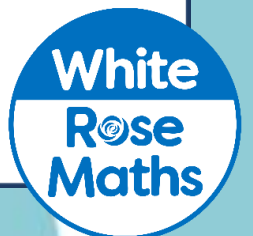


Spring Term

Year 8

#MathsEveryoneCan



	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Proportional Reasoning						Representations					
	Ratio and scale		Multiplicative change		Multiplying and dividing fractions		Working in the Cartesian plane		Representing data		Tables & Probability	
Spring	Algebraic techniques						Developing Number					
	Brackets, equations and inequalities				Sequences	Indices	Fractions and percentages		Standard index form		Number sense	
Summer	Developing Geometry						Reasoning with Data					
	Angles in parallel lines and polygons			Area of trapezia and circles		Line symmetry and reflection	The data handling cycle			Measures of location		

Spring 1: Algebraic Techniques

Weeks 1 to 4: Brackets, Equations & Inequalities

Building on their understanding of equivalence from Year 7, students will explore expanding over a single bracket and factorising by taking out common factors. The higher strand will also explore expanding two binomials. All students will revisit and extend their knowledge of solving equations, now to include those with brackets and for the higher strand, with unknowns on both sides. Bar models will be recommended as a tool to help students make sense of the maths. Students will also learn to solve formal inequalities for the first time, learning the meaning of a solution set and exploring the similarities and differences compared to solving equations. Emphasis is placed on both forming and solving equations rather than just looking at procedural methods of finding solutions.

National curriculum content covered:

- identify variables and express relationships between variables algebraically
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- substitute numerical values into formulae and expressions, including scientific formulae
- understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors
- simplify and manipulate algebraic expressions to maintain equivalence by:
 - collecting like terms
 - multiplying a single term over a bracket
 - taking out common factors
 - expanding products of two or more binomials
- understand and use standard mathematical formulae
- use algebraic methods to solve linear equations in one variable

Week 5: Sequences

This short block reinforces students' learning from the start of Year 7, extending this to look at sequences with more complex algebraic rules now that students are more familiar with a wider range of notation. The higher strand includes finding a rule for the n^{th} term for a linear sequence, using objects and images to understand the meaning of the rule.

National curriculum content covered:

- generate terms of a sequence from either a term-to-term or a position-to-term rule
- recognise arithmetic sequences and find the n^{th} term
- recognise geometric sequences and appreciate other sequences that arise

Week 6: Indices

Before exploring the ideas behind the addition and subtraction laws of indices (which will be revisited when standard form is studied next term), the groundwork is laid by making sure students are comfortable with expressions involving powers, simplifying e.g. $3x^2y \times 5xy^3$. The higher strand also looks at finding powers of powers.

National curriculum content covered:

- use and interpret algebraic notation, including a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$
- use language and properties precisely to analyse algebraic expressions
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- substitute values in expressions, rearrange and simplify expressions, and solve equations

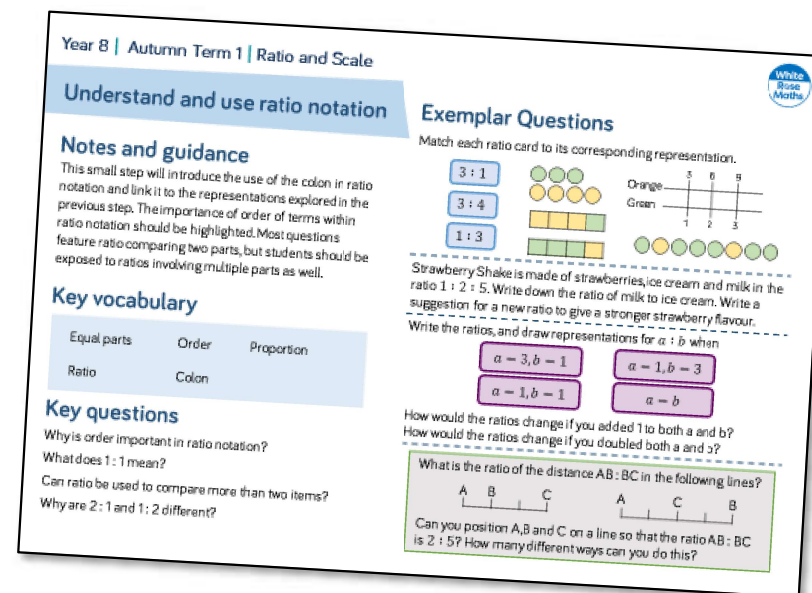
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a “small steps” approach.

As a result, for each block of content in the scheme of learning we will provide a “small step” breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some **brief guidance** notes to help identify key teaching and learning points
- A list of **key vocabulary** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of **key questions** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help **exemplify** the small step concept that needs to be focussed on.



Year 8 | Autumn Term 1 | Ratio and Scale

Understand and use ratio notation

Notes and guidance

This small step will introduce the use of the colon in ratio notation and link it to the representations explored in the previous step. The importance of order of terms within ratio notation should be highlighted. Most questions feature ratio comparing two parts, but students should be exposed to ratios involving multiple parts as well.

Key vocabulary

Equal parts	Order	Proportion
Ratio	Colon	

Key questions

- Why is order important in ratio notation?
- What does 1:1 mean?
- Can ratio be used to compare more than two items?
- Why are 2:1 and 1:2 different?

Exemplar Questions

Match each ratio card to its corresponding representation.

3:1
3:4
1:3

Orange
Green

Strawberry Shake is made of strawberries, ice cream and milk in the ratio 1:2:5. Write down the ratio of milk to ice cream. Write a suggestion for a new ratio to give a stronger strawberry flavour.


Write the ratios, and draw representations for $a:b$ when

$a=3, b=1$
 $a=1, b=1$
 $a=1, b=3$
 $a=b$

How would the ratios change if you added 1 to both a and b ?
How would the ratios change if you doubled both a and b ?

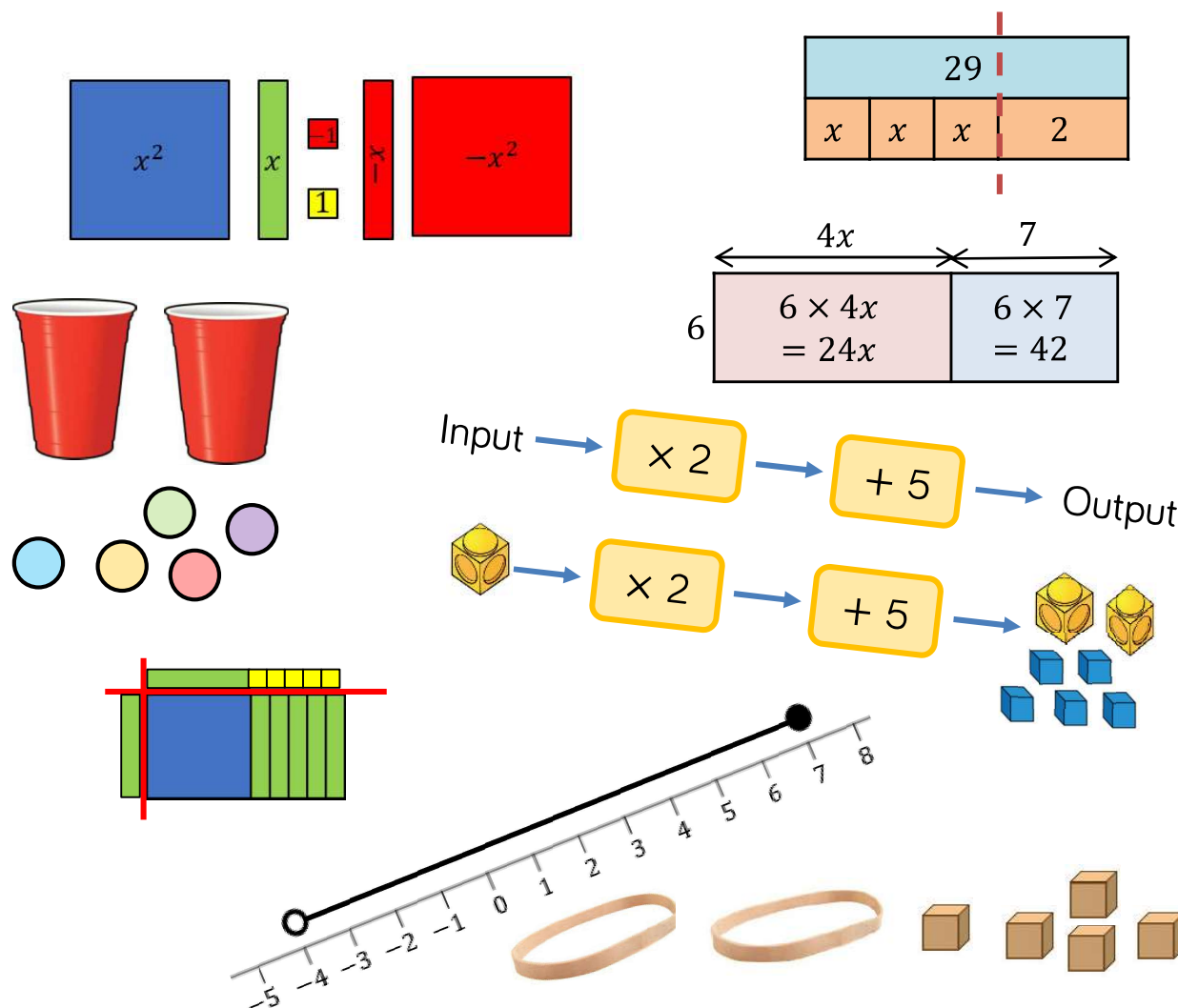
What is the ratio of the distance AB:BC in the following lines?

Can you position A, B and C on a line so that the ratio AB:BC is 2:5? How many different ways can you do this?

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

Key Representations



Here are a few ideas on how you might represent algebraic expressions and the solutions of equations and inequalities.

Cups, cubes and elastic bands lend themselves well to representing an unknown, whereas ones (from Base 10) and counters work well to represent a known number. Be careful to ensure that when representing an unknown, students use equipment that does not have an assigned value – such as Base 10 equipment and dice.

Bar models are useful to support the forming of equations and also help students to make sense of the approach to a solution. Algebra tiles are also very powerful for this and help to make sense of multiplying brackets, as is the area model that also provides a good link to multiplication of two-digit numbers by 1-digit numbers.

Brackets, Equations & Inequalities

Small Steps

- Form algebraic expressions
- Use directed number with algebra
- Multiply out a single bracket
- Factorise into a single bracket
- Expand multiple single brackets and simplify
- Expand a pair of binomials**
- Solve equations, including with brackets
- Form and solve equations with brackets
- Understand and solve simple inequalities

H

H denotes higher strand and not necessarily content for Higher Tier GCSE

Brackets, Equations & Inequalities

Small Steps

- ▶ Form and solve inequalities
- ▶ Solve equations and inequalities with unknowns on both sides H
- ▶ Form and solve equations and inequalities with unknowns on both sides H
- ▶ Identify and use formulae, expressions, identities and equations

H denotes higher strand and not necessarily content for Higher Tier GCSE

Form algebraic expressions

Notes and guidance

This step revises the basic algebraic notation students have met in Year 7. Students may need reminding that \times and \div signs should not appear in algebraic expressions, numbers are written before letters and that e.g. aa is written a^2 . Students could revisit the use of function machines and explore the use of algebraic expressions within any other area that needs revising e.g. probability.

Key vocabulary

Expression	Simplify	Term
Substitute	Coefficient	Equivalent

Key questions

What is the difference between a term and an expression?
 When can/can't an expression be simplified?
 Spot the mistake(s) in this expression e.g. $6ff$, $3a4b$.
 Why are e.g. $q - 4$ and $4 - q$ not equivalent?

Exemplar Questions

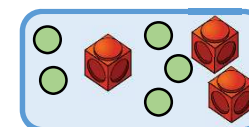
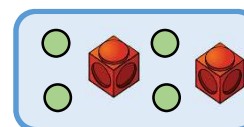
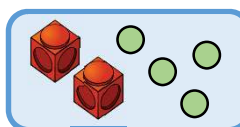
k is a number. Write an expression for the number that is,

- Five more than k
- One third of k
- Four multiplied by k
- Seven less than k
- The difference between k and 10

Give your answers in correct algebraic notation.

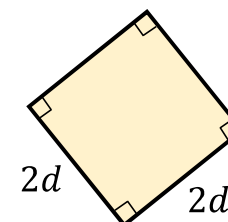
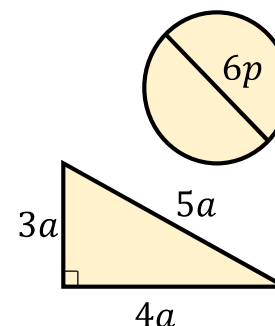
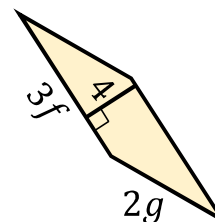
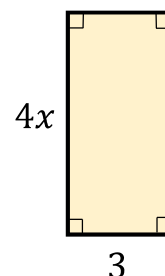
Compare your answers with a partner's, do you have different but equivalent expressions? Check by substituting in values for k and comparing your answers.

On each card, a cube represents $3p$ and a counter represents $2q$. Write an expression for the total of each card, giving your answers as simply as possible.



Repeat if the cube now represents $5p + 2$ and the counter $3q - 1$.

Write simplified expressions for the perimeter and area of each shape.



Use directed number with algebra

Notes and guidance

This step revisits the use of directed number and substitution into algebraic expressions, both of which were covered in Year 7, in preparation for the more complex expressions coming up later in this block. Double-sided counters are very helpful to support understanding of the four operations with directed numbers. Entering negative numbers on a calculator is not always obvious and may need modelling by the teacher.

Key vocabulary

Positive	Negative	Directed
Substitute	Solve	Simplify

Key questions

Why is it not true that 'two minuses make a plus'?

How do we enter...on a calculator?

Which order do we perform operations when substituting numbers into an expression? Why?

Is e.g. $2x^2$ always, sometimes or never the same value as $(2x)^2$?

Exemplar Questions

Work out the value of these expressions when $a = 2$ and $b = -4$

$a + b$

$a - b$

ab

$3ab$

$\frac{a}{b}$

$\frac{b}{a}$

$3a^2$

$a^2 + b^2$

$a^2 - b^2$

$2a - 3b$

Now find the values again this time using $a = -2$ and $b = -4$
Which expressions give the same answer as before? Why?

Mo is answering the question on the card.

Find the value of x^2 when $x = -2.5$

Mo enters -2.5^2 into his calculator and is surprised to get the answer -6.25 as he thinks the answer should be positive.

Discuss why the calculator shows a negative answer.

Solve the equations.

$x + 1.7 = 6.8$

$x - 1.7 = -6.8$

$6.8 = 1.7x$

$\frac{x}{1.7} = -6.8$

$-6.8 = 1.7x$

$1.7x - 5.1 = -6.8$

$6.8 = 2x - 1.7$

Simplify the expressions on the cards.

$3p + 4p - 8p$

$-3p + 4p - 8p$

$-3p - 4p - 8p$

$3 \times -4p$

$-3 \times -4p$

$-3 \times 4p$

$-3 \times -4p \times -2$

Multiply out a single bracket

Notes and guidance

It is useful to represent the expansion of brackets in many forms making links to number work in particular through the use of the area model. As well as including all combinations of $+$ and $-$ signs, examples should include those where the multiplier is a constant e.g. just 5, a variable e.g. just x or more complex e.g. $3a$. Examples involving more than two terms inside the bracket are also useful to include.

Key vocabulary

Expand	Multiply out	Coefficient
Bracket	Identity	Product

Key questions

What does expand mean when we are working with brackets?

What's the link between multiplication and repeated addition?

Is it possible to have three or more terms inside a bracket?

How would this look as a diagram?

