Autumn Term

Year 8

#MathsEveryoneCan

White Rose Maths

WRM – Year 8 Scheme of Learning

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Proportional Reasoning**						**Representations**					
	Ratio and scale		Multiplicative change		Multiplying and dividing fractions		Working in the Cartesian plane			Representing data		Tables & Probability
Spring	**Algebraic techniques**						**Developing Number**					
	Brackets, equations and inequalities				Sequences	Indices	Fractions and percentages			Standard index form		Number sense
Summer	**Developing Geometry**						**Reasoning with Data**					
	Angles in parallel lines and polygons			Area of trapezia and circles		Line symmetry and reflection	The data handling cycle				Measures of location	

Autumn 1: Proportional Reasoning

Weeks 1 and 2: Ratio and Scale

This unit focuses initially on the meaning of ratio and the various models that can be used to represent ratios. Based on this understanding, it moves on to sharing in a ratio given the whole or one of the parts, and how to use e.g. bar models to ensure the correct approach to solving a problem. After this we look at simplifying ratios, using previous answers to deepen the understanding of equivalent ratio rather than 'cancelling' purely as a procedure. We also explore the links between ratio and fractions and understand and use π as the ratio of the circumference of a circle to its diameter. Students following the higher strand also look at gradient in preparation for next half term.

National Curriculum content covered includes:

- make connections between number relationships, and their algebraic and graphical representations
- use scale factors, scale diagrams and maps
- understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction
- divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio
- solve problems involving direct and inverse proportion

Weeks 3 and 4: Multiplicative Change

Students now work with the link between ratio and scaling, including the idea of direct proportion, linking various form including graphs and using context such as conversion of currencies which provides rich opportunities for problem solving. Conversion graphs will be looked at in this block and could be revisited in the more formal graphical work later in the term. Links are also made with maps and scales, and with the use of scale factors to find missing lengths in pairs of similar shapes.

National Curriculum content covered includes:

- extend and formalise their knowledge of ratio and proportion in working with measures and in formulating proportional relations algebraically
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- use scale factors, scale diagrams and maps
- solve problems involving direct and inverse proportion, including graphical and algebraic representations
- move freely between different numerical, algebraic, graphical and diagrammatic representations

Weeks 5 and 6: Multiplying and Dividing Fractions

Students will have had a little experience of multiplying and dividing fractions in Year 6; here we seek to deepen understanding by looking at multiple representations to see what underpins the (often confusing) algorithms. Multiplication and division by both integers and fractions are covered, with an emphasis on the understanding of the reciprocal and its uses. Links between fractions and decimals are also revisited. Students following the Higher strand will also cover multiplying and diving with mixed numbers and improper fractions.

National Curriculum content covered includes:

- consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals and fractions
- select and use appropriate calculation strategies to solve increasingly complex problems
- use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative

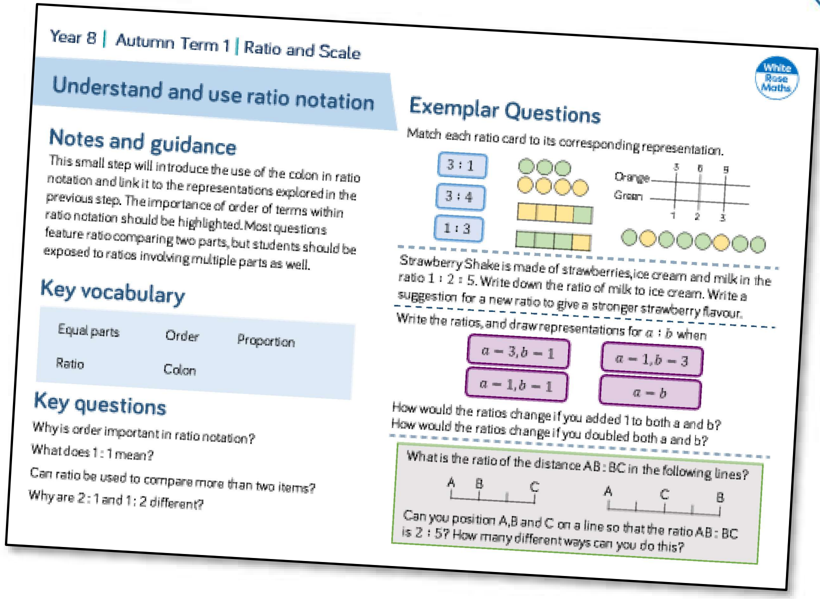
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 8 | Autumn Term 1 | Ratio and Scale

Understand and use ratio notation

Notes and guidance

This small step will introduce the use of the colon in ratio notation and link it to the representations explored in the previous step. The importance of order of terms within ratio notation should be highlighted. Most questions feature ratio comparing two parts, but students should be exposed to ratios involving multiple parts as well.

Key vocabulary

Equal parts	Order	Proportion
Ratio	Colon	

Key questions

Why is order important in ratio notation?
What does 1 : 1 mean?
Can ratio be used to compare more than two items?
Why are 2 : 1 and 1 : 2 different?

Exemplar Questions

Match each ratio card to its corresponding representation.

3 : 1
3 : 4
1 : 3

Orange
Green

Strawberry Shake is made of strawberries, ice cream and milk in the ratio 1 : 2 : 5. Write down the ratio of milk to ice cream. Write a suggestion for a new ratio to give a stronger strawberry flavour.

Write the ratios, and draw representations for $a : b$ when

$a = 3, b = 1$
$a = 1, b = 3$
$a = 1, b = 1$
$a = b$

How would the ratios change if you added 1 to both a and b?
How would the ratios change if you doubled both a and b?

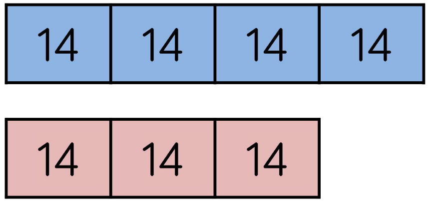
What is the ratio of the distance AB : BC in the following lines?

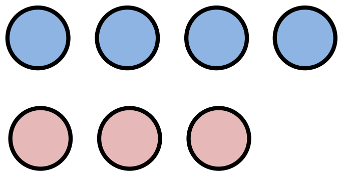
Can you position A, B and C on a line so that the ratio AB : BC is 2 : 5? How many different ways can you do this?

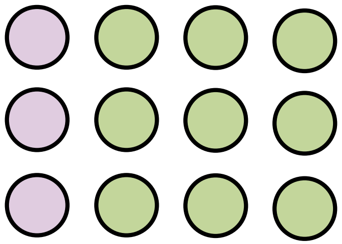
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

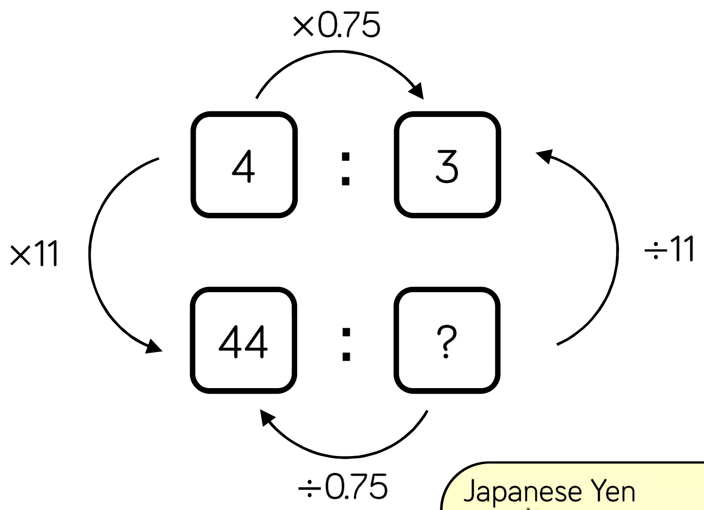
In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

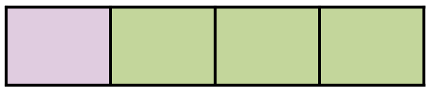
Key Representations

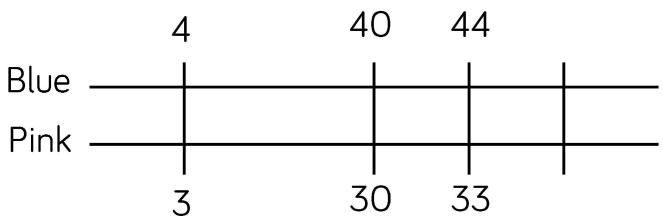


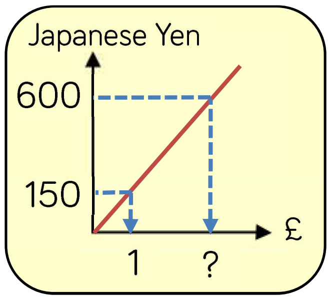












Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Here are a few ideas on how you might represent ratio.

Bar models clearly show the equal parts, which section of the ratio has the largest share and help students to identify which part of the ratio they have been given, or are being asked to find.

Double number lines show the multiplicative nature of ratio and keep track of student's thinking.

White Rose Maths

Ratio and Scale

Small Steps

- Understand the meaning and representation of ratio
- Understand and use ratio notation
- Solve problems involving ratios of the form $1 : n$ (or $n : 1$)
- Solve proportional problems involving the ratio $m : n$
- Divide a value into a given ratio
- Express ratios in their simplest integer form
- Express ratios in the form $1 : n$ H
- Compare ratios and related fractions
- Understand π as the ratio between diameter and circumference
- Understand gradient of a line as a ratio H

H denotes higher strand and not necessarily content for Higher Tier GCSE

Representations of ratio

Notes and guidance

This small step is to ensure that students have a firmer understanding of the meaning of ratio. They should be able to represent ratios pictorially and have knowledge of the language involved. In pictorial representations, it is important to emphasise the equal parts of a ratio.

The link to fractions will be the focus of a later small step.

Key vocabulary

Ratio	Equal parts	For every
Proportion	Relationship	

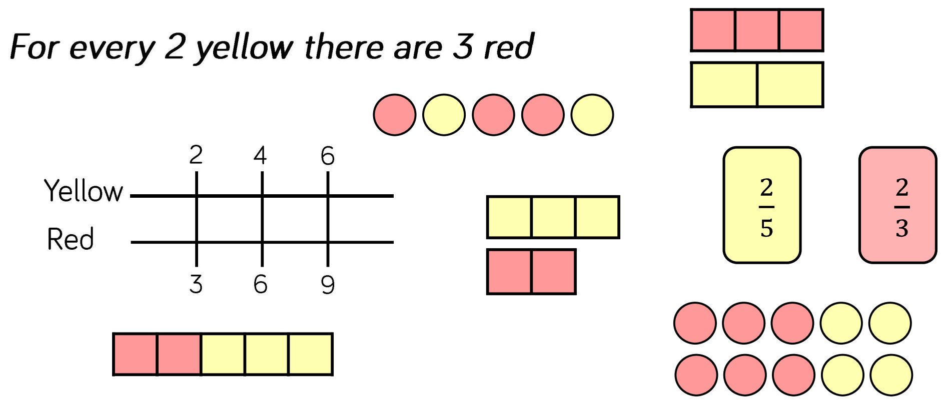
Key questions

What is the purpose of a ratio?

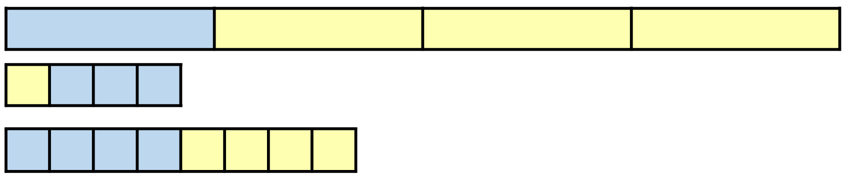
Why should the blocks on a bar model be of equal size when representing ratio?

Exemplar Questions

What is the same and what is different about these representations?



Green paint is made by mixing blue and yellow.
The more blue paint, the darker the green is. Which of these makes the darkest green? How could you make an even darker green?



How could you represent these phrases?

Faisal has twice as many cousins as Mo.	For every 50p you spend, we will donate an extra pound to charity.	1 in every 3 snakes is poisonous.

Understand and use ratio notation

Notes and guidance

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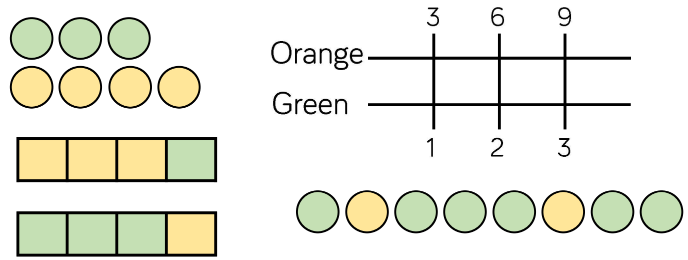

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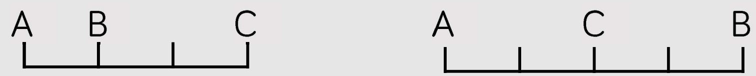


Can you position A,B and C on a line so that the ratio AB : BC is $2 : 5$? How many different ways can you do this?

Solve problems in the ratio $1 : n$

Notes and guidance

In this small step, students will use simple multiplicative reasoning with ratio. In this early stage of ratio learning, n will always be an integer.

For larger values of n, students can be introduced to the advantages of a double number line to support their calculations.

Key vocabulary

Divide	Proportional	Multiply
Part	Double number line	

Key questions

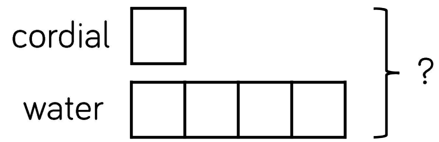
In the ratio $1 : n$, which is the bigger part?

Is bar modelling suitable for all ratios?

How are the ratios $1 : 20$ and $20 : 1$ different?

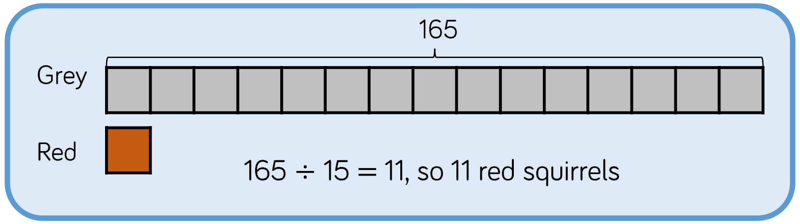
Exemplar Questions

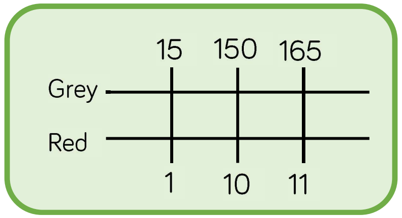
Juice is made using cordial and water in a ratio of $1 : 4$
Use the bar model to work out how much juice will be made with 40 ml of cordial.

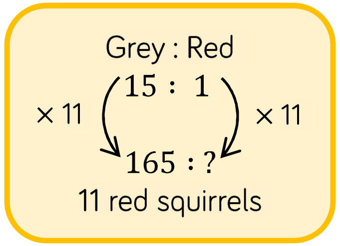


What if there were 40 ml of water?
What if there were 40 ml in total?

The ratio of grey to red squirrels in a forest is $15 : 1$
There are 165 grey squirrels. How many red squirrels are there?
Which of these representations do you prefer and why?







Solve problems in the ratio $m : n$

Notes and guidance

Students will be familiar with terminology of e.g. 'for every 4, there are 3'. from KS2. They will now develop their understanding of ratio alongside formal mathematical notation. Students can explore multiple methods including double number lines, finding multipliers or using bar models and then discuss which is most appropriate to the problem.

Key vocabulary

Proportional	Equal Parts	Multiplier
Placeholder	Units	

Key questions

Can there be more than two amounts in a ratio?

Does adding 1 to each part change the ratio?

How do you set up a bar model for a ratio like 3 : 2?
Does the size of the bars matters?

