Spring Term

Year 7

#MathsEveryoneCan

2019-20

White Rose Maths

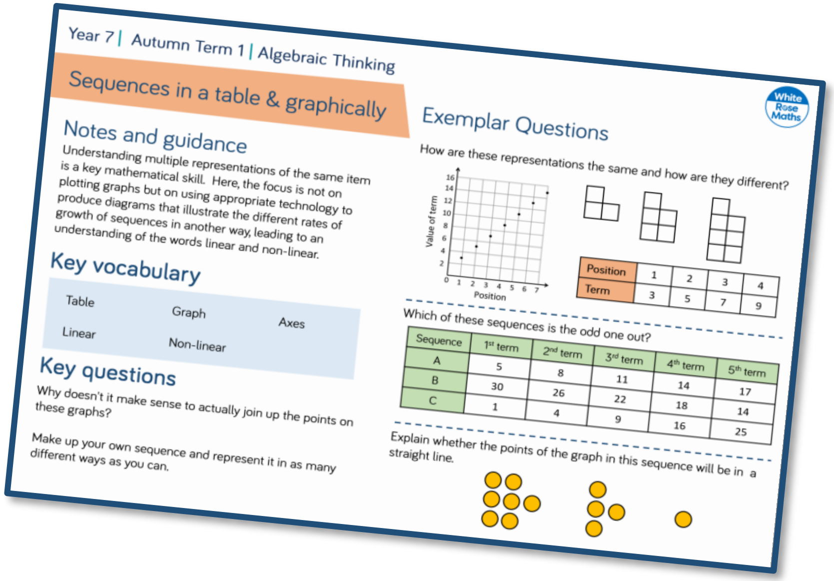
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



Year 7 | Autumn Term 1 | Algebraic Thinking

Sequences in a table & graphically

Notes and guidance

Understanding multiple representations of the same item is a key mathematical skill. Here, the focus is not on plotting graphs but on using appropriate technology to produce diagrams that illustrate the different rates of growth of sequences in another way, leading to an understanding of the words linear and non-linear.

Key vocabulary

Table
Graph
Axes
Linear
Non-linear

Key questions

Why doesn't it make sense to actually join up the points on these graphs?

Make up your own sequence and represent it in as many different ways as you can.

Exemplar Questions

How are these representations the same and how are they different?

Position	1	2	3	4
Term	3	5	7	9

Which of these sequences is the odd one out?

Sequence	1st term	2nd term	3rd term	4th term	5th term
A	5	8	11	14	17
B	30	26	22	18	14
C	1	4	9	16	25

Explain whether the points of the graph in this sequence will be in a straight line.

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol 💡.
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

Supporting resources

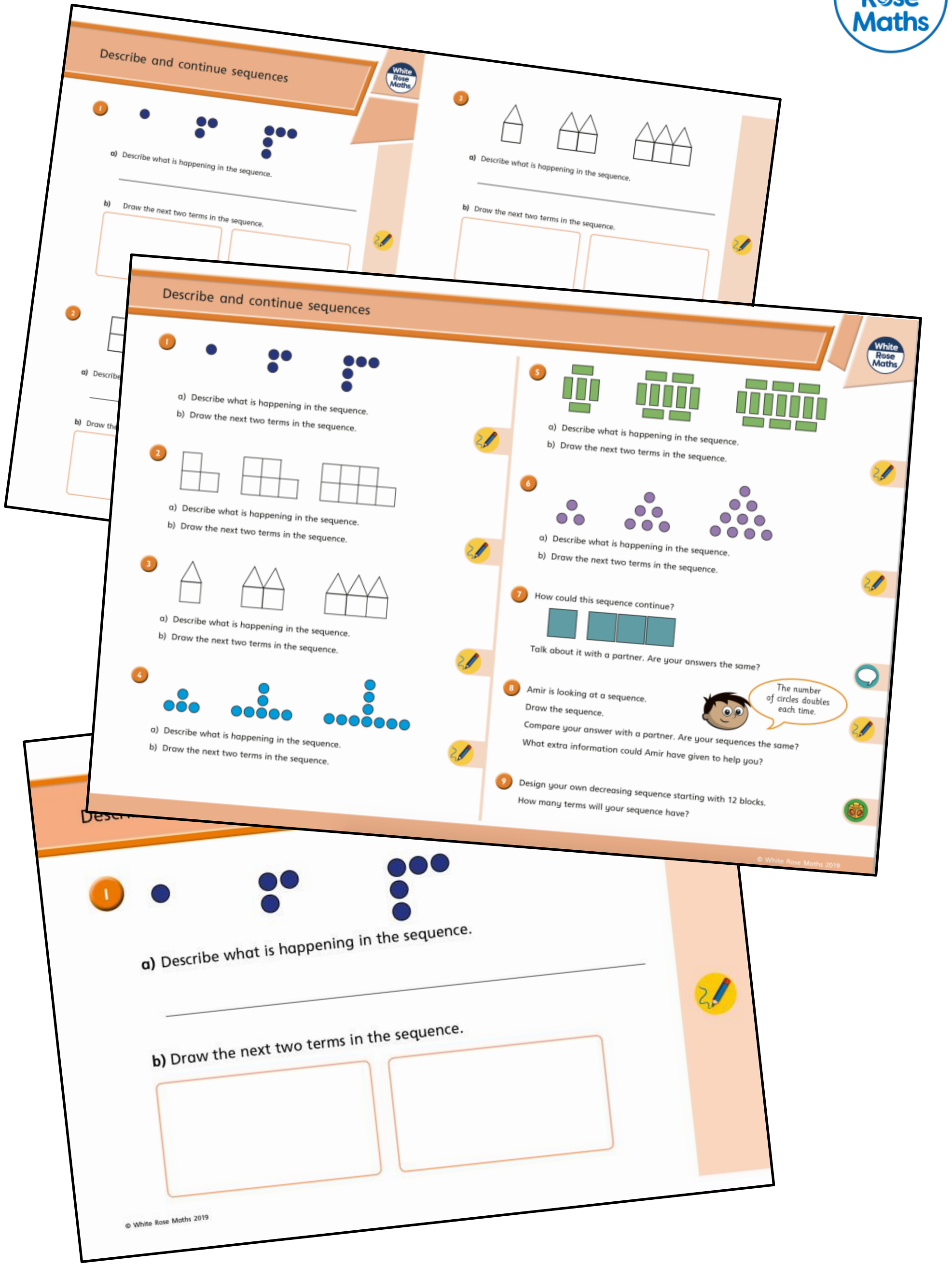
NEW for 2019-20!

We have produced supporting resources for every small step from Year 1 to Year 8.

The worksheets are provided in three different formats:

- Write on worksheet – ideal for children to use the ready made models, images and stem sentences.
- Display version – great for schools who want to cut down on photocopying.
- PowerPoint version – one question per slide. Perfect for whole class teaching or mixing questions to make your own bespoke lesson.

For more information visit our online training and resources centre www.resources.whiterosemaths.com or email us directly at support@whiterosemaths.com



Describe and continue sequences

① a) Describe what is happening in the sequence.
b) Draw the next two terms in the sequence.

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b) Draw the next two terms in the sequence.

⑦ How could this sequence continue?
Talk about it with a partner. Are your answers the same?

⑧ Amir is looking at a sequence.
Draw the sequence.
Compare your answer with a partner. Are your sequences the same?
What extra information could Amir have given to help you?

⑨ Design your own decreasing sequence starting with 12 blocks.
How many terms will your sequence have?

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① a) Describe what is happening in the sequence.
b) Draw the next two terms in the sequence.

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WRM – Year 7 Scheme of Learning

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Algebraic Thinking**						**Place Value and Proportion**					
	Sequences		Understand and use algebraic notation		Equality and equivalence		Place value and ordering integers and decimals			Fraction, decimal and percentage equivalence		
Spring	**Applications of Number**						**Directed Number**			**Fractional Thinking**		
	Solving problems with addition & subtraction		Solving problems with multiplication and division			Fractions & percentages of amounts	Operations and equations with directed number			Addition and subtraction of fractions		
Summer	**Lines and Angles**						**Reasoning with Number**					
	Constructing, measuring and using geometric notation			Developing geometric reasoning			Developing number sense		Sets and probability		Prime numbers and proof	

Spring 1: Application of Number

Weeks 1 & 2: Solving problems with addition & subtraction

The focus for these two weeks is building on the formal methods of addition and subtraction students have developed at Key Stage 2. All students will look at this in the context of interpreting and solving problems, for those for whom these skills are secure, there will be even more emphasis on this. Problems will be drawn from the contexts of perimeter, money, interpreting bar charts and tables and looking at frequency trees; we believe all these are better studied alongside addition and subtraction rather than separately. Calculators should be used to check and/or support calculations, with significant figures and equations explicitly revisited.

National curriculum content covered:
- use formal written methods, applied to positive integers and decimals
- recognise and use relationships between operations including inverse operations
- derive and apply formulae to calculate and solve problems involving: perimeter
- construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts and pictograms for categorical data, and vertical line (or bar) charts for ungrouped numerical data

Weeks 3 to 5: Solving problems with multiplication & division

The rest of the term is dedicated to the study of multiplication and division, so allowing for the study of forming and solving of two-step equations both with and without a calculator. Unit conversions will be the main context as multiplication by 10, 100 and 1000 are explored. As well as distinguishing between multiples and factors, substitution and simplification can also be revised and extended. Again, the emphasis will be on solving problems, particularly involving area of common shapes and the mean. Choosing the correct operation to solve a problem will also be a focus. There will also be some exploration of the order of operations, which will be reinforced alongside much of this content next term when studying directed number.

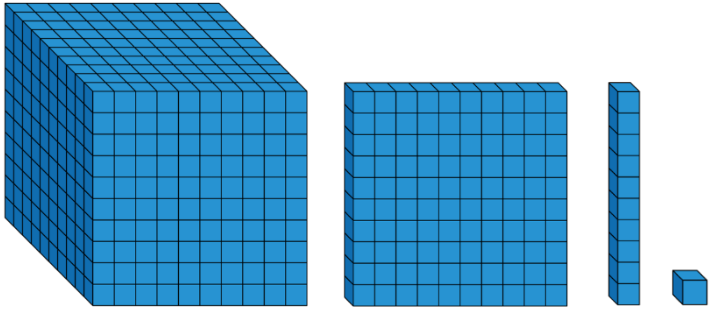
National curriculum content covered:
- use formal written methods, applied to positive integers and decimals
- select and use appropriate calculation strategies to solve increasingly complex problems
- recognise and use relationships between operations including inverse operations
- use the concepts and vocabulary factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple
- change freely between related standard units [time, length, area, volume/capacity, mass]
- derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, and trapezia **(H)**
- substitute numerical values into formulae and expressions, including scientific formulae
- use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement)
- describe, interpret and compare observed distributions of a single variable through: the mean

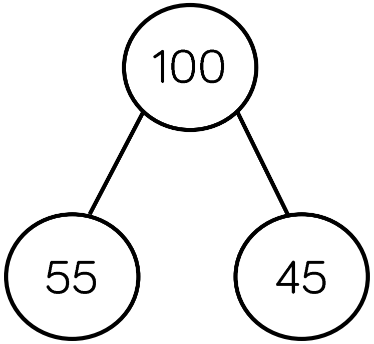
Week 6: Fractions and percentages of amounts

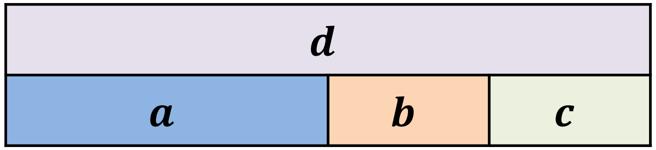
This short block focuses on the key concept of working out fractions and percentages of quantities and the links between the two. This is studied in depth in Year 8

National curriculum content covered:
- use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions
- interpret fractions and percentages as operators

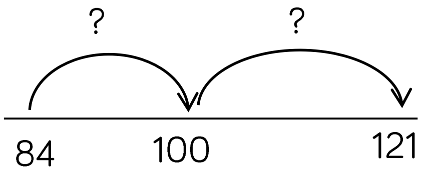
Key Representations

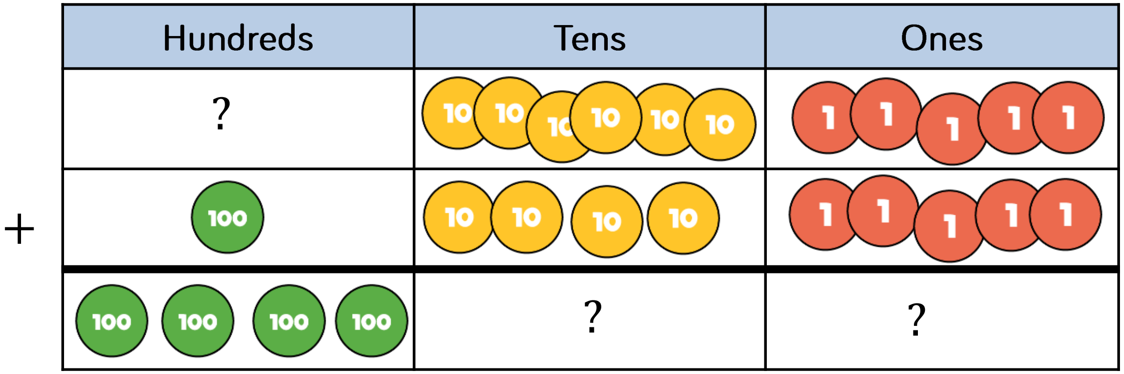






True or False? $a + b = d - c$





Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Number lines are particularly useful for both addition and subtraction and provide a good model of mental methods.

The column methods are sometimes not understood by students and are therefore prone to error. Linking these formal methods to the use of place value counters and/or base 10 blocks illustrating exchanges is very useful.

Addition and Subtraction

Small Steps

- Properties of addition and subtraction
- Mental strategies for addition and subtraction
- Use formal methods for addition of integers
- Use formal methods for addition of decimals
- Use formal methods for subtraction of integers
- Use formal methods for subtraction of decimals
- Choose the most appropriate method: mental strategies, formal written or calculator
- Solve problems in the context of perimeter
- Solve financial maths problems

Addition and Subtraction

Small Steps

- Solve problems involving tables and timetables
- Solve problems with frequency trees
- Solve problems with bar charts and line charts
- **Add and subtract numbers given in standard form** H

H denotes higher strand and not necessarily content for Higher Tier GCSE

Properties of addition & subtraction

Notes and guidance

Students will know from earlier study that addition and subtraction are inverses, and that addition is commutative but subtraction is not. This step reinforces these concepts and the associated language and encourages multiple representations of calculations to deepen understanding. It is useful to extend this to algebraic expressions and also to use the associative law to simplify calculations.

Key vocabulary

Total
Sum
Difference
Number Line
Commutative
Associative
Inverse

Key questions

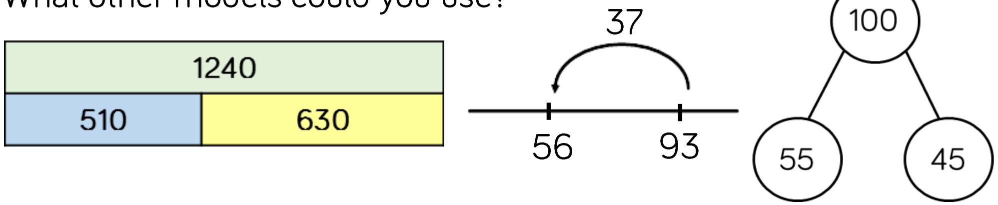
If we know $x = y + z$, what other addition facts do we know? What subtraction facts do we know?

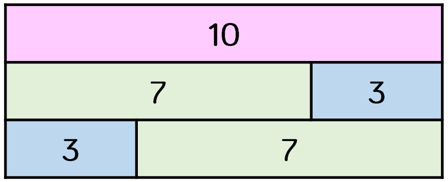
What's the easiest way to add a list of numbers like this: $6 + 8 + 4 + 7 + 2 + 3$?

How could a number line help us to find the difference between, say, 186 and 354?

Exemplar Questions

List all the additions and subtractions that these diagrams show.
What other models could you use?

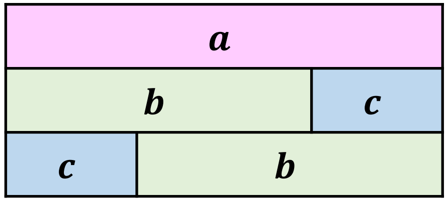




This bar model illustrates that

$$7 + 3 = 3 + 7$$

We say addition is **commutative**.



Generalise the above example using this second bar model.
Is subtraction **commutative**?
Why, or why not?

$$17 + 26 + 14 = 17 + 26 + 14$$
$$(17 + 26) + 14 = 17 + (26 + 14)$$
$$43 + 14 = 17 + 40$$
$$57 = 57$$

This example shows that addition is **associative**.
Which is the easiest way to find the sum of the three numbers?
Why?

Marcel says that addition and subtraction are **inverse operations**.
Use examples and diagrams to explain what this means.

Mental strategies

Notes and guidance

This small step looks at ways students can develop their flexibility and efficiency in mental addition and subtraction calculations. Increased flexibility in their choice of strategy is developed through regular discussion and comparison of different approaches. The use of part-whole models and number lines to illustrate methods will help students' understanding.

Key vocabulary

Bridging
Difference
Compensation
Count On
Partition
Number bonds

Key questions

Make up an example where number bonds to 10 and 100 are useful to perform mental calculations.
How does adding the same number to both parts of a subtraction affect the difference?
Find three ways to mentally calculate $700 - 438$

Exemplar Questions

Here are some ways of working out $78 + 96$

$78 + 96$
$70 + 90 + 8 + 6$
$160 + 14$
174

$78 + 96$
$78 + 90 + 6$
$168 + 6$
174

$78 + 96$
$96 + 78$
$96 + 4 + 74$
$100 + 74$
174

$78 + 96$
$+2 \quad -2$
$80 + 94$
$94 + 80$
174

$78 + 96$
$78 + 100 - 4$
$178 - 4$
174

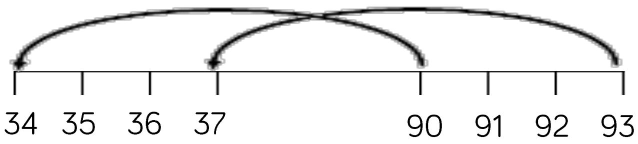
$78 + 96$
$-4 \quad +4$
$74 + 100$
174

$78 + 96$
$78 + 2 + 94$
$80 + 94$
174

Which strategies do you prefer and why?
How would you work out each of these mentally?

$386 + 99$ $57 + 64$ $89 + 254$ $694 + 238$

How does this number line show that $93 - 37 = 90 - 34$?