Exemplar Questions

Annie is working out 6×82 and reasons she can do the same with any number a

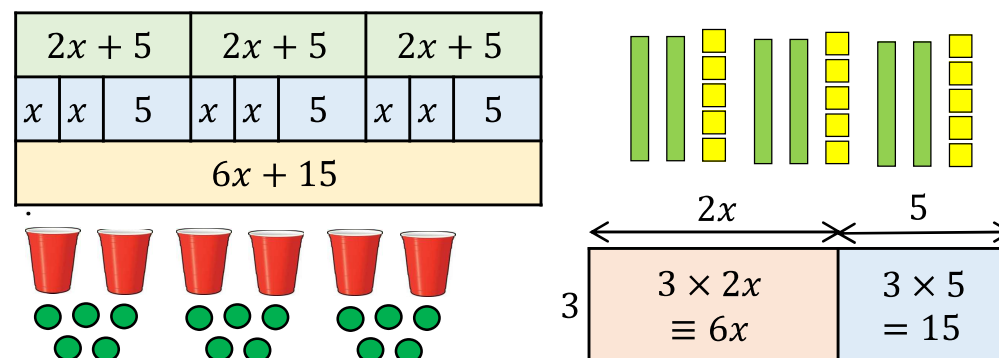
$$\begin{aligned} 6 \times 82 &= 6 \times (80 + 2) \\ &= 6 \times 80 + 6 \times 2 \\ &= 480 + 12 \\ &= 492 \end{aligned}$$

$$\begin{aligned} 6 \times (a + 2) &\equiv 6 \times a + 6 \times 2 \\ &\equiv 6a + 12 \end{aligned}$$

Compare the value of $6 \times (a + 2)$ with the value of $6a + 12$ for different values of a (positive, negative, fractions, decimals)

Do they always have the same value?

Explain how these representations show that $3(2x + 5) \equiv 6x + 15$



Expand these brackets.

$$3(x + 5)$$

$$3(x - 5)$$

$$-3(x + 5)$$

$$-3(x - 5)$$

$$3(5 + x)$$

$$3(5 - x)$$

$$x(x + 5)$$

$$2x(5 - x + y)$$

Factorise into a single bracket

Notes and guidance

Students do not always link factorising expressions with looking for factors of numbers, so it is useful to be explicit about the similarities. This helps to reinforce the language of common factor and highest common factor, and these topics could be revisited during starters. When factorising into a single bracket, again using a variety of signs and types of terms (numerical, algebraic) is useful.

Key vocabulary

Factor	Factorise	Factorise fully
Common	Common factor	HCF

Key questions

Is it useful to have 1 as a common factor? Why/Why not?

Can 0 ever be a factor of an expression?

What do you look for to find the highest common factor of a set of terms?

Is it always true that if you can't halve an expression then the expression doesn't factorise?

Exemplar Questions

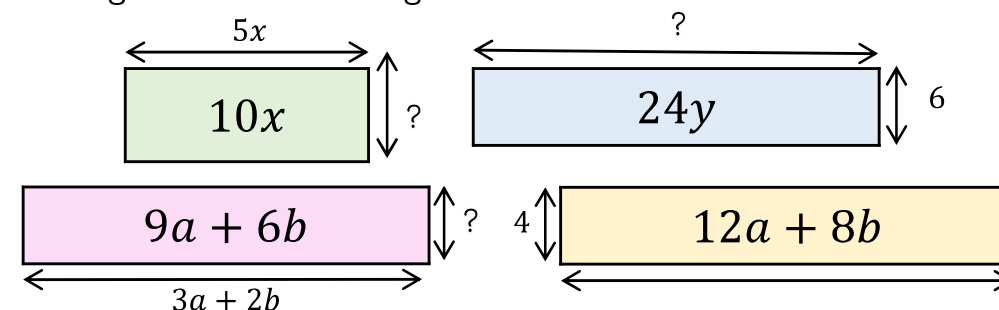
List the factors of the numbers or expressions on each card.

10

18

 $3x$
 x^2
 $2x^2$
 $6xy$

The area and the length of one of the sides is given for each of the rectangles. Find the missing sides.



Complete the factorisations.

$$6x + 9y \equiv 3(\underline{\quad} + \underline{\quad})$$

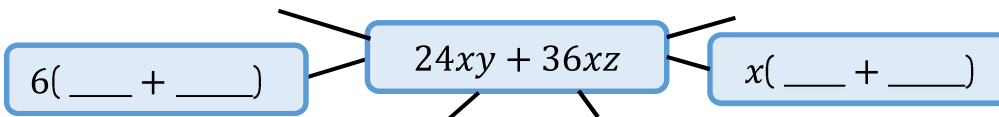
$$4x - 6y \equiv 2(\underline{\quad} + \underline{\quad})$$

$$xy + 7x \equiv x(\underline{\quad} + \underline{\quad})$$

$$a^2 + ab + 6a \equiv \underline{\quad}(a + b + 6)$$

$$12pq - 15qt = \underline{\quad}(4p - 5t)$$

$$20d^2 + 5d \equiv \underline{\quad}(\underline{\quad} + 1)$$



- How many ways can you find to factorise the expression?
- Fully factorise the expression.

Simplify multiple single brackets

Notes and guidance

Students often only expand and simplify expressions of the form $3(x \pm 4) \pm 4(x \pm 5)$ and make errors with shorter expressions like $3 \pm 4(x \pm 5)$. Using concrete manipulatives to 'build' the expressions is a useful way of developing understanding of the difference between similar looking expressions. Careful choice of numbers in examples and exercises, and varying numbers and signs is also helpful.

Key vocabulary

Expression	Simplify	Like terms
Unlike terms	Expand	Equivalent

Key questions

How do we write '1x'?

Is it possible to simplify an expression and end up with the answer 0?

Does the order in which we work out expansions matter?

Exemplar Questions

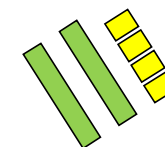
Without expanding the brackets, decide whether you think these expressions will be equivalent or not.

$$3(x + 4) + 2$$

$$2x + 3(x + 4)$$

$$2 + 3(x + 4)$$

$$3(x + 4) + 2$$



Checking by building the expressions using algebra tiles.
Simplify the expressions and compare with your concrete versions.

Ron has made mistakes in both these simplifications.

$$5 + 3(a + 6)$$

$$8(a + 6)$$

$$8a + 48$$



$$5(b - 3) + 2b$$

$$5b - 15 + 2b$$

$$3b - 15$$



Explain Ron's errors and work out the correct answers.

Expand and simplify the expressions.

$$3(5a + 2) + 4(2a + 3)$$

$$3(5a + 2) - 4(2a + 3)$$

$$3(5a + 2) + 4(2a - 3)$$

$$3(5a - 2) - 4(2a + 3)$$

$$3(5a - 2) + 4(2a - 3)$$

$$3(5a - 2) - 4(2a - 3)$$

$$3(5a - 2) - 5(3a - 2)$$

$$3(4a - 2) - 2(6a - 3)$$

Expand a pair of binomials

H

Notes and guidance

The vocabulary binomial (the sum or difference of two terms) and quadratic (an expression where the highest power of the variable is 2) will be new to most students. Concrete and pictorial ways of finding the expansions will support written methods which will be developed over the coming years. Students need to be confident with simplification and dealing with negative numbers for this higher strand step.

Key vocabulary

Binomial	Simplify	Like terms
Unlike terms	Expand	Quadratic

Key questions

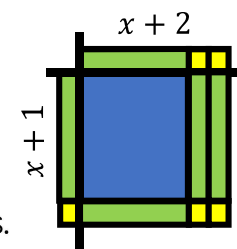
Why do you get four terms when you multiply two binomials?

Why can you simplify some quadratic expressions to three or fewer terms, but not others?

Do simplified quadratics always have three terms?

Exemplar Questions

Explain how the algebra tiles show that $(x + 1)(x + 2) \equiv x^2 + 3x + 2$



Use algebra tiles to expand the expressions.

$$(x + 4)(x + 3)$$

$$(x + 3)(x + 4)$$

$$(x + 4)(x - 3)$$

$$(x + 3)(x - 4)$$

$$(x + 3)^2$$

Here is Tommy's method for working out 62×43 by thinking of the calculation as $(60 + 2) \times (40 + 3)$

\times	60	2
40	2400	80
3	180	6

$$2400 + 180 + 80 + 6 = 2666$$

Complete this adaptation of Tommy's method to work out $(a + 3)(b + 4)$

\times	a	3
b	ab	$3b$
4	—	—

$$ab + 3b + \dots$$

What would be different if Tommy was working out $(a + 3)(a + 4)$?



Annie works out $(2x + 5)^2$ as $4x^2 + 25$

Show that Annie is wrong using:

■ substitution

■ an area model

■ algebra tiles

Solve equations with brackets

Notes and guidance

Solving one-step and two-step equations should be secure before moving on to working with equations with brackets. 'Think of a number' problems are a good introduction (see next step), but students should also deal with equations with non-integer solutions (using a calculator when necessary) to avoid reliance on 'spotting' solutions. Alternate methods should be explored as exemplified in the final question.

Key vocabulary

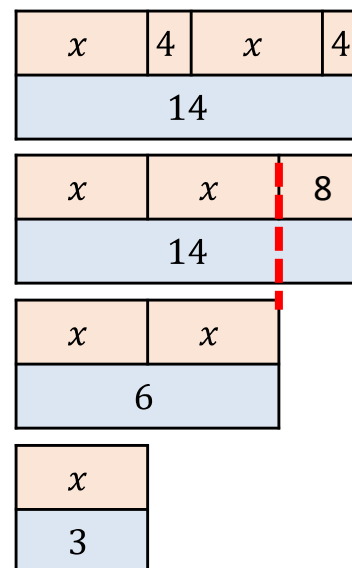
Solve	Equation	Unknown
Coefficient	Expand	Solution

Key questions

Do you have to expand the brackets to solve the equation?
Which order do we need to carry out the steps when solving an equation?
How many solutions will the equation have?

Exemplar Questions

Whitney uses bar models to solve $2(x + 4) = 14$. She explains her steps on the right hand side.



$$2(x + 4) = 14$$

Expand brackets

$$2x + 8 = 14$$

$$-8 \quad -8$$

$$2x = 6$$

$$\div 2 \quad \div 2$$

$$x = 3$$

Solve the equations. $\blacklozenge 4(a + 4) = 60$ $\blacklozenge 10 = 5(b + 1)$
 $\blacklozenge 3(x + 2.7) = 4.5$ $\blacklozenge 12 = 2(x - 3)$ $\blacklozenge 6(e - 1) + 2e = 10$

Compare these solutions of the equation $3(x + 5) = 12$.
Explain the steps in each method. Which approach do you prefer?

$$\begin{aligned} 3(x + 5) &= 12 \\ 3x + 15 &= 12 \\ 3x &= -3 \\ x &= -1 \end{aligned}$$

$$\begin{aligned} 3(x + 5) &= 12 \\ x + 5 &= 4 \\ x &= -1 \end{aligned}$$

Form/solve equation with brackets

Notes and guidance

'Think of a number' problems and flowcharts are good models to support students to distinguish between e.g. $2x + 3$ and $2(x + 3)$. It is worth investing class time developing students' skills in forming equations as the mechanics of solving can be more easily practised as homework. It is also useful to interleave other topics here e.g. forming equations to find missing angles on a straight line, missing probabilities etc.

Key vocabulary

Equation	Side	Form
Solve	Unknown	Check

Key questions

What is different about $2x + 3$ and $2(x + 3)$?

What is the first step you need to think about when forming an equation from a worded problem?

How can we check if the answer to the equation is correct?

Exemplar Questions

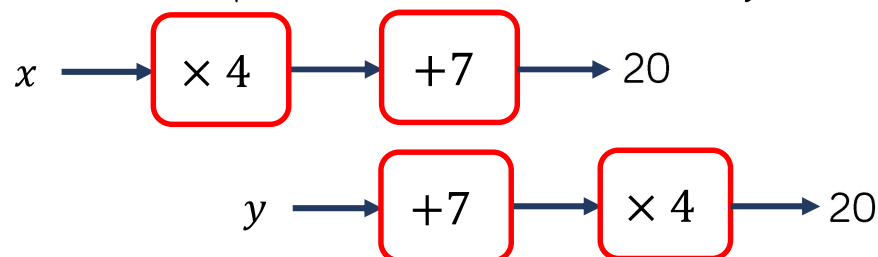
Compare the two number puzzles.

I think of a number.
I add on 6
I double my answer.
Now I'm thinking of 18
What was my original number?

I think of a number.
I double it.
I add on 6 to my answer.
Now I'm thinking of 18
What was my original number?

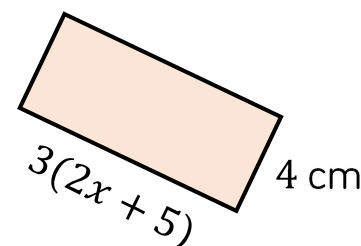
What's the same and what's different? What were the numbers?
Represent the puzzles using concrete materials, pictures and equations to check your answers.

Write and solve equations to find the values of x and y



The area of the rectangle is 72 cm^2

Work out the value of x and hence find the perimeter of the rectangle.



Simple inequalities

Notes and guidance

Students will be familiar with the inequality signs from earlier work on comparison, but solving inequalities and the idea of a solution set (as opposed to a single value) will be new to most. It is worth discussing that e.g. $x > 7$ and $7 < x$ mean the same; reading the inequalities aloud is helpful in determining meaning. Students sometimes replace the given sign with an equals sign; this is error-prone and should be discouraged.

Key vocabulary

Inequality	Satisfy	Solution set
Solve	Greater/less than (or equal)	

Key questions

What's the same and what's different about solving an equation or an inequality?

How many solutions does an inequality have?

How can we check our solution to an inequality is correct?

What values would be useful to test with?

Exemplar Questions

Which of the inequalities does the number 7.5 satisfy?

$x > 7$

$7 < x$

$7 \leq x$

$x < 8$

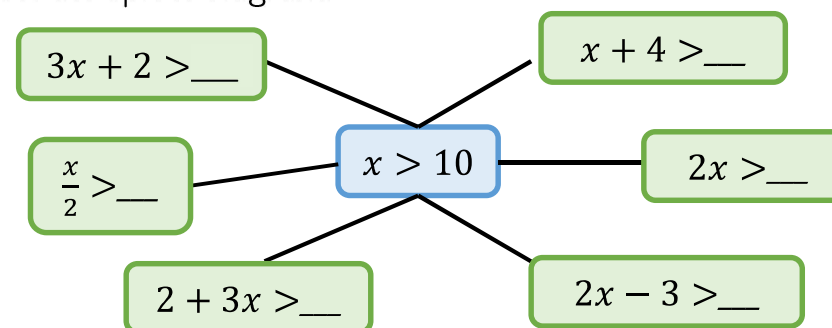
$x \geq 8$

What's the same and what's different about the inequalities?

Mo says "If $x > 10$, then $x + 1 > 11$ "

Explain why Mo is right.

Complete the spider diagram.



If $x > 10$, which of the cards are true and which are false?

$10 < x$

$40 < 4x$



$-x > -10$

Solve the inequalities.

$x + 2 > 7$

$x + 2 > -7$

$x - 2 > 7$

$x - 2 > -7$

$x + 2 < 7$

$x + 2 < -7$

$x - 2 \leq 7$

$x - 2 \geq -7$

$2x + 2 < 7$

$4x + 2 \geq -7$

$3 + 5x \leq 7$

Form and solve inequalities

Notes and guidance

Teacher modelling is again important here, as students often find forming equations/inequalities from given information difficult; class time can be spent just forming the inequalities with the solving left to later in the lesson and/or homework. Consideration needs to be given as to whether the full solution set or only particular integers are required. It is also worth discussing which values could be chosen to test whether the solution set is correct.