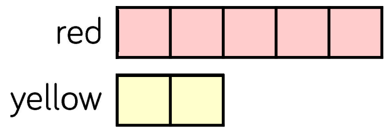
Exemplar Questions

The ratio of men to women in the doctor's waiting room is $4 : 3$
Decide which of these are always, sometimes or never true:

- There are more men than women
- For every 3 men there are 4 women
- There are 7 men and women altogether
- If another man walks in the ratio will change to $5 : 3$

Can you draw a model to support your answers?

A shop orders red and yellow flowers in a ratio of $5 : 2$



One week they order 50 yellow flowers.
How many red flowers do they order?
The following week they order 50 red flowers.
How many yellow flowers do they order?
One week they order 140 flowers altogether.
How many more red flowers than yellow did they buy?

For every 2 hours Jane revises for, she gives herself 30 minutes free time. Which of these ratios represent this situation, and which don't?

$2 : 30$ | $2 : 0.5$ | $2 : \frac{1}{2}$ | $120 : 30$ | 2hrs : 30mins

Divide a value into a given ratio

Notes and guidance

In this small step it is important to expose students to the many combinations of 'sharing in a ratio' questions that can be asked, and not just when the total is given.
Bar modelling gives students a strategy to ensure that they have understood the information and can represent clearly what is known and what is unknown. Varying the ratios for a constant given total is useful.

Key vocabulary

Share	Total	Label
Parts	Ratio	

Key questions

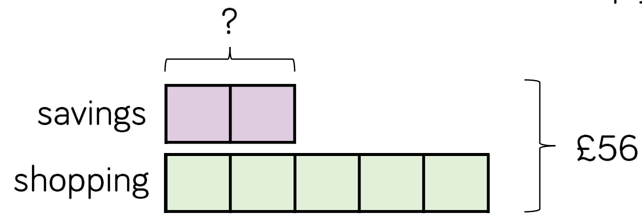
What is the total number of parts?

Where should you label the question mark in your bar model?

What other information does the bar model tell you?

Exemplar Questions

Sam decides to spend his monthly allowance of £56 on savings and shopping in the ratio $2 : 5$
How much does he save? Use the bar model to help you.



How much does he spend on shopping?
Next month he decides to save more, and changes the ratio to $3 : 5$
How much less does he spend on shopping this month?

What is the same and what is different about these questions?

Share £60 in the ratio of $1 : 5$
Share £120 in the ratio of $1 : 5$
Share £120 in the ratio of $2 : 10$
Share £120 in the ratio of $2 : 9 : 1$

A spice mix is made of cayenne pepper and paprika in the ratio $5 : 3$
How much paprika is in 45.44 g of the mix?

Tom, Sam and Harry share some money in the ratio $2 : 3 : 5$
How much does Tom get if the total is £60?
How much does Tom get if Sam gets £60?
How much does Tom get if Harry gets £60?

3 numbers in the ratio $2 : 3 : 7$ have a mean of 48
What is the median of the numbers?

Express ratios in simplest form

Notes and guidance

The concept of simplifying by finding factors will be familiar to students from work on equivalent fractions. Ratios will be simplified to their smallest integer terms. Pictorial or concrete representations should be used to support understanding of the concept. It may be useful to look at the answers to questions in previous steps and simplify these to see the original ratio is obtained.

Key vocabulary

Factors	Equivalent	Divide
Simplify	Common factors	

Key questions

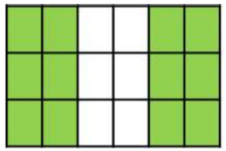
Why are factors useful when simplifying ratio?

What do we mean by 'common factors'?

When might you multiply to simplify a ratio?

Exemplar Questions

Here is the flag of Nigeria. Explain how all these ratios could be used to describe the flag. Which is the most appropriate ratio and why?

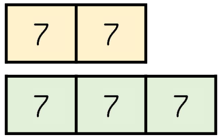


Green : White $2 : 1$

Green : White $6 : 3$

Green : White $12 : 6$

Explain how these show that the ratio $14 : 21$ can be simplified to $2 : 3$



$$14 : 21$$
$$7 \times 2 : 7 \times 3$$
$$2 : 3$$

$$14 : 21 \xrightarrow{\div 7} 2 : 3$$

Which of these ratios are the same?

$16 : 20$ | $8a : 10a$ | $4 : 5$ | $\frac{28}{45} : \frac{35}{45}$

$0.8 : 1$ | $4 \times 10^3 : 5 \times 10^2$

In a field there are 56 female sheep and 16 male sheep.
The farmer wants to keep the same ratio of female to male sheep in a field with only 14 female sheep.
How would simplifying the ratio help to answer this question?

Express ratios in the form $1 : n$ H

Notes and guidance

Here, students will use the simplification techniques from the previous small step to express a unit ratio; final ratios are no longer limited to integers.

It is easier to compare ratio in this format, and helps with later understanding of proportional change and scale factors.

Key vocabulary

Scale	Simplify	Compare
Divide	Units	

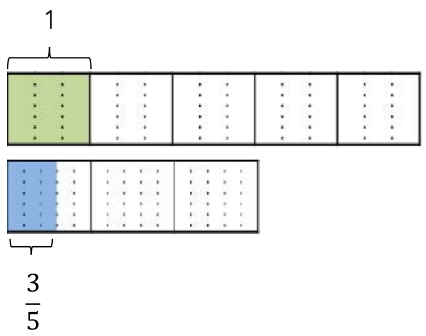
Key questions

If a ratio is simplified to the form $1 : n$, will n always be an integer?
Why is the ratio format $1 : n$ useful for making comparisons?
Which would be larger, a $1 : 200$ scale model or a $1 : 300$ scale model?

Exemplar Questions

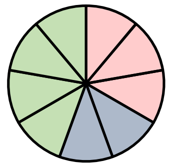
To express the ratio $5 : 3$ in the form $1 : n$, explain how the bar model illustrates this simplification.

Divide each side by 5
$5 : 3$
$5 \div 5 : 3 \div 5$
$1 : 0.6$



The pie chart shows the colours of pixels in a logo.
Complete the sentences:

- For every one red pixel, there are … green pixels.
- For every one green pixel, there are … red pixels.



The ratio of which 2 colours would give the smallest value of n in the ratio $1 : n$?

$11 : 2$ $4 : 9$ $4 : 6$ $6 : 4$

$5 : 2$ $5 : 8$ $4\text{ kg} : 200\text{ g}$

Write these ratios in the form $1 : n$
What do your answers tell you about the different ratios?
Now write the ratios in the form $n : 1$
What's the same and what's different?

Compare ratios and fractions

Notes and guidance

The previous small steps highlighted total number of parts in a ratio, which is looked at again here when finding each part as a fraction of the whole. Students often incorrectly think e.g. the ratio 2 : 3 represents two-thirds of the whole.

Pictorial support or using cubes etc. is helpful here to address this misconception.

Key vocabulary

Total parts	Fraction	Proportion
Denominator	Numerator	

Key questions

What is the same and what is different when we look at a ratio and a fraction?

What's the connection between the sum of the parts of a ratio and its corresponding fraction?

Exemplar Questions

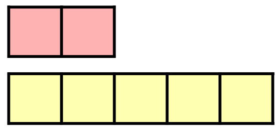
Match each statement to the bar model. How would you model the unmatched statements? What fractions can you see?

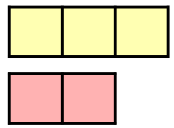
Two fifths are red

Yellow : Red
2 : 3

Two sevenths are yellow

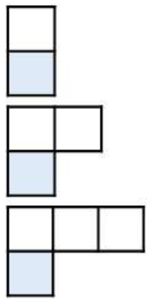
Red : Yellow
2 : 5





There are twice as many sheep to cows in a field.
What fraction of the animals in the field are sheep?
If there were 3 times as many sheep as cows, what fraction of the animals are cows?
If there were the same number of sheep as cows, what fraction of the animals are sheep?

John is writing ratios as fractions. Which column is wrong? Correct and continue his pattern.



B : W	Fraction Blue	W : B	Fraction White
1 : 1	$\frac{1}{2}$	1 : 1	$\frac{2}{1}$
1 : 2	$\frac{1}{3}$	2 : 1	$\frac{3}{1}$
1 : 3	$\frac{1}{4}$	3 : 1	$\frac{4}{1}$

Understand π as a ratio

Notes and guidance

Measuring circumferences and diameters of circular objects helps to establish that the circumference is a multiple of the diameter and to find an approximation for π. Defining π as the ratio of the circumference to the diameter leads to $\pi = \frac{c}{d}$ and then the formula for the circumference.

Students should then practise using this given the diameter or radius of circles, semi-circles etc.

Key vocabulary

Perimeter	Circumference	Constant
Pi (π)	Regular	Diameter

Key questions

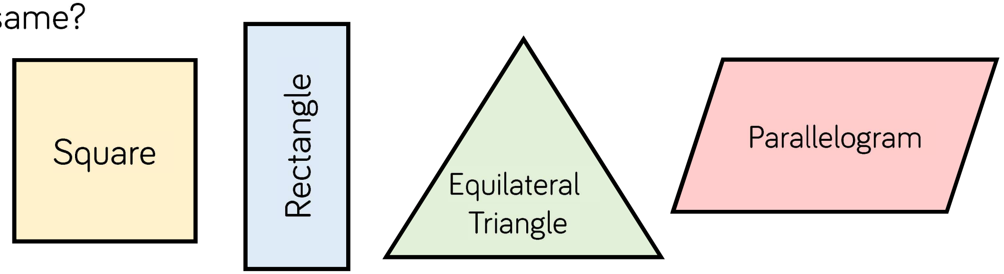
How is a square a rectangle? What makes it unique? Can you explain using ratio?

What's the difference between the radius of a circle and its diameter?

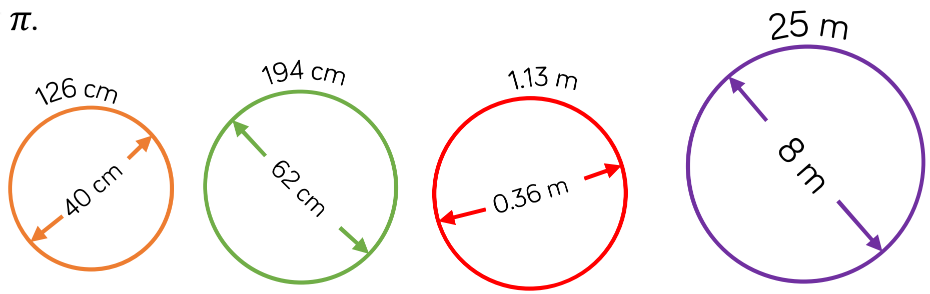
If I triple the diameter of a circle, what happens to its circumference?

Exemplar Questions

Explore the ratio of Width : Perimeter in the form $1 : n$ of the following shapes. For which shapes will the ratios always be the same?



The ratio of diameter : circumference in the form $1 : n$ of a circle is constant. It is $1 : \pi$. Use the circles given to find an approximation for π.



Which of these formulae are correct?

$\boxed{C = \pi d}$ $\boxed{d = \pi C}$ $\boxed{C = 2\pi r}$ $\boxed{\frac{C}{d} = \pi}$

Calculate the circumference of a circle with diameter 0.4 cm

Calculate the perimeter of a semicircle with a diameter 0.4 cm

Understand gradient as a ratio H

Notes and guidance

In this small step, students will formally discuss gradient for the first time, having considered slope only briefly in Year 7 Making the link between gradient and ratio will help deepen understanding and the making of links between many areas of the curriculum, including scale factors, direct proportion graphs etc. At this stage, only positive gradients are considered.

Key vocabulary

Right-angled triangle	Gradient
Slope	Steep

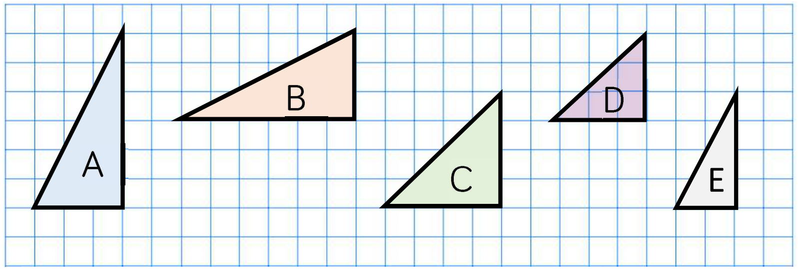
Key questions

What does gradient measure?

What happens to the gradient as a line gets steeper?

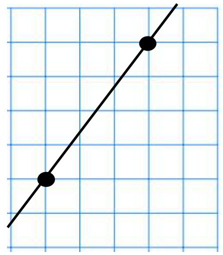
How is the gradient of $\frac{3}{4}$ different to a gradient of $\frac{4}{3}$?

Exemplar Questions



Which of these triangles have the same height to width ratio?
Write the width : height ratio of each triangle in the form $1 : n$
Draw the triangles in ascending order of n. What do you notice?
Draw 3 different triangles with a width to height ratio of 1 : 3

The gradient of this line is $\frac{4}{3}$
Use a triangle and the points on the line to explain why.
How is this different to a gradient of $\frac{3}{4}$?



A line has a gradient of 3. One of the points on the line is (4,2)

Natasha says "to work out another point on the line, add 3 to the y-coordinate for every one that you add to the x-coordinate".

Draw a diagram to see if she is correct. How many points on the line could you find?

Multiplicative Change

Small Steps

- Solve problems involving direct proportion
- Explore conversion graphs
- Convert between currencies
- Explore direct proportion graphs H
- Explore relationships between similar shapes
- Understand scale factors as multiplicative representations
- Draw and interpret scale diagrams
- Interpret maps using scale factors and ratios

H denotes higher strand and not necessarily content for Higher Tier GCSE

Direct proportion

Notes and guidance

In this small step we will explore the fundamentals of direct proportion. Students could think of examples of direct proportion in real life, where as one variable doubles, so does the other. Multiple methods should be explored to give students strategies for the variety of problems that can be posed. Some of these methods are shown in the exemplar questions.

Key vocabulary

Proportion	Ratio	Double
Triple	Linear	Variable

Key questions

How is direct proportion similar to times tables?
If two variable quantities are in direct proportion, what happens if you halve the value of one variable?
What happens if you triple the value of one variable?
Is direct proportion linked to ratio?

Exemplar Questions

5 scoops of ice cream costs £4.50. How much would it cost for:

- 10 scoops
- 8 scoops
- 1 scoop
- 9 scoops

Which of these situations are direct proportion?

'If you double the number of cans, you double the cost'
'Every time you add an extra can the total weight increases by the same'
'If you have 20% fewer cans, you will have 20% less volume'
'If there are 4 times as many people, we will need 4 times as many cans'
'Every extra row of pop is an extra 4 cans'
'If you have no cans, it will cost nothing'

The recipe has been stained.
Use everyone's working out to find the missing information.

Zaib is making 12 muffins
20 : 250 ml
1 : 12.5 ml
12 : 150 ml
150 ml of milk

Carina is making 50 muffins.
50 = '2 and a half lots of 20'
2.5 × 250 = 625 g of sugar

Emma is making 80 muffins.

20	20	20	20

8 eggs

Daniel is making 5 muffins.
20 ÷ 5 = **4**
"I need **4** times less than the recipe
I will use 100g of flour".

Explore conversion graphs

Notes and guidance

This is a particular skill that students will come across in many other topics in school.

For more precise conversions, graph paper should be used practising the use of scales. Students should be encouraged to draw vertical and horizontal lines for the most accurate conversions from their graphs.

Key vocabulary

Linear	Axes	Labelling
Units	Conversion	Approximation

Key questions

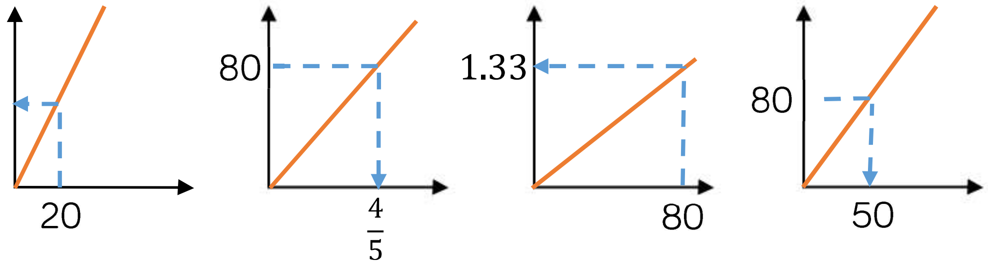
Do all conversion graphs start at the origin?