What strategies would you use to work out these subtractions mentally?

$786 - 299$ $852 - 131$ $81 - 54$ $2000 - 1864$

$97 - 29$ $4378 - 240$ $6502 - 1601$

Formal methods: adding integers

Notes and guidance

For students who are confident with the formal method of addition, this small step will provide practice and revision. Students who find this more challenging should have the opportunity to revisit with concrete materials alongside the formal method to develop their understanding.

Key vocabulary

Column Method	Place Value	Carrying
Exchange	Placeholder	

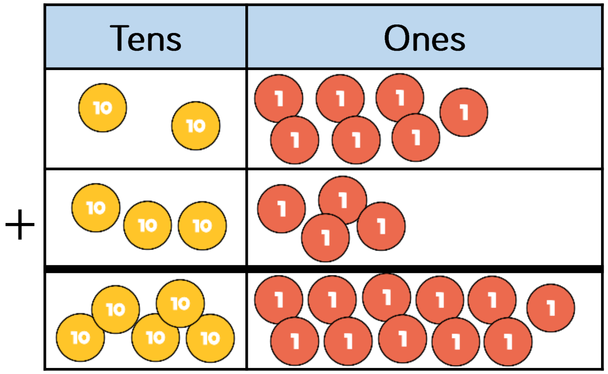
Key questions

Why do we start column addition with the column on the right?

When and why do we exchange in column addition?

Is the column method always the best way to solve an addition problem?

Exemplar Questions



What addition calculation is illustrated here?

What exchange needs to be done to complete the calculation?

Compare this to the formal written method for adding two integers.

Complete these calculations.

	H	T	O
	1	8	7
+	5	4	2

	H	T	O
	2	0	7
+	6	4	3

	H	T	O
	3	8	6
+	2	1	5

What are the similarities and differences between the calculations?

Estimate the answers to these calculations and then use the column method of addition to find the actual answers.

2634 more than 1800

35172 + sixty-seven thousand

485 000 + six hundred and seven thousand

850 000 added to half a million

7648 + 372 + 5063

Formal methods: adding decimals

Notes and guidance

Here students will build on the previous small steps on addition, making use of estimation and the column method paying particular attention to alignment and the use of placeholders. It is also a good opportunity to revisit the meanings of tenths and hundredths and to build on last term's work of decimal and fraction equivalence and earlier work on algebraic substitution.

Key vocabulary

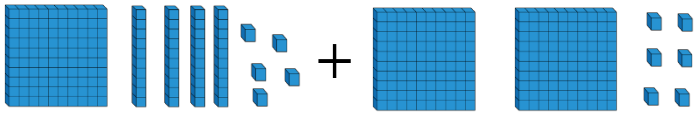
Place value	Decimal point	Equivalence
Place holder	Estimating	Partition

Key questions

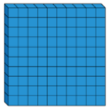
How do we line up decimal addition if one of the numbers is an integer?

What does placeholder mean? Why do we use placeholders?

Exemplar Questions



Write the above representation as an addition using the column method.

Repeat the calculation if  represents 1 instead of 100

What is the same and what is different about your calculations?

Here are 4 ways of using the column method to set up 4.38 + 7.9
Which ones are suitable, and which are not? Why?

4	.	3	8	
	7	.	9	+

4	.	3	8	
7	.	9		+

4	.	3	8	
7	.	9	0	+

4	.	3	8	
7	.	9	+	

Work out the answers to these calculations.

$5.43 + \frac{8}{10}$

$5.43 + \frac{59}{100}$

$5.43 + \frac{3}{4}$

$5.43 + \frac{3}{5}$

Given that $a = 12.6, b = 0.74, c = 20$ and $d = 1.08$, evaluate.

$a + b$

$d + b$

$a + b + c + d$

$a + d + a$

The first term of a linear sequence is 11.3, and the common difference between terms is 4.2
How often will the sequence produce integers?

Formal methods: subtracting integers

Notes and guidance

Following on from pervious steps, the use of the formal method of subtraction needs a good understanding of how and when to exchange e.g. one ten for ten ones. Linking back to concrete and pictorial representations may be necessary for some students. Setting questions in the context of equations and checking by addition will reinforce the concept of inverse operations.

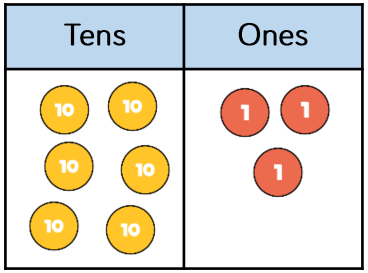
Key vocabulary

Exchange
Placeholder
Difference
Subtraction
Equation
Inverse

Key questions

Why isn't subtraction commutative?
How can we check the answer to a subtraction?
When do we need to perform exchanges when doing a written subtraction?

Exemplar Questions



How would you use place value counters to illustrate $63 - 21$?
How does this compare to the written column method?
Compare the place value and column methods for $63 - 25$

Complete these calculations.

	H	T	O
	6	5	7
–	4	3	2

	H	T	O
	4	2	7
–	2	4	9

	H	T	O
	8	0	4
–	3	1	5

What are the similarities and differences between the calculations?

Use the column method of subtraction to solve these equations. Check your answers using the column method of addition.

$a + 3500 = 8267$

$85172 = b + 27000$

$c = 715\,000 - 67\,800$

$56302 = 28275 + c$

$e + 456\,231 = 1\,00\,0000$

Formal methods: subtracting decimals

Notes and guidance

The clear links to the formal method of subtraction of integers and to the addition of decimals need to be emphasised. In particular, the use of zeros as placeholders is essential. Although the emphasis is on the formal method, it is worth discussing whether alternative methods could or should be used e.g. counting on for change.

Key vocabulary

Place value	Digit	Equation
Placeholder	Difference	Exchange

Key questions

When would it be appropriate to include a hundredths column in a number that is given in tenths?
For what types of subtraction is the formal method most/least useful?

Exemplar Questions

What mistakes have been made in these calculations?

	5	.	8	3
+		2	.	4
	7	.	8	7

	8	.	1	6
–	3	.	5	4
	5	.	4	2

	7	.	6	
–	6	.	5	4
	1	.	1	4

	2	.	3	6
+	3	.	6	7
	5	.	0	3

Work out the correct answers to the calculations.

Solve these equations without using a calculator.

$a + 13.7 = 28.6$

$b - 13.7 = 28.6$

$324 = c + 47.2$

$6.1 = d - 26.97$

Work out the range of the four values a, b, c and d.

Joachim says that to work out £10 – £3.27, you could work out £9.99 – £3.26 instead.
Work out both calculations to show that he is **correct**.
Why does his method work?

Work out the answers to these calculations.

$407 - 126$

$407 - 12.6$

$407 - 1.26$

$6.7 - \frac{1}{5}$

$6.7 - \frac{3}{5}$

$6.7 - \frac{1}{4}$

$6.7 - \frac{3}{4}$

Choosing the appropriate method

Notes and guidance

As well as flexibility in applying methods, students should be encouraged to choose which method to apply in which situation – mental, jottings, formal written, or calculator. The discussion as to which method can draw out, or lead to, understanding of the methods themselves and this is sometimes as powerful as the practice itself.

Key vocabulary

Formal method
Written
Estimate
Jottings
Mental
Calculator

Key questions

How do you decide which method to use to perform a calculation?

Give an example of when a calculator isn't the quickest way to work out an answer.

Exemplar Questions

Estimate the answers to these calculations, and then check your answers using an appropriate method.

$199 + 299$

$£10 - £3.26$

$£3.97 + £4.56$

$685,172 - 491,203$

$0.963 + 0.251$

1.8 million + 5.7 million

Decide whether a mental, written or calculator method would be best for each of the calculations.

- Bashir earned £942.18 one month. He spent £787.40 on rent and bills. How much money did he have left?
- A film starts at 1855 and finishes at 2040
  How long did the film last?
- Mary had £2500 in her savings. She withdrew £850
  How much was left in the bank?
- In 2018, the population of England is 54.79 million. 8,136 million people live in London. How many people live in the rest of England?

Explain why a mental method would be best for these calculations.

$12\,456 + 3999$

$85 + 0.001$

$85 - 0.001$

$12\,456 - 3999$

Solve problems with perimeter

Notes and guidance

Students will be familiar with perimeter from primary school. This small step is an opportunity to revisit the concept and solve addition and subtraction problems in context. This is also an opportunity to revise forming and solving one-step equations and/or simplifying and substituting into expressions.

Key vocabulary

Length	Path	Distance
Units	Edges	Polygon

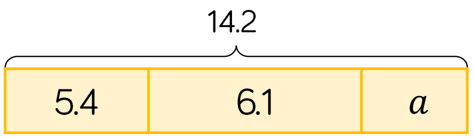
Key questions

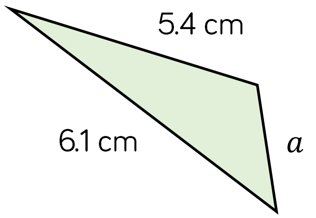
Why is the number of sides on a shape the same as the number of terms in a perimeter addition?

If all the sides of a rectangle are increased by 2 units, how could we know how much the perimeter has increased by?

Exemplar Questions

The perimeter of this shape is 14.2 cm. What is the length of the missing side? How does the bar model help?

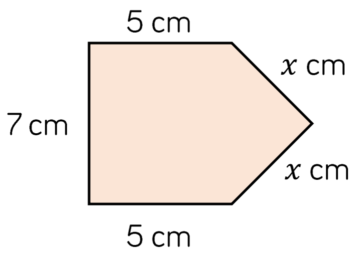




A rectangle has perimeter 20 cm.
If the side lengths are integers, what might the dimensions be?

How many **triangles** with integer side-lengths and a perimeter of 20 cm can be made?
Why is 14 cm, 4 cm, 2 cm **not** a possible combination?

Two sides of an isosceles triangle are 8.7 cm long. If the perimeter of the triangle is 29.2 cm, calculate the length of the third side.



Write an expression for the perimeter of this pentagon.
If the perimeter is 26.4 cm, form and solve an equation to find the value of x.
If instead $x = 4.1$, find the perimeter of the pentagon.

Solve financial problems

Notes and guidance

This small step uses addition and subtraction, particularly in a familiar context whilst also introducing potentially new vocabulary. Students may practise calculator or non-calculator skills as appropriate following previous learning. Estimation and checking answers on a calculator will support entering values some of which are in pounds and some in pence, and interpreting displays such as "14.4"

Key vocabulary

Profit
Debit
Loss
Statement
Balance
Change
Credit
Bill

Key questions

What is the difference between the words credit and debit on a bank statement.?

How do you calculate profit?

Why does a calculator display £12.50 as 12.5?

Exemplar Questions

A bracelet costs £3.99 and a bobble costs £1.29
How much change should there be from £10 If I buy both items?

John spends £112.50 on ingredients and £17.80 on advertising for a cake sale.
He sells all the cakes for a total of £145.12
Does he make a profit or a loss?
How much profit or loss does he make?

Complete the bank statement.

Date	Description	Credit (£)	Debit (£)	Balance (£)
Mar 1	Opening balance			93.68
Mar 3	Gas bill		84.17	
Mar 7	Wages	312.72		
Mar 9	Rent		145.10	

Previous Reading	Current Reading	Unit price
16 851	18 123	12.7p

The table shows part of an electricity bill.
How many units have been used?
If there is a standing charge of £23.56, work out the total bill.

Tables and timetables

Notes and guidance

Reading tables is a key life skill and provides a good context for practising addition and subtraction skills. Calculations with time can create difficulties as students are not used to working with non-decimal contexts. Number lines can be a very valuable support here.

Key vocabulary

- Row
- Hours
- Column
- Minutes
- Entry
- Difference
- Total

Key questions

Does the column method for subtraction work when dealing with time? Why or why not?

Explain how we could use a number line (or time line) to help us with calculations for time.

Is it true that sum of all the row totals in a table equal to the sum of all the column totals? Why or why not?

Exemplar Questions

London			
211	**Cardiff**		
556	493	**Glasgow**	
518	392	177	**Belfast**

This table shows the distance by air between some UK cities.

Hoda flies from London to Belfast and then from Belfast to Cardiff. How far does she fly in total? How much longer is her journey than flying directly from London to Cardiff?

The table below shows part of the results of a survey in a school with 900 students.

	Left-handed	**Right-handed**	**Total**
Girls	34		361
Boys		463	
Total			

Work out the missing numbers in the table. To the nearest whole number, what percentage of the boys are left-handed?

Harton	1005	1045	1130
Bridge	1024	1106	1147
Aville	1051	1133	1205
Ware	1117	1202	1233

Investigate which of the buses shown in this timetable is the quickest or slowest between each pair of towns.

Frequency trees

Notes and guidance

Frequency trees provide a good opportunity for students to practise addition and subtraction in a different context. Although this may be unfamiliar, links can be made to using tables in the previous step , and to the part-whole model. Students can be challenged to create their own frequency tree questions and to investigate the minimum amount of information needed to complete a frequency tree.

Key vocabulary

Frequency	Frequency Tree	Sum
Total	Part-whole	

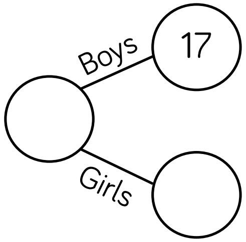
Key questions

Explain the relationships between the numbers in a frequency tree.

When might we have a frequency tree with more than two branches?

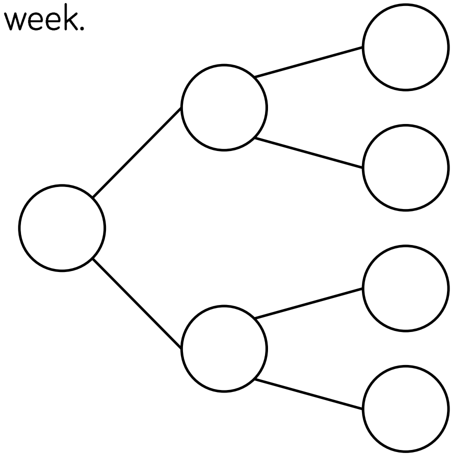
Exemplar Questions

There are 32 students in a class.
17 of the students are boys.
Complete the frequency tree.



80 people took their driving test one week.
45 of the people were men.
28 of the men passed their test.
27 of the women passed their test.
Complete the frequency tree.

How many more men than women did not pass their test?



In a year group of 140 students, 74 are girls.
42 of the students wore glasses, including 23 boys.
Show this information on a frequency tree.

	Men	Women
Suitcase	63	
Rucksack	45	32

The table shows information about luggage carried by 203 passengers on a flight. Complete the table and represent as a frequency tree.

Bar & line charts

Notes and guidance

Students are very familiar with the construction of bar and line charts, so the foci of this small step should be the interpretation of ready-drawn diagrams and linking different forms of charts to tables. As well as opportunities to solve addition and subtraction problems, the notation of scaled axes can be discussed making links with intervals on number lines studied last term.

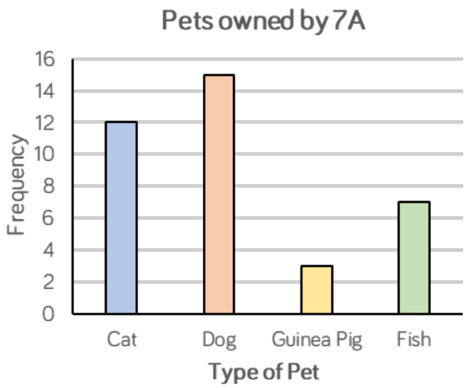
Key vocabulary

Frequency	Axis	Scale
Difference	Dual	Multiple

Key questions

What difference does it make if a bar chart is drawn horizontally rather than vertically?

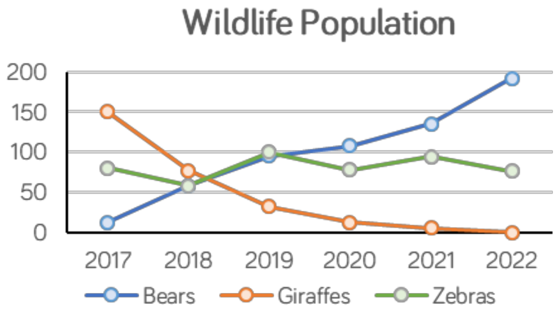
Exemplar Questions



The bar chart shows the number of children in a class of 30 pupils who have various types of pet.
What is the total of the frequencies?
Why is this not total not 30?
What is the difference between the number of pupils with a dog and the number with a guinea pig?
What other questions can you ask?

	11	12	13	Total
Girls	19			44
Boys		13	7	
Total	35	30		

The table shows the ages of girls and boys in a youth group.
Complete the table and represent the information as a multiple bar chart.



The chart shows the expected changes in animal populations in a wildlife park.
Describe the changes, using calculations to justify your findings.

Add & subtract in standard form H

Notes and guidance

In this small step, students will have the opportunity to revisit standard form notation through exploring addition and subtraction, noticing that adding powers is an incorrect approach. It might also be a good opportunity to consolidate knowledge of working with billions and rounding to one significant figure.

Key vocabulary

Standard form	Power	Exponent
Significant figure	Billion	Million

Key questions

Why do you not add/subtract the powers when adding/subtracting numbers written in standard form?

Explain the difference between 10^{-3} and 10^{3}

Exemplar Questions

Round the populations of the three countries shown to 1 significant figure, giving your answers in standard form.

Country	Population	Population rounded to 1sf
New Zealand	4.7 million	
Slovenia	2 100 000	
Djibouti	960 000	
Total		

What is the difference between the totals of the two columns?

Work out each calculation, giving your answer as an ordinary numbers.

$3 \times 10^{5} + 4 \times 10^{4}$

$2 \times 10^{7} - 6 \times 10^{5}$

$7 \times 10^{5} - 6 \times 10^{4} + 8 \times 10^{3}$

$4 \times 10^{-2} + 5 \times 10^{-3}$

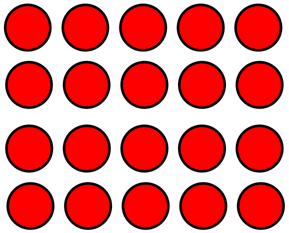
$4 \times 10^{-2} - 5 \times 10^{-3}$

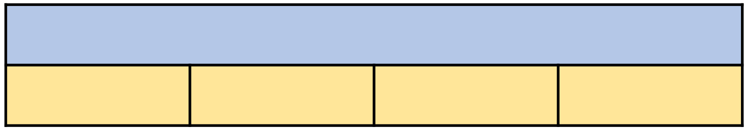
$5 \times 10^{3} + 4 \times 10^{3} = 9 \times 10^{6}$

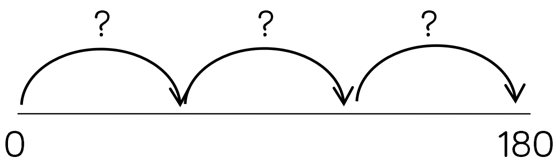
$3 \times 10^{5} + 7 \times 10^{5} = 10 \times 10^{5}$

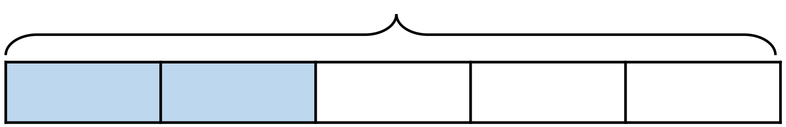
For homework, Jon had to add up pairs of numbers and give the answers in standard form. Explain why he has got both these questions **wrong**.

