with the numbers 1 to 100 on are placed in a bag.
To play the game, you randomly select a raffle ticket.
You choose the winning criteria. Which would you choose?
Justify your answer.

Win a prize for a multiple of 8	Win a prize for a prime number.

Square and triangular numbers

Notes and guidance

This small step provides opportunities for students to spot patterns and follow a line of enquiry.
Concrete resources and pictorial representations are very useful for this. Students should be encouraged to notice that the sum of two consecutive triangle numbers is a square number, and this can be easily shown with a diagram.

Key vocabulary

Triangular Number	Relationship	Investigate
Square Number	Expression	

Key questions

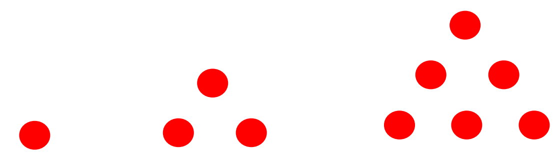
Can a number be both square and triangular?
Why do square numbers always have an odd number of factors?
What's the difference between n^2 and $n + n$?
Can triangular and square numbers be odd or even?

Exemplar Questions

Make the following square numbers using counters.

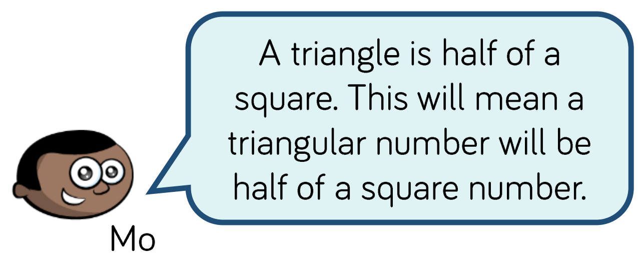
What do you notice about the way square numbers increase.
How many counters will you need for the 10th pattern? 20th pattern?

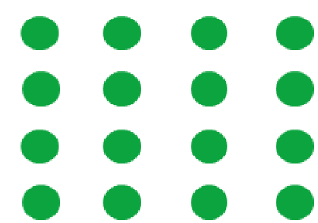
Make the following triangular numbers using counters.



What do you notice about the way triangular numbers increase.
How many counters will you need for the 10th pattern? 20th pattern?

Which triangle numbers can you see within this square number?
Investigate with other square numbers.
What relationships can you find?





Common Factors and HCF

Notes and guidance

Secure table knowledge is beneficial for this step, although it can still be accessed by use of supporting manipulatives. At this stage, we are understanding the idea of HCF rather than exploring algorithms for this. We can extend to looking at common factors of algebraic expressions if appropriate. It is important to include finding the HCF of more than two numbers.

Key vocabulary

Factor	Common Factor	Factorising
Factorise	Highest Common Factor	

Key questions

What number is a common factor of all numbers?
How do we know when we have found the highest common factor?
What do you notice about the HCF of two numbers when one is a multiple of the other?

Exemplar Questions

Draw as many different rectangles as possible, with integer lengths and widths, with an area of:

$12cm^2$ $18cm^2$

Write down the highest common factor of 12 and 18
Use this fact to help work out the highest common factors of these pairs of numbers.

120 and 180 | 6 and 9 | 24 and 36 | 18 and 27

Bob has two pieces of ribbon, one 75 cm long and one 45 cm long. He wants to cut them up into smaller pieces that are all of the same length, with no ribbon left over. What is the greatest length of ribbon that he can make from the two pieces of ribbon?

Keira works out the HCF of some pairs of numbers.

Pair	HCF
12 and 30	6
30 and 60	30
12 and 60	12

Keira says the HCF of 12, 30 and 60 is 30 because it is the highest HCF of the pairs. Do you agree? Why or why not?

Common Multiples and LCM

Notes and guidance

Students will benefit from the modelling of a systematic method of finding the LCM. It is helpful to make the link to finding the lowest common denominator when adding fractions. It is useful to look at the LCM of more than two numbers and if appropriate, algebraic expressions. Emphasis should be placed on language and student explanation to prevent confusion between HCF and LCM.