Is it important to label axes on a conversion graph?

What should the limits of your axes be?

Exemplar Questions

Match the conversion graph to the statement.



Convert width of a square to perimeter

Convert kilometres to miles

Convert centimetres to metres

Convert seconds to minutes

Draw an accurate version of each graph, labelling your axes. Think carefully about the choice of scales.

Draw a conversion graph for converting Fahrenheit to Celsius given the following facts:

- Water freezes at 32 degrees Fahrenheit (32°F)
- Water boils at 212 degrees Fahrenheit
- −40 is the same in Celsius and Fahrenheit

Amir is asked to convert decimals to percentage.
He decides to draw a graph.
What are the features of his graph?
Can he use it to convert fractions to decimals?

Convert between currencies

Notes and guidance

Conversion of currency brings together many of the ideas covered in previous small steps. It is useful to explore the many different methods that can be employed in this topic. Students should be encouraged to estimate their answers before converting to ensure they have a sensible answer. Both calculator and non-calculator should be explored, considering which is appropriate when.

Key vocabulary

Exchange rate	Currency	Conversion
Estimate	Sterling	

Key questions

How is the conversion of pounds to dollars different to dollars to pounds?

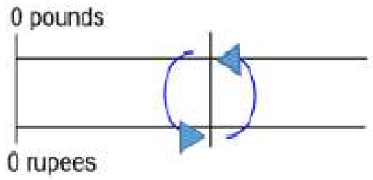
How do conversion rates relate to ratios?

Is converting a currency an example of direct proportion?

Exemplar Questions

£1 = 90 Rupees

Copy and complete the number line.



Alex calculates changing 200 rupees into £

Her answer is £18 000

Does this seem right?

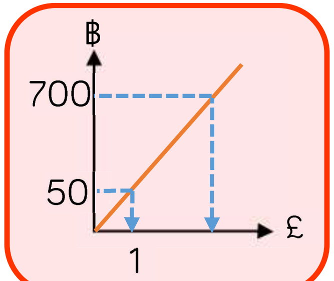
Explain why or why not.

Write a sentence explaining what each of these calculations works out.

400×90 | $400 \div 90$ | 800×90

1 British pound (£) is approximately 50 Thai Baht (฿)

Explain how each of these representations could be used to convert 700฿ into pounds. Why do they all work?



$\times 0.02$

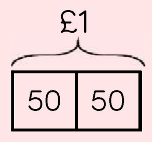
฿ → $\div 100$ → $\times 2$ → £

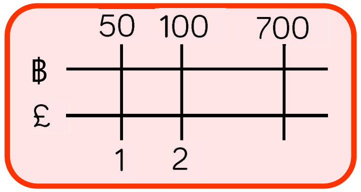
$700฿ \times \left(\frac{£1}{100฿}\right)$

$p = \frac{b}{50}$

where p= number of pounds and b= number of baht

$1 : 50$
$? : 700$





Direct proportion graphs H

Notes and guidance

Encourage students to think of where they might find direct proportion in real life and to represent their ideas as graphs. Comparing these will help to cement some of the main features of the graphs. Remind students of the importance of labelling their axes and finding an appropriate scale. Students can be asked questions involving values outside those in the conversion graph.

Key vocabulary

Rate	Directly proportional	Origin
Constant	Relationship	Linear

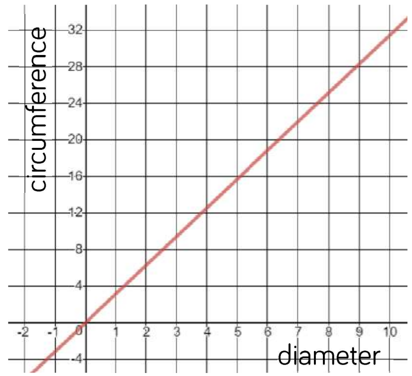
Key questions

Do all direct proportion graphs start at the origin?

How might we use the graph to answer questions that use values beyond those on the axis?

Why is it important to label the axes?

Exemplar Questions



The circumference of a circle is directly proportional to its diameter.

Use the graph to estimate the circumference of a circle with diameter:

- 4 m
- 10 m
- 14 m
- 140 cm
- 0.14 inches

Are these statements true or false?
The graph shows a constant rate of increase.
A circle with a negative diameter has a negative circumference.
All direct proportion graphs go though the origin.

5 scoops of ice cream cost £4.50. Draw a graph that will show the price of up to 40 scoops of ice cream.
How could you use this to find the price difference between 15 and 24 scoops?

If a car is travelling at a *constant* speed, the distance it travels is directly proportional to the time it has been travelling.
Complete the table and draw the graph, describing its key features.

Time (mins)	30	60		114.2
Distance (miles)	18		300	

Ratio between similar shapes

Notes and guidance

Students have already been briefly exposed to similar shapes in their exploration of π and gradient. In this small step we will focus on the fact that corresponding lengths on similar shapes are in the same ratio. Students should be familiar with similar shapes presented in different orientations. Exploration of examples and non-examples is useful here to cement the concept of similar shapes.

Key vocabulary

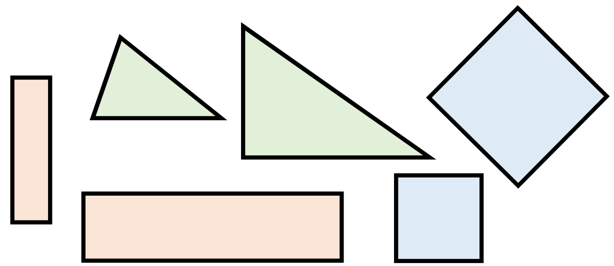
Orientation	Similar	Corresponding
Proportion		

Key questions

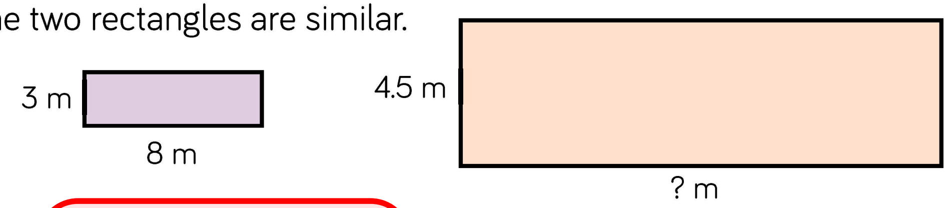
What do you notice about the angles in a pair of similar shapes?
If shapes are not drawn to scale, how can we show they are similar?
How can labelling the vertices be useful with similar shapes?

Exemplar Questions

These shapes are drawn to scale.
Which pairs of shapes are similar?
How can you be sure that they are similar?



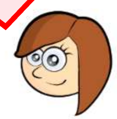
The two rectangles are similar.



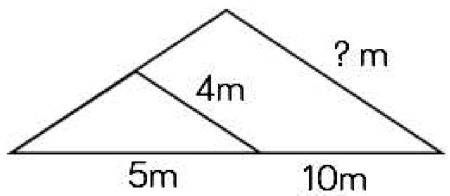
The height has gone up by 1.5 m, so the width of the orange rectangle is 9.5 m.

The ratio of the height of the purple to orange rectangle is 2 : 3

Do you agree with Rosie or Tommy?
Explain your answer.



Two triangles are similar.
Find the missing length.



Understand scale factors

Notes and guidance

Bringing together work on ratio of $1 : n$ and similar shapes, this small step introduces enlargement scale factors, and could be taught in conjunction with the similar shapes small step.

The focus is on length scale factor, not area or volume, though some students may naturally make observations.

Key vocabulary

Scale factor	Enlargement	Object
Image	Length	

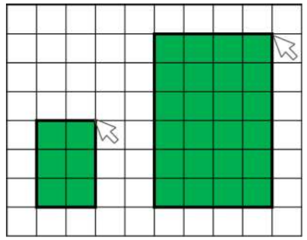
Key questions

How does a scale factor compare to a ratio?
What range of scale factors would make an image smaller?
If the lengths of a shape have tripled, what is the scale factor?

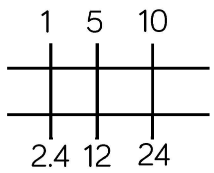
Exemplar Questions

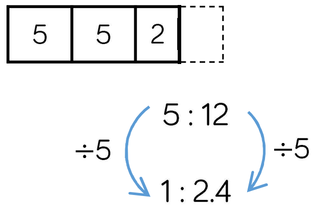
The green rectangle is enlarged on a computer. Which of the statements are true?

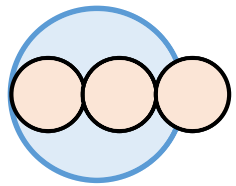
- The height ratio from object to image is 3 : 6
- The width ratio from object to image is 2 : 4
- The ratio of 2 corresponding lengths is 1 : 2
- The image is enlarged by a scale factor of 2
- The image is twice as big as the object.
- The diagonals of the larger rectangle are twice the length of those of the smaller rectangle.



The radius of a circle is enlarged from 5 cm to 12 cm.
How do these images represent the fact that the scale factor is 2.4?







An object is enlarged by a scale factor of 0.5
Which of the following is true? (You could draw a shape and it's enlargement on squared paper to help you)

- The sides of the new image are half as long.
- An enlargement can make shapes smaller.
- The area of the image is half the area of the object.

Draw and interpret scale diagrams

Notes and guidance

Students should explore this step practically by creating and using scale drawings of items from the classroom etc.
The link between scale, scale factors and ratio needs to be made explicit. This could be reinforced by linking back to earlier representations such as the double number line.
Examples of diagrams that are not to scale could be useful to emphasise the key features of scale.

Key vocabulary

Scale factor	Not to scale	Enlargement
Plan	Image	

Key questions

Are scale diagrams always smaller versions of the original?

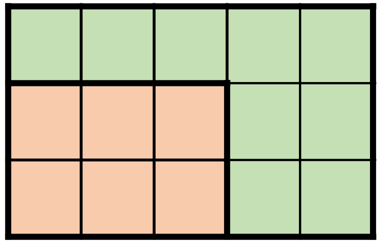
Why is a scale diagram useful?

Describe a method for finding an appropriate scale.

Exemplar Questions

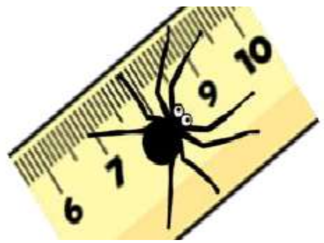
Tommy is planning where to build his house on his land.
He has drawn one idea on some cm squared paper. 1 cm = 3 m

(peach)	Tommy's House
(green)	Tommy's Garden



What is the actual width of his plot of land?
What is the actual perimeter of the house?
What is the actual area of the land left after building the house?

A biologist is drawing a diagram of a house-spider to go in a book.
She wants it to fill a whole A4 page.
What scale factor could she use?
How would the scale factor change if she wanted twice as many spiders on a page?



Samira draws a picture of a car with a scale of 1 : 30
The car is 3 m long. How long is her scale drawing?
The height of the car on the picture is 5 cm.
What is the height of the actual car?

Interpret maps with scale factors

Notes and guidance

Teachers might consider revisiting metric unit conversions before starting this small step. Specifically, students need to be confident in working with large numbers (for example, above 10 000). This small step could be introduced using real-life maps, and the meaning of each scale. Using representations, such as double number lines, will help students to connect this small step to previous ones.

Key vocabulary

Distance   Conversion   Units

Metric

Key questions

What does the scale 1 : 25 000 mean on a map?
Can you express it as a ratio in mixed units?

Would a map with scale 1 : 25 000 need to be bigger or smaller than a map with scale of 1 : 1250 showing the same features?

Exemplar Questions

Explain why each of these describes a scale of 1 : 25 000

1 cm on the map is 25 000 cm in real life.

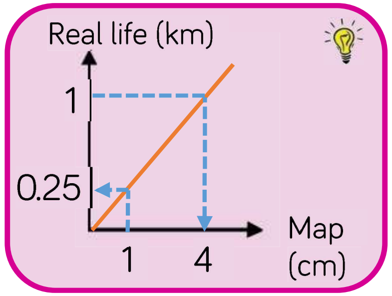
$1 : 2.5 \times 10^4$

1 cm : 25 000 cm

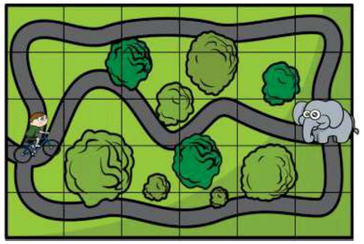
1 cm : 250 m

1 cm : 0.25 km

4 cm on the map is 1 km on the ground.



A pirate sails from her island to find treasure.
She travels 15 km North, turns East and sails 30 km, and then turns North again for the final 40 km to take her to some treasure. Draw a scale map of her journey using a scale of 1 : 500 000
Her parrot flies directly from the island to the treasure.
Use your map to find out how much further the pirate travelled than the parrot.



The scale of this map is 1 : 1250
Each square is 1 cm by 1 cm.
Which is the shortest route for the boy to cycle to his elephant?

Multiplying & Dividing Fractions

Small Steps

- Represent multiplication of fractions
- Multiply a fraction by an integer
- Find the product of a pair of unit fractions
- Find the product of a pair of any fractions
- Divide an integer by a fraction
- Divide a fraction by a unit fraction
- Understand and use the reciprocal
- Divide any pair of fractions

Multiplying & Dividing Fractions

Small Steps

- Multiply and divide improper and mixed fractions H
- Multiply and divide algebraic fractions H

H denotes higher strand and not necessarily content for Higher Tier GCSE

Represent fraction multiplication

Notes and guidance

Repeated addition is used here to help understand the multiplication of fractions. Students will also explore familiar representations of fractions from previous years.
Manipulatives such as paper plates and fraction pieces can be used to demonstrate the multiplications. Paper strips and Cuisenaire rods link well to pictorial representations as well as bar models.

Key vocabulary

Unit fraction	Numerator	Denominator
Product	Repeated addition	

Key questions

When making a representation of a fraction why is it important that each part is equal?

If 3 multilink blocks is 1 whole, what other facts do we know?

How is addition related to multiplication?

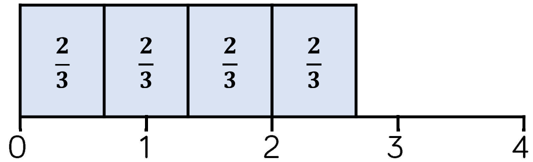
Exemplar Questions

3 paper strips are folded into quarters. One quarter of each is shaded. Which of the statements do you agree with? Why are they correct?

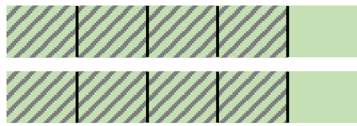
‘This shows $\frac{3}{12}$’ ‘It shows $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$’ ‘The paper shows $3 \times \frac{1}{4}$’

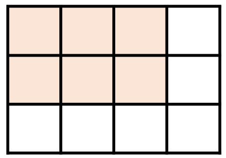
‘I can see three quarters’ ‘It could be on 1 bar model’ 

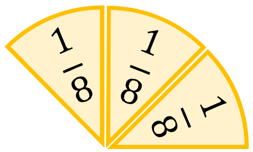
What multiplication do each of the diagrams show?

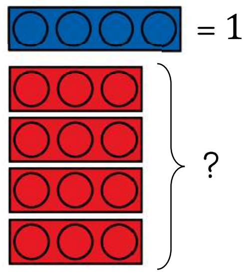


One bead represents $\frac{1}{7}$









Seth says that to work out $7 \times \frac{2}{3}$ on a number line, you just jump 2 forward then 3 back and repeat that 7 times.
Is he correct?

Multiply a fraction by an integer

Notes and guidance

In this small step, students explore and formalise multiplication of a fraction by an integer. Calculations supported with pictorial representations are still encouraged at this stage. Multiple methods will allow students to pick the strategy that best suits the question.
It could be useful to remind students of the word ‘product’ at this stage.

Key vocabulary

Unit fraction	Numerator	Denominator
Product	Repeated addition	

Key questions

How is finding a fraction of an amount the same as multiplying by a fraction?

Does multiplying by a fraction always give an answer less than 1?