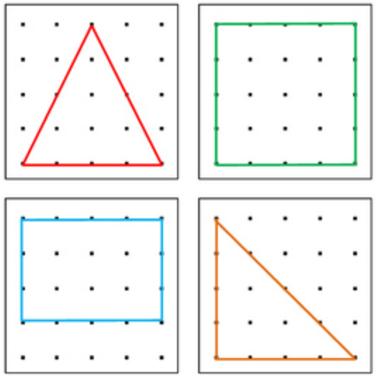
Key Representations

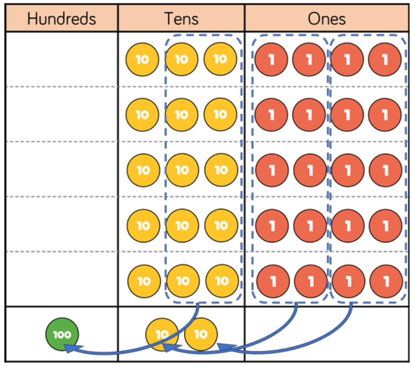


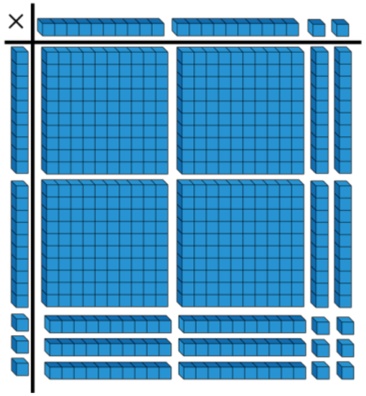












Arrays of counters are very useful to illustrate both multiplication and division, as well as demonstrating the commutativity of multiplication.
Number lines are particularly useful to illustrate the links between multiplication and repeated addition, and division and repeated subtraction.

The column methods are sometimes not understood by students and are therefore prone to error. Linking these formal methods to the use of place value counters and/or base 10 blocks illustrating the result of increasing by factors of 10 are very useful.

Bar models are particularly useful for linking multiplication and division and for calculations involving fractions of amounts.

Multiplication and Division

Small Steps

- Properties of multiplication and division
- Understand and use factors
- Understand and use multiples
- Multiply and divide integers and decimals by powers of 10
- **Multiply by 0.1 and 0.01** H
- Convert metric units
- Use formal methods to multiply integers
- Use formal methods to multiply decimals
- Use formal methods to divide integers
- Use formal methods to divide decimals

Multiplication and Division

Small Steps

- Understand and use order of operations
- Solve problems using the area of rectangles and parallelograms
- Solve problems using the area of triangles
- **Solve problems using the area of trapezia** H
- Solve problems using the mean
- **Explore multiplication and division in algebraic expressions** H

H denotes higher strand and not necessarily content for Higher Tier GCSE

Properties of multiplication & division

Notes and guidance

Students should be reminded of various forms of representing multiplication including those shown and number lines. Discussing scaling models as well as repeated addition would be useful. The inverse nature of multiplication and division should be emphasised, as should the commutativity and associativity of multiplication.

Key vocabulary

Product
Multiply
Divide
Inverse
Quotient
Commutative

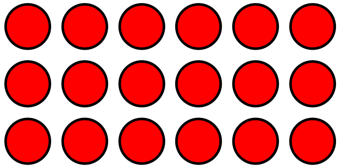
Key questions

If $a = b \times c$ what other multiplication and division facts do we know?

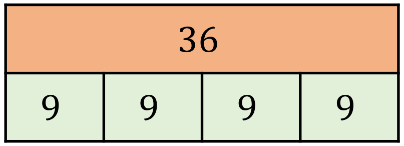
Why is doubling and doubling again the same as multiplying by 4?

Is $\times\ 10$ and then $\div\ 2$ a quick way of multiplying by 5? Find a similar way to divide by 50.

Exemplar Questions



What two multiplications does the array show?
What two divisions does the array show?
Explain how the array shows that multiplication is commutative.
Is division commutative? Why or why not?



Write the fact family for this bar model

Draw a bar models to illustrate these:

$$c \div 3 = d$$
$$5p = g$$

What other facts do your models show?

Write true or false next to each statement.
Explain your reasons for each decision.

- 32 × 4 gives the same answer as 4 × 32
- Lucy says that 125 ÷ 5 is the same as 5 ÷ 125
- 62 = 248 ÷ 4 is another way of writing 248 ÷ 4 = 62

Are these statements always, sometimes or never true?
Give examples to illustrate your decisions.

Multiplication makes numbers bigger

$1 \times y = y$

$a \times b = b \times a$

You cannot multiply or divide by 0

$a \times (b \times c) = (a \times b) \times c$

$a \div b = b \div a$

Understand and use factors

Notes and guidance

This small step is a good opportunity to revisit the concept or check understanding through the use of arrays and area models. It is important to emphasise the need for a systematic approach when recording factors, such as recording factor pairs in ascending order. Explore why a number is not a factor as well as why a number is a factor - again arrays will be helpful here.

Key vocabulary

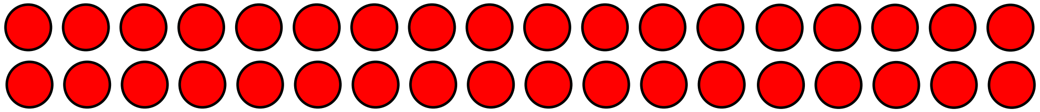
Factor	Array	Venn diagram
Odd	Even	Integer

Key questions

How do you work out the factors of a number?
Which numbers have an odd number of factors? Explain why.
The larger the number the more factors it has. True or false?
Why are factors always integers?

Exemplar Questions

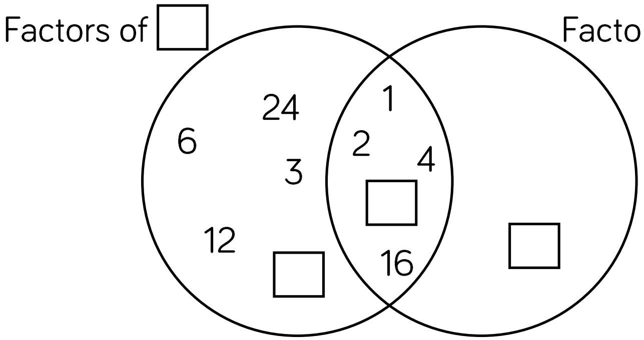
How many different arrays can you make with 36 counters?
For example:



What does this tell you about the factors of 36?
How do you know when you have them all?
Repeat for 24, 17 and 25
What do you notice?

Work out the factors of 30
Explain your method.
What are the factors of 45?
What are the common factors of 30 and 45?
What is their highest common factor?

Here is a part completed Venn diagram containing the factors of two numbers. Work out the missing information.



Understand and use multiples

Notes and guidance

Students need to understand that a multiple of a number is the result of multiplying a number by an integer. Use a bar model to help children see what a multiple looks like. Students may list out the multiples of numbers by multiplying the number by 1, 2, 3 etc… Students need to also be able to work out common multiples of numbers, and also understand the term "lowest common multiple".

Key vocabulary

Multiple

Common

Lowest Common Multiple

Key questions

How do multiples relate to times-table facts?
Is 0 a multiple of every number?
Can multiples be negative?
Do multiples have to be a whole number?
Explain how 18 can be both a factor and a multiple of a number.

Exemplar Questions

Use the diagram to explain why 48 is a multiple of 12

12	12	12	12

Write down 5 other multiples of 12
Write down a multiple of 12 that is greater than 1000.
Explain why 40 is not a multiple of 12

Here is a 50 grid.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Circle all the multiples of 4
Now put a square around all the multiples of 6
What are the common multiples of 4 and 6 less than 50?
What is the lowest common multiple of 4 and 6?
How do you know?

Is this always, sometimes or never true?

> To work out the lowest common multiple of two numbers, you multiply the numbers together.

What is the lowest common multiple of 3, 6 and 9?
What is the next common multiple of 3, 6 and 9?

Multiply & divide by powers of 10

Notes and guidance

Students are first introduced to multiplying and dividing by powers of 10 in KS2. It is important that within this small step teachers check there is conceptual understanding and not just that students rely on a rule or procedure. Using counters and place value grids will help explain that you don't just "add a zero". Particular attention needs to paid to working with decimals.

Key vocabulary

Place value	Ones	Tenths
Hundredths	Multiply	Divide

Key questions

What's the same and what's different about dividing 30 by 10 and 3 by 10?

Why is dividing a number by 10 and then dividing by 10 again the same as dividing the original number by 100?

What's different about multiplying an integer by 10, 100 or 1000 and multiplying a non-integer by 10, 100 or 1000?

Exemplar Questions

Draw counters on each place value grid to show the new number and complete the calculations.

___ × ___ = ___

___ × ___ = ___

What's the same, what's different?

Solve the equations.

$\frac{x}{10} = 5.8$

$100y = 4$

$1.18 \times z = 1180$

Put these calculation cards in order starting with the card that gives the smallest result.

100 × 3.2

320 ÷ 10

3.2 × 1000

3200 ÷ 1000

What is the range of the value of the cards?
What is the median of the value of the cards?

A B C

- B is 10 times bigger than A
- C is 1000 times bigger than A
- What is the value of C ÷ B?

Multiply by 0.1 and 0.01

H

Notes and guidance

Students are already familiar with converting tenths and hundredths between decimal and fractional form, but this next step in understanding can prove challenging. Emphasising the links between fractional and decimal forms is essential. For students following the core strand, it may be best to stick to considering just multiplying by 0.1 To avoid confusion, division by decimals is studied in Year 8

Key vocabulary

Place value	Ones	Tenths
Hundredths	Multiply	Divide

Key questions

What decimal is the same as $\frac{1}{10}$?

How do you find one-tenth of a number?

Explain why $\times\ 0.1$ is the same as $\div\ 10$

Give an example of when multiplication makes a number bigger, and one where it makes a number smaller.

Exemplar Questions

Complete the calculations

$$63 \times 0.1 = 63 \times \frac{1}{10} = 63 \div 10 = \square$$

$$603 \times 0.1 = 603 \times \frac{1}{10} = 603 \div 10 = \square$$

$$603 \times 0.01 = 603 \times \frac{\square}{\square} = 603 \div \square = \square$$

$\times\ 1$	$\div\ 10$
$\times\ 0.1$	$\div\ 100$
$\times\ 0.01$	$\div\ 1$

Match the cards on the left to the equivalent cards on the right.

What card would match with this card?

$\div\ 0.1$

Put the results of these calculations in order, starting with the smallest.

82×0.1	$802 \div 10$	$80.2 \div 100$
8.2×10	$82 \div 100$	80.2×0.01

Convert metric units

Notes and guidance

When students convert metric units they need to understand the different types of metric units - length, mass and capacity. Students need to understand the relative size of these different measures to help them understand the connection between them. This will help them see whether they need to multiply or divide., rather than relying on just remembering; for example 1 m is 100 times as big as 1 cm

Key vocabulary

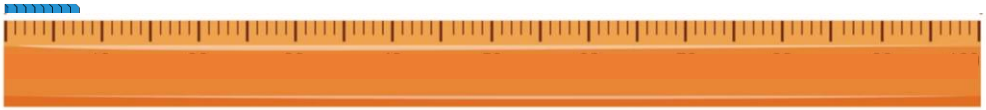
Metric	Milli-	Centi-	Kilo-
Convert	Litre	Gram	Metre

Key questions

What do the words milli-, centi- and kilo- mean?
How do you convert km to m and kg to g? What's the same, what's different?
What do you think a centilitre is? What about a kilolitre?
Do these measurements exist?
Why can you not convert metres to milligrams?

Exemplar Questions

How many ones can you place along a metre stick?
What does this tell you?



Complete each bar model and conversion.

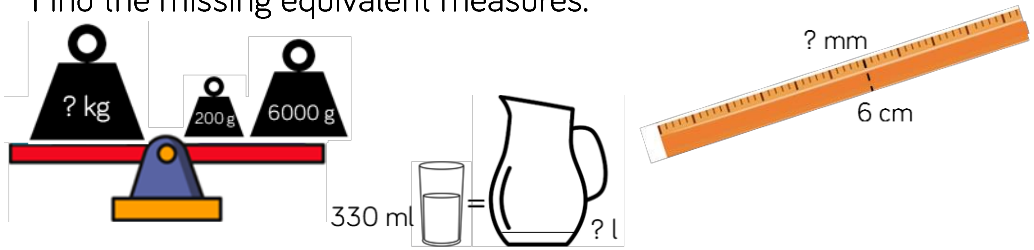
1 km	1 km	1 km	1 km
1,000 m	1,000 m		

4 km = ________ m

1 kg	1 kg	1 kg	1 kg	1 kg	1 kg	$\frac{1}{2}$ kg	
1000 g	1000 g	1000 g					

$6\frac{1}{2}$ kg = ________ g

Find the missing equivalent measures:



Which is the greatest in each pair? How do you know?

20 m or 20 000 cm

3 kg or 30 000 g

0.7 m or 7 cm

0.4 kg or 40 000 mg

60 cl or 6000 ml

Formal methods: multiply integers

Notes and guidance

Students have been exposed to formal methods of multiplication throughout KS1 and KS2, but may not have discussed the conceptual understanding behind each individual method or which method is a more efficient method to use especially when we are using increasingly larger numbers. Revisiting of estimating using rounding to one significant figure is vital here.

Key vocabulary

Multiply	Integer	Product
Efficient	Estimate	

Key questions

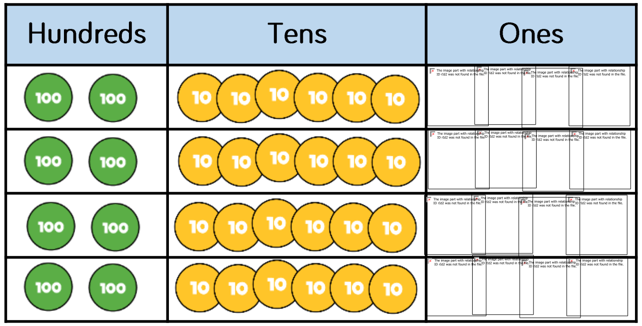
Why would it not be sensible to show 27 × 39 using place value counters?

Is a formal method always the best way to solve a multiplication?

How would you work out 63 × 99?

Why is 36 × 24 ≠ 30 × 20 + 6 × 4?

Exemplar Questions



What multiplication is illustrated here?

What exchanges could you make?

How does this link to the formal column method?

Complete these calculations.

	H	T	O
	1	8	7
×			9

×	100	80	7
9			

	1	8	7
	1	8	7
	1	8	7
	1	8	7
	1	8	7
	1	8	7
	1	8	7
	1	8	7
+	1	8	7

Which is the most efficient method?

Which method is not appropriate for 187 × 56?

By rounding the numbers to one significant figure, estimate the answers to these calculations.

Then use the column method to find the actual answers.

25 times as big as 61

What is the product of 84 and 12?

9 tens and 3 ones multiplied by 235

Formal methods: multiply decimals

Notes and guidance

Students should learn to multiply decimals through using what they have learned about multiplying and dividing by powers of 10. For example when multiplying 0.2 by 0.3 they should think of it as 2 × 3 first then adjust their answer to match the original question. A common mistake here is to think the answer is 0.6. Students should recognise that the calculation has been multiplied by 100 not 10 and therefore the answer should be divided by 100 not 10, hence giving them 0.06

Key vocabulary

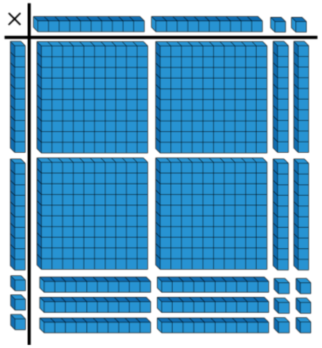
Place value	Adjust	Estimate

Key questions

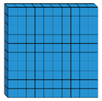
How do you estimate the answer to a decimal multiplication?

Explain why $6.4 \times 24 = 2.4 \times 64$. Tell me three more multiplications using these digits that have the same answer.

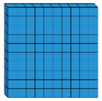
Exemplar Questions



What does the diagram represent

if  is equal to 100?

What does the diagram represent

if  is equal to 1?

Work out 17 × 8. Use your answer to write down the answers to

- 1.7×8
- 0.17×8
- 0.8×17
- 0.8×0.17

Compare these ways of calculating 43×9.9

- Work out 43×99 and divide the answer by 10
- Work out 43×10 and 43×0.1 and subtract the answers

Find the missing number in the calculations using the fact that

58 × 113 = 6554

5.8 × 113 = ☐ 5.8 × ☐ = 65.54 ☐ × 1.13 = 0.6554

Here is a rule for generating a sequence.

Multiply the previous number by 1.6 then add 5

The second term of the sequence is 15

What is the difference between the third and fourth terms of the sequence?

Formal methods: divide integers

Notes and guidance

Students have studied both short and long division at KS2. In this small step they will revisit the formal method of short division, and also consider strategies to simplify complex divisions e.g. $8808 \div 24$ as $8808 \div 6$ and then divide the answer by 4. Problems should be chosen so that answers with remainders and with decimals are appropriate.

Key vocabulary

- Place value
- Divisor
- Dividend
- Quotient
- Remainder

Key questions

The quotient is 7. Make up some questions.
The quotient is 23. Make up some questions.
How do you do about estimating the answer to a division calculation?
Is it possible to divide an integer by a larger integer? Why or why not?

Exemplar Questions

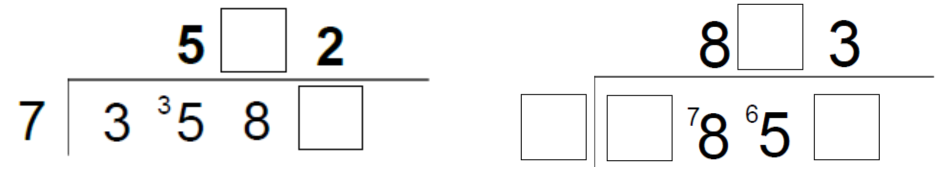
Complete the calculations.