Key vocabulary

Solution	Inequality	Form
Solve	Unknown	Check

Key questions

Which way round will the inequality sign point in this question? Why?
 What does integer mean? How does this change the question?
 How can we check our solution to an inequality is correct?
 What values would be useful to test with?

Exemplar Questions



Whitney

Three more than double my number is greater than 10

Write an inequality and solve it to find the possible range of values for Whitney's number.

What is the smallest integer Whitney could be thinking of?

Annie has £100

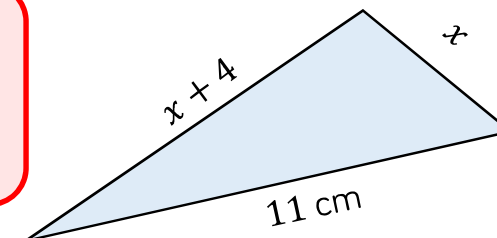
She wants to buy three T-shirts and a jumper.

The jumper costs £45, and she doesn't have enough money to buy everything she wants.

What can be worked out about the price of the T-shirts?

Explain why you cannot make a triangle with three sides of lengths 4 cm, 5 cm and 12cm.

This triangle had sides x , $x + 4$ and 11 cm.
 Work out the range of possible values of x .



Unknowns on both sides

H

Notes and guidance

Students should be familiar with the 'balance' method of solving equations that lends very well to equations of this form. It is important that students consider three-term as well as four-term equations, and as ever deal with a variety of letters and positions. As well as the bar model illustrated, cups and counters is a good model for unknowns on both sides, alongside the abstract method so students make sense of the method for more difficult equations.

Key vocabulary

Equation	Balance	Side
Solve	Unknown	Check

Key questions

How can we check our solution to an equation is correct?
When solving a four-term equation, why is it better to deal with the letters before the numbers?
Do we always start solving equations by subtracting something from both sides? Why or why not?

Exemplar Questions

x	x	x	x	12
x	x	18		

Write down the equation shown by the bar model.

x	x	x	x	12
x	x	18		

Write down the new equation if the two left-most x s are removed from the bars. Work out the value of x .

Use the bar model to help you complete the workings to find the value of y .

$$\begin{aligned}
 5y &= 3y + 15 \\
 -3y &\quad -3y \\
 2y &= 15 \\
 \text{etc.}
 \end{aligned}$$

y	y	y	y	y
y	y	y	15	

Solve the equations.

$$5x + 1 = 71$$

$$5x + 1 = 7x$$

$$5x + 1 = 2x + 7$$

$$17 = 4x - 3$$

$$2x = 4x - 3$$

$$2x + 1 = 4x - 3$$

What's the same what's different about how you approach them?

Complex equations/inequalities H

Notes and guidance

Building on the earlier steps of this block, students can explore equations and inequalities with brackets and unknowns on both sides. It is important that students only move on to this step when they have a good understanding of the earlier material. With complex equations and inequalities it is more important than ever to check the answer by substituting back into the original problem.

Key vocabulary

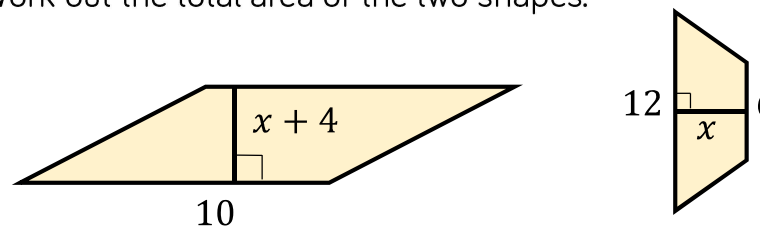
Equation	Inequality	Form
Solve	Unknown	Check

Key questions

Can you think of an equation with more than one solution?
 Can an inequality have more than one solution?
 Describe the steps you need to take to solve.....
 Does the order of the steps matter? Why?
 How do you form an equation from a worded problem?
 What do you need to decide first?

Exemplar Questions

The area of the parallelogram is twice the area of the trapezium.
 Work out the total area of the two shapes.



Dora and Amir are both given the same starting number.



Dora

I triple the number and add on seven

I add two to the number and then multiply by 4



Amir

Dora's answer is less than Amir's.
 Is it possible that the starting number was negative?
 If so, give an example.

Verify, by substitution, that $x = 3$ is the solution to the equation.

$$7x + 3(2x - 4) = 4(2x + 4) - 2(3x - 8)$$

Now solve the equation algebraically.

Esther adds together three consecutive even numbers.
 Her total is less than 80. Use an algebraic method to work out the greatest of Esther's three numbers.

Identify algebraic constructs

Notes and guidance

In this step, students have the opportunity to practise distinguishing between expressions, equations, formulas (or formulae) and identities. It is particularly important that students know that an identity is true for all values of the variable(s) and includes the symbol \equiv , whereas an equation can be solved to find particular values. A formula is distinguished from an equation as it can be used to find particular values of the subject.

Key vocabulary

Expression	Identity	Formula
Equation	Equivalent	Variable
Subject	Substitute	

Key questions

What do we mean by an expression?
 How could you change an expression into a formula or an equation? What symbol do we use to show an identity?
 Can an equation have more than one variable?
 What formulas do you use in other subjects?

Exemplar Questions

What's the same and what's different about the cards?

$$2(a + b)$$

$$P = 2(a + b)$$

$$2(a + b) \equiv 2a + 2b$$

Which of these formulas do you recognise?

Explain the meaning of all the variables in the formulas.

$$C = \pi d$$

$$P = 2l + 2w$$

$$A = \frac{1}{2}bh$$

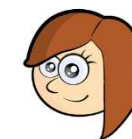
$$P = 4s$$

$$S = \frac{D}{T}$$

$$A = lw$$

Investigate how the value of the subject of the formula changes as the other variables change.

An equation is anything with an equals sign.



Rosie

Explain why Rosie is wrong.

Which of these cards show equations? What do the other cards show?

$$3a + 2(a + 5) = 25$$

$$v = u + at$$

$$5x + 6x \equiv 11x$$

$$b \times b \times b \equiv b^3$$

$$10 = \frac{p}{2} - 3$$

$$6 - m = 2$$

$$\frac{1}{2}(a + b)h$$

$$a^2 + b^2 = c^2$$

$$6 - m = F$$

Sequences

Small Steps

- Generate sequences given a rule in words
- Generate sequences given a simple algebraic rule
- Generate sequences given a complex algebraic rule
- Find the rule for the n^{th} term of a linear sequence**

H

H denotes higher strand and not necessarily content for Higher Tier GCSE

Sequences from rules in words

Notes and guidance

Building on from year 7, students revisit the idea of forming a sequence given a rule in words. They should now be able to deal with more complex multi-step rules, and operations such as cubing and rooting. This step is a good chance to revisit the vocabulary of sequences, and students should also be able to use correct language to fully describe a given simple sequence. Exploring Fibonacci sequences is worthwhile.

Key vocabulary

Sequence	Position	Term
Linear	Non-linear	Fibonacci
Difference	Constant	Term-to-term

Key questions

What's the name for a sequence where there is a constant difference between successive terms?

What would the graph of such a sequence look like?

What information do you need to give to fully describe a sequence? Why is e.g. 'it goes up in 3s' not enough?

Exemplar Questions

Compare these sequences by working out the first five terms.

Sequence A

The first term is 10
Each term is four greater than the previous term.

Sequence B

The first term is 10
Each term is four smaller than the previous term.

Sequence C

The first term is 10
Each term is four multiplied by the previous term.

Which of the sequences are linear and which are not?

Describe each of these sequences.

Sequence D 10, 15, 20, 25, 30...

Sequence E 10, 10, 10, 10, 10...

Sequence F 10, 4, -2, -8, -14..

Work out the first five terms of each sequence. What do you notice?
For which sequence can you easily work out the 100th term?

Double take 5

The first term is 7
Each term is five less than double the previous term.

Square add 1

The first term is 10
Each term is one more than the square of the previous term.

Take from 15

The first term is 4
Each term is the result of subtracting the previous term from 15

Investigate the Fibonacci sequences.

1, 1, 2, 3, 5, 8...

3, 7, 10, 17, 27...

Sequences from algebraic rules

Notes and guidance

As well as providing practice in substitution, this step provides plenty of opportunity for students to develop their reasoning. They can observe the behaviour of the linear sequences in preparation for the later higher step of finding the rule, and solve equations to determine whether a number is a term in a sequence or not by considering if the solutions are integers. Similarly, they could also practice forming and solving inequalities.

Key vocabulary

Algebraic	Integer	Non-integer
Substitute	Linear	Non-linear

Key questions

How can you tell by looking at the rule for the n^{th} term of a sequence whether it is linear or not?

Is it possible for n to take non-integer values? Why or why not?

How can we form an equation to see if the number is in the sequence?

Exemplar Questions

Find the value of these expressions when $n = 1, 2, 3$ and 100

$$7n + 4$$

$$20 - 3n$$

$$\frac{n}{2} - 1$$

$$n^2 + 1$$

Work out the first five terms of the sequences given by these rules.

$$3n - 1$$

$$3n + 5$$

$$4n + 5$$

What connections do you see between your sequences and their algebraic rules?



Rosie

None of the terms in the sequence given by $5n + 2$ will end in 0

Is Rosie correct? How do you know?

A sequence is given by the rule $3n + 7$

- Work out the 45th term of the sequence.
- Form equations to determine which, if any, of these numbers are in the sequence. 113 213 313
- Form an inequality to find the position of the first term in the sequence that is greater than 1000
- Is the sequence linear? How do you know?

Complex algebraic rules

Notes and guidance

Students explored simple algebraic sequences in Year 7. They have since looked at more complex expressions involving squares, cubes and brackets in much more detail and so this step allows them to practice their substitution skills in the context of sequences; they may need reminders as to the behaviour of directed number. As well as the examples shown, students could also explore fractions e.g. $\frac{n}{n+3}$

Key vocabulary

Algebraic	Bracket	Expand
Substitute	Linear	Non-linear

Key questions

What is the difference between how we work out e.g. $3n^2$ and $(3n)^2$? How do you know?

Do we need to expand the brackets first in order to substitute e.g. $n = 5$ into an expression like $2(n + 3)$?

Exemplar Questions

Work out the first five terms of each of the sequences given by the rules on the cards. What's the same and what's different?

$$n^2$$

$$(n - 1)^2$$

$$(n + 1)^2$$

$$(1 - n)^2$$

Dora is working with the sequence given by the rule $n(n + 1)$

$$1^{\text{st}} \text{ term, } n = 1 \text{ so } n(n + 1) = 1 \times (1 + 1) = 1 \times 2 = 2$$

$$2^{\text{nd}} \text{ term, } n = 2 \text{ so } n(n + 1) = 2 \times (2 + 1) = 2 \times 3 = 6$$

Complete the working to find the first six terms.

Dexter is using counters to make the triangle numbers.



Continue Dexter's pattern and work out the first six triangle numbers.



What is the connection between Dora's rule and the triangle numbers?

Compare the sequences given by the pairs of rules.

$$3(n + 1)$$

$$3n + 1$$

$$2n^2$$

$$(2n)^2$$

$$5(2 - n)$$

$$n(2 - n)$$

$$3n$$

$$n^3$$

Finding the algebraic rule

H

Notes and guidance

This higher step should only be completed when students are comfortable with using rules for the n^{th} term and distinguishing between linear and non-linear sequences; this will be revisited in Year 9 and KS4. The aim is for students to understand the connection between the sequence and the associated multiplication table. Linking the sequences to pictures helps bring understanding to the rules.

Key vocabulary

Rule	Term-to-term	Position-to-term
Linear	Non-linear	Coefficient

Key questions

What does n represent here?

How can you tell the sequence is linear?

What is the constant difference in this sequence?

How does this relate to the coefficient of n ?

How do the e.g. $3n$ and the $+1$ relate to the pattern?

Exemplar Questions

Work out the first five terms of the sequences give by these rules.

$4n$

$4n + 3$

$4n - 1$

$4n + 7$

Compare the sequences to the 4 times table. What do you notice?

Match these sequences and rules, working out the missing number.

Sequence A 6, 10, 14, 18...

$4n - 2$

Sequence B 1, 5, 9, 13...

$4n + 2$

Sequence C 9, 13, 17, 21...

$4n + 5$

Sequence D 2, 6, 10, 14...

$4n - \underline{\hspace{1cm}}$

Which of these sequences follow a rule of the form $3n + \underline{\hspace{1cm}}$ or $3n - \underline{\hspace{1cm}}$? Why or why not?

A 4, 7, 10, 13...

B 3, 6, 10, 15...

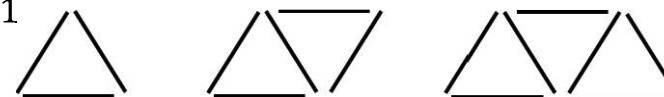
D 1, 4, 7, 10, ...

C 30, 27, 24, 21...

E 13, 16, 19, 22, ...

Find the rules for the n^{th} term of the sequences that are of the form $3n + \underline{\hspace{1cm}}$ or $3n - \underline{\hspace{1cm}}$

The rule for the number of sticks needed to make the n^{th} triangle in this pattern is $2n + 1$



Why does the number of sticks go up two each time you add a triangle? Why is there a “+1” in the rule?

Indices

Small Steps

- ▶ Adding and subtracting expressions with indices
- ▶ Simplifying algebraic expressions by multiplying indices
- ▶ Simplifying algebraic expressions by dividing indices
- ▶ Using the addition law for indices
- ▶ Using the addition and subtraction law for indices
- ▶ **Exploring powers of powers**

H

H denotes higher strand and not necessarily content for Higher Tier GCSE

+/- expressions with indices

Notes and guidance

Students sometimes mix up adding and subtracting when dealing with expressions involving indices; this step is to clarify the need for like terms in order to be able to add and subtract terms. Students may need reminding of the word 'coefficient' and the convention that we don't usually use 1 as a coefficient. Using manipulatives helps to explain why e.g. $2x^2 + 3x^2 \equiv 5x^2$ rather than $5x^4$

Key vocabulary

Expression	Simplify	Term
Coefficient	Index/Indices	Power(s)

Key questions

What is the difference between a term and an expression?

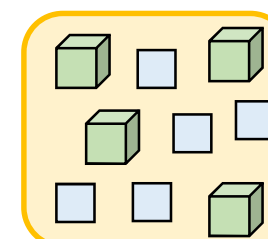
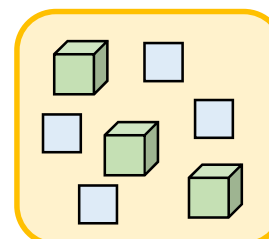
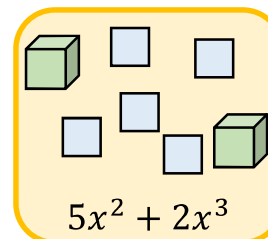
When are terms 'like terms'?

When can/can't an expression be simplified?

Why don't we usually write '1x' or '0x'?

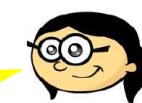
Exemplar Questions

Each square represents x^2 and each cube represents x^3 .
Write expressions for each card. The first one is done for you.



Jack

I think the first one should be $2x^3 + 5x^2$



Annie

I think the first one should be $7x^5$

Do you agree with either student? Explain why or why not.

Simplify the expressions on the cards.

$$3x^2 + 2x^2$$

$$4x^3 + 5x^3$$

$$6x^3 - 2x^3$$

$$8x^2 - 3x^2$$

$$2x^3 + 7x + 5x^3 + 3x$$

$$5x^3 + 6x^2 - 4x^2 - 2x^3$$

Check Dexter's indices homework.

Which ones are definitely right/wrong?

Which ones might people disagree about?