Key vocabulary

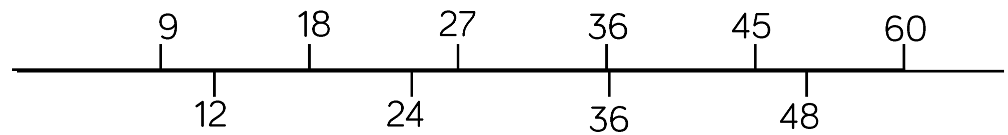
Common Multiple	Product
Lowest Common Multiple	Multiple

Key questions

Why will the product of two numbers be a common multiple of the numbers?
When is the LCM of a set of numbers not the same as their product?
Can the HCF and LCM of a pair of numbers be the same?

Exemplar Questions

Marcel uses number lines to find the LCM of 9 and 12



Explain how Marcel uses this to find the LCM of 9 and 12
Find the LCM of the following numbers.

6 and 8 | 6 and 15 | 6, 8 and 15

Marcel notices that the LCM of 6, 8 and 15 is 5 times as large as the LCM of 6 and 8. Explain why this is true.

Explain how the LCM can be used to compare the size of the following pairs of fractions:

$\frac{3}{5}$ and $\frac{7}{10}$ | $\frac{7}{9}$ and $\frac{5}{6}$ | $\frac{7}{12}$ and $\frac{11}{20}$

What other ways could you use to compare the size of the fractions?

At a bus stop, Bus A arrives every 4 minutes and bus B arrives every 6 minutes. Bus A and B both arrive at 10am.
At what time to Bus A and Bus B arrive together next?
Bus C arrives every 10 minutes.
How many times per hour do buses A, B and C arrive at the same time?

White Rose Maths

Product of prime factors

Notes and guidance

A key concept to cover here is that all non-prime positive integers can be written as a product of prime factors, and that this product is unique. Index form is not required at this stage, but could be explored if appropriate. Students may need help with the language associated with this step, including the command word 'express'. The 'factor tree' method should be distinguished from the familiar additive part-whole model.

Key vocabulary

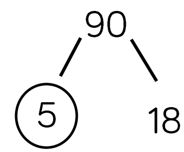
Factor	Prime Factor	Factorise
Product	Express	

Key questions

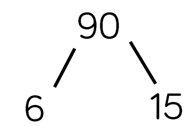
Is there more than one way to factorise 12?
Is there more than one way to express 12 as a product of prime factors? Does the order of the factors matter?
What happens when you find the prime factorisation of a prime number?

Exemplar Questions

Tom knows that $90 = 5 \times 18$
He starts his prime factor tree:



Amir knows that $90 = 6 \times 15$
He starts his prime factor tree:



Complete both trees. What do you notice about your final answers?

List the factors of 80

Express 80 as a product of its prime factors.

Factorise 80

What's the same and what's different about each question?
Which of the questions can you answer in more than one way?
Why is it only possible to express the product of 80 in terms of its prime factors in one way only?

Liam is investigating the prime factorisation of some numbers.
He lists his results so far in a table. Copy and complete the table.

Number	Prediction	How do you know?
12	$2 \times 2 \times 3$	$12 = 4 \times 3$ and $4 = 2 \times 2$ so $12 = 2 \times 2 \times 3$
24	$2 \times 2 \times 3 \times 2$	24 is double 12, so $24 = (2 \times 2 \times 3) \times 2$
72		
36		
144		

Use a prime factor trees to check your answers.

Venn Diagrams to find HCF/LCM

Notes and guidance

Identifying the intersection on a Venn diagram as common elements in both sets reinforces the idea of common factors. This then supports understanding of the calculation for the highest common factor. Finding any common multiples of two numbers using a Venn Diagram, and finally working out a method to calculate the lowest common multiple, supports understanding of the structure underlying this method.

Key vocabulary

Highest Common Factor	Union/Intersection
Lowest Common Multiple	Prime Factors

Key questions

$60 = 2 \times 2 \times 3 \times 5$ $\quad$ $168 = 2 \times 2 \times 2 \times 3 \times 7$

Why don't we write 2, 2, 2, 2, 3, and 3 in the intersect on the Venn diagram?

Why is $2 \times 2 \times 3$ the HCF of the two numbers?

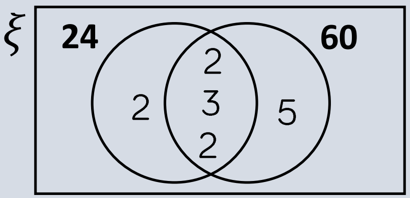
Why is $2 \times 2 \times 2 \times 3 \times 5 \times 7$ the smallest of the common multiples?