- 135 ÷ 3
- 136 ÷ 3
- 137 ÷ 3
- 138 ÷ 3
- 139 ÷ 3
- 140 ÷ 3

What's stayed the same?
What's changed?
What generalisations can you make?

Predict what would happen when you divide six consecutive numbers by 4 then test if you are correct by completing your calculations.

Find the missing numbers in these calculations.



To divide a number by 18 you can use the rule:

Divide the number by 6 then divide that answer by 3

Use the rule to work out the answer to 387 ÷ 18
Why do you think the rule works? Which other one digit numbers could you have used instead of 6 and 3?

Formal methods: divide decimals

Notes and guidance

The previous step looked at dividing integers by integers. This step builds on this by extending to dividing decimals by integers. Dividing decimals by decimals is covered in Year 8. Students should use the formal method for division used earlier to divide a decimal by an integer. Remind students that a division may be written as a fraction too.

Key vocabulary

Place value

Divisor

Dividend

Quotient

Decimal

Key questions

How do you know $341 \div 2$ will not have an integer answer?

Explain why 341 is the same as 341.0 or 341.00

What type of equations are solved using division? Tell me three examples.

Exemplar Questions

Use formal methods to solve the equations.

$3a = 411$ $\quad$ $3a = 41.1$ $\quad$ $3a = 4.11$ $\quad$ $3a = 0.411$

$6b = 72.6$ $\quad$ $4c = 0.9$ $\quad$ $12d = 96.9$ $\quad$ $e = \frac{36.8}{8}$

Garnel is thinking of a number.
When Garnel multiplies his number by 8 he gets 12.48
What number did Garnel start with?

Complete the missing numbers.

Make up a similar problem of your own.

$$\begin{array}{r} 0\ \square\ 4\ .\ \square \\ 8\overline{)\,\square\ {}^{1}9\ {}^{3}\square\ .\ {}^{4}0} \end{array}$$

---

16 text books cost £61.60 in total. Compare these methods of finding the cost of one text book.

£$61.60 \div 8 \div 2$ $\quad$ £$61.60 \div 4 \div 4$ $\quad$ £$61.60 \div 2 \div 2 \div 2 \div 2$

The books weigh 9.28 kg in total.
Find the weight of three of the books.

Explain your different approaches to perform these calculations.

$34.7 \div 8$ $\quad$ 34.7×8 $\quad$ 34.7×0.8

Order of operations

Notes and guidance

Students have met the order of operations at KS2 but some may be reliant on rules such as BIDMAS and may have misconceptions about when it is correct to work from left to right e.g. 10 – 3 + 5 should be 7 – 5 = 2 but is often incorrectly performed as 10 – 8 "because you have to do addition before subtraction".

Key vocabulary

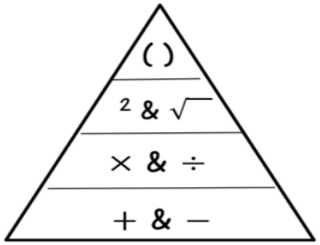
Order

Operation

Priority

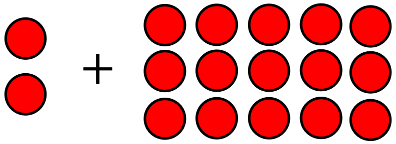
Key questions

Why is multiplication done before addition?
Why do multiplication and division have equal priority?
Explain how this diagram helps you remember which operations come before others.



Exemplar Questions

Explain how these counters show 2 + 3 × 5



What is the answer?
Which part of the calculation did you do first?

Underline the first part of the calculation you will do.
Then work out each calculation.

3 + 5 × 8	3 × 5 – 8	3 – (5 – 8)
72 + 5 × 2	5 × 2 + 72	72 + (5 × 2)

What mistake has been made in each of the calculations?

18 – 10 ÷ 2 = 4

10 – 2 + 4 = 4

What should the correct answer be?

Find the missing numbers

_____ × 3 + 8 = 22

19.8 – 5 + ____ = 11.3

126^2 – _____ = 74

____ × 4 + 10 ÷ ____ = 17

Explain two ways you could work out the answer to this calculation.

39 × 2 + 2 × 17

Is one method more efficient that the other(s)?

Area of rectangles & parallelograms

Notes and guidance

Students should explore the connection between the area of a rectangle and parallelogram. They should see how the area formulae for both shapes are related. You might want to use squared or dotty paper with students so that they can understand why the area of a 6 × 4 rectangle is 24 cm^2 and it will help them see that the area of a parallelogram is base × perpendicular height.

Key vocabulary

Base

Parallelogram

Perpendicular height

Parallel

Key questions

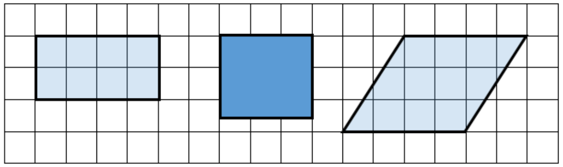
What is the same? What's different about finding the area of a rectangle and parallelogram?

Draw a rectangle with an area of 20 cm^2. Draw a parallelogram with an area of 20 cm^2. Now draw more. What do you notice?

"If the area of the two rectangles are equal, then the perimeters are equal." Always, never or sometimes true?

Exemplar Questions

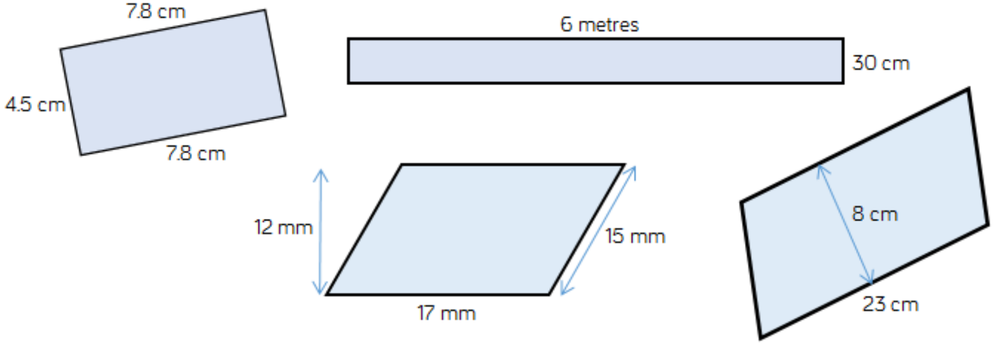
Find the area of each of these shapes.



A rectangle has area 34 cm^2 . Find its width if its length is:

- 17 cm
- 8 cm
- 4 cm

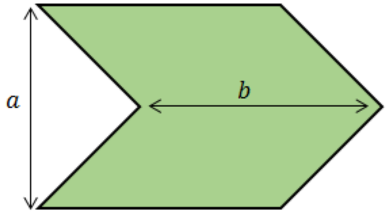
Calculate the area of these shapes.



How many rectangles can you find with an area of 60 cm^2 if the length and width are both integers?

Which rectangle has the greatest perimeter?

Explain why the area of this shape is ab.



Area of triangles

Notes and guidance

The focus of this small step is more on solving problems as they have met the area of a triangle previously. Remind students that the area of a triangle can be found by multiplying the base and perpendicular height and then dividing the answer by 2. Help them understand why they divide by 2 by showing a squares, rectangles and parallelograms divided into two equal sized triangles.

Key vocabulary

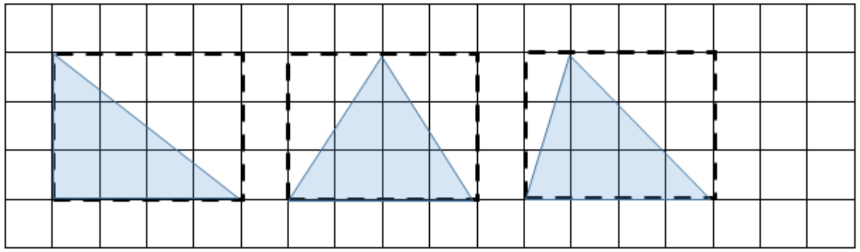
Base
Parallelogram
Perpendicular height
Parallel

Key questions

Explain/show why you need to divide by 2 to find the area of triangle.
What is meant by the perpendicular height?
How do you work out the area of a triangle when the units are different?
How can you show any triangle is half of a parallelogram?

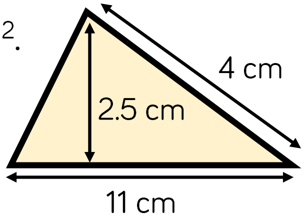
Exemplar Questions

Find the area of these triangles.



What do you notice? Why is this the case?

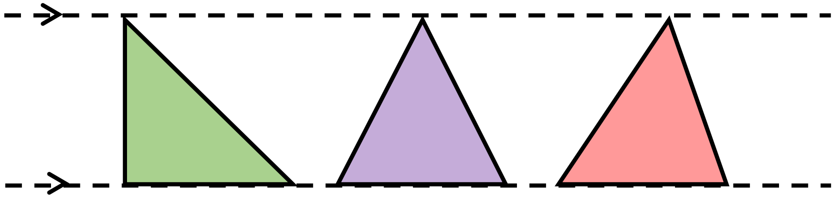
Max says the area of this shape is 22 cm^2.
Explain why Max is wrong.
How can he work out the area of the triangle?



The area of a triangle is 50 cm^2.
What is the height of the triangle if the base and height are equal?
Work out the height of the triangle if the length of the base is

- 10 cm
- 20 cm
- 50 cm
- 1 cm

The base of each of these triangles is equal.
Explain why the area of the triangles is equal.



Area of trapezia

Notes and guidance

Ask students to find the area of a right trapezium. Students will probably divide the shape into a rectangle and a triangle and then add up the answers. Consider then replacing the sides with letters to be able to find a formula for the general area of the trapezium. Explore other trapezia including isosceles, acute and obtuse examples. Students should always be encouraged to find the area of the a trapezium using the formula.

Key vocabulary

Trapezium Perpendicular height Parallel

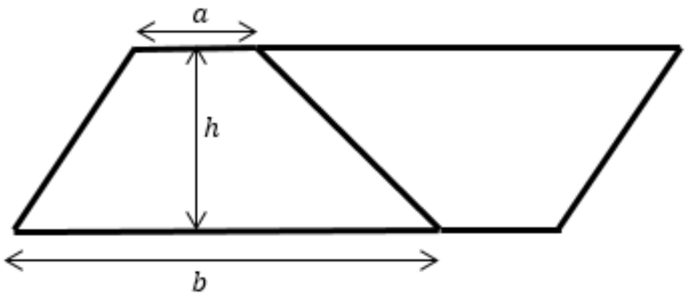
Key questions

What is a trapezium? What are the properties? How many different types of trapezia can you draw/make?

How could you find the area of this trapezium? Can you prove that the area of a trapezium is always $\frac{1}{2}(a+b)h$?

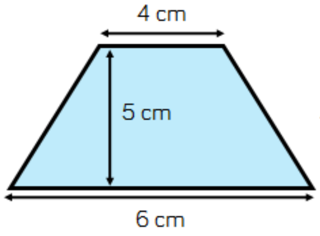
Why is it more efficient to use the formula for find the area rather than dividing it into other shapes?

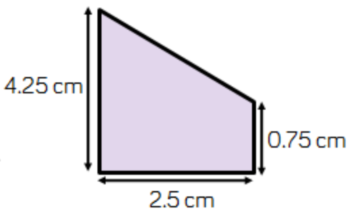
Exemplar Questions

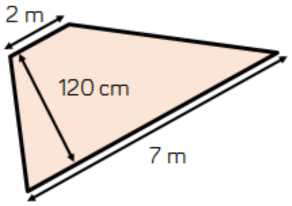


The diagram shows two identical trapezia. Explain why the area of the trapezium is given by $\frac{1}{2} \times (a+b) \times h$

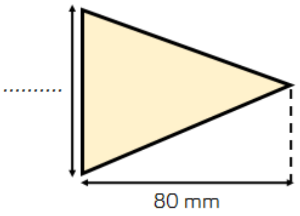
Calculate the area of each shape.

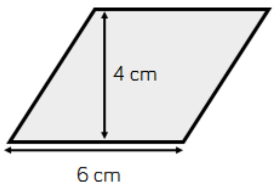


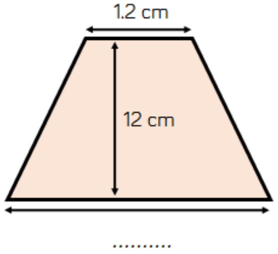




All these shapes have the same area.
Find the lengths of the missing sides.







Solve problems involving the mean

Notes and guidance

Students should understand that the mean of a set of number is an example of an average. The mean gives an idea of the central value. It is important for students to understand visually what happens when you find the mean and how the set of numbers "average out". This will help students when they have to find missing numbers.
Consider also with some students extending finding the mean of a set of numbers that have been summarised in a table.

Key vocabulary

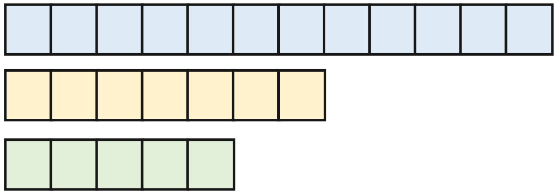
Mean Average Median Range

Key questions

Can you show visually what happens when you find the mean of a set of numbers?
Do you know any other measures of average?
When might you use the mean over the median? When might it be better to use the median rather than the mean?
If you know the mean of a set of numbers, how do you find their total?

Exemplar Questions

Alex has 12 cubes, Bilal has 7 cubes and Carlos has 5 cubes.



What is the mean number of cubes?
Use cubes to help you.
How would you move the cubes to show that the mean is 7?

Find the mean of these sets of numbers.

5, 7, 11, 29, 29 12.8 kg, 32.5 kg, 19.7 kg, 84.6 kg

What do you notice about the mean of all these sets of numbers?

◆	10	10	10	10	10
◆	10	10	10	5	15
◆	50	0	0	0	0
◆	26	20	2	1	1

Write down more sets of 5 numbers that have the same mean.

The mean of these numbers cards is 12
What is the missing number?

19 18 7 ?

The mean of a different 4 cards is 6
The median of the cards is 7
What could the cards be?

Multiplication & division with algebra H

Notes and guidance

Students will already be familiar with substitution into expressions from their study of algebra in the Autumn term. This step builds on this, and gives them the opportunity to look at more complex expressions involving repeated letters and more than one letter. Division of algebraic terms is often neglected, but should be taught alongside multiplication emphasising the inverse nature of the operations.

Key vocabulary

Coefficient
Simplify
Quotient
Term
Expression

Key questions

Why is it possible to simplify $2a \times 3b$ but not $2a + 3b$?

The area of a rectangle is $6xy$. What might the lengths of the sides be?

Why do we write $a \times 2$ as $2a$ instead of $a2$?

Exemplar Questions

If $x = 7$ and $z = 3$, calculate the value of the following expressions:

$$\frac{4x}{z} \qquad z(3x + 4z) \qquad \frac{3}{4}x \qquad x^z$$

Put these expressions in ascending order of size.
Could you change the values of x and z to change the order of the value of the expressions?

Simplify the following sets of expressions. What's the same and what's different?

2×6
$2a \times 6$
$2 \times 6b$
$2a \times 6b$

$15a^2 \div 3$
$15a^2 \div a$
$15a^2 \div 3a$
$15a^2 \div 3a^2$

$20cd \div 4$
$20cd \div 4d$
$20cd \div 4c$
$20cd \div 4cd$

Make up some more sets of related calculations.

Jim says all three of these expressions are the same as they're just the same numbers and letters in a different order. Do you agree?

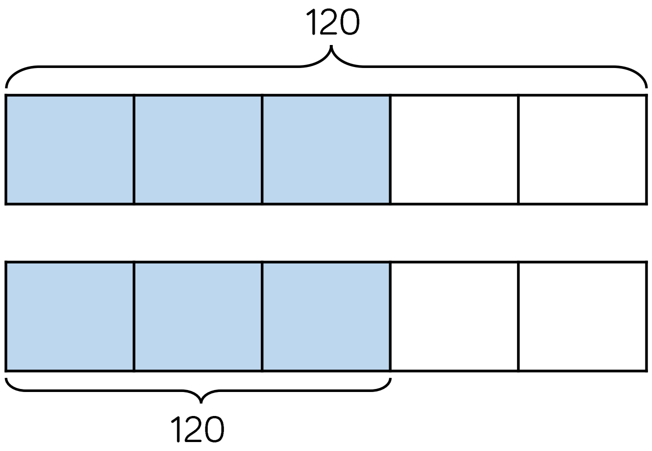
$4ab$ $\quad ab4$ $\quad a4b$ $\quad 2a2b$ $\quad 4ba$

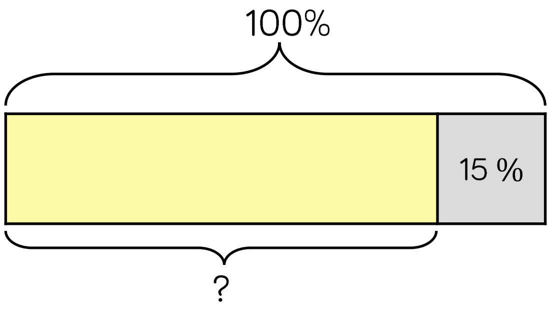
Match the sets of cards that show equivalent expressions

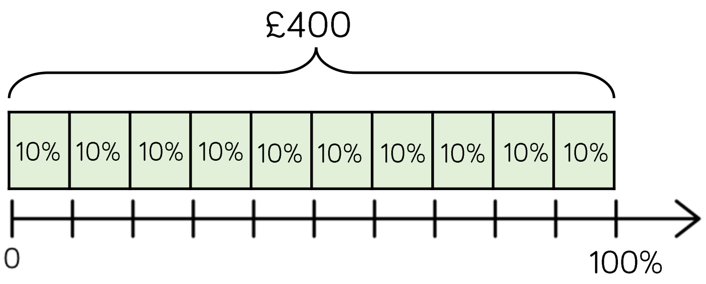
$\frac{24de}{2e}$ $\quad 12d^3$ $\quad 6d^2 + 6d^2$ $\quad 48d \div 4$ $\quad \frac{36d^2}{3}$

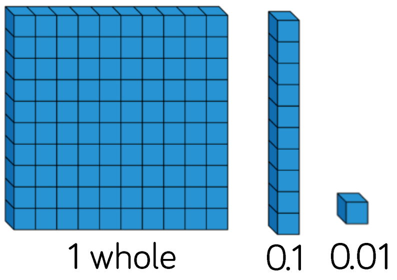
$2d \times 6d$ $\quad d \times 3 \times 4d$ $\quad 4d^2 \times 3d$ $\quad 3d \times 4$

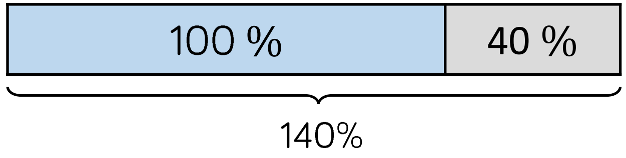
Key Representations

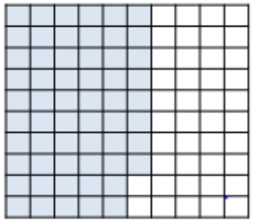












Making links between concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Bar models are particularly useful when working with fractions of amounts. The need for equal parts is emphasised and carefully labelling will help support distinguishing between problems where the whole or where a part is known.