Dexter's homework

$$5a^2 + 2a^2 \equiv 7a^2$$

$$4b^3 - b^3 \equiv 4$$

$$4c^2 + 2c^2 - 6c^2 \equiv 0c^2$$

$$9d^3 - 6d^3 - 2d^3 \equiv 1d^3$$

$$5e^3 + 6e^2 - 2e^3 + e^2 \equiv 3e^3 + 7e^2$$

Multiply expressions with indices

Notes and guidance

Students should already be aware of the conventions for simplifying expressions like $3 \times a$, $b \times 4$ and $c \times c$ from their work in Year 7; this step builds on this to include terms with more than one letter and several letters/numbers by considering the factors of each term. The formal rules of indices are dealt with later in this block, but within this step students should deal with squares, cubes and their products.

Key vocabulary

Multiply	Product	Power
Index/Indices	Expand	Simplify

Key questions

What does the word 'index' mean?

What is the result of multiplying x^2 by x ? And then multiplying by x again? And again?

What is your strategy for multiplying e.g. $3a^2b$ and $5ab^3$?

What do you look at first? Then what?

Exemplar Questions

Complete the working.

$$\begin{aligned} 5a \times 3b \\ \equiv 5 \times a \times 3 \times b \\ \equiv 5 \times 3 \times a \times b \\ \equiv \end{aligned}$$

$$\begin{aligned} 7p \times 4q \\ \equiv 7 \times _ \times 4 \times _ \\ \equiv 7 \times 4 \times _ \times _ \\ \equiv \end{aligned}$$

Match the expressions on the cards with their simplified forms.

$3a \times 2$

$a \times b$

$a \times 2b$

$3a \times 2b$

$2ab$

$6a$

$6ab$

ab

Correct Tommy's answer.

$3d \times 4d \equiv 12dd$



Tommy

Which of these is the correct simplification of $2t^3 \times 3t^2$?

$5t^5$

$5t^6$

$6t^5$

$6t^6$

Expand the brackets and simplify as far as possible.

$3x(y + z) + 5y(z + 2x)$

$5pq(p + q) - 2q^2(p + p^2)$

$6a \times 3b \times 2a + 5ab(3b - 2a)$

Divide expressions with indices

Notes and guidance

This step will reinforce students' understanding of algebraic notation, particularly the use of fractional form to represent division. This is helpful here as the fractional form can help students identify the common factors more easily, and links to writing fractions in simplest form. Students may need to be reminded that it is expected to give answers in the form e.g. $\frac{y}{2}$ rather than involving decimals such as $0.5y$

Key vocabulary

Numerator	Denominator	Factor
Common factor	Coefficient	Simplify

Key questions

What is the difference between a term and an expression?

When can/can't an expression be simplified?

Exemplar Questions

Write these fractions in their simplest form.

$$\frac{15}{60}$$

$$\frac{27}{60}$$

$$\frac{5 \times 7 \times 11}{7 \times 11 \times 13}$$

$$\frac{2 \times 3 \times 11}{3 \times 7 \times 11}$$

$$\frac{24}{60}$$

$$\frac{80}{360}$$

$$\frac{2 \times a \times b}{5 \times a}$$

$$\frac{3 \times c \times d}{3 \times d \times d}$$

By thinking about the factors of the numerators and denominators, simplify these fractions.

$$\frac{6xy}{2}$$

$$\frac{6xy}{x}$$

$$\frac{6xy}{2x}$$

$$\frac{6xy}{3y}$$

$$\frac{6x^2}{2x}$$

$$\frac{6x^2y}{3xy}$$

Work out the divisions.

$$18 \div 3$$

$$18a \div 3$$

$$18a \div 3a$$

$$18ab \div 3$$

$$18ab \div 3b$$

$$36ab \div 3ab$$

$$24a^2 \div 6a$$

$$30a^2b \div 6ab$$

$$30a^2b \div 5ab^2$$

Compare the solutions to the calculation $3ab \times 4bc \div 24abc$.

$$2c$$

$$0.5c$$

$$\frac{c}{2}$$

Which one(s) do you agree with? Justify your answer.

The addition law for indices

Notes and guidance

Through experimentation, students usually quickly see that multiplying terms of the form a^m and a^n gives the result a^{m+n} . Nonetheless, the sight of a multiplication sign often results in errors like $2^6 \times 2^2 = 2^{12}$ and it is helpful to include and discuss examples like this, and also noting the rule does not apply to different bases e.g. $2^3 \times 3^4 \neq 6^7$. Likewise, the convention of writing x rather than x^1 can result in errors.

Key vocabulary

Base	Index/Indices
Power	Exponent

Key questions

What is the difference between a base and an index?
How can you simplify the multiplication of two terms involving indices if they have the same base?
Can you use the same rule if the bases are different?
Why is e.g. $a^6 \times a = a^7$ when there is no index on the second term?

Exemplar Questions

Complete the calculations on the cards.

$$\begin{aligned} 3^2 \times 3^4 \\ = (3 \times 3) \times (3 \times 3 \times 3 \times 3) \\ = 3^? \end{aligned}$$

$$\begin{aligned} 5^3 \times 5^5 \\ = (5 \times \dots) \times (5 \times 5 \times \dots) \\ = 5^? \end{aligned}$$

In the same way, work out $2^5 \times 2^3$ and $10^6 \times 10^6$, giving your answers as single terms.

- What connections do you see between the questions and the answers?
- Predict the answers to $7^4 \times 7^8$ and $6^{11} \times 6^{12}$
- Compare your answers with a partner's & discuss your findings

Explain why Dexter is wrong.

$$2^5 \times 2^6 = 4^{11}$$



Dexter

Work out the missing values.

$$2^7 \times 2^3 = 2^{\square}$$

$$a^7 \times a^3 = a^{\square}$$

$$a^7 \times a^9 = a^{\square}$$

$$2^4 \times 2^{\square} = 2^8$$

$$a^3 \times a^{\square} = a^8$$

$$a^{\square} \times a^3 = a^9$$

$$2^6 \times 2 = 2^{\square}$$

$$a^5 \times a = a^{\square}$$

$$a^x \times a^x = a^{16}$$

+/- laws for indices

Notes and guidance

This step develops from the last, illustrating that dividing expressions of the form a^m and a^n gives the result a^{m-n} . Common errors include not realising that a is the same as a^1 and mistakenly treating the exponent as 0. It is worth noting the difference between writing e.g. $6^5 \div 6^3$ as a single power and evaluating the result of $6^5 \div 6^3$. It is useful to mix up questions to include \times , \div and both operations.

Key vocabulary

Base	Index/Indices
Power	Exponent

Key questions

What is the difference between a base and an index?
How can you simplify the multiplication of two terms involving indices if they have the same base?
Can you use the same rule if the bases are different?
Why is (e.g.) $a^6 \div a = a^5$ when there is no index on the second term?

Exemplar Questions

Complete the calculations on the cards.

$$\begin{aligned} 2^6 \div 2^2 &= \frac{2^6}{2^2} \\ &= \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2} \\ &= 2^{\square} \end{aligned}$$

$$\begin{aligned} 3^7 \div 3^3 &= \frac{3^7}{3^3} \\ &= \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3} \\ &= 3^{\square} \end{aligned}$$

In the same way, work out $6^5 \div 6^3$ and $9^8 \div 9^4$, giving your answers as single terms.

- What connections do you see between the questions and the answers?
- Predict the answers to $7^{10} \div 7^6$ and $10^{12} \div 10^2$
- Compare your answers with a partner's & discuss your findings



Whitney

10 \div 5 is 2, so
 $2^{10} \div 2^5 = 2^2$

Explain why Whitney is wrong.

Work out the missing values.

$$2^7 \div 2^3 = 2^{\square}$$

$$a^7 \div a^3 \equiv a^{\square}$$

$$a^8 \div a^2 \equiv a^{\square}$$

$$2^4 \times 2^3 \div 2^6 = 2^{\square}$$

$$b^8 \div b^4 \times b^{\square} \equiv b^4$$

Exploring powers of powers

H

Notes and guidance

In this higher strand step, it is again quite easy to establish that $(a^b)^c = a^{bc}$, but this can sometimes then get confused with the addition of indices rules and lead to errors. It is useful for students to look at questions involving all three operations and consider the meaning of the calculations (and what they might look like if written in full) rather than relying on memorisation.

Key vocabulary

Base	Index/Indices	Power
Exponent	Product	

Key questions

How would you start solving an index question that involves more than one operation?

Will $(a^b)^c$ be the same as, or different from $(a^c)^b$? Why?

Why do we need to be careful with expressions like $(5x^4)^3$?

Exemplar Questions

Complete the calculations on the cards.

$$(3^4)^2 = 3^4 \times 3^4 = 3^?$$

$$(5^6)^3 = 5^6 \times _ \times _ = 5^?$$

Mo makes a conjecture.



Mo

When you raise a term with a power to another power, you multiply the indices
e.g. $(6^3)^4 = 6^{3 \times 4} = 6^{12}$

Test Mo's conjecture by checking his example and testing three more of your own. Do you agree with Mo?

Work out the missing values.

$$(2^7)^3 = 2^{\square}$$

$$(2^3)^7 = 2^{\square}$$

$$(5^5)^5 = 5^{\square}$$

$$(a^4)^{\square} \equiv a^8$$

$$(a^4)^{\square} \equiv a^{16}$$

$$(a^{\square})^5 \equiv a^{40}$$

Dani thinks $(2x^4)^3 \equiv 6x^{12}$. What mistake has Dani made?

Solve the equations.

$$2^x \times 2^4 = 2^{12}$$

$$2^{12} \div 2^y = 2^3$$

$$(2^2)^z = 2^{12}$$

$$3^5 \times 3^6 \div 3^a = 3^{20}$$

$$3^{14} \div (3^b)^3 = 3 \times 3^3 \times 3$$

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Proportional Reasoning						Representations					
	Ratio and scale		Multiplicative change		Multiplying and dividing fractions		Working in the Cartesian plane		Representing data		Tables & Probability	
Spring	Algebraic techniques						Developing Number					
	Brackets, equations and inequalities				Sequences	Indices	Fractions and percentages		Standard index form		Number sense	
Summer	Developing Geometry						Reasoning with Data					
	Angles in parallel lines and polygons			Area of trapezia and circles		Line symmetry and reflection	The data handling cycle			Measures of location		

Spring 2: Developing Number

Weeks 1 and 2: Fractions and Percentages

This block focuses on the relationships between fractions and percentages, including decimal equivalents, and using these to work out percentage increase and decrease. Students also explore expressing one number as a fraction and percentage of another. Both calculator and non-calculator methods are developed throughout to support students to choose efficient methods. Financial maths is developed through the contexts of e.g. profit, loss and interest. The higher strand also looks at finding the original value given a percentage or after a percentage change.

National Curriculum content covered includes:

- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics
- work interchangeably with terminating decimals and their corresponding fractions
- define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%
- interpret fractions and percentages as operators

Weeks 3 and 4: Standard Index Form

Higher strand students have already briefly looked at standard form in year 7 and now this knowledge is introduced to all students, building from their earlier work on indices last term. The use of context is important to help students make sense of the need for the notation and its uses. The higher strand includes a basic introduction to negative and fractional indices.

National Curriculum content covered includes:

- use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 and distinguish between exact representations of roots and their decimal approximations
- interpret and compare numbers in standard form $A \times 10^n$, $1 \leq A < 10$, where n is a positive or negative integer or zero

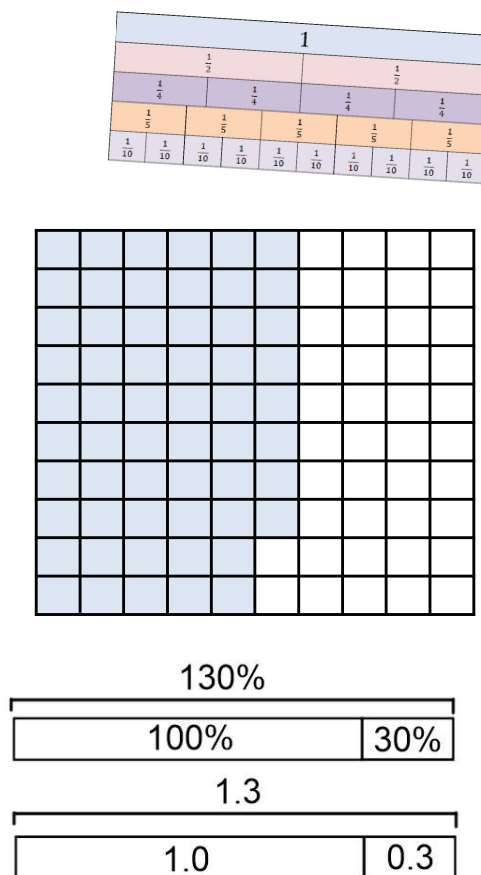
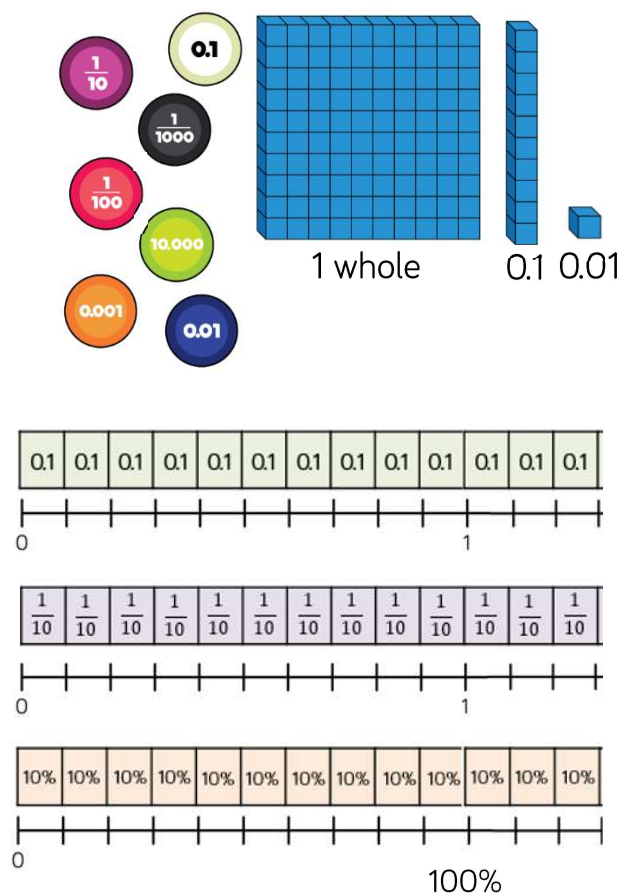
Weeks 5 and 6: Number Sense

This block provides a timely opportunity to revisit a lot of basic skills in a wide variety of contexts. Estimation is a key focus and the use of mental strategies will therefore be embedded throughout. We will also use conversion of metric units to revisit multiplying and dividing by 10, 100 and 1000 in context. The higher strand will extend this to look at the conversion of area and volume units, as well as having an extra step on the use of error notation. We also look explicitly at solving problems using the time and calendar as this area is sometimes neglected leaving gaps in student knowledge.

National Curriculum content covered includes:

- use standard units of mass, length, time, money and other measures, including with decimal quantities
- round numbers and measures to an appropriate degree of accuracy [for example, to a number of decimal places or significant figures]
- use approximation through rounding to estimate answers and calculate possible resulting errors expressed using inequality notation $a < x \leq b$
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately

Key Representations



Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Number lines are a useful way of assessing whether children understand the size of a fraction, decimal or percentage. Extending the number line above 1 is an option for some students.

Bar models and paper strips can be folded to represent different fractions, decimals or percentages and are particularly useful when making comparisons. Bar models are particularly useful to show when an amount has increased above or decreased below 100%

Number lines can be used to find original amounts for specific given percentage change problems.