Exemplar Questions

Mo is working out $3 \times \frac{2}{7}$. His friends are helping him, here is what they say. Which comment do you think is most helpful and why?

The answer will be less than one

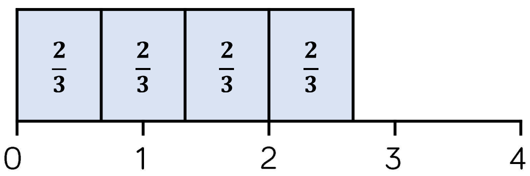
3 lots of 2 is 6
So 3 lots of 2 *sevenths* is 6 *seventh*.

It’s just the same as $\frac{2}{7} + \frac{2}{7} + \frac{2}{7}$

It’s the same as $\frac{3 \times 2}{7}$

Divide 3 by 7, then multiply it by 2

This bar model shows that $4 \times \frac{2}{3} = \frac{8}{3}$. It also shows $\frac{8}{3} = 2\frac{2}{3}$



Use the bar model to work out $3 \times \frac{2}{3}$ $\quad 8 \times \frac{2}{3}$ $\quad \frac{2}{3} \times 5$ $\quad 1.5 \times \frac{4}{3}$

What is the same and what is different about each pair of calculations and their answers? Use bar models to help explain.

$3 \times \frac{2}{5}$	$2 \times \frac{3}{5}$
$3 \times \frac{2}{5}$	$3 \times \frac{2}{7}$
$3 \times \frac{2}{5}$	$6 \times \frac{1}{5}$

Product of unit fractions

Notes and guidance

This step gives students the chance to understand the underlying mathematics of multiplying any fractions together. When folding paper, remind students that each side of the original shape has a unit length of 1. This links it to grid method multiplication and clearly shows the size of the product of unit fractions is always smaller than one.

Key vocabulary

Denominator	Product	Square
Whole	Unit fraction	

Key questions

Does multiplying always make numbers larger?

Why will the product of two unit fractions always have 'one' as a numerator?

How many other fractions can we find by folding paper in different ways?

Exemplar Questions

A square napkin is folded in half vertically, then into thirds horizontally.

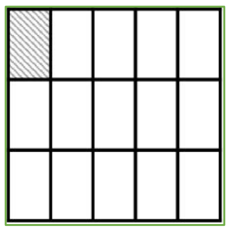
- How many parts will there be when it is unfolded fully?
- How does this help to work out $\frac{1}{3} \times \frac{1}{2}$?
- Does the napkin have to be square?

Use a piece of A4 paper as the napkin to investigate this.

Whitney has worked out $\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$

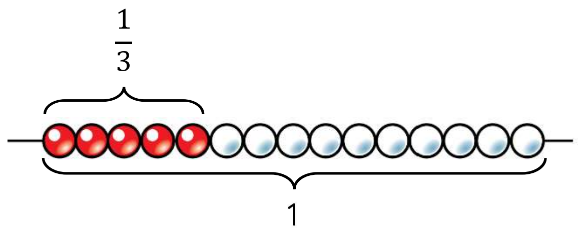
How do the following link to her calculation?

A third of one fifth is three times smaller than one fifth.



$\frac{1}{3} \div 5$

$\frac{1}{5} \div 3$



Find the missing numbers. Do any have more than one answer?

$\frac{1}{5} \times \frac{1}{6} = \frac{1}{?}$

$\frac{1}{3} \times \frac{1}{?} = \frac{1}{30}$

$\frac{1}{?} \times \frac{1}{?} = \frac{1}{30}$

Product of any fractions

Notes and guidance

This small step will look at the multiple ways in which the students might approach finding the product of any two fractions, allowing students to come up with their own conjectures for 'quick methods.'

Again, using familiar concrete and/or pictorial representations from previous steps will support abstract understanding.

Key vocabulary

Non-unit fraction	Commutative
Numerator	Denominator

Key questions

How can multiplying by a fraction be expressed as a multiplication and a division?

Is it always, sometimes or never appropriate to convert fractions to decimals before multiplying?

Exemplar Questions

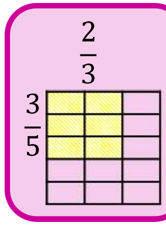
Which method is the most effective to work out $\frac{2}{3} \times \frac{3}{5}$?

$\frac{2 \times 3}{3 \times 5} = \frac{6}{15} = \frac{2}{5}$

$\frac{2 \times 3}{3 \times 5} = \frac{2 \times 3}{5 \times 3} = \frac{2}{5}$

Rewrite the question as $2 \times 3 \times \frac{1}{3} \times \frac{1}{5}$

Find a third of $\frac{3}{5}$, then double it.



Put the following in ascending order.

$\frac{1}{5} \times \frac{3}{8}$ $\quad$ $\frac{2}{5} \times \frac{3}{8}$ $\quad$ $-\frac{1}{15} \times \frac{9}{16}$ $\quad$ $\frac{2}{15} \times \frac{15}{16}$ $\quad$ $\left(\frac{3}{5}\right)^2$

Tommy says that to multiply fractions you just need to multiply the numerators and multiply the denominators.
Explain whether this method would be suitable for the following.

$\frac{3}{8} \times \frac{8}{7}$	$0.2 \times \frac{40}{81}$	$\frac{5}{7} \times \frac{3}{11}$
Three-quarters of 40%	$\frac{2}{3} \times \frac{3}{4} \times \frac{32}{56}$ of 2	$\frac{3}{4} + \frac{4}{10}$

Divide an integer by a fraction

Notes and guidance

In this small step, students understand the link between multiplying and dividing integers to multiplying and dividing fractions. A fact family with integer values will be intuitive but students may want to ask more questions when the fact family involves division of fractions. Demonstrations with bar models and fraction strips will help explain this.

Key vocabulary

Unit fraction	Whole
Quotient	Denominator

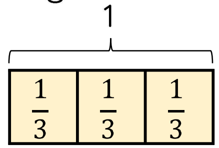
Key questions

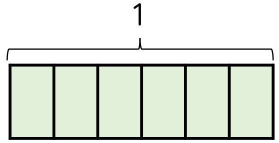
How many unit fractions make a whole?

When we divide, is the answer always smaller than the dividend?

Exemplar Questions

Use the diagrams to help complete the sentences.





'There are … thirds in 1 whole.'
'Therefore, there are … thirds in 4 wholes.'

$1 \div \frac{1}{3} =$ $\quad 4 \div \frac{1}{3} =$ $\quad 40 \div \frac{1}{3} =$

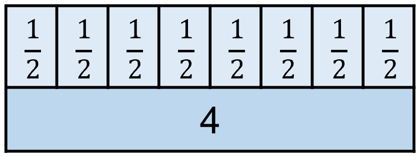
'There are 6 … in 1 whole.'
'Therefore, there are 30 … in … wholes.'

$1 \div \frac{1}{?} = 6$ $\quad ? \div \frac{1}{6} = 30$ $\quad ? \div \frac{1}{6} = 15$

Work out the following.

- $4 \div \frac{1}{2}$
- $4 \div \frac{1}{3}$
- $4 \div \frac{1}{4}$
- $4 \div \frac{1}{5}$
- $4 \div \frac{1}{13}$
- $8 \div \frac{1}{13}$
- $8 \div \frac{1}{n}$
- $a \div \frac{1}{n}$

Complete the facts shown in the representation below.



$4 \div 8 =$

$1 \div \frac{1}{2} =$

$2 \div \frac{1}{2} =$

$4 \div \frac{1}{2} =$

Are there any other facts you can see?

Divide a fraction by a unit fraction

Notes and guidance

As work with fractions becomes more abstract, it is useful to get students to reason their solutions. The questions asked in this small step will revolve around reasoning rather than procedure.

The language of dividing, ‘How many … in …?’, will help students to estimate answers before formally giving them.

Key vocabulary

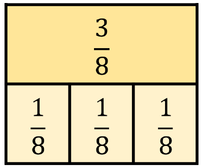
Divide	Estimate
Numerator	Denominator

Key questions

Shade in three quarters of a square. Count the number of quarters. How does this show $\frac{3}{4} \div \frac{1}{4}$?

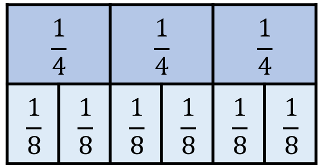
When we divide, the quotient is always smaller than the dividend. True or False?

Exemplar Questions

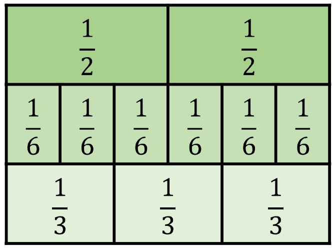


How many eighths are there in three-eighths?

Complete $\frac{3}{8} \div \frac{1}{8} =$

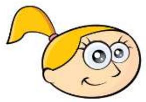


How many eighths are there in one-quarter?
How many eighths are there in three-quarters?
Complete $\frac{3}{4} \div \frac{1}{8} =$



Use the equivalent fraction wall to calculate:

- $\frac{1}{2} \div \frac{1}{6}$
- $\frac{1}{3} \div \frac{1}{6}$



Using the wall, I can see that one-and-a-half thirds fit into one-half.
This means that $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$

Is Eva right? Explain your answer.

Complete the statements using <, > or =

$\frac{3}{5} \div \frac{1}{5}$ ◯ $\frac{3}{5} \div \frac{1}{6}$

$\frac{1}{2} \div \frac{1}{5}$ ◯ $\frac{1}{4} \div \frac{1}{5}$

$\frac{1}{5} \div \frac{1}{3}$ ◯ $\frac{3}{5} \div \frac{1}{9}$

White Rose Maths

Understand and use the reciprocal

Notes and guidance

Here students will learn through investigation that the division of a number is equivalent to the multiplication by its reciprocal. They should be able to find the reciprocal of fractions and decimals and use these to answer questions on division.

They should also understand that a number multiplied by its reciprocal is always 1

Key vocabulary

Reciprocal	Convert

Key questions

How would you find the reciprocal of a decimal?

Can we find the reciprocal of zero?

What do you notice about $\frac{1}{5} \times 5$? Try multiplying another number by its reciprocal. Is this true for all numbers?

Why is dividing by a fraction the same as multiplying by its reciprocal?

Exemplar Questions

Find the reciprocal of the following.

$2 \qquad \frac{1}{2} \qquad \frac{2}{5} \qquad 0.4 \qquad a \qquad \frac{2}{a}$

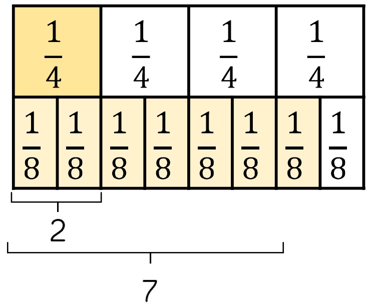
Work out the following sets of calculations. What do you notice?

$3 \div \frac{1}{4}$	$3 \div 4$	$8 \div \frac{1}{5}$	$8 \div 5$
3×4	$3 \times \frac{1}{4}$	8×5	$8 \times \frac{1}{5}$

Dora reasons that $\frac{1}{4} \div \frac{7}{8}$ must give an answer of less than 1

Use the picture to explain why $\frac{1}{4} \div \frac{7}{8} = \frac{2}{7}$

How does this show that $\frac{1}{4} \div \frac{7}{8}$ is the same as $\frac{1}{4} \times \frac{8}{7}$?



Use the idea of division being the same as multiplying by the reciprocal to calculate.

$2 \div \frac{1}{5} \qquad 2 \div \frac{2}{5} \qquad \frac{3}{5} \div \frac{2}{5} \qquad \frac{3}{10} \div \frac{2}{5} \qquad \frac{1}{5} \div 2 \qquad 0.4 \div \frac{1}{2}$

Divide any pair of fractions

Notes and guidance

Students should now have developed their reasoning and so have many methods available for dividing fractions. This small step develops the concepts further so they can understand the division of any pair of fractions.
Encourage students to think about efficient methods depending on the question instead of relying solely on procedure.

Key vocabulary

Simplify	Factors
Denominators	

Key questions

Why is a common denominator useful when dividing fractions?

Can there be a remainder when dividing by fractions?

Can a fraction be a factor of another fraction?

Exemplar Questions

Explain each of these methods for finding $\frac{2}{5} \div \frac{3}{4}$

$$\frac{2}{5} \div \frac{3}{4} = \frac{8}{20} \div \frac{15}{20} = \frac{8}{15}$$

$$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3} = \frac{8}{15}$$

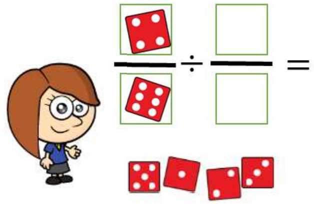
$$\frac{2}{5} \div \frac{3}{4} = \frac{0.4}{0.75} = \frac{40}{75} = \frac{8}{15}$$

$$\frac{2}{5} \div \frac{3}{4} = x$$
$$\frac{2}{5} = \frac{3}{4}x$$
$$\frac{2}{5} \times 4 = 3x$$
$$\frac{8}{5} \div 3 = x$$
$$\frac{8}{15} = x$$

Calculate the answers to the following, thinking carefully about which method to use.

$\frac{8}{9} \div 0.9$	$\frac{63}{35}$ divided by $\frac{70}{175}$	How many times does $\frac{4}{100}$ fit into 0.39	$\frac{5}{6}y = \frac{35}{48}$

Eva and Rosie are playing a game with 4 dice arranged in a calculation. If the answer is a whole number you win a point.



What numbers could Eva roll to score a point?

Here is Rosie's roll. Can she score a point?

Improper and mixed fractions H

Notes and guidance

Teachers might introduce this by using a bar model to make links between repeated addition, multiplication and improper and mixed fractions. Being able to visualise this, helps to develop 'number sense' around multiplying and dividing fractions. It could be useful to highlight the common misconception that integer values cannot be multiplied and divided separately is worth noting.

Key vocabulary

Unit fraction	Repeated addition	Whole
Numerator	Denominator	

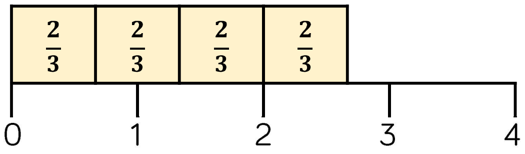
Key questions

Count up in thirds starting from 0
Did you count up in improper fractions or mixed fractions?

Is it easier to multiply and divide fractions as improper or mixed?

Exemplar Questions

How many different written representations can you find these for this bar model?



Alex is trying to work out $6\frac{4}{5} \div 2\frac{2}{5}$. What has Alex done wrong?

$6 \div 2 = 3$

$\frac{4}{5} \div \frac{2}{5} = 2$

So the answer is $3 + 2 = 5$

Work out the following multiplications.

$2\frac{1}{4} \times 3$ $\quad$ $2\frac{1}{4} \times 4$ $\quad$ $5 \times 2\frac{1}{4}$

$5\frac{5}{8} \times 1\frac{7}{9}$ $\quad$ $5\frac{5}{8} \times 1\frac{5}{9}$ $\quad$ $5\frac{5}{8} \times 1\frac{5}{7}$

Work out the following divisions.

$1 \div \frac{2}{5}$ $\quad$ $1 \div 2\frac{2}{5}$ $\quad$ $2 \div 2.4$

$4\frac{3}{8} \div 2\frac{1}{7}$ $\quad$ $2\frac{1}{7} \div 4\frac{3}{8}$ $\quad$ $\frac{22}{7} \div 4.375$

White Rose Maths

Algebraic Fractions

H

Notes and guidance

There have been opportunities throughout these small steps for students to generalise a method algebraically. Here we will look at calculations that involve algebraic fractions. Students should be encouraged to reason every step of their solution.

Pictorial representations, such as bar models, may still be appropriate when introducing this small step.

Key vocabulary

Generalise	Cancel	Term
Expression	Simplest Form	

Key questions

How many different ways can you write a quarter of x?

Can you have an improper algebraic fraction?

Can we use repeated addition for multiplying algebraic fractions?

Exemplar Questions

Imran is working out $\frac{2x}{5} \div \frac{x}{5}$

He says the answer is 2 without doing any working out.

How does he know this?