A number line or a bar can also help to visualise percentages. This is especially helpful when working with e.g. multiples of 10%, 20% or 25%. They can also be useful to relate to the whole e.g. showing that to work out 90% of a quantity you can just subtract 10% form the whole rather than multiply 10% by 9

Fractions & Percentages of Amounts

Small Steps

- Find a fraction of a given amount
- Use a given fraction to find the whole and/or other fractions
- Find a percentage of a given amount using mental methods
- Find a percentage of a given amount using a calculator
- **Solve problems with fractions greater than 1 and percentages greater than 100%** H

H denotes higher strand and not necessarily content for Higher Tier GCSE

Fractions of amounts

Notes and guidance

Students should have met finding fractions of an amount throughout primary school. This step provides an opportunity for students to consolidate their understanding and attempt increasingly difficult problems. In order to aid understanding students should be able to represent and see the problem with a bar model They could use comparison bar models to look at e.g. one-third of 90 and two-thirds of 45

Key vocabulary

Fraction
Denominator
Equivalent
Whole
Numerator

Key questions

How do you work out $\frac{3}{5}$ of a number?
Draw a diagram to explain why your method works.

What's the same and what's different about these two questions?

$\frac{2}{3}$ of 60 = ☐ $\frac{2}{3}$ of ☐ = 60

Why is one third of 90 equal to two-thirds of 45?

Exemplar Questions

Use the bar model to help you work out $\frac{2}{5}$ of £95

Work out: $\frac{1}{8}$ of 720 lbs $\frac{3}{8}$ of 720 lbs $\frac{5}{9}$ of 8.19 km $\frac{11}{10}$ of 120 kg

Ron bakes 280 cookies on Monday.
On Tuesday he bakes $\frac{1}{8}$ as many more cookies.
How many cookies did he bake altogether over the two days?

Tommy and Whitney each make a tower made up of red and blue bricks. They each use the same number of blue bricks.

- $\frac{3}{8}$ of Tommy's tower is made up of blue bricks.
- $\frac{1}{3}$ of Whitney's tower is made up of blue bricks.
- Tommy uses 48 red bricks.

How many bricks are there in Tommy's tower?

Sort these cards into pairs with equal values. What do you notice?

$\frac{1}{2}$ of 30	$\frac{2}{3}$ of 60	$\frac{3}{8}$ of 160	$\frac{1}{4}$ of 60
$\frac{6}{7}$ of 210	$\frac{3}{4}$ of 80	$\frac{1}{3}$ of 120	$\frac{2}{7}$ of 630

Find the whole

Notes and guidance

Bar models are again a useful tool for 'working backwards' to find the whole given a particular fraction, either unit or non-unit. As with the previous step, they help make sense of the process involved rather than attempting rote memorisation of division/multiplication by the numerator and denominator. Once the whole is found, other fractions can be easily found. Students can be challenged by considering fractions of increasingly complex expressions.

Key vocabulary

Fraction

Equivalent

Numerator

Denominator

Whole

Original

Key questions

How can I work out a number if I know a fraction of the number?

What's different about these questions?

- What number is half of 12?
- 12 is half of what number?

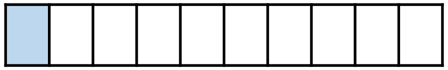
Exemplar Questions

Half of a number is 24. What is the number?

One-third of another number is 24. What is this number?

One-tenth of a third number is 24

What is the third number?

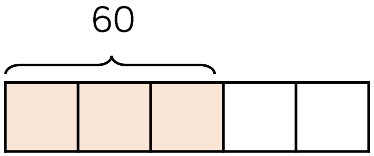


$\frac{3}{5}$ of a number is 60

What is $\frac{1}{5}$ of the number?

What is the number?

Work out the whole number if

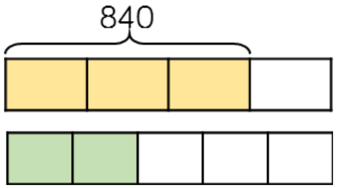


- $\frac{2}{3}$ of the number is 60
- $\frac{3}{4}$ of the number is 60
- $\frac{5}{6}$ of the number is 60
- $\frac{5}{12}$ of the number is 60

Dexter spends two-fifths of his money on a book. He has £15 left. How much money did he have to start with?

$\frac{3}{4}$ of a number is 840

What is $\frac{2}{5}$ of the number?



$\frac{2}{3}$ of an expression is $12x$. What is the expression?

Percentages of amounts: Mental

Notes and guidance

Students should have met finding percentages of an amount in KS2. They are likely to have focused on finding multiples of 5% and 10%, and many will be used to 'build-up' methods from key percentages. It is worth exploring alternative methods and discussing when which method would be appropriate e.g. 95% is best found by subtraction from the whole.

Key vocabulary

Place value

Percent

Percentage

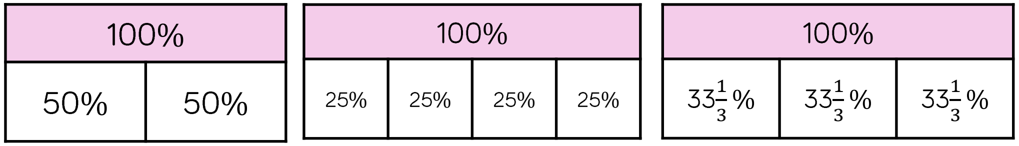
Key questions

Why is it that you divide by 10 to find 10% of a number, but you don't divide by 20 to find 20% of a number?

If you know 10% of a number, what other percentages can you easily work out?

Find as many ways as you can to work out 60% of 45

Exemplar Questions

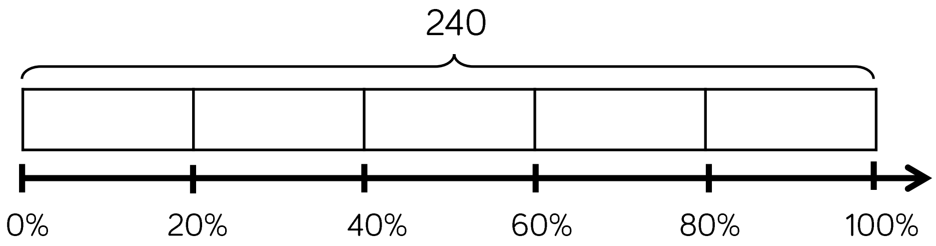


Use the bar models above to explain how you work out:

- 50% of 30
- 50% of 80
- $33\frac{1}{3}\%$ of 90
- 50% of 24
- 50% of 120
- $66\frac{2}{3}\%$ of 18

Draw a bar model that shows you how to work out 10% of a number.
What other percentages can you then work out?

What percentages of 240 can you easily work out from this model?



Compare these (or other) methods to find 45% of a number e.g. 60

10% × 4 + 5%

20% + 25%

50% – 5%

Find the missing numbers in these calculations.
Could there be more than one answer?

20% of ☐ = ☐ of 200

20% of ☐ = 25% of ☐

Percentages of amounts: Calc

Notes and guidance

It is unlikely that students will have used a calculator to find percentages and this is a good opportunity to explore the variety of methods available, including the percentage button. In particular, students should consider when a calculator method is preferable to a mental method. This is a good step to discuss real life percentage problems such as interest rates, commission charges etc.

Key vocabulary

Place value
Decimal
Percent
Convert
Percentage
Equivalent

Key questions

When is it easier to use a mental method rather than a calculator?

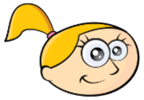
How do you know how to interpret the display on a calculator?

What does the % button on your calculator do?

Exemplar Questions

Jack, Eva and Dora are working out 37% of £680

37% = 0.37, so I did 0.37 × 680



I did 680 ÷ 100 to work out 1% and then multiplied the answer by 37

I did 37 ÷ 100 × 680

Which method do you prefer? Can you find another way?
All three calculators gave the answer 251.6, what is the 6 worth in this number?

Use a calculator to work out these calculations.
What do you notice about your answers? Can you generalise?

34% of 47

47% of 34

Amir puts £300 into a bank account.
At the end of the year, he received 4% interest. How much is this?
How much would he receive if the interest rate was 4.3%?

Mo works out 17% of £84.10 on his calculator.

"It says 14.297.
Does this mean £14, £14.29 or £14.30?"

Which answer would you choose?

Percentages over 100%

H

Notes and guidance

As students understand percentages as 'out of a hundred' there is often confusion about going over 100%. It is therefore a worthwhile discussion as to when it is and isn't appropriate to have percentages over 100%. Bar models can then support finding the total percentage and the decimal conversion. This step is covered again in the core strand in Year 8.

Key vocabulary

Fraction	Equivalent	Numerator
Denominator	Whole	Original

Key questions

Can 110% of the class be absent on one day?

If the price of an item increases by 60%, what percentage is the new price of the old price?

Can a price increase/decrease by 180% or 200%?

Exemplar Questions

Which of these statements make sense, and which are impossible?

I'm going to give 150% effort

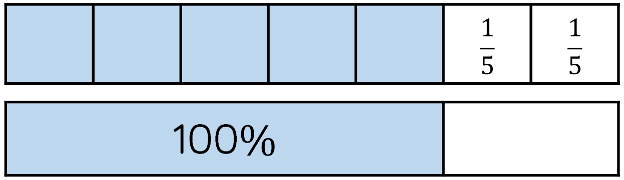
My pocket money has gone up by 150% Woo hoo!

My pocket money has gone down by 150%

The population of Mathtown today is 150% of its population last year.

A company's profits grew by $\frac{2}{5}$ between 2018 and 2019

What percentage of the 2018 profit is the 2019 profit?



Match the equivalent cards.

160%

$\frac{8}{5}$

$\frac{5}{2}$

$\frac{7}{4}$

1.75

2.5

175%

250%

1.6

Spring 2: Directed Number and Fractional Thinking

Weeks 1 to 3: Directed number

Students will only have had limited experience of directed number at primary school, so this block is designed to extend and deepen their understanding of this. Multiple representations and contexts will be used to enable students to appreciate the meaning behind operations with negative integers rather than relying on a series of potentially confusing "rules". As well as exploring directed number in its own right, this block provides valuable opportunities for revising and extending earlier topics, notably algebraic areas such as substitution and the solution of equations; in particular students will be introduced to two-step equations for the first time in this block.

National curriculum content covered:

- select and use appropriate calculation strategies to solve increasingly complex problems
- use the four operations, including formal written methods, applied to integers, both positive and negative
- recognise and use relationships between operations including inverse operations
- use square and square roots
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately
- substitute numerical values into formulae and expressions, including scientific formulae
- understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors
- simplify and manipulate algebraic expressions to maintain equivalence
- understand and use standard mathematical formulae

Interleaving/Extension of previous work

- use conventional notation for the priority of operations
- forming and solving linear equations, including two-step equations

Weeks 4 to 6: Fractional thinking

This block builds on the Autumn term study of "key" fractions, decimals and percentages. It will provide more experience of equivalence of fractions with any denominators, and to introduce the addition and subtraction of fractions. Bar models and concrete representations will be used extensively to support this. Adding fractions with the same denominators will lead to further exploration of fractions greater than one, and for the Core strand adding and subtracting with different denominators will be restricted to cases where one is a multiple of the other.

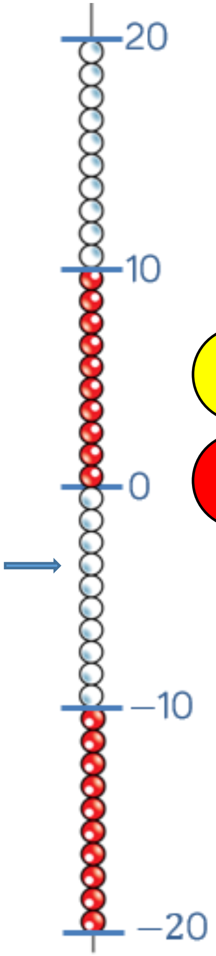
National curriculum content covered:

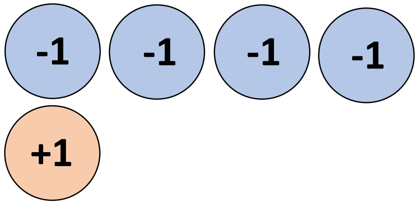
- move freely between different numerical, graphical and diagrammatic representations [for example, equivalent fractions, fractions and decimals]
- express one quantity as a fraction of another, where the fraction is less than 1 and greater than 1
- order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, , ≤, ≥
- select and use appropriate calculation strategies to solve increasingly complex problems
- use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative
- work interchangeably with terminating decimals and their corresponding fractions

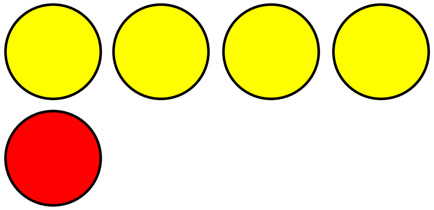
Interleaving/Extension of previous work

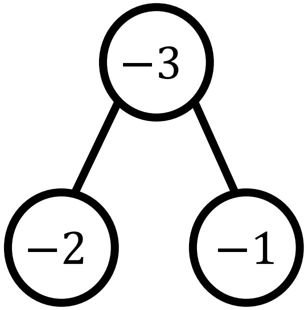
- finding the range and the median
- substitution into algebraic formulae
- forming and solving linear equations, including two-step equations

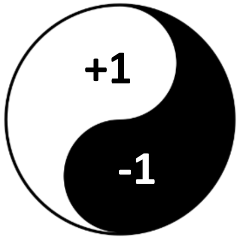
Key Representations

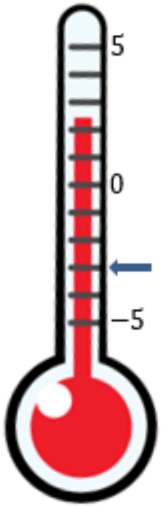


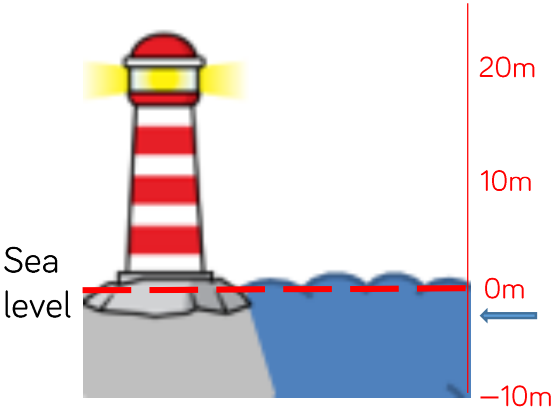


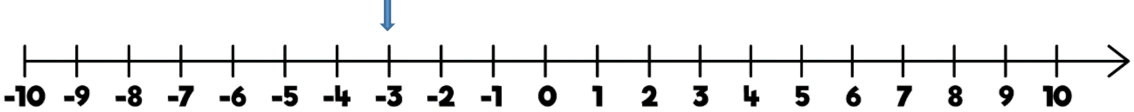












Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

When dealing with directed numbers, it is important to use both horizontal and vertical number lines. The vertical will be familiar from experience of temperature. It is preferable to refer to numbers below zero as e.g. "negative three" rather than "minus three" to try and avoid confusion between numbers and operators and the common misuse of language is a good discussion point.

Directed Number

Small Steps

- Understand and use representations of directed numbers
- Order directed numbers using lines and appropriate symbols
- Perform calculations that cross zero
- Add directed numbers
- Subtract directed numbers
- Multiplication of directed numbers
- Multiplication and division of directed numbers
- Use a calculator for directed number calculations
- Evaluate algebraic expressions with directed number
- Introduction to two-step equations

Directed Number

Small Steps

- Solve two-step equations
- Use order of operations with directed numbers
- **Understand that positive numbers have more than one square root** H
- **Explore higher powers and roots** H

H denotes higher strand and not necessarily content for Higher Tier GCSE

Representations of directed number

Notes and guidance

Students should recognise and use negative numbers in a variety of different representations, including real-life contexts and more abstractly with concrete manipulatives and written notation. Students should be introduced to the reflective nature of positive and negative numbers on the number line e.g. knowing -4 and 4 are equidistant from 0. To avoid confusion "-4" should be read as "negative 4" etc.

Key vocabulary

Positive	Negative	Reflection
Symmetric	Sea level	

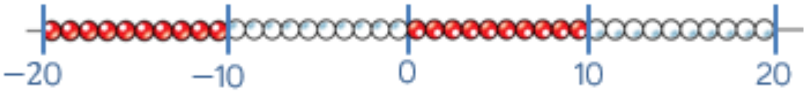
Key questions

How far is -3 from zero? How far is 3 from 0? How are they different?

What does this tell us about positive and negative numbers? (If using bead strings, they can be moved to emphasise the reflection about 0)

Exemplar Questions

Find the following pairs of numbers on the bead string.
What do you notice about each pair?

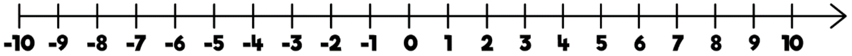


- −10 and 10
- −3 and 3
- −17 and 17
- Does 0 have a matching number?

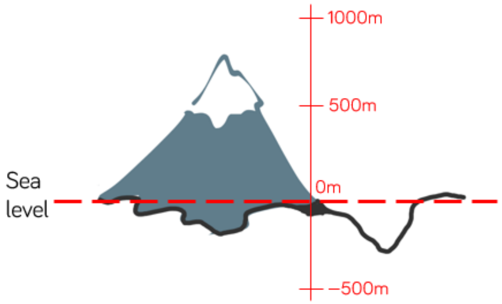
If 🔴 = −1 and 🟡 = 1, what is the total of each below?

- 🔴🔴🔴🔴🔴 =
- 🟡🟡🟡 =
- 🟡🟡🟡🟡🟡🟡 =
- 🟡🟡🟡🟡🟡 =
- 🔴🔴🔴 =
- 🔴🔴🔴🔴🔴🔴 =

Label them on the number line.