Fractions and Percentages

Small Steps

- ▶ Convert fluently between key fractions, decimals and percentages R
- ▶ Calculate key fractions, decimals and percentages of an amount without a calculator R
- ▶ Calculate fractions, decimals and percentages of an amount using calculator methods R
- ▶ Convert between decimals and percentages greater than 100%
- ▶ Percentage decrease with a multiplier
- ▶ Calculate percentage increase and decrease using a multiplier
- ▶ Express one number as a fraction or a percentage of another without a calculator
- ▶ Express one number as a fraction or a percentage of another using calculator methods

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered in Year 7

Fractions and Percentages

Small Steps

- ▶ Work with percentage change
- ▶ Choose appropriate methods to solve percentage problems
- ▶ **Find the original amount given the percentage less than 100%** H
- ▶ **Find the original amount given the percentage greater than 100%** H
- ▶ **Choose appropriate methods to solve complex percentage problems** H

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered in Year 7

Fluently convert F, D & P

R

Notes and guidance

This small step revises year 7 work on mental conversion of key fractions, decimals and percentages. Use of diagrams such as the 100 square, and number lines to compare these will help to secure understanding; bead strings are also useful. Students should be confident in articulating their methods and using them to compare different forms e.g. which is larger $\frac{3}{5}$ or 65%

Key vocabulary

Fraction	Decimal	Percentage
Equivalent	Denominator	Numerator

Key questions

Why do we use all three representations of fractions, decimals and percentages?
Explain why one third is not the same as 0.3 or 30%
Can you draw a diagram to show the meaning of 0.7?
Which is greater in value 0.5 or 50%?

Exemplar Questions

72% of the Earth's surface is covered by water. Tick all answers below which represent the percentage of earth which is not covered by water.

0.28

 $\frac{56}{200}$

0.72

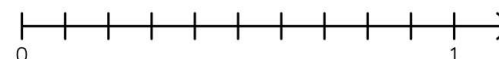
 $\frac{36}{50}$
 $\frac{7}{25}$

Complete the statements using $<$, $>$ or $=$

1) 0.37 $\frac{3}{8}$ 3) 0.4 4%

2) 0.35 $\frac{3}{5}$ 4) 0.6 60%

Label on the number line $\frac{4}{5}$, 0.7 and 75%



In a bag, $\frac{2}{5}$ of the counters are red. 0.15 of the counters are green.
The rest of the counters are blue. What percentage of the counters are blue?



Rosie says that $\frac{1}{4}$ is equivalent to 25% so $\frac{1}{8}$ is 12.5%

Use this information to write $\frac{3}{8}$ as a percentage and a decimal.

Huan thinks $\frac{1}{3} = 30\%$. Prove that Huan is wrong.

Calculate F, D & P mentally

R

Notes and guidance

Students will have visited finding fractions and percentages of amounts during year 7. This step will provide a further opportunity to consolidate their understanding and revisit key ideas and supporting diagrams such as the bar model.

Decimal multiplication can sometimes cause confusion, but using their knowledge of conversions and starting with $0.1 \times \dots = \dots \div 10$ and building from this is helpful.

Key vocabulary

Fraction	Decimal	Percentage
Equivalent	Denominator	Numerator

Key questions

Explain how to find $\frac{3}{7}$ of an amount.

Is it possible to find $\frac{6}{5}$ of a number? If so, how?

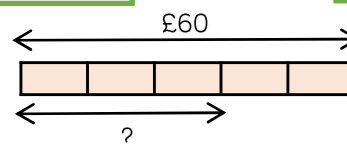
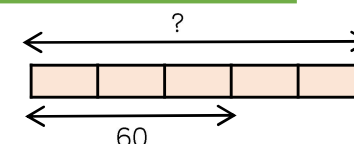
Explain why is it that when we divide an amount by 10 it gives 10%, but if you divide by 20 it does not give 20%?

Is it true that 45% of 60 is equal to 60% of 45?

Does this work for other pairs of numbers?

Exemplar Questions

What is the same and what is different about the calculations for the questions below?

Find $\frac{3}{5}$ of £60

 $\frac{3}{5}$ of a number is 60. What is the number?


Show that the values of these calculations are all equal.

0.1×300

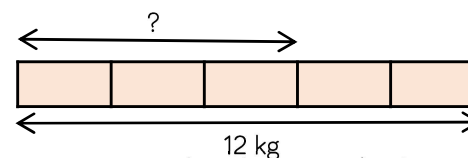
20% of 150

$\frac{3}{5}$ of 50

25% of 120

$\frac{5}{6}$ of 36

0.04×750



What might the question for the diagram be? How many questions can you find? Include fractions, decimals and percentages.

Match the cards that are equal in value.

$\frac{2}{5}$ of 30

15% of 40

100% of 20

0.3×40

$\frac{7}{7}$ of 12

$\frac{1}{8}$ of 160

F, D & P with calculator

R

Notes and guidance

Teachers should model the use of calculators so students gain awareness of efficient methods and using estimation before calculating. Comparison of the fraction and percentages keys will be useful. When solving problems, students will have access to a calculator but may still need access to supporting tools, such as the bar model, to compliment their understanding.

Key vocabulary

Fraction key	Decimal	Percentage
Estimate	Rounding	Conversion

Key questions

How do you use the percentage key on your calculator?
How does this compare to using decimal equivalents?

How do you use the fraction key on your calculator?

What keys could you press to find 23% of 45?

Exemplar Questions

Teddy and Mo are asked to calculate 35% of 150 cm.
Which of their methods do you prefer and why?

Mo

$$35 \div 100 = 0.35$$

$$0.35 \times 150 = 52.5 \text{ cm}$$

Teddy

$$150 \div 10 = 15 \text{ cm}$$

$$10\% = 15 \text{ cm} \quad 5\% = 7.5 \text{ cm}$$

$$30\% = 45 \text{ cm}$$

$$30\% + 5\% = 45 \text{ cm} + 7.5 \text{ cm}$$

$$35\% = 52.5 \text{ cm}$$

Rosie is working out 37% of £2800

She estimates the answer as $0.4 \times £3000 = £1200$

Is this a good way of estimating? Why or why not?

Estimate and then find the answers to the calculations on the cards.

23% of 800

2.3% of 800

68% of 600

18.5% of 61

The calculations below are used to find $\frac{3}{8}$ of £16000

How many other different ways can you find to calculate $\frac{3}{8}$ of £16000?

 $16000 \times 3 \div 8$
 0.375×16000
 $37.5\% \times 16000$

Jack says that 27% of 500 is the same as 54% of 1000
Show that Jack is wrong using a calculator and using a diagram.

Convert D & P both $<$ and $> 100\%$

Notes and guidance

Students should already be fluent in converting between decimals and percentages up to 100% and now explore the equivalence of percentages above 100%. This will support later use of multipliers for percentage increase. Physical resources and pictures, particularly the hundred square are very useful. It is good to link e.g. $130\% = 100\% + 30\%$ to the decimal addition $1 + 0.3$

Key vocabulary

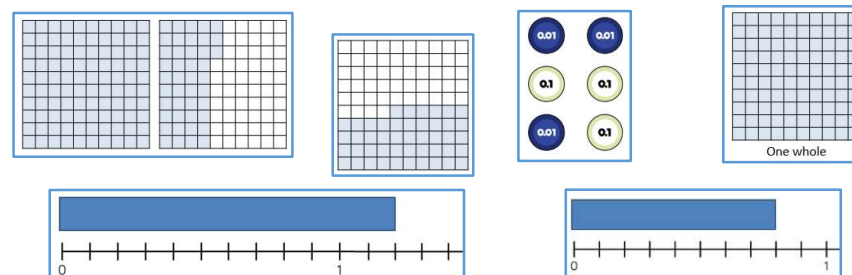
Fraction	Decimal	Percentage
Equivalent	Hundredth	Tenth

Key questions

Why is 0.3 the same as 30% and not 3%?
 Is it possible to have a percentage greater than 100%?
 How might 140% look like as a decimal multiplier?
 Why does multiplying a decimal by 100 give you an equivalent percentage?
 How can you order mixed decimals and percentages?

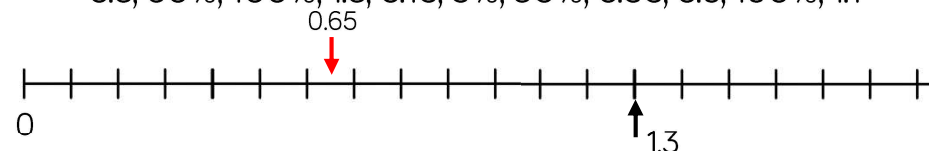
Exemplar Questions

Write down the percentage shown by each diagram.



Use the given information to add these decimals and percentages to the number line. Label each one.

0.3, 60%, 100%, 1.3, 0.15, 5%, 30%, 0.55, 0.9, 190%, 1.1



Match the equivalent decimals and percentages. Write the equivalent percentage and decimal for any cards that are not paired up.

—	0.08	0.80	8%	18%
1.8	0.8%	—	80%	0.8

Fill in the blanks.

99% = —
 100% = 1.00
 —% = 1.01

—% = 0.9
 100% = 1.00
 110% = —

Percentage decrease: multipliers

Notes and guidance

For percentage decrease, students will need to understand that they are subtracting the given percentage from 100%. This concept should be represented using bar models and number lines to help reinforce how to find the correct multiplier. This should also avoid the misconception of e.g. multiplying by 0.2 to find a 20% decrease.

Key vocabulary

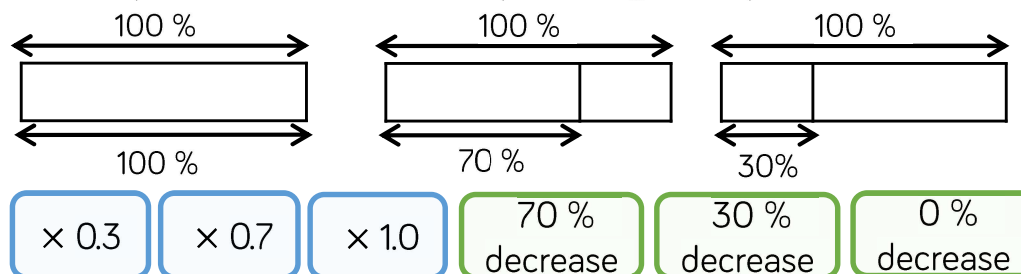
Decimal	Percentage	Reduce
Equivalent	Decrease	Multiplier

Key questions

Why is decreasing by 46% the same as finding 54%?
 If I am multiplying by 0.2 why is this an 80% decrease?
 What mistakes might happen if we are decreasing by 1.5%?
 What happens if I decrease an amount by 0%?
 What does the word 'discount' mean?

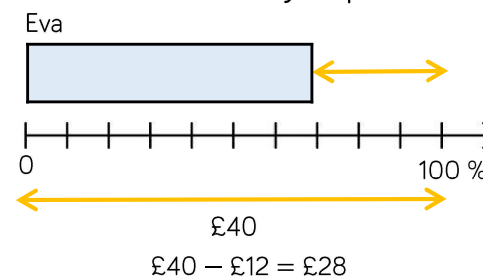
Exemplar Questions

Match up the bar model with the percentage multiplier and statement.



In a sale, 30% is taken off all prices. Eva, Mo and Alex are calculating the sale price of a shirt that costs £40 before the sale.

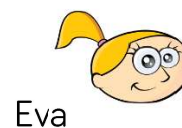
Which method do you prefer? Why?



Mo
 $100\% - 30\% = 70\%$
 $0.7 \times 40 = 28$

Alex
 $10\% = £4$
 $30\% = £12$
 Sale price = $£40 - £12 = £28$

Explain why Eva is wrong.
 What should the multiplier be?



To reduce a number by 45% the multiplier is 0.45



What is the multiplier for a 10% reduction followed by another 10% reduction?

Increase & decrease: multipliers

Notes and guidance

Students build on the last two steps using multipliers above one to increase an amount by a given percentage.

It is worth discussing the similarities and differences between percentage increase and decrease and mixing questions so that students are thinking carefully rather than just using a procedure. Starting with a bar representing 100% can help access worded problems.

Key vocabulary

Multiplier	Decimal	Percentage
Equivalent	Increase	Growth

Key questions

When increasing an amount by a given percentage, how do we calculate the multiplier?

What is the percentage increase if you double a number?

Will a number always get bigger if we increase it by a given percentage?

Can you represent this question with a bar model?

Exemplar Questions

Match the multiplier with the correct percentage statement.

1.3

30% increase

2.4

0.8

40% increase

1.4

140% increase

92% decrease

20% decrease

0.65

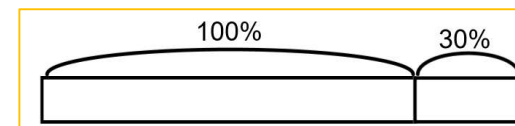
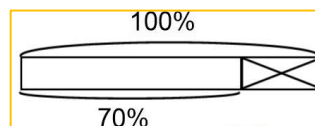
0.08

35% decrease

Dexter earns £30 a week for his paper round.

His employer gives him a 30% pay rise.

Which of the bar models shows this?



Work out Dexter's new wage.

Aisha earns £35000 a year. Her boss offers her a pay rise of 6% a year, but a rival employer offers to pay her £180 more per month.

Which offer should she accept to get the most money?

Alex increases 30 g by 20%

She then decreases her answer by 20%

Dora says she will have less than her original amount of 30 g

Alex disagrees. Who is correct? Justify your answer.

Express as a % : Non-calculator

Notes and guidance

As a first step on the way to expressing one number as a percentage of another, students will firstly explore writing one number as a fraction of another. In this step, the focus will be to support students to express fractions as percentages where the fraction denominators are factors or multiples of 100. This is another good opportunity to make links to probability and simple conversions.

Key vocabulary

Express	Fraction	Percentage
Equivalent	Factor	Multiple

Key questions

Why can we convert quarters, fifths and tenths easily to a percentage but not thirds?
 Why can't we compare a mark out of 20 and a mark out of 25 directly? What are the factors of 100?
 Is it possible to convert fortieths to hundredths?
 Why or why not?

Exemplar Questions

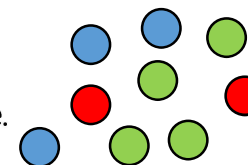
Tommy saves £13 of his £20 pocket money each week.
 He gives £3 to his sister.

What fraction of his pocket money does he have left to spend?
 What percentage of his pocket money does he have left to spend?

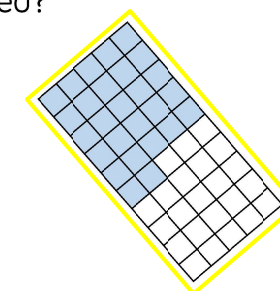
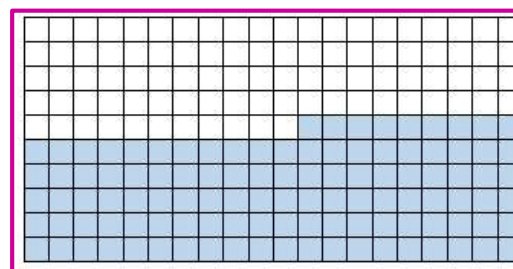
Eva has a bag of 200 counters. 34 counters are red, 46 are blue and the rest are green.

What proportion of the counters are green?
 Give your answer as a fraction and a percentage.
 She takes out the blue counters.

By what fraction has the number of counters in the bag been decreased by? Express this fraction as a percentage.



Which shape has the larger percentage shaded?



A bag contains green and red counters in the ratio 12 : 13
 What percentage of the counters are green?

Express as a % : Calculator

Notes and guidance

Building on from the previous step, students are asked to consider a number as a percentage of another both from fractions that can be converted mentally and those that are best converted using a calculator. To keep the focus on conversion rather than rounding, it might be best to give non-exact answers to the nearest whole number percentage; this skill may need revising in starters.