His next question is $\frac{2x}{5} \div \frac{5}{x}$

Will the same method work? Why or why not?

Work these out, giving your answers in their simplest form.

$\frac{2}{5} \times \frac{w}{r}$	$\frac{2w}{5} \times \frac{w}{r}$	$\frac{2w}{5} \times \frac{w}{r} \div 2$
$\frac{3}{5w} \times \frac{w}{r}$	$\frac{3r}{5w} \times \frac{w}{3r}$	$\frac{3r}{5w} \times \frac{2w}{3r^2} \times \frac{1}{m}$
$\frac{4}{5w} \div \frac{r}{w}$	$5 \times \frac{4}{5w} \div \frac{r}{w}$	$5 \div \frac{4}{5w} \div \frac{r}{w}$

What is the same and what is different about these pairs of calculations?

$\frac{a}{2} \times \frac{4a}{3}$	$\frac{2}{a} \times \frac{4a}{3}$
$\frac{3}{b} \div \frac{b}{5}$	$\frac{b}{3} \div \frac{b}{5}$
$\frac{c^2}{4}$	$\left(\frac{c}{4}\right)^2$
$(d+5) \div 2d$	$(d+5) \div \frac{2}{d}$

Autumn 2: Representations

Weeks 1 to 3: Working in the Cartesian Plane

Building on their knowledge of coordinates from KS2, students will look formally at algebraic rules for straight lines, starting with lines parallel to the axes and moving on to the more general form. They can explore the notions of gradient and intercepts, but the focus at this stage is using the equations to produce lines rather than interpretation of m and c from a given equation; this will be covered in Year 9. Use of technology to illustrate graphs should be embedded. Appreciating the similarities and differences between sequences, lists of coordinates and lines is another key point. Students following the higher strand may also explore non-linear graphs and mid-points of line segments.

National curriculum content covered:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- develop algebraic and graphical fluency, including understanding linear (and simple quadratic) functions
- make connections between number relationships, and their algebraic and graphical representations
- substitute numerical values into formulae and expressions
- recognise, sketch and produce graphs of linear functions of one variable with appropriate scaling, using equations in x and y and the Cartesian plane

Weeks 4 and 5: Representing data

Students are introduced formally to bivariate data and the idea of linear correlation. They extend their knowledge of graphs and charts from Key Stage 2 to deal with both discrete and continuous data.

National curriculum content covered:
describe, interpret and compare observed distributions of a single variable through: appropriate graphical representation involving discrete, continuous and grouped data

- construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data
- describe simple mathematical relationships between two variables (bivariate data) in observational and experimental contexts and illustrate using scatter graphs
- use language and properties precisely to analyse probability and statistics

Weeks 6: Tables and Probability

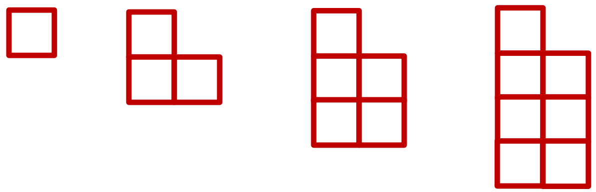
Building form the Year 7 unit, this short block reminds students of the ideas of probability, in particular looking at sample spaces and the use of tables to represent these.

National curriculum content covered:

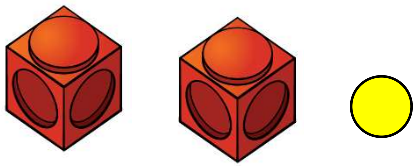
- record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness, fairness, equally and unequally likely outcomes, using appropriate language and the 0-1 probability scale
- generate theoretical sample spaces for single and combined events with equally likely, mutually exclusive outcomes and use these to calculate theoretical probabilities
- use language and properties precisely to analyse probability and statistics

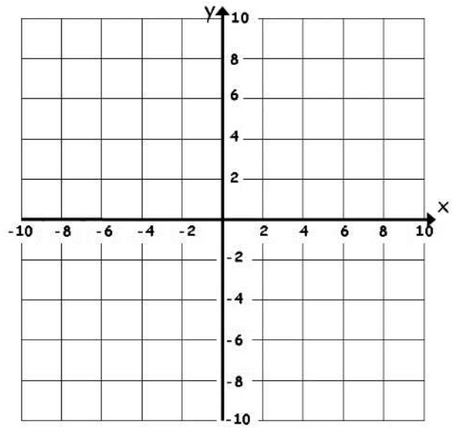
Key Representations

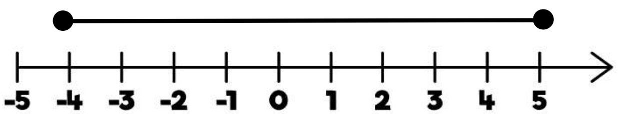
Input → $\times 2$ → -1 → Output



x	1	2	3	4
y	1	3	5	7







In these small steps, axes that allow the use of all four quadrants should be used. It is important that children see a wide range of axes, for example, those with different scales. If appropriate, counters and small cubes can be used to demonstrate a coordinate before it is marked onto the grid.

When introducing the midpoint of a line segment, teachers might start with looking at numbers that are half way along a number line (tape measures may be of use here).

Making links between sequences, co-ordinates and the equation of a straight line help students to make sense of these relationships and therefore children should have opportunities to explore the pictorial representations of these to aid investigation and enhance understanding.

White Rose Maths

Working in the Cartesian Plane

Small Steps

- Work with coordinates in all four quadrants
- Identify and draw lines that are parallel to the axes
- Recognise and use the line $y = x$
- Recognise and use lines of the form $y = kx$
- Link $y = kx$ to direct proportion problems
- **Explore the gradient of the line $y = kx$** H
- Recognise and use lines of the form $y = x + a$
- Explore graphs with negative gradient ($y = -kx, y = a - x, x + y = a$)

H denotes higher strand and not necessarily content for Higher Tier GCSE

Working in the Cartesian Plane

Small Steps

- Link graphs to linear sequences
- Plot graphs of the form $y = mx + c$
- **Explore non-linear graphs** H
- **Find the midpoint of a line segment** H

H denotes higher strand and not necessarily content for Higher Tier GCSE

Coordinates in all four quadrants

Notes and guidance

Students build on their KS2 knowledge of the coordinate plane, created by the intersection of two number lines in 2-D space, developing their understanding of the x-axis, y-axis and the origin. Students should be given the opportunity to draw their own axes and need careful support in labelling these correctly. Students should be able to label each quadrant from 1^{st} to 4^{th}

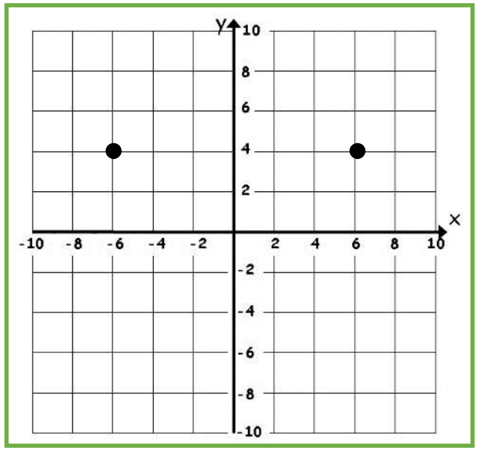
Key vocabulary

Quadrant	Coordinates	Horizontal
Vertical	Axis	Origin

Key questions

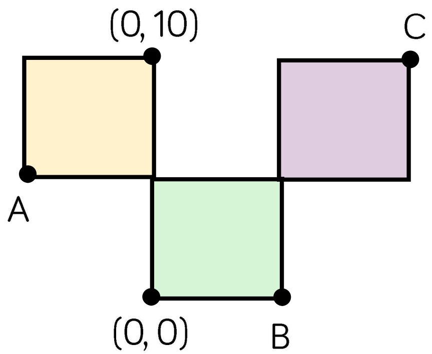
What is the same and what is different about the points with coordinates $(a, 0)$ and $(-a, 0)$?
Why are coordinates $(a, 0)$ and $(0, a)$ different?
Why do the order of the numbers in a coordinate matter?
Describe how you read and plot a coordinate.
Where is the origin?

Exemplar Questions



Write down the coordinates of the points highlighted.
By plotting one further point, create an isosceles triangle and work out its area.
Investigate other types of triangles that can be created by plotting one other point.

Plot the coordinates $(1, -4), (7, 3), (-4, 3), (9, -5)$ on a coordinate grid.
Which two coordinates are on the same line?
Which coordinate is in the second quadrant?



Three identical squares are shown.

Work out the coordinates of the points A, B and C.

Explain your strategy.

Lines parallel to the axes

Notes and guidance

Students are introduced to the idea of a straight line as an infinite set of points with a common feature. Care needs to be taken to ensure they understand that lines parallel to the x-axis have equations of the form $y = a$ and similarly $x = a$ will be parallel to the y-axis. This should be revisited regularly in starter activities to aid retention. The term ‘parallel’ might need revision here.

Key vocabulary

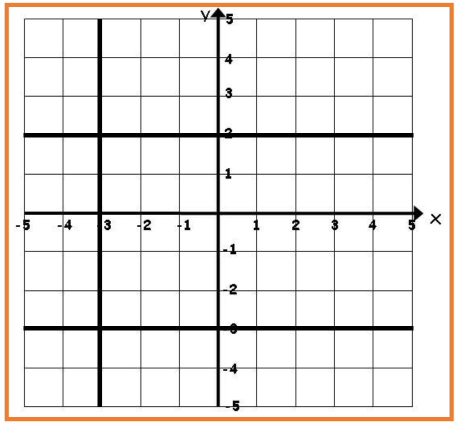
Parallel	Straight line	Vertical
Horizontal	Equation	Graph

Key questions

Give an example of an equation of a line that is parallel to the x-axis/y-axis.
Is the line $3 = x$ the same as the line $x = 3$? What about the line $x - 3 = 0$?
Why is the line $x = 0$ different from the x-axis?
Will the lines $x = \cdots$ and $y = \cdots$ ever meet? Why or why not?

Exemplar Questions

Plot and join the points with coordinates $(4, 1)$, $(4, -3)$ and $(4, 5)$
What do you notice? Write down another coordinate on the line.
How many points are on the line in total? How do you know?
Why do you think we call this line $x = 4$?



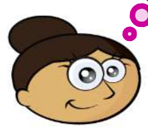
Write down the equations of the lines shown.

Label the lines with their equations. Draw the line $x = 4$ onto the grid.

Write down the coordinates of the points where the lines intersect.

The line $y = 3$ is parallel to the y-axis.

The line $y = 3$ is parallel to the x-axis.



Dora

Who do you agree with? Use a diagram to help explain why.

Amir

Recognise and use the line $y = x$

Notes and guidance

This step is to help students understand the line $y = x$, the first ‘diagonal’ line they will formally study. It should be explicitly covered that the line will only form a 45 degree angle with the axes if the scales are equal on both axes.

If appropriate with students following the Higher strand, compare and contrast with the line $y = -x$

Key vocabulary

Axis	Diagonal	Straight line
Origin	Scale	Graph

Key questions

Is the graph $y = x$ the same as the graph $x = y$?

How many points lie on the line $y = x$? Why?

Why are the scales of the axes important when plotting graphs?

Exemplar Questions

Plot the points given in the table. Give the coordinates of three more points on the line.

x	-4	-1.5	3	$5\frac{1}{2}$
y	-4	-1.5	3	$5\frac{1}{2}$

Discuss as a class what the equation of the line should be.

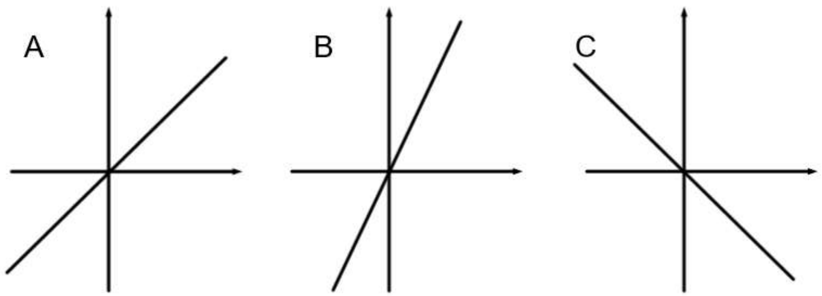
Which of the following points will lie **on** the line $y = x$?
Which of the others lie above the line $y = x$, and which lie below?

$(19, 19)$ $(-10, -9 - 1)$ $(8, 7)$ $(7, 8)$

(a, a) $(0.3, 0.3)$ $(b \times 2\, , b + b)$ $(6, -6)$

Whitney

Whitney thinks graph A and graph C could be $y = x$, but graph B can’t be because it’s not at 45 degrees to the axes. Do you agree? Why?



Recognise and use lines $y = kx$

Notes and guidance

This step builds on the understanding of the line $y = x$ by introducing k and highlighting its affect on the steepness of a line. Lines with equation $y = kx$, make up a family of straight lines through the origin. Students can use graphical software to explore how increasing/decreasing the value of k affects the graph. It is important to vary coordinates and scale values beyond an integer.

Key vocabulary

Multiple	Steep	Linear	Substitute
Table	Slope	Scale	Axes

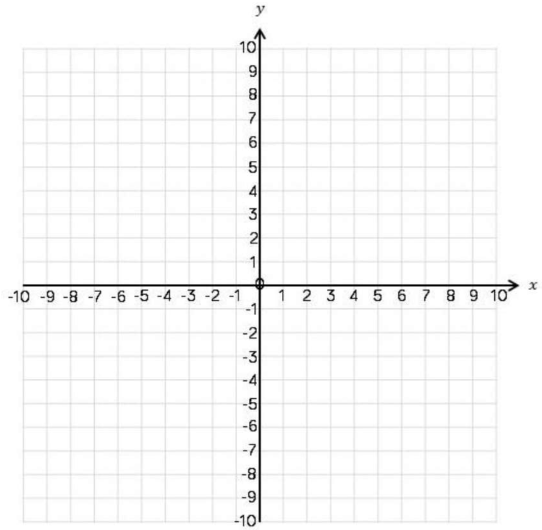
Key questions

How can you recognise a line of the form $y = kx$?
What's the same and what is different about the lines $y = kx$ and $y = x$?
What effect does increasing or decreasing the value of k have on lines with equations in the form $y = kx$?
Do all lines with equations in the form $y = kx$ form a straight line and go through the origin? Why or why not?

Exemplar Questions

Tommy is completing a table of values to plot the graph $y = 2x$
Complete the table and plot the graph using the axes provided.

x	-3	-2	-1	0	1	2
$y = 2x$	-6				2	



Using the same axes, plot the graph of $y = x$

What is the same and what is different about the two lines?

Which graph is steeper and why?

Using the same method as Tommy, plot the following two graphs.

$y = 4x$

$y = \frac{3x}{2}$

Why is it important to consider which x values to use? How will this impact on the scale of your y-axis?
What is the same and what is different about the two lines you have drawn?

Direct proportion using $y = kx$

Notes and guidance

Teachers might introduce this using a familiar context, for example, 'if one apple costs 20p, how much would 2 apples cost, 3 apples, 0 apples etc.' This helps to introduce the idea of a constant multiplier and hence, direct proportion.

Teachers should illustrate direct proportion using different representations, for example, tables, graphs and equations.

Key vocabulary

Linear	Proportion	Scale
Unitary	Multiplier	Direct

Key questions

How would you know if a straight line or a table of values represents direct proportion? What are the key features?
What is a conversion graph and how can information be obtained from it to answer questions?
Why do direct proportion graphs always start at $(0, 0)$?

Exemplar Questions

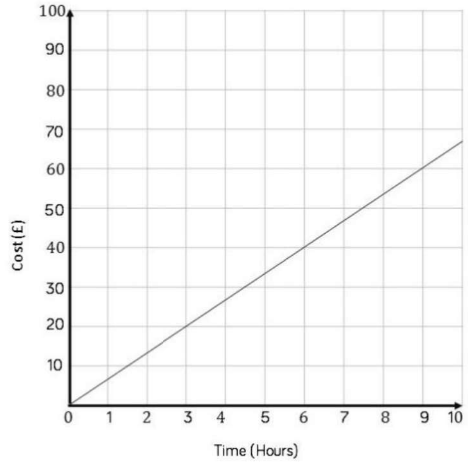
No of Pencils	1	2	3	4	5	6	10	40
Cost		36p						

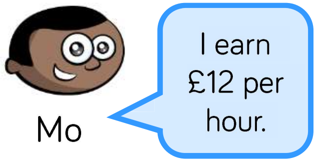
Two pencils cost £0.36, use this to complete the table.
What would the cost of 8 pencils be? What about 100 pencils?
What do you notice with the numbers in the table, what is the link?