What is the approximate height of the mountain?
How deep is the valley?



Order directed numbers

Notes and guidance

In this small step, students practise ordering directed numbers. Order is established using both vertical and horizontal number lines,. The appropriate symbols are then used for comparison. Students should practise ordering negative fractions and decimals on a number line, as well as integers.

Key vocabulary

Ascending	Descending	Smaller/bigger than
Positive	Negative	Greater/less than

Key questions

Is ordering temperatures from hottest to coldest, putting them in ascending or descending order?

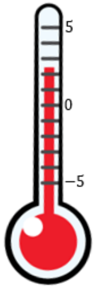
Where would $+\frac{1}{4}$ be on the number line? Is it closer to 0 or 1? How does this help us to put $-\frac{1}{4}$ on the number line?

Between which two consecutive integers does −1.5 lie?

Exemplar Questions

Put the following temperatures in order from coldest to hottest.

2°C, −1°C, −7°C, 0°C, 12°C

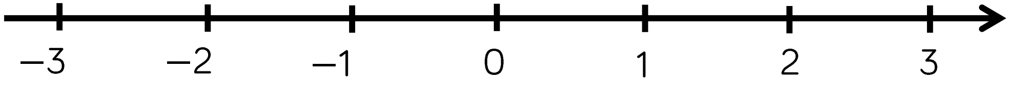


Complete the statements using > or <

- −2°C ○ −6°C
- −6°C ○ 4°C
- −9°C ○ 0°C
- −26°C ○ −24°C
- 15°C ○ −115°C
- −6°C ○ −6.2°C

Use the number line to help you put these numbers in ascending order.

- $\frac{1}{4}$
- −1.5
- $-\frac{1}{4}$
- $-1\frac{3}{4}$
- −1
- 2.5



Perform calculations that cross zero

Notes and guidance

Students can explore number pairs that add to 0 e.g. $-5 + 5$ to show that one negative and one positive of the same magnitude "cancel each other out". Students can use number lines to support adding and subtracting through partitioning: e.g. $-8 + 12 = -8 + 8 + 4 = 4$. A number line is also useful to illustrate the difference between two numbers e.g. -3 and $+4$.

Key vocabulary

- Negative
- Decrease
- Positive
- Difference
- Increase

Key questions

How could you use the number line to help perform this calculation?

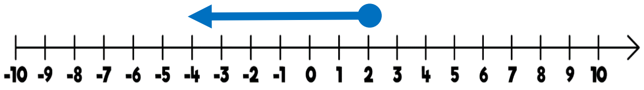
What is $4 - 4$? What is $-4 + 4$? What do you notice?

How is $-3m + 5m$ different from $-3 + 5$?

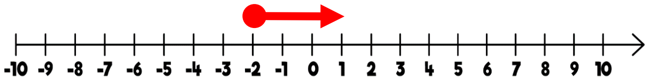
How are they the same?

Exemplar Questions

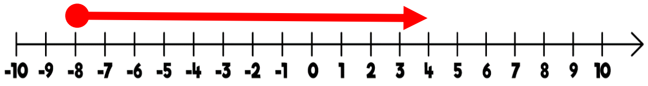
Complete the equation for each representation.



$2 - \boxed{} = -4$



$-2 + \boxed{} = \boxed{}$



$\boxed{} + \boxed{} = \boxed{}$

Work out the missing numbers.

$4 - 8 = \boxed{}$

$2 - \boxed{} = -8$

$-3 + 23 + 30 = \boxed{}$

$-2 + \boxed{} = 42$

- The temperature in Moscow is −3°C at 8am. By 2pm the temperature has gone up by 10°C. What is the temperature at 2pm?
- The temperature in Cardiff is 4°C at 4pm. At 8pm, the temperature has dropped by 5°C. What is the temperature at 8pm?
- The temperature in Paris is 2°C at 3pm. At 3am the temperature is −3°C. What is the temperature difference?

Simplify the expressions.

- $3m - 8m$
- $-3d + 8d$
- $-3w + 3w$

Add directed numbers

Notes and guidance

Students can use double sided counters to model adding negative and positive numbers. Introducing zero pairs will be helpful for both addition/subtraction of directed numbers and help with the use of partitioning e.g. $6 + -4$ as $2 + 4 + -4 = 2 + 0 = 2$. Students may then generalise that adding a negative number is equivalent to a subtraction, although the emphasis should be on understanding the calculation rather than memorising rules.

Key vocabulary

Add	Negative	Minus
Subtract	Partition	Zero pair

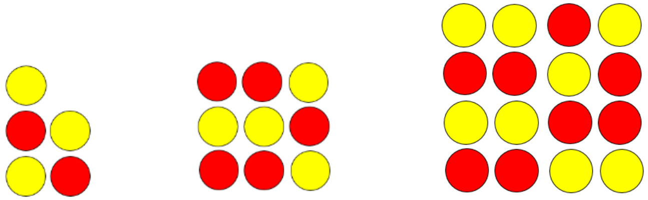
Key questions

Why is adding a negative the same as subtracting?
Why is $100 + -102$ an easy calculation despite the large numbers? How does partitioning help?
Give an example to show the statement "Two negatives make a positive" is wrong.

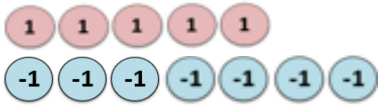
Exemplar Questions

By making zero pairs, what is the total value of each set of counters?

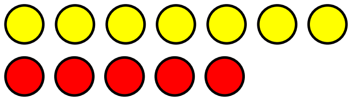
🔴 $= -1$
🟡 $= 1$



Complete the calculations using the concrete manipulatives.
What does each counter represent?



$5 + -7 =$



$7 + -5 =$

How would you model $-7 + -5$?

Sort the number cards into pairs so that each pair has the same total.

−5 −9 −3 3 −11 1

Simplify the expressions by collecting like terms.

- $5a + -12a$
- $-5a + -12a$
- $-5a + 12a$

Subtract directed numbers

Notes and guidance

Students can explore sequences of equations in order to generalise and gain a stronger understanding of this concept. Another useful approach is to have a collection of mixed double-sided counters and see what happens to the total when some/all of the negative counters are removed. Avoid phrases such as "two negatives make a positive" as this leads to misconceptions such as "−1 − 2 = +3 ".

Key vocabulary

Subtract

Negative

Minus

Key questions

Using the manipulatives, what happens to the total when I take away 2 negatives?

What happens when the lowest score is removed? Does the total increase or decrease?

What happens when you subtract a negative number from a positive total? How can you represent this visually?

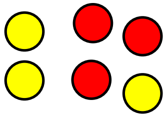
Exemplar Questions

Complete the sequence of questions on the left, and then answer the questions on the right.

$3 - 3 =$
$3 - 2 =$
$3 - 1 =$
$3 - 0 =$
$3 - (-1) =$
$3 - (-2) =$
$3 - (-3) =$

$5 - 8 =$
$5 - (-8) =$
$-5 - (-8) =$
$-5 - 8 =$
$0 - 8 =$
$0 - (-8) =$

Find the totals of these sets of counters. 🔴 = −1 🟡 = 1



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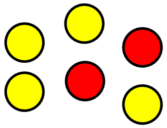

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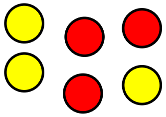


=

How does the total change if you remove a red counter?
How does the total change if you remove a yellow counter?

In a singing competition, four judges give a competitor a score between −10 and 10 points. The scores given are as follows:

8 7 5 −3

- What is the total score?
- What is the mean score?
- If the lowest score is taken away, what is the new total score?

Multiplication with directed numbers

Notes and guidance

Students can use jumps on a number line and manipulatives to model multiplication with directed numbers. Drawing a carefully labelled bar model can also help (see example in the next step). The result of multiplication of two negatives can be justified with continuing patterns within a multiplication grid. It may be useful to teach this and the next step concurrently.

Key vocabulary

- Product
- Multiply
- Commutative
- Inverse

Key questions

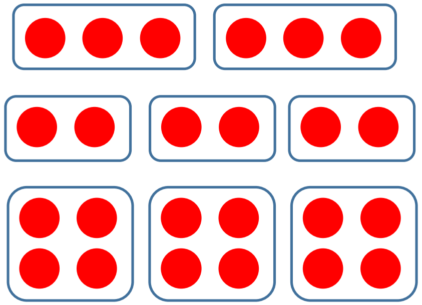
How could we use the number line to answer this question?

If 3 × −2 = −6, what is −3 × −2? How do you know?

Why is $-3 \times 5a$ equal to $3 \times -5a$?
What other calculations give the same answer?

Exemplar Questions

If ● = −1, write calculations for the manipulatives below.



$-3 \times \square = \square$ and $2 \times \square = \square$

$-2 \times \square = \square$ and $3 \times \square = \square$

$-4 \times \square = \square$ and $3 \times \square = \square$

×	−2	−1	0	1	2
−2					
−1					
0					
1					
2					

Complete the multiplication grid and use it to answer the following questions.

- 2 × −1 =
- −2 × 1 =
- −1 × −2 =
- −1 × −2 =

Calculate:

- 3 × 2 =
- −3 × 2 =
- 3 × −2 =
- −3 × −2 =
- 10 × −12 =
- −10 × 1.2 =
- 100 × −7.13 =
- −100 × 0.713 =

Multiplication and division

Notes and guidance

Students can use jumps on a number line and manipulatives to model multiplication with directed numbers. Drawing a carefully labelled bar model can also help (see example in the next step). The result of multiplication of two negatives can be justified with continuing patterns within a multiplication grid. It may be useful to teach this and the next step concurrently.

Key vocabulary

Product
Multiply
Commutative
Inverse

Key questions

How could we use the number line to answer this question?

If 3 × −2 = −6, what is −3 × −2? How do you know?

Why is $-3 \times 5a$ equal to $3 \times -5a$?
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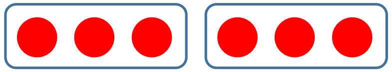
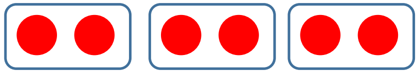
Exemplar Questions

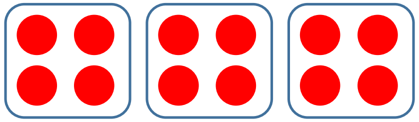
If ● = −1, write calculations for the manipulatives below.



−6 ÷ ☐ = ☐ and −6 ÷ ☐ = ☐

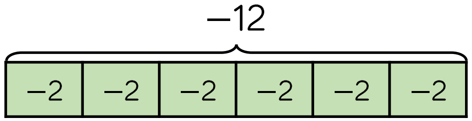


−6 ÷ ☐ = ☐ and −6 ÷ ☐ = ☐



−12 ÷ ☐ = ☐ and −12 ÷ ☐ = ☐

Complete the equations represented by the diagrams.

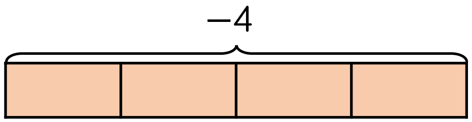


−2 × ☐ = ☐

☐ × −2 = ☐

−12 ÷ ☐ = ☐

−12 ÷ ☐ = ☐



☐ × ☐ = ☐

☐ × ☐ = ☐

☐ ÷ ☐ = ☐

☐ ÷ ☐ = ☐

- If we know −3 × −2 = 6,
  we also know:
  6 ÷ −2 = _____
  6 ÷ −3 = _____

- If we know −5 × −8 = _____,
  we also know:
  _____ ÷ −5 = _____
  _____ ÷ −8 = _____

Use a calculator for directed number

Notes and guidance

The main reason for this step is to develop is to develop students' calculator proficiency. Students should be introduced to the ± button through teacher modelling. Students could also be introduced to the fraction button as an alternative to the division button.

Key vocabulary

Calculator

Sign change

±

Fraction button

Key questions

Explain how to use the ± on a calculator. How is it different from the – button?

What is the difference between -2.3^2 and $(-2.3)^2$

If there is no sign written in front of a number, is it positive or negative?

Exemplar Questions

Compare the calculations using <, > or =

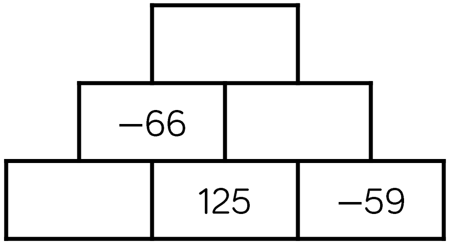
$17 - -0.5 + -2.7$ ○ $17 - (2.7 - 0.5)$

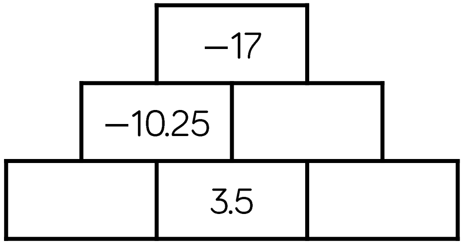
$(-2.3)^2 \times -1.38$ ○ $-2.3^2 \times -1.38$

$\frac{116.5+-8.9}{-2}$ ○ $116.5 + -8.9 \div -2$

What's the same and what's different about the pairs of calculations?

Complete the addition pyramids.





–5 7 –2 10

Using **each** number card and any operations, can you make each of the target numbers? Can you find more than one way?

20 250 42 –40

Evaluate algebraic expressions

Notes and guidance

This small step continues to build on students' use of the order of operations, now through substitution. As in the previous small step, students should be encouraged to take care in organising their recording of work, ensuring they have substituted accurately and maintained the correct order of calculations throughout. Correct use of brackets around negative numbers should be modelled.

Key vocabulary

Substitute

Expression

Order of operations

Key questions

How do we substitute values into an expression?

What is the correct order of operations?

Why is it useful to put negative numbers in brackets when substituting?

Exemplar Questions

Evaluate the expressions by substituting the values $a = 5$, $b = -3$, $c = -1$ and $d = 0$

- $a - b$
- $3(a - b)$
- $-3(a - b)$
- bd
- $\frac{bd}{2}$
- c^2
- $2c$
- $2c - b^2$

Using the same values of a, b, c and d, write an algebraic expression that gives the values.

1 10 –10 55

What mistake has Tommy made?

Substitute $x = 3$ and $y = -5$ into the expression $x - y^2$

$= 3 - - 5^2$

$= 3 + 5^2$

$= 28$

How could he make sure he doesn't make this mistake in future?

Substitute $m = -4$ and $n = -7$ into the expressions, then place in ascending order.

$3m$, $-3m$, $2n$, $-2n$, $5m + 3n$, $5m - 3n$

Introduction to two-step equations

Notes and guidance

Students have met one-step equations and these should be revised in order to move on to two-step equations. Practice of one-step equations can now of course include ones with negative solutions. Students could use concrete manipulatives, such as cups and counters and bar models, to represent the ideas pictorially. These should be used alongside written calculations.

Key vocabulary

Solve	Equation	Balance
Solution	Function machine	Zero pair

Key questions

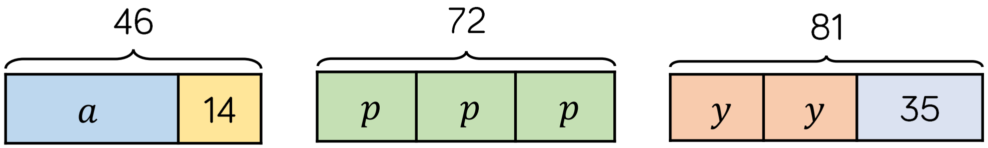
How do you know if an equation can be solved in one step or more than one step?

Can the solution to an equation be a negative number?

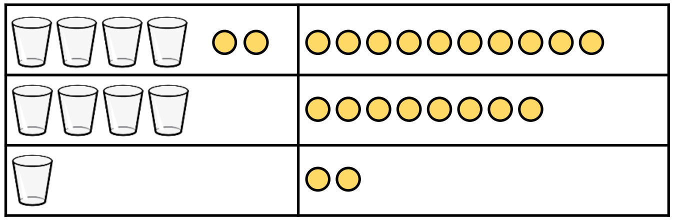
How does a bar model help you to decide what step to take first when solving a multi-step equation?

Exemplar Questions

Use the bar model to write an equation and solve it to find the unknown value.

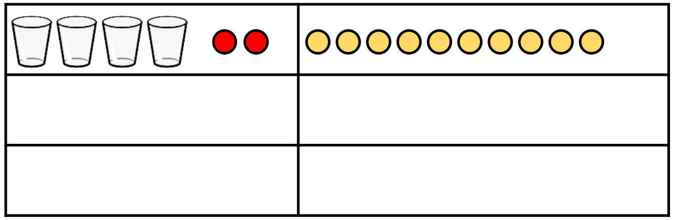


How does the diagram connect to the calculation?



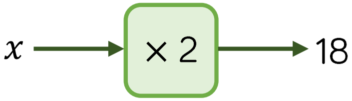
$$\begin{array}{ccc}4x+2&=&10\\ -2&&-2\\ 4x&=&8\\ \div 4&&\div 4\\ x&=&2\end{array}$$

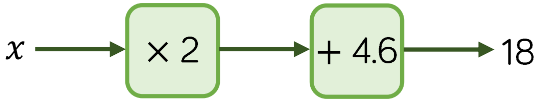
What's the same and what's different about these calculations?
Complete the representation and calculation.



$$4x-2 = 10$$

Write and solve an equation for each function machine.





Solve the equations.

$4m = 96$ $4m = -96$ $g + 46 = 91$ $g + 46 = 11$

Solve two-step equations

Notes and guidance

Students continue to develop their understanding of solving equations in this small step, which includes more negative number work and negative solutions. There are opportunities to consider how varying the signs, coefficients and operations in an equations affects its solution. Students should continue to use bar models and concrete representations as appropriate.

Key vocabulary

Solve

Equation

Balance

Positive/negative solution

Key questions

What is the same and what is different about these questions and answers?

When is it most useful to use a bar model for a two-step equation?

How do you know the order of steps to take to solve an equation?

Exemplar Questions

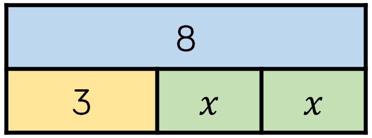
Explain why each equation has the same solution.

$2a + 5 = 1$ $\quad 4a + 10 = 2$ $\quad 2 = 10 + 4a$

$2a - 1 = -5$ $\quad 10 = 2 - 4a$ $\quad -2a - 5 = -1$

Find the solution and write three more equations with the same solution.

Use the bar model to solve $8 - 2x = 3$



Solve the equations.