Key vocabulary

Fraction	Decimal	Percentage
Equivalent	Round	Integer

Key questions

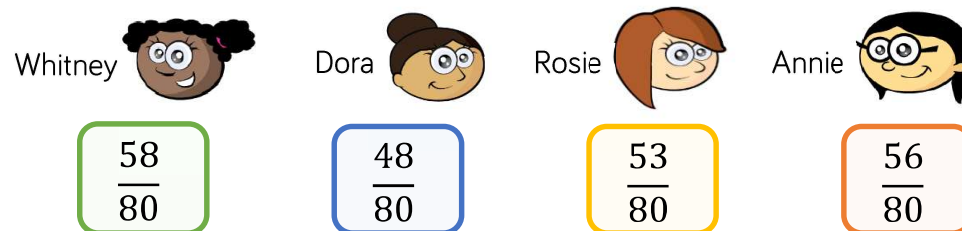
Why might we need a calculator to calculate the percentage of a test mark out of 30, but not for a mark out of 50?

How do we use a calculator to convert a fraction to a decimal and then to a percentage?

Is it possible to work out e.g. 70 as a percentage of 65?

Exemplar Questions

Here are the marks from a test.



Convert the marks to percentages.

Why are some of the percentages integers and others not?

In a local election 1389 out of 6000 residents vote.

Jack is working out the percentage of the village that voted.



Jack

$\frac{1389}{6000} = 0.2315$, so that's 23.15%
15 is more than 5 so that's 24%
to the nearest whole number.

Explain why Jack has rounded this incorrectly.

The attendance of three classes one Friday was:

- Class X had 3 people missing out of 29
- Class Y had 4 people missing out of 31
- Class Z had 2 people missing out of 30

Work out the percentage attendance of each class, giving your answers to the nearest whole percent.

Which class had the highest percentage attendance?

Work with percentage change

Notes and guidance

Students continue to express one number as a percentage of another, this time in the context of change. Good contexts to consider include percentage profit and loss and interest to remind students of these words. It is also useful to look at situations that can be worked out using both calculator and non-calculator methods allowing the students to choose the most appropriate method.

Key vocabulary

Profit	Loss	Interest	Change
Original	Invest	Numerator	Denominator

Key questions

What's the difference between profit and loss?
How can you represent this percentage change question on a bar model?
Why is it important to identify the original amount before doing the calculation for percentage change questions?

Exemplar Questions

Amir sells his mobile phone for £240
He paid £480 for the mobile phone when it was new.
Has he made a profit or loss?
What percentage profit or loss has he made?

240%

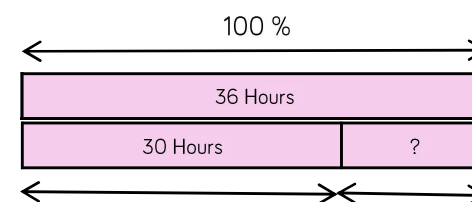
480%

50%

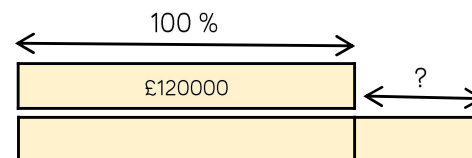
100%

200%

When new, Dani's phone battery would last 36 hours between charges. Now it only lasts 30 hours between charges.
Calculate the percentage decrease in battery life.



Jack buys a house for £120000
He sells the house one year later for £135000
What is the percentage profit that Jack has made?



Brett invests £80000 in a new business.
At the end of the year he is paid back £81140
What is his percentage return on his investment?
Alex invests £60000 in a saver account that pays 2.5% interest per year. Show that Alex's investment earns more than Brett's, and work out by how much.

Choose appropriate methods

Notes and guidance

In this step students will use all the skills gained from the previous steps to apply to various percentage problems. It is worth investing time in analysing and discussing what questions are being asked and how to choose methods, to avoid students rushing into an inappropriate procedure. In particular, students need to decide whether a question asks for finding a percentage or express as a percentage.

Key vocabulary

Original	Percentage	Increase
Decrease	Profit/Loss	Express

Key questions

Describe the different calculation processes involved in these questions.

How can you represent this on a bar model?

What is the same and what is different in these questions?

What type of percentage question is this problem?

How can you tell?

Exemplar Questions

What's the same and what's different?

Work out 30% of £70

Increase £70 by 30%

What percentage of £70 is £30?

Eva scores 60% on a test.

Which of the cards could have been her score?

14 out of 20

54 out of 90

31 out of 50

20 out of 30

9 out of 15

33 out of 55

The sign shows the cash price of a TV set and a monthly payment plan.

How much more does it cost altogether to buy the TV set using the monthly payment plan? Express this as a percentage of the cash price.

Pay £1200 today

OR

Pay a 15% deposit
+ 12 payments of £99

Ron bakes some cakes.

He pays £15 altogether for the ingredients.

He sells 10 of the cakes, but makes a loss of 20% overall.

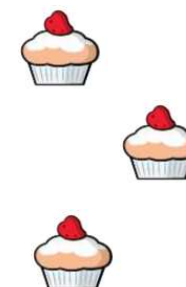
How much did Ron charge for each cake?

Ron reviews his pricing strategy and bakes some more cakes.

He again pays £15 altogether for the ingredients.

This time he sells 23 of the cakes for 90p each.

What percentage profit did Ron make this time?



Find original less than 100%

H

Notes and guidance

It is useful to concentrate on questions where a calculator is not required so students can interpret rather than just follow a procedure. Common errors include finding the given percentage of the given number rather than working backwards towards an original. Bar models are a useful model as they show both the reduction and the remainder providing a strong visual clue as to how to find the original.

Key vocabulary

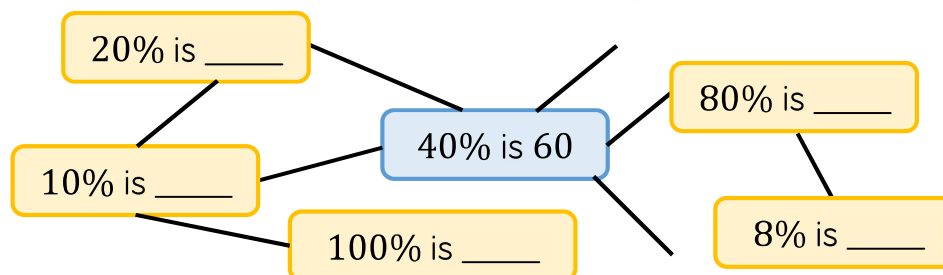
Original	Percentage	Reverse
Equivalent	Multiple	

Key questions

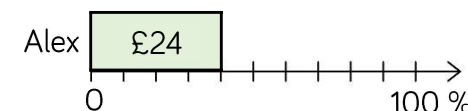
Is the original value greater than or less than the given amount? What percentage is the original amount?
How can we represent this using a bar model?
From the percentage given, what other percentages can we easily work out?
How can we build on these to find 100%?

Exemplar Questions

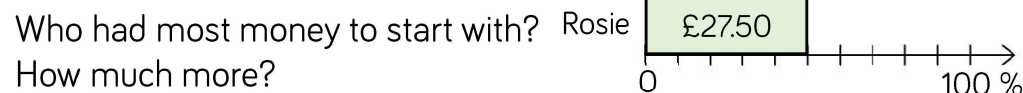
40% of a number is 60. What other facts can you find?



After spending 60% of her money, Alex has £24 left.



After spending 50% of her money, Rosie has £27.50 left.



Who had most money to start with?
How much more?

Amir is comparing the sale price of t-shirts from two different shops. In which shop was the t-shirt originally more expensive?

Drimark
20% off!
Now only
£44

N & S
30% off!
Now only
£42

Whitney flips a coin and gets heads 45% of the time. She gets heads 54 times. How many times did she flip the coin?



Find original more than 100% H

Notes and guidance

This step is closely linked with the previous one and once again a heavy emphasis should be placed on adding the percentage increase to 100%. This will enable students to understand what percentage the value they are given represents. Common misconceptions include students finding the percentage increase of the given amount and subtracting it from the given amount.

Key vocabulary

Original	Percentage	Reverse
Equivalent	Increase	

Key questions

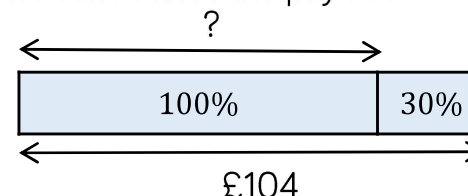
Is the amount given more or less than the new amount?

How can we represent this on a bar model?

What is the same and what is different between these two bar models?

Exemplar Questions

After a 30% pay rise, Eva earns £104 a week.
How much did she earn before the pay rise?



Valued Added Tax (VAT) is charged at 20% in the UK.
Complete this calculation to find the cost of the jacket without VAT.



$\div 12$	$\begin{array}{c} \curvearrowright \\ \curvearrowleft \end{array}$	120%	£192	$\begin{array}{c} \curvearrowright \\ \curvearrowleft \end{array}$	\div _____
\times _____	$\begin{array}{c} \curvearrowright \\ \curvearrowleft \end{array}$	10%	£ _____	$\begin{array}{c} \curvearrowright \\ \curvearrowleft \end{array}$	\times _____
		100%	£ _____		

After a 20% change in price, a games console now costs £288
What was the original price if the change was a 20% increase?
What was the original price if the change was a 20% decrease?



Annie has some sweets.

Teddy gives her some sweets and she now has 50% more.

Rosie gives her some sweets and she now has an extra 40%

Annie now has 63 sweets. How many did she have originally?



Complex percentages

H

Notes and guidance

This is another opportunity for students following the higher strand to practise interpretation of questions so that they can choose the correct method. They should look at a variety of situations including the 'reverse' percentage questions just studied mixed with percentage increase, decrease, finding a percentage and expressing as a percentage.

Key vocabulary

Original	Percentage	Reverse
Express	Decrease	Increase
Profit/Loss		

Key questions

How can you represent this problem using a bar model?
How can you tell if a question involves finding an amount before a percentage change? How does this affect your approach to the question?

Exemplar Questions

Mo buys a rare comic for £120 and sells it again for £170
Compare these methods to work out his percentage profit.

Method 1

$$170 - 120 = 50$$

$$\frac{50}{120} = 0.41666 \approx 42\%$$

Method 2

$$\frac{170}{120} = 1.41666 \approx 142\%$$

$$142\% - 100\% = 42\%$$

After a 18% pay rise, Dora's salary is £38350
Which of these calculations will give her original salary?

$$£38350 \div 0.82$$

$$£38350 \times 1.18$$

$$£38350 \times 0.18$$

$$£38350 \div 1.18$$

$$£38350 \times 0.82$$

Write a question that could be solved with each of the other calculations.

Ms Rose bought a house in 2012 for £120000
She sold the house five years later making a profit of 60%
How much did she sell the house for?



ABCD is a rectangle.
The lengths of the sides AB and BD are in the ratio 5 : 4
What percentage of the perimeter of the rectangle is side AC?

Standard Form

Small Steps

- Investigate positive powers of 10
- Work with numbers greater than 1 in standard form
- Investigate negative powers of 10
- Work with numbers between 0 and 1 in standard form
- Compare and order numbers in standard form
- Mentally calculate with numbers in standard form
- Add and subtract numbers in standard form
- Multiply and divide numbers in standard form
- Use a calculator to work with numbers in standard form

H denotes Higher Tier GCSE content

R Denotes “review step” – content should have been covered in Year 7

Standard Form

Small Steps

▶ Understand and use negative indices

H

▶ Understand and use fractional indices

H

H denotes Higher Tier GCSE content

R Denotes “review step” – content should have been covered in Year 7

Positive powers of 10

Notes and guidance

Using experimentation students will explore powers of ten. This recaps and builds on students' understanding from the indices unit and the work they did in year 7. It is useful to explore why $10 \times 10^3 \neq 10^3$ as this is a common misconception. Discussion around the benefits of writing numbers in powers of ten can be demonstrated using large numbers such as 1 billion = $1000\ 000\ 000 = 10^9$

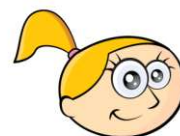
Key vocabulary

Base	Index/Indices	Power
Exponent		

Key questions

How many times bigger than 1000 is 10^8 ?
 Why are (e.g.) $(10^2)^3$ and $10^2 \times 3$ different?
 Is there a simpler way to write (e.g.) 10000×100000 ?
 What calculations could give an answer of (e.g.) 10^{12} ?

Exemplar Questions



$10 \times 10 \times 10 = 10^3$ and
 $10 \times 10 \times 10 \times 10 = 10^4$
 Therefore this must mean that
 $10 \times 10 \times 10 \times 10 \times 10 = 10^5$

- What connections do you see in Eva's examples?
- Use the examples to help work out what is meant by 10^7
- How can we use this understanding to work out $10^7 \times 10$?
- What is the meaning of 10^{20} ?

Fill in the blanks.

$$10^3 \times 10^3 = 10^{\square} \times 10^5$$

$$10^{6+4} = 10^3 \times 10^{\square}$$

Solve the equations.

$$10^6 \times b = 10^{10}$$

$$100a = 10^9$$

$$10^4 \times c = 10^{12}$$

$$100 = \frac{10^6}{d}$$

$$\frac{10^6}{e} = 10^3$$

$$\frac{10^6}{f} = 10^6$$



5^3 is the same as $\frac{10^3}{2}$

Convince me that Amir is incorrect.

Standard form with numbers > 1

Notes and guidance

Students will now write large numbers in standard form. Students should be exposed to correct examples in the form $A \times 10^n$ where A is a number between 1 and 10 and n is an integer. It is important to look at how standard form works rather than just counting zeros. Teachers can deepen this understanding by looking at non-examples such as 0.8×10^4 and $4 \times 10^{0.8}$.

Key vocabulary

Base	Index/Indices	Power
Exponent	Standard (index) form	

Key questions

What is one gigabyte (1 GB) written in standard form?
 What is the same and what is different about how 75 000 and 70 000 are written in standard form?
 Why is it more efficient to write 4×10^{50} in standard form rather than as an ordinary number?

Exemplar Questions

Fill in the blanks.

$$\begin{aligned} 300000 \\ &= 3 \times 10000 \\ &= 3 \times 10^4 \end{aligned}$$

$$\begin{aligned} 6000000 \\ &= 6 \times \square \\ &= 6 \times 10^{\square} \end{aligned}$$

$$\begin{aligned} 70000 \\ &= \square \\ &= \square \end{aligned}$$



5000 is the same as 5×10^3 and 6000 is 6×10^3

5500 must be 5.5×10^3 as it is half way between 5000 and 6000



- How would 5200 be written in standard form?
- How would 52 000 be written in standard form?
- Write 5.2×10^6 as an ordinary number

Fill in the blanks.

$$\begin{aligned} 40000 &= 4 \times 10^4 \\ \square &= 4 \times 10^3 \\ 400 &= \square \end{aligned}$$

$$\begin{aligned} \square &= 4.2 \times 10^2 \\ 425 &= \square \\ \square &= 4.25 \times 10^3 \end{aligned}$$



Neptune is 2.8 billion miles from the sun.

- Write this number in standard form.
- Write 2.8 billion miles in kilometres in standard form.

Negative powers of 10

Notes and guidance

During this step students will look at decreasing powers of ten then investigate what happens if you get to 1 and below. Time should be spent discussing and also investigating 10^0 and explore misconceptions such as $10^0 = 0$ and $10^{-2} = -100$. Students should also be confident working between standard form, decimals and fraction equivalences.

Key vocabulary

Base	Index/Indices	Power
Exponent	Negative	Place value

Key questions

How many different ways can you write 0.001?

How could you show 10^{-2} on a place value grid?

What is the value of 10^0 ? What is the value of 8^0 ?

What is x^0 for any value of x ?

Exemplar Questions

Fill in the blanks.

$$10^3 = \square$$

$$10^2 = 100$$

$$\square = 10$$

$$10^0 = \square$$

$$\square = \frac{1}{10^1} = \frac{1}{10} = 0.1$$

$$10^{-2} = \frac{1}{10^2} = \square = 0.01$$

$$10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = \square$$



Investigate Dora's theory.
Is she correct?

If 10^0 is 1 does that mean that any number to the power of zero is 1?