How can we find a larger common multiple?

Exemplar Questions

The Venn Diagram shows the prime factors of 24 and 60

- Why does 2 appear twice in the intersection?
- Why does 5 not appear in the circle representing 24?
- How does the intersection help us find the all the common factors, and so the HCF of 24 and 60?
- How can we use the diagram to find the LCM of 24 and 60?



$24 = 2 \times 2 \times 2 \times 3$

$60 = 2 \times 2 \times 3 \times 5$

Express 105 and 120 as products of their prime factors.
Use a Venn diagram to find the HCF and LCM of 105 and 120.
Use your answers to work out:

The HCF of 105 and 60 $\quad$ The HCF of 1050 and 120
The LCM of 105 and 240 $\quad$ The LCM of 105 and 12

Mo expresses two numbers as a product of their prime factors: $\quad 30 = 2 \times 3 \times 5 \quad 36 = 2 \times 2 \times 3 \times 3$

Mo said: "I can use these prime factors to calculate the lowest common multiple of 30 and 36: $2 \times 3 \times 5 \times 2 \times 2 \times 3 \times 3 = 1080$"

Mo

Explain Mo's mistake and find lowest common multiple of 30 & 36

Make and test conjectures

Notes and guidance

Conjectures arise when students notice a pattern that hold for many cases. Students will already have made many conjectures e.g. predicting next terms in sequences, square and triangular numbers etc. Provide opportunities for students to explore the concept of a conjecture by using examples where several conjectures emerge and can be tested.

Key vocabulary

Conjecture	Explain	Relationship	True
False	Proof	Demonstration	

Key questions

How many examples do you need to prove that a conjecture is always true?
Convince me that your conjecture is always true. Give me a mathematical reason.
What's meant by proof?
Why is proof different from demonstration?

Exemplar Questions

Sort these conjectures into: always true, sometimes true, never true.

Birds can fly.

Odd + Odd = Even

1, 2, 4….
The numbers in the sequence are doubling each time.

To find the area of a shape, you multiply length by the width.

In an equilateral triangle, each angle is 60°

$a \times b = b \times a$

Here is a 5-term Fibonacci sequence.

1, 1, 2, 3, 5

Add together the first and last terms in the sequence.
What do you notice about the relationship between this, and the middle term of the sequence?
Repeat with several other 5-term Fibonacci sequences.
Is this result always true? Can you use counters or cubes to prove it?

Sarah finds the HCF of 12 and 18 is 6
She also finds the LCM of 12 and 18 is 36
She notices that $6 \times 36 = 12 \times 18 = 216$
She conjectures that that the product of the HCF and LCM of two numbers is always the same as the product of the numbers themselves. Investigate Sarah's conjecture.

Counterexamples

Notes and guidance

Students should understand the word counterexample as an example that shows a conjecture is false. It is often easier to disprove a conjecture than prove a conjecture, as only one counterexample is needed. It is useful to reinforce the importance of not making assumptions from a limited number of cases. The already familiar 'always sometimes or never true' activities help here.

Key vocabulary

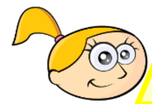
Conjecture	Always	Systematic	Never
Sometimes	Assumption	Counterexample	

Key questions

How many counterexamples do we need to disprove a conjecture?

Is it important to be systematic when looking for a counterexample? Why? What strategies could you use?

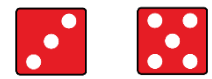
Exemplar Questions



Eva

> $4^2 = 16$
> So, it must be true that if I square a number, the result is always greater than the number I start with.

Write down a counterexample to show that this conjecture is not always true.



Saffi rolls two dice.
She subtracts the scores on the two dice and makes the conjecture:

> The difference between the scores on two dice is always even.

Do you agree, or can you find a counterexample? She conjectures again. Is this conjecture true? If not, find a counterexample.

> If the total of the scores on the two dice is even, then the difference between the scores on two dice is also even.

Ali works out the perimeter and area of this square.
Perimeter = 24 cm
Area = 36 cm^2
He thinks "The perimeter of a square can never be equal to its area".
Do you agree? Justify your answer.