$2x + 3 = 8$ $\quad 2x - 3 = 8$ $\quad \frac{x}{2} - 3 = 8$ $\quad 8 - \frac{x}{2} = 3$

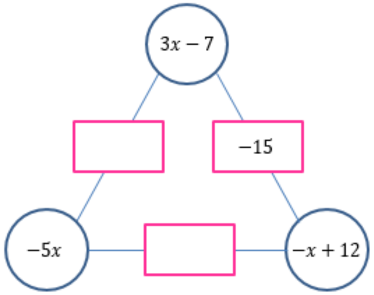
Solve the equations.
Which are most suited to be represented with a bar model?

$5x + 3 = 28$ $\quad 5x + 28 = 3$

$\frac{x}{5} - 3 = 28$ $\quad \frac{x - 3}{5} = 28$

The value in the rectangle is the total of the expressions in the circles on either side.

Complete the missing numbers in the rectangles.



Use order of operations

Notes and guidance

Students build on their understanding of the order of operations, now including negative numbers. Students should be encouraged to pay careful attention to their recording of solutions. Discussion of common misconceptions is useful here. A reminder about commutativity should help students to understand why e.g. multiplication and division are of equal priority.

Key vocabulary

Order of operations	Indices	Brackets
Commutative	Priority	

Key questions

What does it mean when there is a number directly in front of a bracket e.g. $3(6+4)$?

What's the difference between $(-6)^2$ and -6^2?

Does a negative number change the order of operations?

Exemplar Questions

Which is the correct answer? What have the others done wrong?

Student 1	Student 2	Student 3
$(5-3^2)\div 8$	$(5-3^2)\div 8$	$(5-3^2)\div 8$
$=2^2\div 8$	$=5-9\div 8$	$=(5-9)\div 8$
$=4\div 8$	$=5-1.125$	$=-4\div 8$
$=0.5$	$=3.875$	$=-0.5$

Calculate. Show each step of your working.

- $21+18\div -3$
- $\frac{21+18}{-3}$
- $-3\times 5+8-7$
- $3(5+8)-7$
- $-6^2+14\times 2$
- $(-6)^2+14\times 2$
- $-3+4^2$
- $(-3+4)^2$

Substitute $n=1$, $n=2$, $n=3$ and $n=4$ into the expressions. Are either of the sequences linear?

$3(n-5)$

n	1	2	3	4
Output				

$3(n-5)^2$

n	1	2	3	4
Output				

Understand square roots

Notes and guidance

Students should be secure on what a square number is before this small step e.g. by using manipulatives to show why they are called square numbers. Students can logically come to the conclusion that positive numbers have more than one square root by exploring ideas from previous small steps, such as finding the square numbers in the multiplication grid shown.

Key vocabulary

Square
Positive
Square root
Negative
Inverse
Power

Key questions

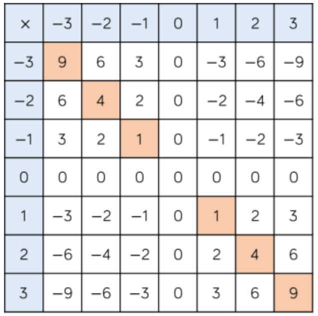
What is a square number?

What is the inverse of multiplication/squaring a number?

What is the difference between $(-5)^2$ and -5^2?

Does 5 have a square root?

Exemplar Questions



×	−3	−2	−1	0	1	2	3
−3	9	6	3	0	−3	−6	−9
−2	6	4	2	0	−2	−4	−6
−1	3	2	1	0	−1	−2	−3
0	0	0	0	0	0	0	0
1	−3	−2	−1	0	1	2	3
2	−6	−4	−2	0	2	4	6
3	−9	−6	−3	0	3	6	9

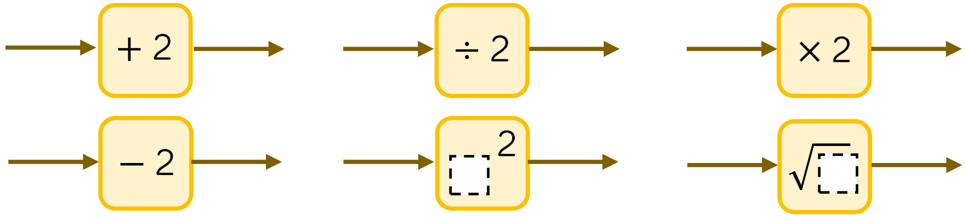
What do you notice about the numbers across the diagonal that have been shaded?

Group the calculations and their answers.

3^2 $(-2)^2$ 4 $(-3)^2$ 2^2

1^2 9 $(-1)^2$ 1

If 9 is the **output** for each function machine, what could the input be?
Is there more than one possible answer?



Calculate.

$\sqrt{4}$ $\sqrt{16}$ $\sqrt{100}$ $\sqrt{225}$

Between which integer values would $\sqrt{45}$ lie?
Complete the inequalities.

$\square < \sqrt{45} < \square$ or $\square < \sqrt{45} < \square$

The area of a square is 169 cm². What could the side length be?

Explore higher powers and roots H

Notes and guidance

Students continue to further their understanding of powers by extending their knowledge of square and cube numbers. If appropriate, extend to look at higher powers. Understanding roots as the inverse operation will help understanding of powers. Students need to be taught that a radical without a number ($\sqrt{\ }$) means square root.

Key vocabulary

- Power
- Root
- Indices
- Exponent
- Inverse

Key questions

What does cube mean?

How do you raise a number to the fourth power?

How do you find roots and powers on your calculator?

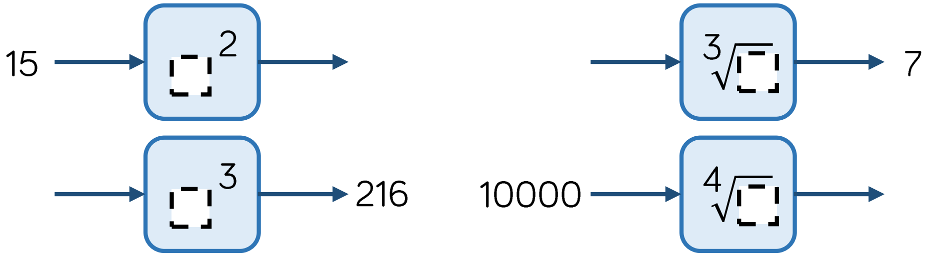
If a number has two square roots, does it have three cube roots?

Exemplar Questions

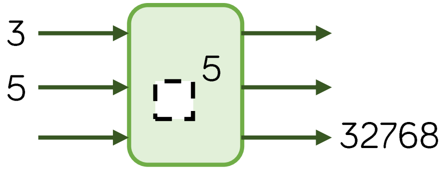
Work out the calculations.
What do you notice about the digit in the ones column?

$3^2 = 3 \times 3 =$	$4^2 =$	$5^2 =$
$3^3 =$	$4^3 =$	$5^3 =$
$3^4 =$	$4^4 =$	$5^4 =$
$3^5 =$	$4^5 =$	$5^5 =$

Use a calculator to complete the function machines.



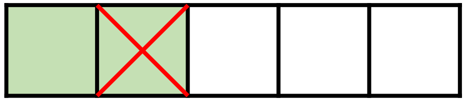
Use a calculator to complete the function machine.

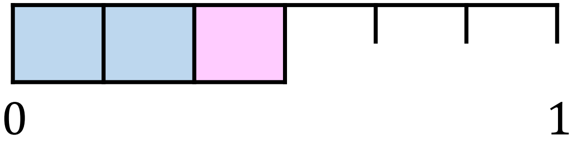


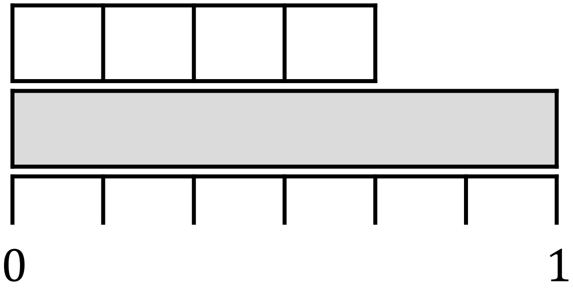
What do you notice about the answers to the calculations?

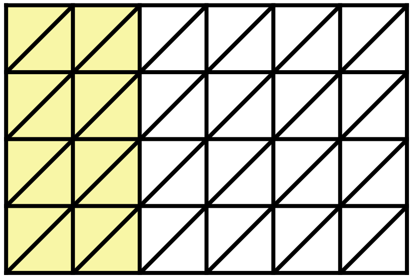
256^1	16^2	4^4	2^8
729^1	27^2	9^3	3^6

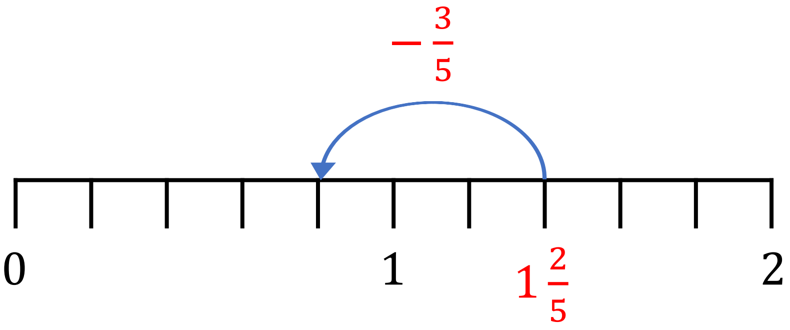
Key Representations



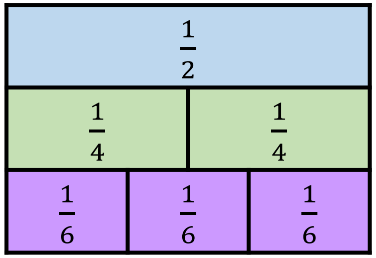








$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{12}{36} = \frac{16}{48}$$



Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Here are a few ideas for how you might represent adding and subtracting fractions. Cuisenaire rods are a useful way to introduce adding and subtracting fractions alongside the pictorial representations, such as the bar model and number line.

Fraction tiles and pattern blocks can be useful in exploring equivalent fractions.

The number line is particularly useful for converting an improper fraction to a mixed number, and more generally to reinforce the fraction's position on the number line.

Fractional Thinking

Small Steps

- Understand representations of fractions
- Convert between mixed numbers and fractions
- Add and subtract unit fractions with the same denominator
- Add and subtract fractions with the same denominator
- Add and subtract fractions from integers expressing the answer as a single fraction
- Understand and use equivalent fractions
- Add and subtract fractions where denominators share a simple common multiple
- Add and subtract fractions with any denominator
- Add and subtract improper fractions and mixed numbers

H denotes higher strand and not necessarily content for Higher Tier GCSE

Fractional Thinking

Small Steps

- Use fractions in algebraic contexts
- Use equivalence to add and subtract decimals and fractions
- **Add and subtract simple algebraic fractions** H

H denotes higher strand and not necessarily content for Higher Tier GCSE

Representations of fractions

Notes and guidance

Students should be presented with, and be expected to represent, fractions in many ways to ensure conceptual understanding of what a fraction is and flexibility between forms. Emphasis should be placed on the need for equal parts, which can be explored and made explicit through the exemplar questions. Number lines can help reinforce that a fraction is a number with a position on the number line.

Key vocabulary

Equal parts
Congruent
Divide
Denominator
Numerator

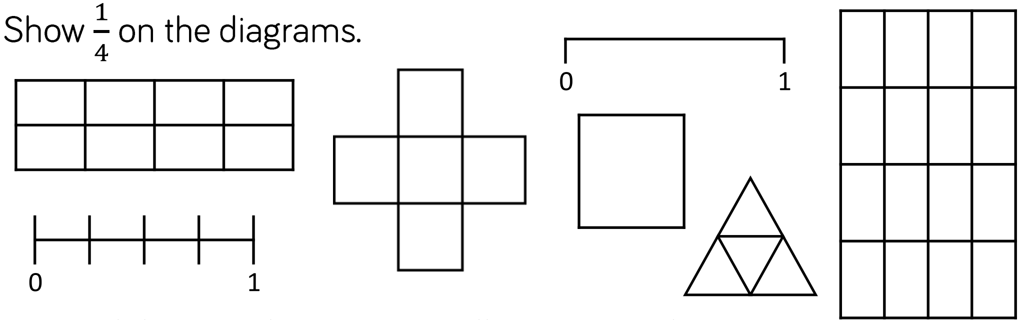
Key questions

How do you know each part is equal when they look different?

Where would this fraction be on a number line? How else can you represent this fraction?

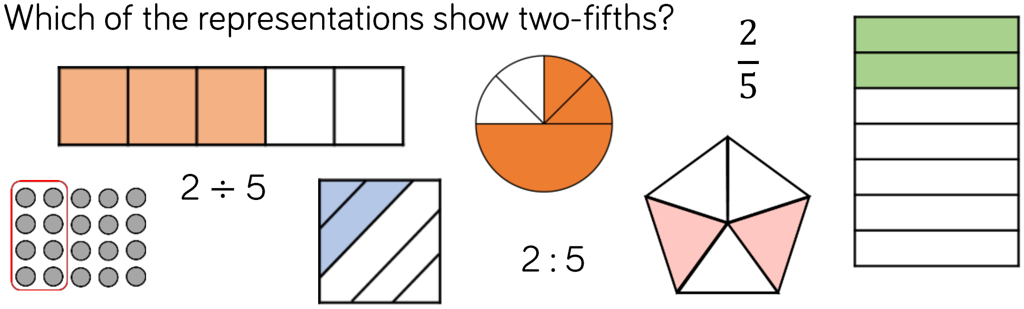
Exemplar Questions

Show $\frac{1}{4}$ on the diagrams.



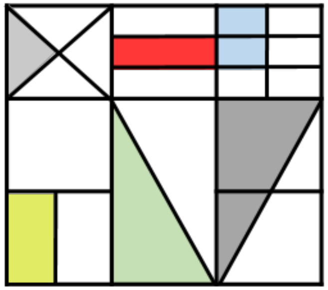
Which did you find the most challenging? Explain why.

Which of the representations show two-fifths?



Explain your reasons.
What are you assuming when answering this question?
Why might someone think that all of these represent two-fifths?

Complete the sentence.



- $\frac{1}{18}$ of the shape is coloured ________.
- The colours ______ and ______ are equal in size. They both show $\frac{\square}{\square}$

Convert mixed numbers

Notes and guidance

Students need to understand conceptually what a mixed number is. A common misconception is that a fraction is part of a whole one, so it is necessary to reinforce that fractions can be greater than one. Bar models and number lines are helpful to build conceptual understanding of how many wholes there are and what fractional part is remaining.

Key vocabulary

Ascending
Positive
Descending
Negative
Smaller/bigger than
Greater/less than

Key questions

How many _________ are there in a whole?

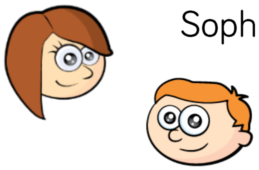
Is (e.g. $\frac{5}{4}$) greater than one or less than one? How do we know?

Why is it called a 'mixed' number?

Why is it called an 'improper' fraction?

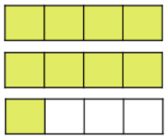
Exemplar Questions

Always, sometimes, or never true?
'A fraction is smaller than one'.



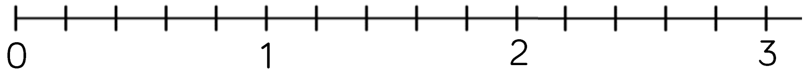
Sophie says that this diagram shows $2\frac{1}{4}$

Ron says that it shows $\frac{9}{4}$

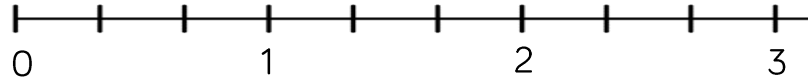


Who is correct? Explain your answer.

Show $\frac{8}{5}$ on the number line, and write it as a mixed number.



Show $2\frac{2}{3}$ on the number line, and write it as an improper fraction.



What's the same and what's different about the number lines?

Complete the statements.

$1\frac{\square}{3} = \frac{5}{\square}$

$\frac{13}{\square} = 3\frac{\square}{\square}$

$\frac{17}{\square} = \square\frac{1}{\square}$

$b\frac{c}{a} = \frac{\square}{\square}$

Is there more than one answer to any of these questions?

Add and subtract unit fractions

Notes and guidance

In this small step, students build conceptual understanding of what it means to add and subtract fractions. The emphasis is on adding and subtracting unit fractions only, so bar models or number lines split into the same number of parts as the denominator will be useful representations.. The common misconception of adding both the numerators and denominators should be addressed here.

Key vocabulary

Unit fraction	Denominator	Equal parts
Whole	Numerator	Multiple

Key questions

How many ______ make a whole?

What happens when you subtract a unit fraction from the same unit fraction?

Would the answers to these questions be different if we performed the operations in a different order?

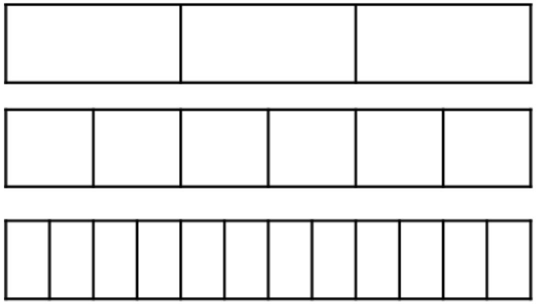
Exemplar Questions

Complete the following calculations:

$\frac{1}{3} + \frac{1}{3} = \square$

$\frac{1}{6} - \frac{1}{6} = \square$

$\frac{1}{12} + \frac{1}{12} - \frac{1}{12} = \square$

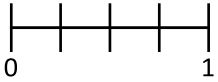


Evaluate the following:

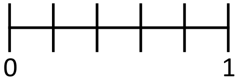
$\frac{1}{5} + \frac{1}{5} = \square$ $\frac{1}{5} - \frac{1}{5} = \square$ $\frac{1}{5} + \frac{1}{5} - \frac{1}{5} = \square$ $\frac{1}{5} + \frac{1}{5} - \frac{1}{5} - \frac{1}{5} = \square$

Complete the missing denominators. Show your answers on the number line.

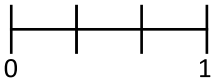
$\frac{1}{\square} + \frac{1}{\square} = \frac{2}{4}$



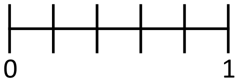
$\frac{1}{\square} - \frac{1}{\square} = \frac{0}{5}$



$\frac{1}{\square} + \frac{1}{\square} + \frac{1}{\square} = \frac{3}{3}$



$\frac{1}{\square} + \frac{1}{\square} = \frac{2}{5}$



How many fifths will be added together to make one whole?
Use a diagram to justify your answer.
How many sixths or sevenths do you need to make a whole?
Can you generalise?