Match the equivalent cards and write a different card for each.

$$\frac{1}{10^3 \times 10^0}$$

$$\frac{1}{10000}$$

$$\frac{1}{10}$$

$$\frac{1}{10^4}$$

$$0.01$$

$$10^{-1}$$

$$10^{-5} \times 10^3$$

$$10^{-3}$$

Complete the statements using $<$, $>$ or $=$

$$10^3 \square 10^4$$

$$10^{-3} \square 10^4$$

$$10^3 \square 10^{-4}$$

$$10^{-3} \square 10^{-4}$$

$$10^1 \square 10^{-2}$$

$$10^1 \square 10^2$$

Numbers between 0 and 1

Notes and guidance

Once negative powers are understood, students can explore the patterns and connections between decimal numbers and standard form. They should also be exposed to similar questions that have different answers to deepen their understanding e.g. comparing 8.9 and 8.09 written in standard form. Misconception such as $3 \times 10^{-1} = -0.3$ should also be explored and addressed.

Key vocabulary

Base	Index/Indices	Power
Exponent	Negative	Place value

Key questions

What is the same and what is different about (e.g.) 3×10^{-4} and 3×10^4 ?
Explain why (e.g.) 4×10^{-3} is greater than 5×10^{-4} .
Are negative powers of 10 always, sometimes or never negative numbers?

Exemplar Questions

Fill in the blanks.

$$5 = 5 \times 1 = 5 \times 10^0$$

$$0.5 = 5 \times 0.1 = \boxed{}$$

$$0.05 = 5 \times 0.01 = \boxed{}$$

$$0.006 = \boxed{} = \boxed{}$$

$$4.5 = \boxed{} = 4.5 \times 10^0$$

$$\boxed{} = 4.5 \times 0.1 = 4.5 \times 10^{-1}$$

$$0.045 = 4.5 \times 0.01 = \boxed{}$$

$$0.0045 = \boxed{} = \boxed{}$$

Which of the following cards correctly represents 0.0003?

$$3 \times 10^4$$

$$\frac{1}{3000}$$

$$\frac{1}{3^4}$$

$$\frac{1}{30000}$$

$$3 \times 10^{-4}$$



0.0302 is the same as
 3.2×10^{-2}

I think 0.0302 is 3.02×10^{-2}



- Discuss which statement is correct?
- Can you represent 0.504 on a place value grid?

Match the cards of equal value.

$$4.05 \times 10^{-3}$$

$$5.4 \times 10^{-2}$$

$$0.045$$

$$0.00405$$

$$0.054$$

$$4.05 \times 10^{-2}$$

$$5.4 \times 10^{-1}$$

$$5.04 \times 10^{-2}$$

$$0.54$$

$$0.0504$$

$$0.0405$$

$$4.5 \times 10^{-2}$$



Give three examples of ordinary numbers that could round to 5.6×10^{-3} . What degree of accuracy are you rounding to?

Order numbers in standard form

Notes and guidance

Students will order numbers given in words, standard form and ordinary form. Strategies for comparing numbers, such as considering the exponent of 10 as an initial check should be discussed. Students should revisit ordering decimal numbers to tease out any misconceptions that may be a barrier to ordering numbers in standard form. Access to place value grids would be useful here.

Key vocabulary

Base	Index/Indices	Power
Exponent	Negative	Place value

Key questions

How can we compare a fraction, a decimal and a number written in standard form? What could you do to make it easier?

What do you look at first when comparing numbers written in standard form? Why?

Exemplar Questions



3×10^4 is less than 4×10^3

Show that Tommy is wrong.

A 6×10^7 , 6.3×10^7 , 6.03×10^7 and 6.18×10^7

B 6.13, 7.31, 6.31, 6.301 and 6.013

C 6×10^4 , 6.9×10^3 , 6.8×10^{-1} and 6.7×1^6

- Order the numbers on each card from smallest to largest.
- Which card is easier to write in size order A, B or C?

Complete the statements using $<$, $>$ or $=$

$$5.6 \times 10^6 \bigcirc 6 \times 10^5$$

$$1 \times 10^0 \bigcirc 1 \times 0$$

$$30 \times 300 \bigcirc 9.5 \times 10^{-3}$$

$$0.4 \bigcirc 4 \times 10^{-1}$$

$$71000 \bigcirc 7.1 \times 10^5$$

$$4.1 \times 10^{-4} \bigcirc \frac{1}{4100}$$



Arrange the following cards in ascending order.

$$\frac{1}{7200}$$

$$7.1 \times 10^{-4}$$

One seventh

$$\frac{7}{7000}$$

$$70 \times 10^{-3}$$

$$7.2 \times 10^1$$

$$7.5 \times 10^0$$

$$0.73$$

Standard form: Mental calculations

Notes and guidance

Using simple numbers, students will complete mental calculations where one number in standard form is multiplied or divided by an integer. The result may no longer be in standard form. A key focus of this step is correcting answers such as 24×10^8 by using the fact that $24 = 2.4 \times 10^1$ and completing the calculation using index laws $2.4 \times 10^1 \times 10^8 = 2.4 \times 10^9$

Key vocabulary

Base	Index/Indices	Power
Exponent	Negative	Commutative

Key questions

Why isn't (e.g.) 200×10^6 in standard form? How could rewriting 200 help us?

Explain how 0.2×10^4 and 0.2×10^{-4} can be written in standard form. What is the same and what is different?

Why is $6 \times (5 \times 10^3)$ more difficult than $4 \times (2 \times 10^3)$?

Exemplar Questions

Fill in the blanks to complete the calculations.

$$\begin{aligned} 3 \times (2 \times 10^6) \\ = (3 \times \square) \times 10^6 \\ = 6 \times \square \end{aligned}$$

$$\begin{aligned} (7 \times 10^{-4}) \div 2 \\ = (7 \div \square) \times 10^{-4} \\ = 3.5 \times \square \end{aligned}$$

Use similar strategies to work out the other cards.

$$2 \times (2.5 \times 10^6)$$

$$(9.3 \times 10^{-4}) \div 3$$

Find and correct Alex's mistake.



$$\begin{aligned} (8 \times 10^7) \times 2 &= 16 \times 10^7 \\ &= 1.6 \times 10^1 \times 10^7 = 1.6 \times 10^{17} \end{aligned}$$



Amir works out $2 \times 10^8 \div 4$ and gets the answer 0.5×10^8 . He realises his answer is not in standard form and thinks of two possible solutions. Which one is correct? Why?

$$5 \times 10^9$$



$$5 \times 10^7$$

Complete the calculations, giving your answers in standard form.

$$\begin{aligned} 3 \times (6 \times 10^6) \\ = 3 \times 6 \times 10^6 \\ = 18 \times 10^6 \\ = 1.8 \times 10^1 \times 10^6 \\ = 1.8 \times \square \end{aligned}$$

$$\begin{aligned} (7 \times 10^4) \times 1000 \\ = 7 \times 10^4 \times 10^{\square} \\ = 7 \times \square \end{aligned}$$

$$\begin{aligned} (2 \times 10^{-5}) \div 5 \\ = 0.4 \times 10^{-5} \\ = 4 \times 10^{\square} \times 10^{-5} \\ = 4 \times 10^{\square} \end{aligned}$$

+ and – numbers in standard form

Notes and guidance

Students will compare strategies for addition and subtraction without a calculator. There is a risk of just adding the numbers and adding the powers separately and students may prefer to always convert to ordinary numbers. Even when the powers of 10 are the same, there can be problems such as $3 \times 10^4 + 8 \times 10^4 = 11 \times 10^4$ where the answer needs changing as covered last step.

Key vocabulary

Base	Index/Indices	Power
Exponent	Negative	Commutative

Key questions

Is it easier to add the numbers as they are or convert them to ordinary numbers first?

What do we do if the total isn't in standard form?

What is (e.g.) $10^{-3} + 10^3$ as an ordinary number?

Exemplar Questions

Mo and Dora are working out $7 \times 10^3 + 2 \times 10^3$
Decide which method you prefer and discuss why.



$$\begin{aligned} &= 7000 + 2000 \\ &= 9000 \\ &= 9 \times 1000 \\ &= 9 \times 10^3 \end{aligned}$$



$$\begin{aligned} &= (7 + 2) \times 10^3 \\ &= 9 \times 10^3 \end{aligned}$$

Decide whose method you would use for the following questions.

$$6 \times 10^{-3} + 1.5 \times 10^3$$

$$7 \times 10^5 - 5 \times 10^5$$

$$8.2 \times 10^{-6} - 1.2 \times 10^{-6}$$

$$9.6 \times 10^5 - 3.2 \times 10^4$$

Dora is working out $8 \times 10^4 + 9 \times 10^4$ and gets 17×10^4
She says her answer is not in standard form.

What could she do?



Fill in the blanks.

$$6 \times 10^{\square} + 2.3 \times 10^3 = 6.23 \times 10^4$$

$$6 \times 10^4 \square 2.3 \times 10^{\square} = 2.9 \times 10^5$$

$$6 \times 10^{\square} + 2.3 \times 10^{\square} = 8.3 \times 10^{-6}$$

\times and \div numbers in standard form

Notes and guidance

Students will explore the use of commutativity to multiply and divide numbers given in standard form. Their earlier work on indices and dealing with answers like 30×10^7 should have prepared them for this step. It is helpful to include various forms of the same questions such as

$$(4 \times 10^5) \div (2 \times 10^6) \text{ and } \frac{4 \times 10^5}{2 \times 10^6}$$

Key vocabulary

Base	Index/Indices	Power
Exponent	Negative	Commutative

Key questions

How many different ways can you write (e.g.) $(3 \times 10^4) \times (2 \times 10^4)$?

Describe the steps you need to take to multiply/divide a pair of numbers in standard form.

When can we write a division as a fraction?

Exemplar Questions

Amir is working out $(3.2 \times 10^2) \times (2 \times 10^4)$

Because multiplication is commutative I can write the question like this to help me $(3.2 \times 2) \times (10^2 \times 10^4)$

He then works out $(8 \times 10^6) \div (4 \times 10^3)$

I can write this as $(8 \div 4) \times (10^6 \div 10^3)$

Complete Amir's calculations.

Rosie and Whitney are calculating $(5 \times 10^6) \div (2 \times 10^3)$

Here are their answers. Can you spot what mistakes they have made?



$$2.5 \times 10^2$$

$$3 \times 10^3$$



Which question does not belong?

$$(2 \times 10^8) \times (1.6 \times 10^{-12})$$

$$(2.8 \times 10^{-4}) + (4 \times 10^{-5})$$

$$(4 \times 10^{-4}) - (8 \times 10^{-3})$$

$$(9.6 \times 10^5) \div (3 \times 10^9)$$

Standard form using a calculator

Notes and guidance

All four operations will be explored, and students can further their knowledge of calculators to use the memory and exponent functions. Alternative methods to the answer to the same question could be shown on the calculator, such as using the fraction button for division. The use of emulator software, if available, would be helpful.

Key vocabulary

Standard form	Exponent	Power
Scientific notation	SCI/EXP	

Key questions

Explain how to input (e.g.) 2.4×10^5 on a calculator. What would be different inputting 2.4×10^{-5} ?

What button on your calculator converts an answer into standard form.

How do you round a number in standard form to 1/2/3 significant figures?

Exemplar Questions

Rosie uses a calculator to work out some calculations using standard form.



Some of my answers are already written in standard form and some are not...

- Use your calculator to find an example of multiplication, division, addition and subtraction where the answer is given in standard form and one is not.
- Compare your examples with a partner.
- What is the largest number you can type into your calculator before it automatically changes to standard form?

Use the calculator to complete the calculations on the cards where $a = 3.2 \times 10^4$ and $b = 2.1 \times 10^{-3}$

Give your answers in standard form to 3 significant figures.

$$b^3$$

$$a \div b^3$$

$$2a \div b^2$$

$$a^2 - 2b$$

$$a^2 + 2b$$

$$(a - 2b)^2$$

The average human body can produce 3 million red blood cells every second.

How many red blood cells does the average human body produce in one year?

Give your answer in standard form.

Understand negative indices

H

Notes and guidance

Students will build on their understanding of negative powers of 10 to explore negative indices generally. Common misconceptions around negative powers, such as $5^{-2} \neq -25$ and $5^{-2} \neq -\frac{1}{25}$, should be discussed. It is worth spending time exploring the patterns and linking to inverse operations to deepen conceptual understanding.

Key vocabulary

Power	Negative	Exponent
Reciprocal	Zero	

Key questions

Will a number raised to a negative power always, sometimes or never have a negative value?
How does working out negative powers relate to the subtraction law for dividing indices?
How do you enter negative powers on a calculator?

Exemplar Questions

Teddy is exploring negative indices. Fill in the blanks to complete his investigation.

Every time I decrease the power of 5 by 1
I am dividing the previous answer by 5



$$5^{-1} = 1 \div 5 = \frac{1}{5^1} = \frac{1}{5}$$

$$5^{-2} = 1 \div 5^2 = \frac{1}{\boxed{}} = \frac{1}{\boxed{}}$$

$$5^{-3} = 1 \div \boxed{} = \frac{1}{\boxed{}} = \frac{1}{\boxed{}}$$

Mo, Alex, Rosie and Tommy are working out 2^{-3}
They all have a different answer.

Mo

−6

Alex

$\frac{1}{-8}$

Rosie

$\frac{1}{8}$

Tommy

$\frac{1}{-6}$

- Who is correct?
- Explain the mistakes that have been made by the others.

Sort the cards in ascending order. Check using a calculator.

1^{-1}

2^{-2}

2^3

3^2

3^{-2}

2^{-3}

1^{-10}

Understand fractional indices

H

Notes and guidance

Here students will begin working with fractional indices, finding the square roots and the cube roots of numbers. Powers such as two-thirds are not covered. Misconceptions such as dividing the number by 2 or 3 should be discussed. This step also presents a good opportunity to revisit previous steps of learning by ordering numbers which have fractional and negative powers, as well as revisiting the power of 0.

Key vocabulary

Fraction

Reciprocal

Root

Exponent

Key questions

How does the addition law for indices help us work out the meaning of “to the power half”?

Give an example to show “to the power half” is not the same as “divide by 2”?

Exemplar Questions

Whitney is working out $16^{\frac{1}{2}} \times 16^{\frac{1}{2}}$

She writes $16^{\frac{1}{2}} \times 16^{\frac{1}{2}} = 16^{\frac{1}{2} + \frac{1}{2}} = 16^1$



If $16^{\frac{1}{2}}$ multiplied by itself is 16^1 then that must mean that a $16^{\frac{1}{2}}$ is equivalent to $\sqrt{16}$

Use Whitney's reasoning to find the values of $25^{\frac{1}{2}}$ and $49^{\frac{1}{2}}$

Annie and Mo are working out $27^{\frac{1}{3}}$



The answer is 9

The answer is 3



Who is correct? How do you know?

Put the following cards in descending order of size.

 $64^{\frac{1}{3}}$
 10^3
 $64^{\frac{1}{2}}$
 $100^{\frac{1}{2}}$
 $1000^{\frac{1}{3}}$
 64^1
 10^{-3}
 10^0


Dexter says he can represent 8 in 3 different ways.

 $64^{\frac{1}{2}}$
 2^3
 $512^{\frac{1}{3}}$

He does the same again with a different number.

One of his cards is

 $216^{\frac{1}{3}}$

What could the other cards be?

Number Sense

Small Steps

▶ Round numbers to powers of 10, and 1 significant figure

R

▶ Round numbers to a given number of decimal places

▶ Estimate the answer to a calculation

▶ **Understand and use error interval notation**

H

▶ Calculate using the order of operations

R

▶ Calculate with money

▶ Covert metric measures of length

▶ Convert metric units of weight and capacity

H denotes higher strand and not necessarily content for Higher Tier GCSE

R denotes “review step” – content should have been covered at KS3

Number Sense

Small Steps

- ▶ Convert metric units of area H
- ▶ Convert metric units of volume H
- ▶ Solve problems involving time and the calendar

H denotes higher strand and not necessarily content for Higher Tier GCSE
R denotes “review step” – content should have been covered at KS3

Round to powers of 10 and 1 sf R

Notes and guidance

This small step revises and extends KS2 and Year 7 content. It is important when rounding to avoid the phrase “round down” as this can lead to misconceptions. The use of number lines is very helpful to decide which number to round to, including when rounding to 1sf. Discussing when to round to a particular degree of accuracy is also useful.