This conversion graph shows the cost and time it takes to paint a room.

What is the cost of painting a room that takes 3 hours?

How long would it take to paint room costing £120?





Create a table of values showing the number of hours Mo worked and his earnings.
Draw the straight line graph that represents these values. What is the equation of this line?

Use this equation to work out how much Mo earns in 37.5 hours.

Gradient of the lines $y = kx$ H

Notes and guidance

Teachers might introduce this using real-life examples of gradients (e.g. pictures of hills and mountains). Students are then introduced to finding the gradient using right-angled triangles on the straight line. This can be extended to finding the equation for the gradient given two points (without the straight line being drawn onto a graph). Linking both gradient to k and direct proportion is useful here.

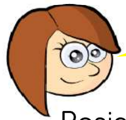
Key vocabulary

Steepness	Difference	Gradient
Straight line	Vertical	Horizontal

Key questions

What does the gradient of a line represent?
How do we know if one line is steeper than another?
Does it matter which right-angled triangle we choose on the straight line when we are calculating the gradient?
What does a gradient of zero mean?
How can working out the gradient of a line help in direct proportion calculations?

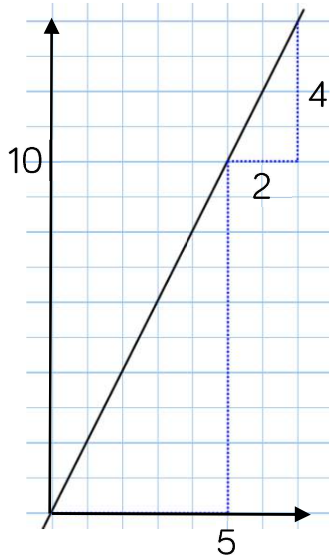
Exemplar Questions



Rosie: The gradient of this line is $\frac{4}{2}$

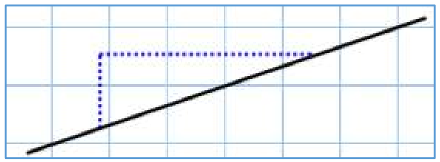
Jack: The gradient of this line is $\frac{10}{5}$

Mo: The equation of the line is $y = 2x$. The gradient is 2

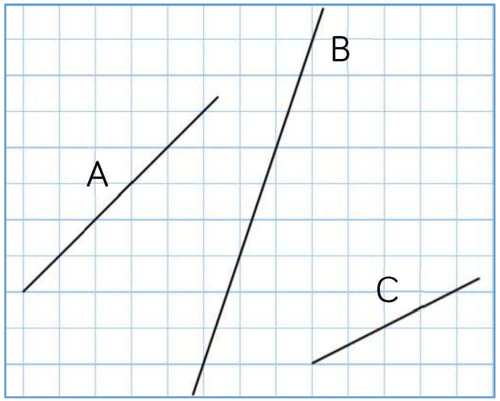


Who is correct and why?

Amir has drawn a right-angled triangle onto the line. What could Amir change about his right-angled triangle to help him work out the gradient?
Work out the gradient of this line.



Using right-angled triangles, calculate the gradients of these lines. Place them in order of steepness.



Lines of the form $y = x + a$

Notes and guidance

Students now consider the impact of adding a constant to the line with equation $y = x$. Students should be encouraged to explore the effect this has on the straight line by generating tables of values and plotting these.
Using a dynamic geometry package supports whole class discussion on this.

Key vocabulary

Equation	Input	Output
Intercept	Linear	Straight line

Key questions

What is the same and what is different about the line $y = x$ and the line $y = x - a$?
What is the gradient of the line $y = a + x$?
What about $y = x + a$?
Is $a - x = y$ the same line as $x + y = a$? Explain.
Explain how you could check that you have plotted the line $y = x + a$ correctly. What could you look for?

Exemplar Questions

Complete the table of values to generate a set of coordinates and plot the graph of $y = x + 2$

x	-3	-2	-1	0	1	2
y						

Generate further tables of values and draw the graph of $y = x - 2$ and $y = x + 5$ on the same axis.
What is the same and what is different about the graphs?

Match each graph with its equation. Explain your strategy.

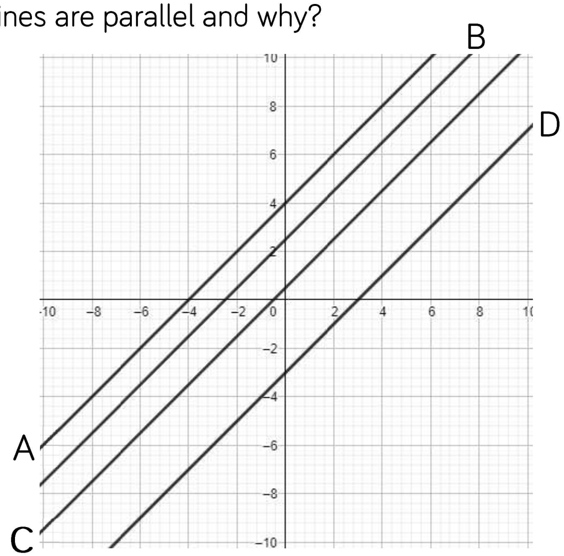
- What do these lines have in common?
- Where do these lines cross the x-axis and the y-axis?
- Which lines are parallel and why?

$y = x + 4$

$y = 2.5 + x$

$y = x - 3$

$x + \frac{1}{2} = y$



Graphs with negative gradients

Notes and guidance

Teachers might start by checking student confidence in using a negative multiplier. The concept of a negative gradient could be introduced by discussing what the gradient of a ski slope might be. Students can then draw straight line graphs where the line has a negative gradient. Equations should be given in different forms (e.g. $y = a - x$, $x + y = a$). Calculating the gradient when it is negative is not expected at this stage.

Key vocabulary

Negative	Gradient	Steepness
Incline	Ratio	Slope

Key questions

What's the same and what's different about the straight lines represented by the equations $y = kx$ and $y = -kx$?
How can you identify whether a straight line, plotted on a graph, has a negative or positive gradient?
How can you identify the type of gradient (positive or negative) of a line by just looking at the equation of the line?

Exemplar Questions

Complete the table of values to generate coordinates and plot the graph $y = -x$

x	-3	-2	-1	0	1	2
y		2				

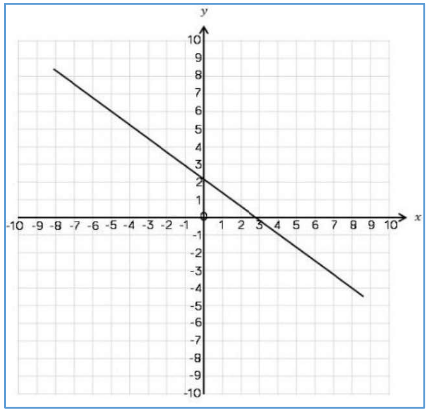
What is the same and what is different about the straight lines represented by the equations $y = x$ and $y = -x$?
Now plot the graph of $y = -3x$ and $y = -\frac{3x}{2}$
What do you notice?

For each of the following equations:

- Use x values from -3 to 3, generate a table of values for x and y
- Plot each straight line onto a separate graph.
- Label each line with the equation and then state whether the gradient is positive or negative. How can you see this from your graph?

$y = 3 - x$ $\quad$ $y + x = 5$ $\quad$ $y - x = 2$

True or False?
The line has a positive gradient.
When x is 0, y is 2
When y is 3, x is 0
When x is a negative number, y is a positive number.
x is always less than y
As x increases, y decreases.



Linking graphs to sequences

Notes and guidance

Students link prior knowledge of sequences with linear equations and their respective graphs. Using pictorial sequences, function machines, coordinates and tables of values, children can explore relationships between these, establishing important connections.
Teachers should highlight how coordinates on a straight line relate to the position and term in a sequence.

Key vocabulary

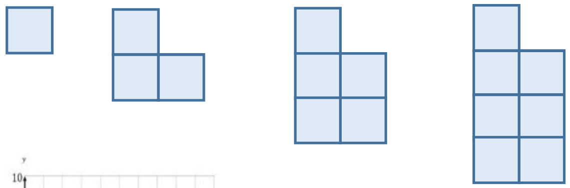
Difference	Sequence	Descending	Linear
Equation	Multiple	Ascending	

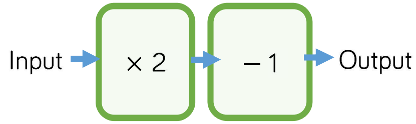
Key questions

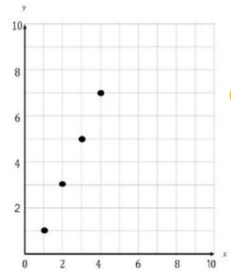
What's the same and what's different about linear graphs and linear sequences? How could we label the axis on a the graph to show the position of a term in the sequence?
Will the gradient of the straight line representing a descending linear sequence be positive or negative? Explain your answer.

Exemplar Questions

What is the same and what is different about these representations?







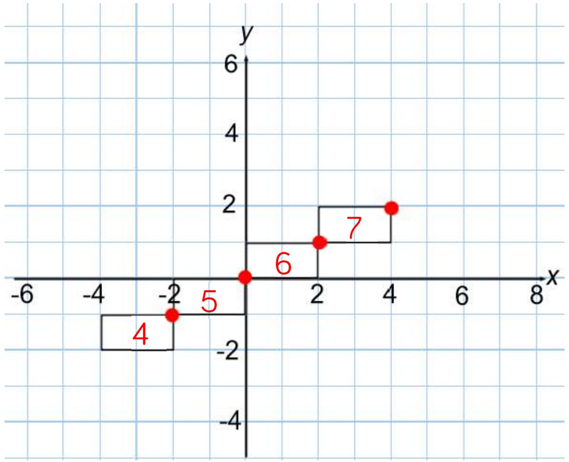
$y = 2x - 1$

(1, 1) (2, 3) (3, 5) (4, 7)

Position	1	2	3	4
Term	1	3	5	7

Write down the coordinates of the points highlighted in each rectangle. What patterns do you notice?

Write down the coordinates of the top right hand vertex of the 3^{rd} and 8^{th} rectangle in the pattern.



The equation of the line which goes through the marked coordinates is $y = \frac{1}{2}x$. How does this relate to the marked coordinates and the patterns you have noticed?

Plotting $y = mx + c$ graphs

Notes and guidance

Students further develop their understanding of equations of straight lines using the general form of $y = mx + c$

This step focuses purely on students becoming familiar with plotting graphs and generating coordinates from a table of values using $y = mx + c$

Interpretation of the equation will be covered in later steps.

Key vocabulary

Gradient	Linear	Integer	Input
Output	Substitution	Table of Values	

Key questions

Why is it a good idea to use three coordinates when plotting a straight line graph?

Can you use non-integer x values in your table to generate your set of coordinates?

Can you extend your straight line outside of the range of values in your table? Explain your answer.

Exemplar Questions

On the same axes, draw the graphs of the following equations by completing the table of values. Discuss key features of each graph.

$y = 3x - 1$

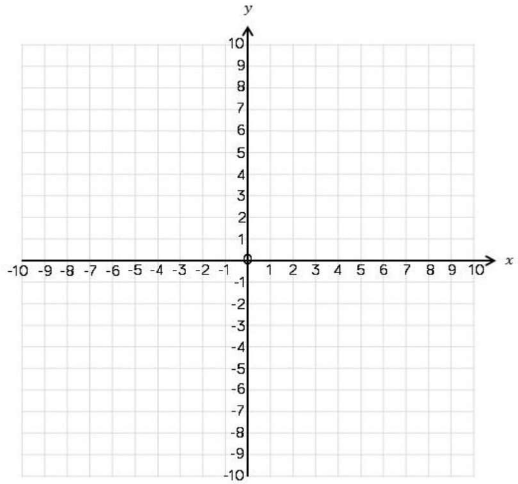
x	-3	0	3
y			

$y = \frac{1}{2}x + 3$

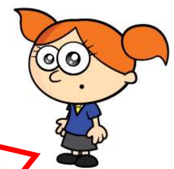
x	-6	0	6
y			

$y = -2x + 6$

x	-2	0	2
y			



Plotting the graphs can help identify any errors in your table.



Alex

Comment on Alex's statement. What does she mean?

Match each coordinate with the equation of the line it could lie on.

$(-2, 8)$ $(5, 4\frac{1}{4})$ $(6, 3.5)$

$y = 10 + x$ $\frac{x+1}{2} = y$ $y = \frac{x}{4} + 3$

Exploring non-linear graphs H

Notes and guidance

Students are introduced to plotting and identifying non-linear graphs. Students will need guidance on how to draw a smooth curve to join the coordinates. It's helpful to discuss why it's inappropriate to join the coordinates with a straight line. Teachers may want to start with x^n where $n > 1$, e.g. $y = x^2$, $y = x^3$ etc.
Further investigation of non-linear graphs can be supported using graphical computer software.

Key vocabulary

Equation	Linear	Curve
Non-linear	Symmetrical	

Key questions

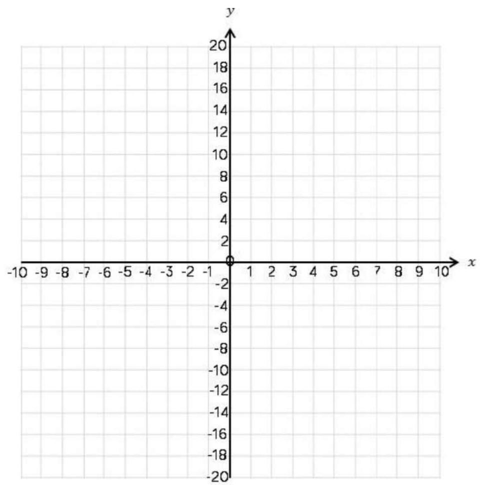
Describe the differences between a linear and a non-linear graph.
How can you use the equation of the graph to determine whether it is linear?
How do we work out the scale for our axes?

Exemplar Questions

Continue completing Dora's table of values to generate a set of coordinates and plot the graph of $y = x^2$ using the axis provided. Describe key features of the graph, including the scale used on the axes.

Dora

x	-3	-2	-1	0	1	2	3	4
y					1			16



Substituting a with numbers, investigate the following graphs using graphical computer software.

$y = ax^2$

$y = ax^3$

$y = \frac{a}{x}$

Which of these equations will produce a non-linear graph. Why?
Check your answer using graphical computer software.

$y = x - \frac{7}{2}$

$y - 4 = 0.5x$

$y = \frac{3}{2}x$

$y = x^2 + 3$

$y = \frac{4}{x}$

$x^3 + y = 5$

Midpoint of a line segment

H

Notes and guidance

Students firstly consider midpoints on number lines.
This allows them to consider the most efficient method of finding a midpoint. Students then build on this to find the coordinates of a midpoint of a line segment, finding a general rule. Finally, they might explore finding the starting point or end point of a line segment given the midpoint.

Key vocabulary

Midpoint	Equidistant	Segment
Difference	Mean	

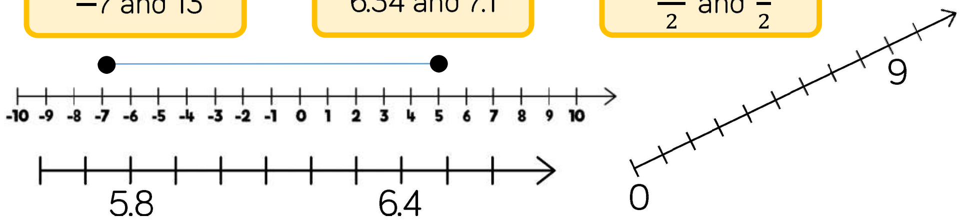
Key questions

What does the word equidistant mean?
How can you work out a midpoint? Is there more than one way?
If given the coordinates of the midpoint, and of the starting point of the line, how can you work out the coordinates of the endpoint of the line?