+/− fractions - same denominator

Notes and guidance

This small step helps to reinforce the idea of adding and subtracting a given number of equal parts. This will help students understand the need for a common denominator in the later small step. Cuisenaire rods, bar models and number lines are useful representations to use alongside the abstract calculation. Conversion between mixed numbers and improper fractions is revisited.

Key vocabulary

Denominator	Numerator	Mixed number
Whole	Addition	Subtraction

Key questions

How many ______ make a whole?

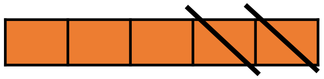
If I have three-fifths and I take away two of those fifths, how many fifths do I have now?

Is it possible to have a negative fraction? Where would it be on the number line?

Exemplar Questions

Use the bar model to work out: $\frac{2}{7} + \frac{3}{7}$

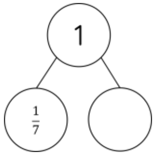
Use this bar model to complete the following $\frac{\square}{5} - \frac{2}{\square} = \frac{\square}{\square}$

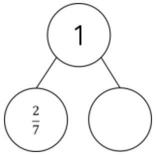


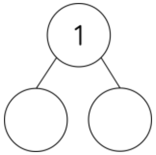
Represent the calculations pictorially and work out each answer:

$\frac{2}{5} + \frac{3}{5}$ $\frac{2}{4} + \frac{3}{4} - \frac{1}{4}$ $\frac{2}{4} + \frac{3}{4} + \frac{2}{4}$ $\frac{7}{23} - \frac{3}{23} - \frac{4}{23}$ $\frac{3}{5} - \frac{4}{5}$

How many different ways can you make a whole using sevenths?

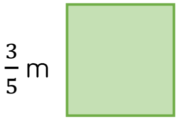


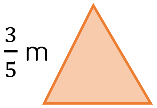


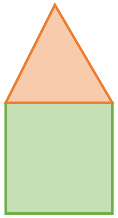


The following equilateral triangle and square are put together to make the shape of a house as shown.

What is the total perimeter of the house?







What is the term-to-term rule for the following sequences?

$\frac{1}{3}, 1, 1\frac{2}{3}, 2\frac{1}{3}, 3, \ldots$ $4\frac{1}{5}, 3\frac{3}{5}, 3, 2\frac{2}{5}, 1\frac{4}{5}, \ldots$

What would the next two terms for each sequence be?

Are the sequences linear or geometric?

+/− fractions from integers

Notes and guidance

Students begin by subtracting a fraction from one whole. They then can use partitioning to subtract from other integers e.g. $4-\frac{2}{5}=3+1-\frac{2}{5}=3\frac{3}{5}$

Students should continue to use bar models and number lines to both support their thinking and conceptual understanding.

Key vocabulary

Integer

Whole

Partition

Subtract

Key questions

How many ______ are there in a whole?

How can a number line or diagram be used to represent this calculation?

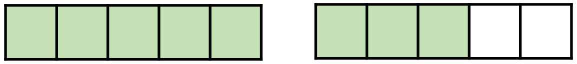
How does partitioning help us to subtract fractions from integers?

Exemplar Questions

Calculate the following. Draw a bar model to show your answer.

◆ $1-\frac{3}{5}$ ◆ $1-\frac{4}{7}$ ◆ $1-\frac{7}{10}$ ◆ $1-\frac{12}{17}$

Complete the number sentences below that are represented by the diagram.



$1+\frac{\square}{\square}=\frac{\square}{\square}$

$2-\frac{\square}{\square}=\frac{\square}{\square}$

Work out the answers to the questions below.
Use bar models to help you.

$4+\frac{5}{9}$ $4-\frac{5}{9}$ $6+\frac{2}{3}+\frac{1}{3}$ $6-\frac{3}{5}-\frac{2}{5}$

Work out the missing fractions.

$4+\frac{\square}{\square}=4\frac{2}{5}$ $4-\frac{\square}{\square}=3\frac{3}{5}$ $6+\frac{\square}{\square}+\frac{\square}{\square}=6\frac{5}{7}$

$7-\frac{\square}{\square}-\frac{\square}{\square}=5\frac{5}{8}$ $6+\frac{\square}{\square}-\frac{\square}{\square}=5\frac{8}{9}$

James, Simon and Chris ordered two pizzas to share.
James ate $\frac{5}{8}$ of a pizza, Simon ate $\frac{7}{8}$ of a pizza and Chris ate the rest.
How much did Chris eat?

Equivalent fractions

Notes and guidance

Students will have some experience of equivalent fractions from their work in the Autumn term and at KS2. The relationship between the numerator and the denominator with unit fractions should be explored, as in the first exemplar question. The relationships between the numerators and denominators of two equivalent fractions should also be explored.

Key vocabulary

Equivalent
Numerator
Denominator
Multiple

Key questions

How do we find a fraction that is equivalent to a given fraction?

Which is the greater/smaller fraction? (e.g. $\frac{3}{4}, \frac{6}{8}$)

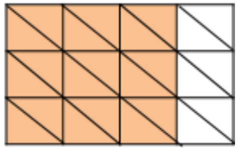
How many fractions can you find are there that are equivalent to one-half? How many are there altogether?

Exemplar Questions

How many equivalent fractions can you find for $\frac{1}{2}$?

How many equivalent fractions can you find for $\frac{1}{3}$?

What is the relationship between the numerator and the denominator in each set of equivalent fractions?



How many ways can you express the fraction shown?

Mo says $\frac{10}{20}$ is twice a big as $\frac{5}{10}$. Do you agree? Explain your answer.

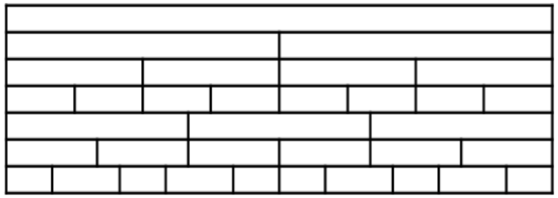
Use a bar model to show these equivalences.

$\frac{2}{3} = \frac{4}{6}$ $\frac{2}{3} = \frac{6}{9}$ $\frac{2}{3} = \frac{8}{12}$

Explain the relationship between each pair of equivalent fractions and how this connects to the bar model.

Write down as many equivalent fractions pairs that you can find in this fraction wall.

For example: $\frac{3}{6} = \frac{1}{2}$



+/− fractions – common multiples

Notes and guidance

Students should build from their understanding of lowest common multiple and adding and subtracting fractions with the same denominator.

An explicit connection should be made to the earlier small step and how finding a common denominator aids in addition and subtraction of fractions.

Key vocabulary

Lowest Common Multiple

Common denominator

Equivalent

Key questions

Why do we need a common denominator to add fractions?

Why is $\frac{1}{10} + \frac{7}{10}$ easier to calculate than $\frac{1}{10} + \frac{7}{15}$?

Is it possible to subtract a larger fraction from a smaller one e.g. $\frac{1}{4} - \frac{1}{2}$?

Exemplar Questions

What is the lowest common multiple of each of the pairs of numbers?

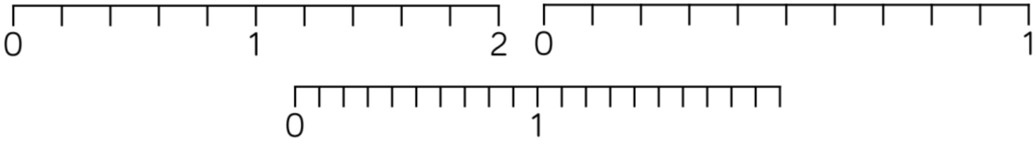
3 , 12 15 , 20 6 , 9 12 , 18

Always, sometimes, or never true?

'You can find the lowest common denominator of a pair of fractions by multiplying their denominators'.

Use number lines to show your calculations:

$\frac{5}{6} - \frac{1}{3}$ $\frac{3}{5} + \frac{7}{10}$ $\frac{2}{3} + \frac{1}{6} - \frac{9}{12}$



What is $\frac{3}{10} + \frac{3}{5}$?

A $\frac{6}{15}$

B $\frac{9}{10}$

C $\frac{6}{10}$

D $\frac{45}{50}$

Draw a diagram to convince me your answer is correct.
Is there more than one correct answer?
Can you identify what the mistake is in each of the wrong answers?

Jane has 3 bars of chocolate.
She gives $\frac{3}{5}$ of a bar to one friend , $\frac{7}{10}$ of a bar to another and $1\frac{1}{5}$ of a bar to another friend.
How much of the chocolate does she have left for herself?

+/− fractions – any denominator

Notes and guidance

In this small step, students will now need to use equivalent fractions for both fractions in order to calculate. They will use their knowledge from the previous steps to extend to add and subtract fractions with any denominator.

Pictorial representations such as fraction walls will help understanding.

Key vocabulary

Lowest common multiple

Common denominator

Equivalent

Key questions

What's the same and what's different about the way we approach $\frac{1}{8}+\frac{3}{4}$ and $\frac{1}{6}+\frac{3}{4}$?

Why don't we always multiply two numbers to find their lowest common multiple?

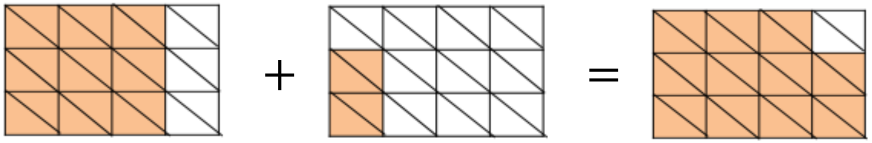
How would approach adding and subtracting with mixed numbers?

Exemplar Questions

What is the lowest common multiple of each of the following pairs?

8 , 12   6 , 12   16 , 24   6 , 9   4 , 10

How does this diagram represent $\frac{3}{4}+\frac{1}{6}=\frac{11}{12}$?



By using equivalent fractions, show your steps to calculate the answer numerically. Can you find more than one way to do this?

Calculate the following. Write your answer as a mixed number where possible. Give your answers in their simplest form.

$\frac{1}{5}+\frac{1}{3}$ $\frac{4}{5}+\frac{2}{3}$ $\frac{4}{5}-\frac{2}{3}$

$\frac{3}{4}+\frac{4}{10}$ $\frac{8}{9}-\frac{3}{7}$ $\frac{3}{5}+\frac{5}{8}-\frac{7}{10}$

Using the number cards and addition and subtraction only, make the totals. You may use the cards only once for each calculation.

$\boxed{\frac{1}{2}}$ $\boxed{\frac{2}{3}}$ $\boxed{\frac{1}{4}}$ $\boxed{\frac{5}{6}}$ $\boxed{\frac{1}{12}}$

- $\frac{1}{6}$
- $\frac{1}{3}$
- $\frac{5}{12}$
- 1
- 0

+/− fractions – improper & mixed

Notes and guidance

Students should explore different ways of adding and subtracting mixed numbers. so they can be flexible when choosing methods.

If students are confident with directed number, then using negative fractions can be introduced as a model for subtraction. It is best to keep the denominators small whilst learning this step.

Key vocabulary

- Commutative
- Common denominator
- Mixed number
- Improper fraction

Key questions

Which method is most efficient for this question and why?

Is it possible to have a negative fraction? Where would this be on the number line?

What could we do if we need to add a negative fraction to a positive integer?

Exemplar Questions

Rosie and Mo have both correctly answered the same question, but in different ways. Explain each method. Can you show them on a bar model?

Rosie's Method

$$2\frac{1}{3} - 1\frac{2}{3} = \frac{7}{3} - \frac{5}{3} = \frac{2}{3}$$

Mo's Method

$$2\frac{1}{3} - 1\frac{2}{3} = 2\frac{1}{3} - 1 - \frac{2}{3} = 1\frac{1}{3} - \frac{2}{3} = \frac{2}{3}$$

Which do you think is more efficient? Why?

$$11\frac{3}{4} + 5\frac{7}{8}$$

$$11\frac{3}{4} - 5\frac{7}{8}$$

Is it more efficient to convert the mixed numbers to improper fractions before adding/subtracting? Or, should I add/subtract my integers first, before the fractions?

Use Whitney's methods to calculate the answers.
What would your advice to Whitney be?

Teddy thinks the difference between the answers is 11.68

Is Teddy right? Explain your answer.

Work out the missing numbers in this linear sequence

?, $3\frac{1}{3}$, $5\frac{5}{6}$, ?

Fractions in algebraic contexts

Notes and guidance

This small step will give students the opportunity to interleave the previous unit of algebraic thinking in the context of fractions, further deepening students' understanding of both. Substitution, sequences, function machines and solving are all explored within the exemplar questions.

Key vocabulary

Sequence Substitute Solve Equation

Linear Geometric Inverse Expression

Key questions

How do we substitute numbers into an expression?

Is it possible to substitute fractions into expressions?

What is the inverse operation of _________ ?

How can you tell if a sequence is linear or not?

Exemplar Questions

If $p = 4$ and $d = 6$, work out the values of these expressions.

$$\frac{1}{p} + \frac{1}{d} \qquad p - \frac{5}{d} \qquad \frac{1}{p^2} - \frac{1}{p} \qquad \frac{p}{d} + \frac{d}{p}$$

Write the first five terms for the sequence given by the rule $\frac{2n}{5}$

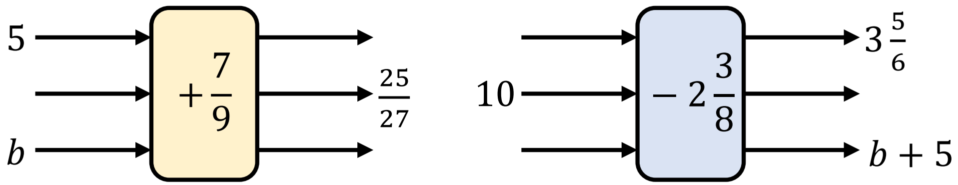
What's the term-to-term rule of the sequence?

Is the sequence linear or geometric?

What would the 100th term of the sequence be?

How often are the terms in the sequence integers?

Find the missing inputs and outputs for the following function machines:



Solve the equations

$$k - \frac{5}{8} = 2 \qquad \frac{5}{8} + y = 2 \qquad 5\frac{1}{5} = 2g - \frac{4}{5} \qquad \frac{r}{4} - \frac{7}{5} = -\frac{2}{5}$$

$+/-$ fractions and decimals

Notes and guidance

This step gives the students an opportunity to revisit fraction and decimal equivalence in the context of addition and subtraction, reinforcing all the skills involved.

Students should be encouraged to estimate before they calculate in order to avoid misconceptions

e.g. $0.5 + \frac{6}{10} = 0.11$

Key vocabulary

- Place value
- Tenths
- Hundredths
- Decimal
- Equivalent

Key questions

How could a number line help with addition and subtraction of fractions and decimals?

If we know $\frac{1}{4} = 0.25$, how could this help us find $\frac{1}{8}$?

Which fractions would be more difficult to give your answer in decimal form?

Exemplar Questions

Using your knowledge of place value, calculate the following. Give your answer in both fractional and decimal form.

$\frac{6}{10} + 0.3$ $\quad$ $0.6 + \frac{4}{10}$ $\quad$ $0.5 + \frac{6}{10}$ $\quad$ $1.1 - \frac{7}{10}$

$0.63 + \frac{7}{100}$ $\quad$ $\frac{7}{100} + 0.07$ $\quad$ $\frac{7}{10} + 0.07$ $\quad$ $\frac{79}{100} + 0.21$

Change the fractions to decimals.

$\frac{1}{4} = \boxed{}$ $\quad$ $\frac{1}{5} = \boxed{}$ $\quad$ $\frac{1}{8} = \boxed{}$

Use this knowledge to complete these statements **in decimal form:**

$\frac{3}{4} + \boxed{} = 1$ $\quad$ $\boxed{} + \frac{2}{5} = 1$ $\quad$ $1 = \boxed{} + \frac{5}{8}$

Calculate the following. In each case, decide whether it is easier to work in fractions or decimals.

$\frac{2}{5} + 0.25$ $\quad$ $\frac{2}{3} + 0.375$ $\quad$ $\frac{5}{7} + 0.75$

$\frac{3}{4} - 0.125$ $\quad$ $\frac{1}{5} + 0.25 - 0.2$ $\quad$ $2\frac{1}{3} - 1.7$

$5\frac{3}{4} + 2.4$ $\quad$ $3\frac{4}{5} + 2.4 - 6\frac{1}{5}$ $\quad$ $2.1 - 5\frac{3}{5}$

+/− algebraic fractions

H

Notes and guidance

Students will further deepen their understanding of fractions within the context of algebra. They should compare adding expressions with fractions to adding those integer form. This step is intended as an introduction to the idea, so fractions should be kept simple rather than dealing with complex multi-term possibilities.

Key vocabulary

Simplify
Like terms
Collect
In terms of
Common denominator

Key questions

What's the same/different about e.g. $\frac{1}{2}a$ and $\frac{a}{2}$?

What does 'in terms of m' mean? Is it possible to get a numeric answer?

How would I do this if algebra were not involved? Now how would I do this algebraically?

Exemplar Questions

Match the equivalent expressions.

$\frac{1}{x}+\frac{1}{x}$ $\quad \frac{1}{4}x+\frac{1}{4}x$ $\quad \frac{2x}{5}$ $\quad \frac{2}{x}+\frac{3}{x}$ $\quad \frac{3}{4}x$

$\frac{2}{x}$ $\quad \frac{x}{5}+\frac{x}{5}$ $\quad \frac{1}{2}x$ $\quad \frac{5}{x}$ $\quad \frac{1}{2}x-\frac{1}{4}x$ $\quad \frac{1}{4}x$

$\frac{3}{x}$ $\quad \frac{1}{2}x+\frac{1}{4}x$

Write $\frac{3}{m}+\frac{4}{m}$ as a single fraction in terms of m :

For what integer values of m are these statements true?

- $\frac{3}{m}+\frac{4}{m}>1$
- $\frac{3}{m}+\frac{4}{m}<1$
- $\frac{3}{m}+\frac{4}{m}=1$

Simplify the following expressions.

$\frac{1}{2}a+\frac{1}{4}a$ $\quad \frac{a}{2}-\frac{a}{4}$ $\quad \frac{1}{a}+\frac{1}{a}+\frac{1}{a}$ $\quad \frac{3a^2}{8}+\frac{a^2}{2}$ $\quad \frac{5}{a}+\frac{1}{2a}$

Solve the equations.

$\frac{1}{x}+\frac{2}{x}=1$ $\quad \frac{1}{y}+\frac{2}{y}+\frac{3}{y}=1$ $\quad \frac{4}{x}+\frac{2}{x}=3$