Exemplar Questions

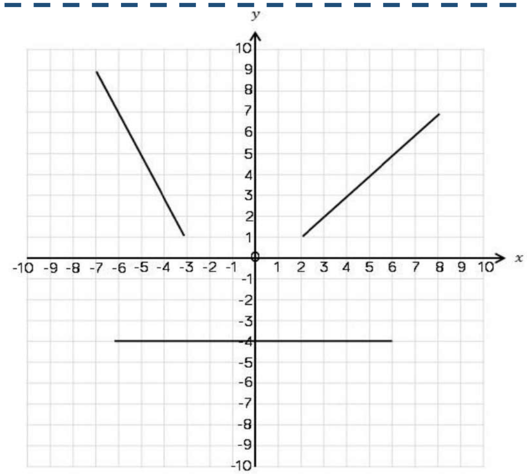
Find the number halfway between each of the pairs shown.
Discuss your strategies.

-7 and 13 | 6.34 and 7.1 | $\frac{15}{2}$ and $\frac{5}{2}$



Calculate the midpoint of each pair of coordinates.

(6, 8) and (6, 20) | (4, 7) and (12, 19) | (2, 7) and (−2, 15)



Work out the midpoint of the following line segments.

Using the same coordinate grid, draw a line with a midpoint of (0, 1)

What are the end points?
How many different pairs of end points could there be?

A line segment starts at (2, 5). It has a midpoint of (6, 1)
What is the end point of the line?

Representing Data

Small Steps

- Draw and interpret scatter graphs
- Understand and describe linear correlation
- Draw and use line of best fit (1) & (2)
- Identify non-linear relationships
- Identify different types of data
- Read and interpret ungrouped frequency tables
- Read and interpret grouped frequency tables
- Represent grouped discrete data
- Represent continuous data grouped into equal classes
- Represent data in two-way tables

Draw and interpret scatter graphs

Notes and guidance

Students need to be confident in drawing and labelling axes, scaffolding this may be necessary. A wide range of examples should be used (include numbers less than 1 and bigger than 1000). Examples of pairs of variables that are appropriate/not appropriate to represent using a scatter graph should be discussed.

Key vocabulary

Variable	Relationship	Origin
Scale	Coordinate	Axis
Increase	Decrease	

Key questions

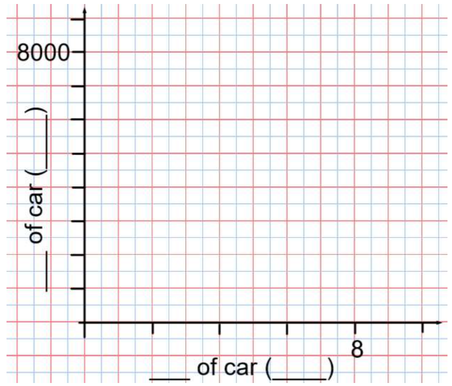
How do we use the data to generate coordinates?
Does it matter if the data points are not in size order?
How do we know how long to draw our axes?
How do we know what scale to use on our axes?
Which labels do we need to place on our graph?

Exemplar Questions

The table shows the age and value of a car.

Age of Car (Years)	2	4	6	8	10
Value of Car (£s)	7500	6250	4000	3500	2500

Complete the pair of axes.
Now use the data in the table to generate coordinates and plot them on the graph.
Complete the sentence,
As the age of the car ________,
the value of the car ________.
Do you think this will always be true? Explain your answer.



For each of the following, decide whether it is appropriate to use a scatter diagram to represent the data. If it is appropriate, sketch what the scatter diagram might look like. If it isn't appropriate, explain why.

Colour of car and make of car

Cost per mile and distance travelled

Number of ice creams sold and temperature during the day

Distance a student lives from school and height of student

Linear correlation

Notes and guidance

In this small step, teachers support students to recognise positive and negative correlation. They also consider the strength of this correlation. Students are also able to decide if there is no correlation, or non-linear correlation. Even when there is non-linear correlation, it is still possible that there is a relationship between the variables and students may need support in describing this.

Key vocabulary

Relationship	Correlation	Positive
Negative	Strong	Weak

Key questions

How can you tell if correlation is positive or negative?
How is correlation useful to us? Can you give some real-life examples?
What's the same and what's different about positive and negative correlation? Can you give some real-life examples for each?

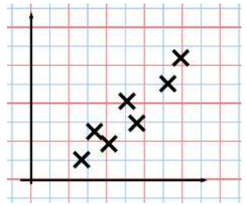
Exemplar Questions

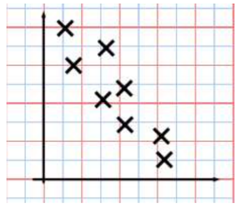
Match one description to each graph. Give two other example descriptions that could fit each graph.

Height and weight of 5 to 18 year-olds

Amount of petrol in tank and distance travelled

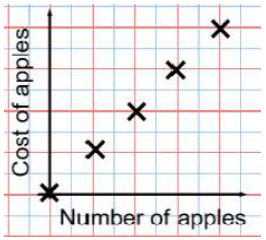
Average time watching TV and size of TV

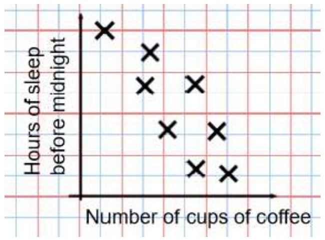


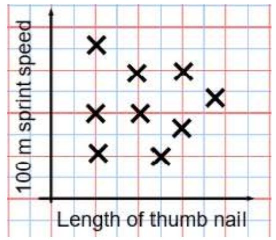


Complete the sentences.
As one variable ___________, the other variable also ___________.
This relationship is called ___________ correlation.
As one variable ___________, the other variable ___________.
This relationship is called ___________ correlation.

For each graph below, state the type of correlation shown.
Describe the relationship between the two variables.







Draw and use line of best fit (1)

Notes and guidance

Teachers should check student misconceptions around lines of best fit: it doesn't have to go through the origin, it doesn't have to go through all of the points, it isn't always drawn from bottom left to top right. Students need to understand that there are approximately the same number of points above the line as below it. Also explore why the line of best fit is straight rather than curved.

Key vocabulary

Line of best fit	Origin	Estimate

Key questions

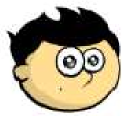
True or false:

- The line of best fit has to go through the origin
- The line of best fit goes through as many points as possible
- The line of best fit extends across the whole graph

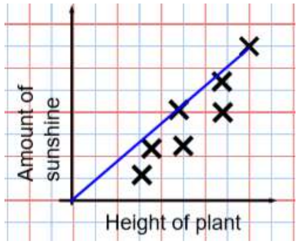
Why do you need the line of best fit in order to make a good estimate? How can you show your method for estimating on the graph?

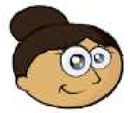
Exemplar Questions

Jack and Dora are both drawing a line of best fit.
Whose method is better? Explain why.

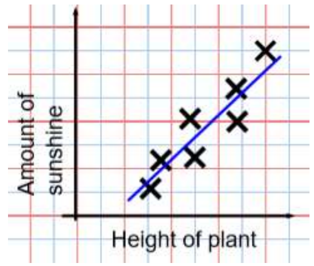


Jack





Dora



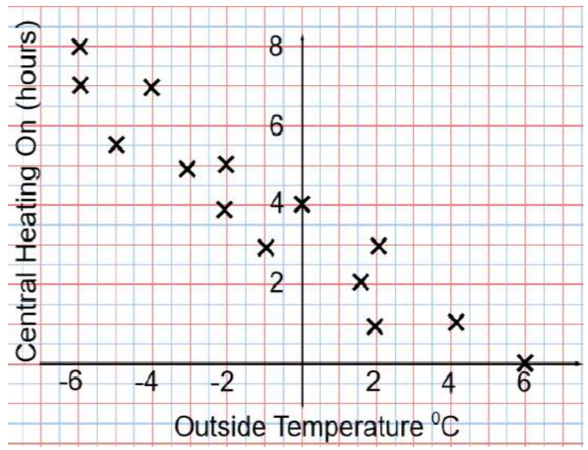
Jack has joined the point representing the tallest plant with the origin.

Dora has wiggled her ruler around until there are roughly the same number of points on each side of the line.

Describe the correlation.

Explain why it's appropriate to draw a line of best fit on this graph. Draw a line of best fit on the graph.

Use your line of best fit to estimate how long the central heating is on for when the outside temperature is 0.5°C



Draw and use line of best fit (2)

Notes and guidance

Students should always show how they arrive at an estimate, by drawing additional lines on the graph.

The term 'extrapolation' will need explaining so that students are aware of why it isn't always sensible to make an estimate which is outside of the range of data presented.

Students are also introduced to outliers.

Key vocabulary

Line of best fit	Origin	Estimate
Straight	Extrapolate	Outlier

Key questions

What does 'extrapolate' mean?

Why might it be a risk to make an estimate outside of the range of your data?

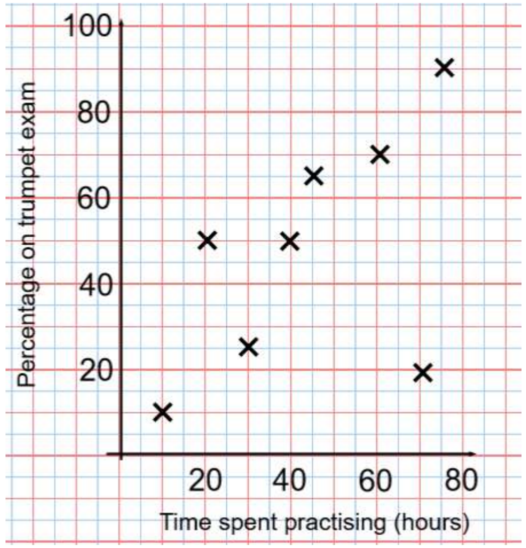
Exemplar Questions

Tommy has practised playing his trumpet for 70 hours. He wants to use the graph to estimate what percentage he will get in the exam.

He looks at the graph and estimates that he will get 20%. What mistake has Tommy made?

Use a line of best fit to estimate the percentage Tommy will get in the exam.

Identify an outlier.



Draw a scatter graph for the following data.

Height of plant (cm)	95	70	80	40	50	25
Width of plant (m)	0.85	0.25	0.8	0.45	0.45	0.2

Circle the outlier. Why is it an outlier?
Describe the relationship between the height of the plant and the width of the plant.
Draw on a line of best fit. Why isn't it sensible to estimate the height of a plant if the width is 2 m?

Identify non-linear relationships

Notes and guidance

Students are also able to decide if there is no correlation, or non-linear correlation.

Even when there is non-linear correlation, it is still possible that there is a relationship between the variables and students may need support in describing this.

Key vocabulary

Non-linear	Outlier	Variable

Key questions

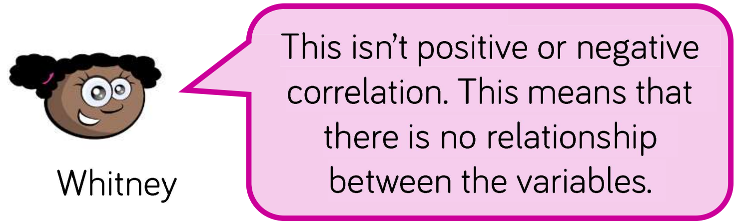
Is there a relationship between the data?

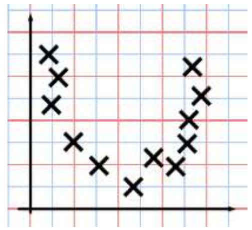
Is it linear or non-linear? How do you know?

What does non-linear mean?

Draw different representations of non-linear scatter graphs and add on possible labels for the axes.

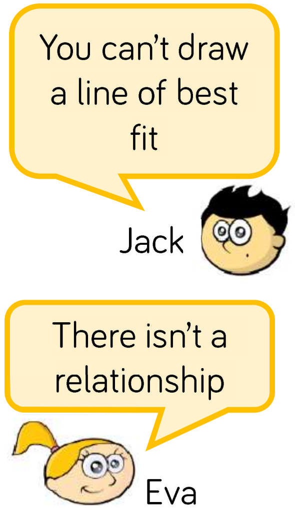
Exemplar Questions

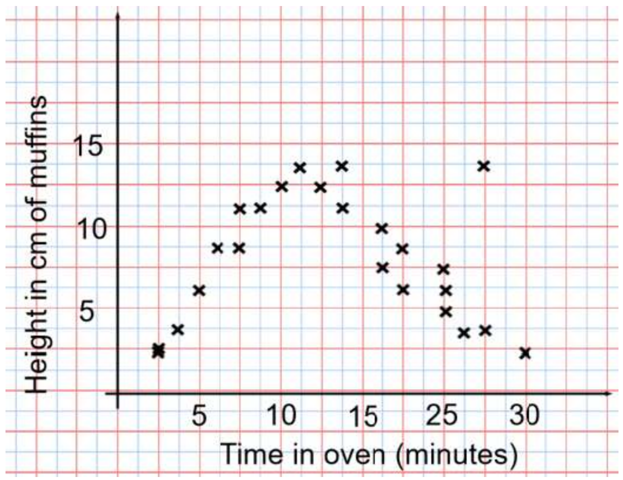




Explain why Whitney is wrong.

Jack's baking some muffins with his friend Eva. They construct this graph.





Jack's right and Eva is wrong. Explain why.
One muffin doesn't fit the pattern. What is the height of this muffin and how long has it been in the oven for?

Identify different types of data

Notes and guidance

In this small step, students are introduced to discrete and continuous data. Teachers could explain continuous data as 'measurements' and discrete data as 'counts'. Qualitative data is also introduced. It's important to establish that there are different data types and that we need to know these so that we can use appropriate graphs and calculations to represent them.

Key vocabulary

Discrete	Continuous	Measured
Counted	Qualitative	Quantitative

Key questions

How can we recognise discrete, continuous and qualitative data? Give me examples of each type.

Why do we need to know about different types of data?

Why do we sometimes have a gap between bars on a bar chart?

Exemplar Questions

Sort the statements into discrete and continuous data.
Two of the statements don't belong in either category, why?

Discrete Data:
E.g. Number of children on a bus

Continuous Data:
E.g. Heights of children on a bus

Number of school buses

Speed of school buses

Age of a person

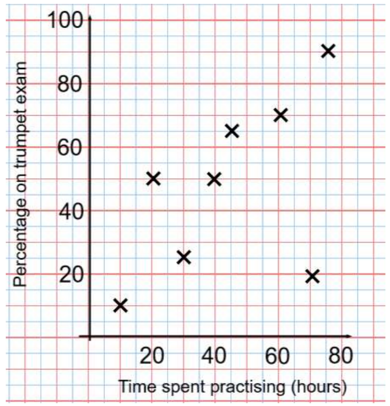
Make of mobile phone

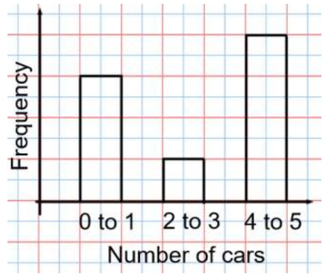
Cost of apples

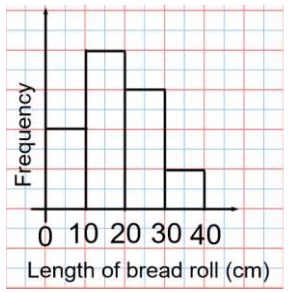
Favourite colour

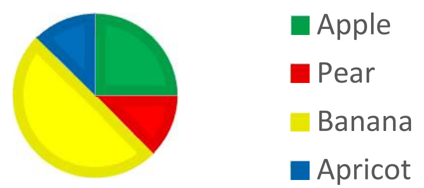
Explain the difference between the types of data to your partner.
Add an example of your own to each category.

For each graph and chart, decide what type of data is being represented, discrete, continuous or qualitative.









Ungrouped frequency tables

Notes and guidance

Students understand 'frequency' by counting the number of items in a given list, and by completing tables. They interpret data by using the information from the table to answer questions in context.
Teachers may want to do a class survey (for example, number of siblings) and calculate the total (by subtotalling each row in the table). Ensuring children work with '0' in either column of the table is key.

Key vocabulary

Frequency	Ungrouped
Total	Subtotal

Key questions

What does the word frequency mean?
What type of data is best represented by ungrouped frequency tables?
How can I calculate subtotals in my frequency table?
Do I still need the row if the frequency is 0?

Exemplar Questions

Dexter asks 10 children in his class how many siblings they have.
Here is his list: 4, 1, 0, 2, 1, 2, 0, 1, 2, 2

Dexter doesn't think he needs the row in his table for 3 siblings.

Is he right? Explain your answer.

Complete the frequency table.

Number of siblings	Frequency
0	
1	3
2	
3	
4	1

Country	Frequency
Poland	11
Spain	24
Greece	34
Pakistan	18
Ghana	1

The table shows results from a survey about which country people had visited.

- How many people took part in the survey?
- How many more people visited Greece than Spain?
- How many fewer people visited Ghana than Pakistan?

Goals Scored	Frequency	Total number of goals
5	4	
6	3	6 x 3 = 18
7	0	
8	1	
		Total Goals =

The table shows the number of goals scored in netball matches over the weekend.

Complete the table to work out the total number of goals scored.

Read & interpret grouped tables

Notes and guidance

This step might be taught in conjunction with the following step. Students start by exploring when it is and isn't appropriate to use an ungrouped frequency table. They then consider sensible class boundaries for grouped frequency tables. Teachers pose different questions related to the grouped frequency table so that students become familiar with reading and interpreting them.

Key vocabulary

Grouped	Tally	Range
Group	Frequency	Equal

Key questions

When would a grouped frequency table be more appropriate to use than an ungrouped table?

How can we work out how to group the data?

Why is it useful to have groups of equal size?

Exemplar Questions

Annie has completed a survey about sparrows. She wants to put her data into a table.

Number of sparrows spotted by different people in her class:
14, 7, 2, 18, 16, 15, 4, 3, 8, 1, 19, 5

Why would an ungrouped frequency table be unsuitable?
What groups would be suitable to use in your table? Explain why.

Number of sparrows	Frequency
6 - 10	2
11 – 15	

Use Annie's data to complete this grouped frequency table.

Look at the number in the circle. Write a full sentence to say what this number tells you.

Number of books	Frequency
0 – 10	2
11 –	3
21 – 30	
- 40	1

A group of 15 children were asked how many books they had in their house.
When results were put in a table, the pen leaked to leave blotches. What numbers are beneath the blotches?

Tommy thinks that 1 person had 40 books in their house.

Mo thinks that 1 person had 35 books in their house.
What do you think?

Alex thinks that the range of the number of books could be as much as 40 or as little as 21. Is she right? Explain why.

Represent grouped discrete data

Notes and guidance

Teachers might start by checking students understand the difference between grouped and ungrouped frequency tables. In this small step, the focus is on discrete data. Populating grouped frequency tables from different types of sources, such as a list of data or a set of written information regarding each group, supports understanding.

Key vocabulary

Grouped	Frequency	Discrete
Class	Class Boundary	Estimate

Key questions

Why do we have gaps between the classes in a discrete grouped frequency table?

If presented with a completed grouped frequency table, do we know the actual data items represented within each group?

Exemplar Questions

Mo is investigating the cost of TVs. He starts putting his data in a grouped frequency table. He has 10 more prices to add into his table:
£279, £120, £249, £239, £280, £299, £169, £150, £199, £299
Use these prices to complete the table.

Cost of TV (£)	Tally	Frequency
101 – 150	~~||||~~ ||	
151 – 200	~~||||~~ ~~||||~~ |	
201 – 250	~~||||~~	
251 - 300	|||	

How many TVs cost more than £150?

What's the maximum possible range for the cost of a TV?

The table shows how many babies were born in a hospital every day in February. Complete the table using the information provided.

Number of babies	0 – 2	3 - 5	6 - 8	9 - 11	12+
Number of days	7				

- On 5 days, 6 to 8 babies were born
- On 13 days, less than 5 babies were born
- 12 or more babies were born on 1 more day than 6 – 8 babies

Represent continuous data

Notes and guidance

Students may need a reminder of the difference between discrete and continuous data.

The idea of rounding continuous data is now explored as this links to why we use inequality signs when writing class boundaries. Students need reminding of the meaning of inequality signs.

Key vocabulary

Grouped	Tally	Less than/Equal to
Greater than	Discrete	Continuous

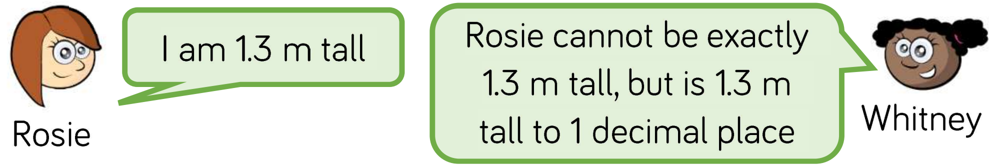
Key questions

Why is there a gap between the groups when the table represents discrete data?

Why isn't there a gap for continuous data?

When would a grouped frequency table be more appropriate to use than a non-grouped table?

Exemplar Questions



Explain why Whitney is correct.
Why do we need to round continuous data?

Match each inequality to the statement that describes it.
The first one has been done for you.

$0 < x \leq 10$	x is greater than or equal to 0 but less than or equal to 10
$0 \leq x \leq 10$	x is greater than 0 but less than 10
$0 < x < 10$	x is greater than 0 but less than or equal to 10
$0 \leq x < 10$	x is greater than or equal to 0 but less than 10

Eva records the weights of some eggs and records the results in this grouped frequency table. The last egg she weighed was 50 g.

Which group should she place it in? Amend the table to show the correct frequency for this last egg.

x Weight(g)	Frequency
$40 < x \leq 50$	1
$50 < x \leq 60$	3
$60 < x \leq 70$	5
$70 < x \leq 80$	1

Eva says that 90% of the eggs are less than or equal to 70 g in weight. Is she right?

Represent data in two-way tables

Notes and guidance

Students start with concrete or pictorial representations to help them understand the structure and purpose of a two-way table. Teachers should then ensure that students can find the correct piece of information from their table through questioning.

Topics such as fractions, percentages and ratio are easily interleaved into this small step.

Key vocabulary

Frequency

Totals

Ratio

Fraction

Percentage

Key questions

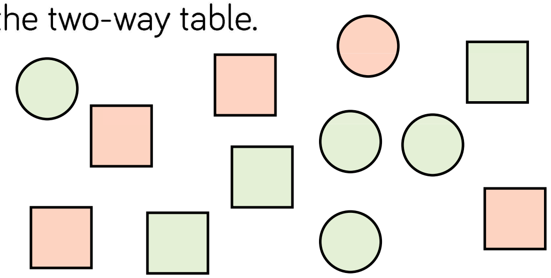
How do we know what each number in each cell means?

Why do we use columns and rows?

Why does the overall total only appear once in our table?

Exemplar Questions

A game has circle and square pieces. Count the pieces and complete the two-way table.



	Squares	Circles
Green	3	
Red		

Dora did a survey of her class about whether they preferred running or swimming. She recorded her results in this two-way table.

How many more girls prefer running to boys?

How many more children prefer swimming to running?

Draw a frequency tree to represent this data.

	Boys	Girls	Totals
Running	6	9	15
Swimming	13	12	25
Totals	19	21	40

There are 24 chocolates in a box. $\frac{1}{3}$ are dark chocolate and the rest are milk chocolate. Of these, some have a soft centre and the rest have a chewy centre. 5 of the milk chocolates have a chewy centre. 25% of the dark chocolates have a soft centre.

Complete the two-way table.

Write down the ratio of soft chocolates to chewy chocolates.

	Soft		
Milk			
Totals			

Tables and Probability

Small Steps

- Construct sample spaces for 1 or more events
- Find probabilities from a sample space
- Find probabilities from two-way tables
- Find probabilities from Venn diagrams
- **Use the product rule for finding the total number of possible outcomes** H

H denotes higher strand and not necessarily content for Higher Tier GCSE

Constructing sample spaces

Notes and guidance

In Year 7 students were introduced to the concept of an experiment, event and outcome. Building on this knowledge, students now consider sample spaces, listing all possible outcomes in an experiment. Sample spaces are usually denoted by the letter S and can be written using set notation, { }. Emphasis should be placed on a systematic approach to listing all possible outcomes.

Key vocabulary

Outcomes	Sample space	Set
Probability	Systematic	Chance

Key questions

What is a sample space and how can you ensure you have listed all possible outcomes in your sample space?

Why is being systematic important when listing outcomes?

How can you determine what method or type of sample space diagram to use?

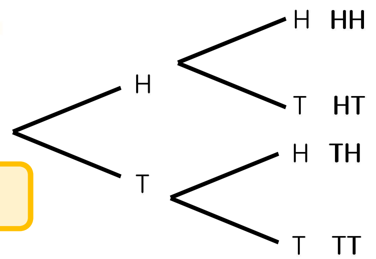
Exemplar Questions

Two coins are flipped at the same time. Compare and contrast what is the same and what is different about the representations below?

	H	T
H	HH	HT
T	TH	TT

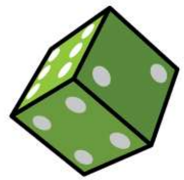
$S = \{TT, TH, HT, HH\}$

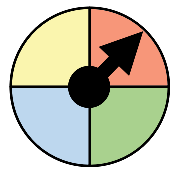
HH, HT, TH, TT



A spinner is spun and a fair die is rolled at the same time. Complete the table listing all the possible outcomes.

	1	2	3	4	5	6
R	1R					
G		2G				
B						
Y						6Y





Eva is working out all the possible combinations she can have for her lunch. Continue listing all her combinations.

Meal Deal £2
Drink: *Water or Juice*
Sandwich: *Tuna, Cheese or Ham*
Fruit: *Apple or Banana*

Drink	Sandwich	Fruit
W	T	A

White Rose Maths

Probability from sample space

Notes and guidance

Students build on their understanding of creating sample spaces by working out probabilities of events. There is introduction of the notation P(event), articulated as the probability of an event happening. It is important to emphasise the different ways probabilities can be represented and to consider when events are and are not equally likely.

Key vocabulary

Sample space	Probability	Event
Equally likely	Unbiased	P(event)

Key questions

What does P(event) mean?
Is it possible to write a probability as 'out of' or as a ratio? Why not?
What are the equivalent different ways of writing a probability?
Can probabilities be simplified? Why/Why not?

Exemplar Questions

Two dice are thrown. Mo thinks that the probability of getting 2 sixes must be $\frac{2}{12}$

Mo is wrong.
Explain why.

There are two dice. Each dice has 6 numbers. This means that there are 12 numbers altogether. So, the probability of getting two sixes must be 2 out of 12

Continue completing the table for rolling two regular dice and adding the numbers together.

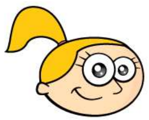
+	1	2	3	4	5	6
1	2	3				7
2						
3						
4						
5						
6						

Work out,
- P(total is even)
- P(6 or 7)
- P(Number>4)
- P(0)
- P(prime number)
- P(square number)

These probabilities should be out of 36 as that's the total.

Eva has worked out all the possible outcomes when flipping two coins. Check her sample space. Is she correct? Why/Why not?

$S = \{TT, TH, HH\}$



The probability of getting a tail and a head when flipping two coins is $\frac{1}{3}$

Why is Eva incorrect? Justify your answer.

Probability from two-way tables

Notes and guidance

This step focuses on using data in two-way tables to find probabilities.

Students should be given guidance on which total to use when answering questions including discussion around how probabilities can be represented e.g. whether it is appropriate to simplify fractions.

Key vocabulary

Two-way table	Probability
Sample	Denominator

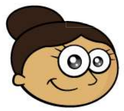
Key questions

How can a two-way table be used to calculate a probability?

How do you decide which row or column to look at?

How do you design a two-way table?

Exemplar Questions



Dora

The probability a Year 8 student has a school dinner is $\frac{18}{49}$

Whitney

No! The probability a Year 8 student has a school dinner is $\frac{18}{100}$

Complete the table and decide whether you agree with Whitney or Dora.

What is the probability that a year 9 student has a packed lunch?

	Year 8	Year 9	Total
School Dinner		26	44
Packed Lunch	31		56
Total		51	

The following table shows how 200 children travelled to school.

	Car	Bus	Walk	Total
Boys	36	46	21	103
Girls	44	29	24	97
Total	80	75	45	200

Work out the probability that a child travels to school by car.
Calculate the probability that a girl walks to school.

- Females are more likely to own a cat than males.
- The probability that a person has a dog or a cat is $\frac{1}{2}$

Construct your own two way table to represent this information.

Probability from Venn diagrams

Notes and guidance

Students build on the link between interpreting Venn diagrams and finding probabilities. They should be familiar with drawing and interpreting Venn diagrams from Year 7 but may need reminding. They should be encouraged to consider which region or regions are included in the event described and which regions are not included.

Key vocabulary

Set	Intersection	Event
And/Or	Union	Region

Key questions

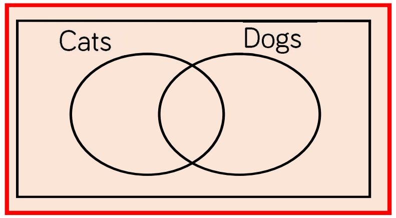
How do the words 'and/or' relate to set notation and regions on a Venn diagram?

Why do we start with the intersection of sets when adding information to a Venn diagram?

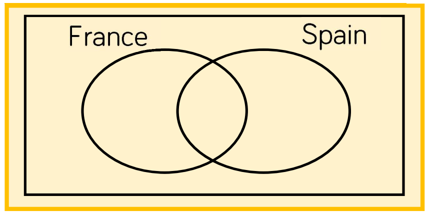
Exemplar Questions

The Venn diagram shows how many students in a class own cats, dogs or both. A student is picked at random from the class. Find:

- P(They own a cat but not a dog)
- P(They own a cat)
- P(They own a dog)
- P(They do not own a dog)
- P(They own both a cat and a dog)
- P(They own neither a cat nor a dog)



100 people were surveyed about countries they had visited.
30 had visited France, 25 had visited Spain and 12 had visited both France and Spain. Use a Venn diagram to show this information. One person is chosen from the survey to win a prize.
Find the probability the winner had visited neither France nor Spain.



In a group of 45 people, 15 belong to a cricket club, 18 belong to a tennis club and 9 belong to both a cricket and a tennis club.
Draw a Venn diagram to represent this information.
A person is chosen at random from this group.
Find the probability that this person:

- belongs to a cricket and a tennis club
- belongs to a cricket or tennis club
- does not belong to a cricket club
- does not belong to either a cricket or a tennis club
- belongs to a tennis club but not a cricket club.

Product rule & total outcomes H

Notes and guidance

Students are introduced to the product rule to find total arrangements. Students start by considering how many choices they have for each place in their list. They need to consider situations where repeats are or are not allowed. Students could also experience opportunities of designing lists given the number of possible arrangements.

Key vocabulary

Total	Possibilities	Outcomes
Product	Table	Order

Key questions

How can you find the total number of arrangements without listing each one?

Is commutativity important when working out the total number of arrangements? Why/Why not?

How can factors help when finding lists that have a specified number of arrangements?

Exemplar Questions

Eva is trying to work out all the different arrangements you can have when buying her meal deal. Comment on her calculation.

Meal Deal £2
Drink: *Water or Juice*
Sandwich: *Tuna, Cheese or Ham*
Fruit: *Apple or Banana*

$2 \times 3 \times 2 =$ 12 Possibilities

In how many different ways can you order the letters C, D, E and F? List all the possible orders.


Dora

$4 \times 4 = 16$
There are 16 possibilities

$4 \times 3 \times 2 \times 1 = 24$
There are 24 possibilities


Amir

Who is right, Dora or Amir? Why?

Here is a set of 5 cards. How many different arrangements are there which include all 5 cards? What strategy did you use to work this out?

A	B	C	1	2

Alex is opening a café. She wants to give her customers 20 possibilities when buying a meal deal.
Write down a meal deal menu that she could use.
Compare you answers as a class.
What's the same and what's different about your menus?