Key vocabulary

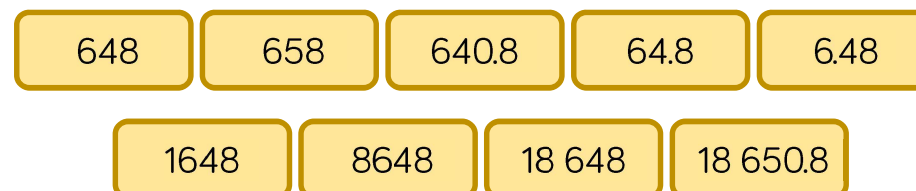
Round	Significant	Power
Nearest	Integer	Number line

Key questions

How can you tell how many significant figures a number has? How do you identify the most significant?
 What's the same and what's different about rounding to the nearest (e.g.) hundred or thousand?
 Can 0 be an answer when rounding a number?

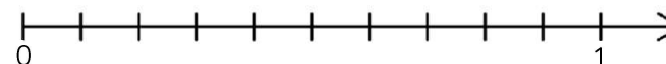
Exemplar Questions

Round the numbers on the cards to the nearest hundred.



Would it better to round some of the numbers to a different degree of accuracy? Explain why or why not.

Show the approximate position of 0.392 on the number line.



Round 0.392 to the nearest integer.

Round 0.392 to one significant figure.

A newspaper reports a lottery win of £20 million.

The size of the win has been rounded to one significant figure.

What is the least possible size of the win?

What is the greatest possible size of the win?

Complete the statements using $<$, $>$ or $=$

12 000 rounded to 1sf ☐ 9650 rounded to the nearest thousand

885 rounded to 1sf ☐ 799.9 rounded to the nearest hundred

24.6 rounded to 1sf ☐ 25.2 rounded to the nearest integer

Round to decimal places

Notes and guidance

Although rounding to decimal places is covered in KS2, it is not explicitly covered in our Year 7 scheme. Students may need reminding of the similarities and differences between rounding to decimal places and rounding to significant figures. It is useful to start by rounding to the nearest integer and then introducing one, two (and more only if appropriate) decimal places.

Key vocabulary

Round	Number line	Decimal point
Decimal place	Integer	Nearest

Key questions

How many figures does (e.g.) 36.514 have after the decimal point? To how many decimal places is it given? What's the same and what's different about rounding (e.g.) 31.57 to 1 significant figure and rounding it to 1 decimal place?

Exemplar Questions

Amir and Rosie are rounding π to 1 decimal place.

$$\pi = 3.14159\dots$$



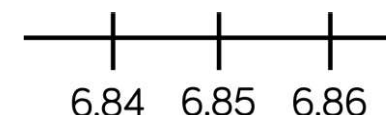
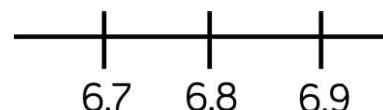
14 159 is much bigger than 5, so the answer is 3.2

π is less than half way between 3.1 and 3.2, so the answer is 3.1



Who do you agree with? Why?

Mark the position of 6.847 on these two number lines.



What is 6.847 rounded to 1 decimal place?

What is 6.847 rounded to 2 decimal places?

e is an important number in advanced mathematics.

$$e = 2.71821818\dots$$

What is e rounded to the nearest integer?

What is e rounded to 1 decimal place? 2 decimal places? 3 decimal places? 4 decimal places? 5 decimal places?

Explain why rounding to one decimal place is the same as rounding to the nearest tenth.

Estimate the answer to a calculation

Notes and guidance

Students will learn to find the estimate to a calculation by rounding the numbers to 1 significant figure and performing the calculation on the simpler numbers obtained. It is worth exploring other strategies and considering whether you can tell if the estimate will be larger or smaller than the actual answer. Using all four operations and powers/roots is also recommended.

Key vocabulary

Significant figure	Estimate	Round
Over/underestimate	Root	Power

Key questions

Why is it useful to make an estimate before doing a calculation?

If both numbers you use when estimating the answer to a calculation are larger than the original numbers, will your estimate be an overestimate or underestimate?

Exemplar Questions

By rounding each number to the nearest integer, find estimates to the calculations.

$$8.7 + 2.9$$

$$8.7 - 2.9$$

$$8.7 \times 2.9$$

$$8.7 \div 2.9$$

Round 21.88 to 1 significant figure.

Use your answer to estimate the answer to these calculations.

$$21.88^2$$

$$21.88^3$$

$$600 \div 21.88$$



$$\frac{21.88}{40} \approx 2$$

$$\frac{21.88}{40} \approx \frac{1}{2}$$



Who do you agree with? Why?



I know $\sqrt{84}$ is greater than 9 and less than 10

Use your knowledge of square numbers to explain why Mo is correct. Do you think $\sqrt{84}$ will be closer to 9 or 10? Why?

Estimate the answers to these calculations by rounding the numbers to 1 significant figure.

$$913 \times 6.42$$

$$82.6 \times 35.1$$

$$6.2 \div 1.9$$

In each case, explain if it is possible to tell if the estimate is an overestimate or an underestimate.

Error interval notation

H

Notes and guidance

This Higher strand step builds on the inequality notation covered earlier in the year to formally represent the upper and lower bounds of a single number that has been rounded to a given degree of accuracy. It is helpful to revise the notation first and to consider the similarities and differences in error intervals for discrete and continuous measures.

Key vocabulary

Round	Significant	Discrete
Continuous	Bound	Integer

Key questions

What is the smallest number that rounds to (e.g.) 16 to the nearest integer? Why isn't 16.4 the largest number that rounds to 16 to the nearest integer?

What's the difference between $<$ and \leq ? How does this affect how we write error intervals?

Exemplar Questions

a is an integer.

For each of the cards, write down the possible values of a .

$$9 \leq a \leq 12$$

$$9 < a \leq 12$$

$$9 < a < 12$$

$$9 \leq a < 12$$

$x = 4$ to the nearest integer.



x could be any number from 3.5 to 4.4

Annie is incorrect. Explain why.

Which of these inequalities represents the possible values of x ?

$$3.5 < x \leq 4.5$$

$$3.5 \leq x \leq 4.5$$

$$3.5 \leq x < 4.5$$

When rounded to two decimal places a number is 7.24

Which of these could have been the original number?

7.234

7.244

7.2445

7.2454

7.2354

Write down the error interval for the number.

A number n , rounded to two significant figures is 3800

Write down the error interval for n .

How would the error interval change if n had been rounded to:

three significant figures?

four significant figures?

Use the order of operations

R

Notes and guidance

Here we build on KS2 and Year 7 content to look at the order of operations in increasingly complex situations. It is useful to include formats involving fraction lines to represent division. Examples including roots as well as powers may be included. Comparing answers with those obtained from calculators is useful for both developing use of calculator skills as well as checking.

Key vocabulary

Operation	Order	Priority
Power	Root	Index/Indices

Key questions

Why do (e.g.) $11 + 7 - 4$ and $11 - 4 + 7$ have the same answer?

Which pairs of operations have equal priority in calculations?

Will (e.g.) $\sqrt{9} + \sqrt{16}$ and $\sqrt{9 + 16}$ have the same answer?

Why or why not?

Exemplar Questions



$$\begin{aligned} 10 - 3 + 5 \\ = 10 - 8 \\ = 2 \end{aligned}$$

$$\begin{aligned} 10 - 3 + 5 \\ = 7 + 5 \\ = 12 \end{aligned}$$



Do you agree with Whitney or Dexter? Why?

Put these cards in order of their value, starting with the smallest

$$3 + 4^2$$

$$(3 + 4)^2$$

$$3^2 + 4$$

$$3^2 + 4^2$$

Without working out the calculations, which cards are equal in value?

$$64 + \frac{38}{2}$$

$$\frac{1}{2} \times 64 + 38$$

$$(64 + 38) \div 2$$

$$\frac{64 + 38}{2}$$

$$38 \div 2 + 64$$

$$38 + 64 \div 2$$

$$64 + 38 \div 2$$

Which of these cards will have a negative value? How do you know?

$$3 - 4 + 5$$

$$3 - 4 \times 5$$

$$3 \times 4 - 5$$

$$-3 \times (4 - 5)$$

Add brackets to the calculations to make them correct.

$$6 + 4 \times 7 - 3 = 22 \quad 6 + 4 \times 7 - 3 = 40$$

$$6 + 4 \times 7 - 3 = 67$$

What is the value of $6 + 4 \times 7 - 3$ if there are no brackets?

Calculate with money

Notes and guidance

This small step provides a good opportunity to revisit other topics such as percentages, fractions and ratio in the context of money, and to maintain fluency with non-calculator methods as dependent on the needs of the class. Interpreting calculator displays can also be checked. It is a good opportunity to remind students of the vocabulary of financial mathematics.

Key vocabulary

Change	Deposit	Interest
Debit	Credit	Balance

Key questions

How do you use a calculator to find a percentage of an amount?

What's the difference between credit and debit?

How many decimal places should I round to when doing a calculation with money in pounds? What if the calculator shows an answer like 6.7 pounds?

Exemplar Questions

Ron buys a magazine for £2.95 and a drink for 75p
He finds the total cost on his calculator and the display shows:

77.95

Use estimation to prove that Ron's answer must be wrong.

What mistake did Ron make?

Find the change he should receive if he pays with a £10 note.



Were £1.75
Now 87 p
Half Price!

Are the chocolates actually half price?
Explain why or why not.

Estimate the cost of buying a box of chocolates for each student in a class of 32

💡 Find the difference between your estimate with the actual cost.

Express the difference as a percentage of the actual cost.

Huan and Filip share some money in the ratio 5 : 2

How much money is there in total if:

- Huan gets £30?
- Filip gets £30?
- The difference between their shares is £30?

Eva has a bank balance of £84.35

She wants to pay a 20% deposit on a new tablet that costs £435

Can she afford the deposit and still be in credit in the bank?

Convert metric measures of length

Notes and guidance

This small step reviews and extends Year 7 content to look at more complex conversions. It provides a good context in which to revisit area formulae to help cement these in students' minds, and to look again at multiplication and division by powers of ten. It is useful to make connections with the prefixes kilo, milli etc. also used in the next step.

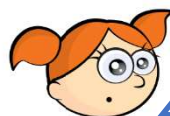
Key vocabulary

Metric	Metre	Prefix
Kilo	Milli	Centi

Key questions

What is the difference between the prefixes kilo and milli?
 Why do we need two prefixes that both mean 1 000?
 How do you know whether to multiply or divide when converting metric units?
 Why is (e.g.) 6.4 cm not equal to 6.40 mm?

Exemplar Questions



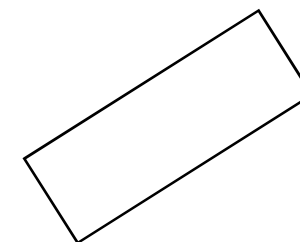
Milli is short for million, so 1 m = 1 000 000 mm

Use the facts that
 $1 \text{ m} = 100 \text{ cm}$ and
 $1 \text{ cm} = 10 \text{ mm}$
 to prove that Alex is wrong.

A rectangle has length 3 m and width 60 cm.

Work out:

- The perimeter of the rectangle in cm
- The perimeter of the rectangle in m
- The area of the rectangle in m^2
- The area of the rectangle in m^2



A circle has radius 85 cm.

Find the circumference of the circle, in metres, giving your answer to three significant figures.

By converting the measurements on the cards to metres, put these lengths in order of size starting with the smallest.

3.6 km

36 cm

360 cm

36 km

36 000 km

A parallelogram of base 2 m has the same area as a triangle of base 60 cm and perpendicular height 400 mm.

Work out the perpendicular height of the parallelogram.

Convert units of weight and capacity

Notes and guidance

This step emphasises the connections between conversions of all the metric units to establish the consistency of meaning of milli- kilo- etc. As well as performing the calculations, there is opportunity to discuss which unit is suitable to measure which item as many students may not be aware of this and see the activity as purely abstract.

Key vocabulary

Round

Significant

Key questions

What is the difference between multiplying an integer and a number with decimal places by 10/100/1 000?

What's the difference between a kilogram and a kilometre
How do you know whether to multiply or divide when converting metric units?

Exemplar Questions

What do all these have in common?

___ ml in a litre

___ mm in a metre

___ mg in a gram

g

kg

mg

tonnes

What would be a sensible unit to measure the mass of each item?

 an ant

 a hamster

 a cow

 a lorry

List some other items that are suitable to measure with each unit.

Complete the statements using $<$, $>$ or $=$

0.4 litres ___ 400 ml

40 g ___ 0.4 kg

0.04 tonnes ___ 4 kg

40 cl ___ 400 ml

40 kg ___ 40 000 g

4 cg ___ 0.4 g



Are these statements true or false? Correct any false statements.

 $x \text{ kg} \equiv 1000x \text{ g}$
 $y \text{ g} \equiv 10y \text{ cg}$
 $z \text{ cl} \equiv 10z \text{ ml}$
 $a \text{ cl} \equiv \frac{a}{10} \text{ litres}$
 $b \text{ g} \equiv \frac{b}{1000} \text{ kg}$
 $c \text{ ml} \equiv \frac{c}{10} \text{ cl}$

Convert metric units of area

H

Notes and guidance

It is worth explicitly challenging the misconception that as $1 \text{ cm} = 10 \text{ mm}$ then $1 \text{ cm}^2 = 10 \text{ mm}^2$. You can do this by exploring the area of a square of length 1 cm and comparing it to the area of a square of length 10 mm. Links could be made to previous steps including rounding when converting. This can be revisited later in the year when the area of a circle is first met.

Key vocabulary

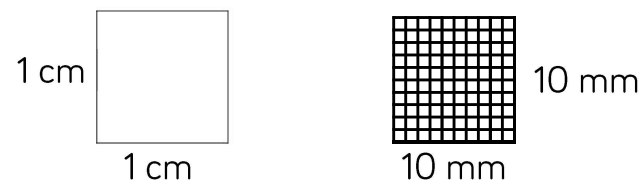
Area	Perpendicular	Units
Square units	Dimensions	

Key questions

Why is it that (e.g.) $1 \text{ cm}^2 \neq 10 \text{ mm}^2$?
 How do we find the area of a...? What happens to all the dimensions if we change them from (e.g.) m to cm?
 Why can't we multiply 30 cm by 5 m without converting first?

Exemplar Questions

Explain how the diagrams show that $1 \text{ cm}^2 \neq 10 \text{ mm}^2$



Put these areas in order of size starting with the smallest.

3.6 cm²

306 mm²

63 mm²

330 mm²

36 mm²

630 mm²

6.3 mm²

36 cm²

A rectangular playground measures 8 m by 17 m.

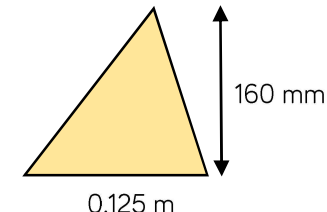
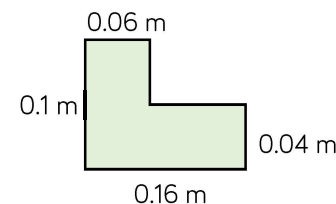
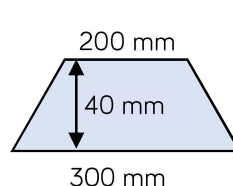
Find the area the playground giving your answer in m².

Now change the lengths to cm and work out the area the playground again, giving your answer in cm².

Use your answers to complete the statement.

1 m² = ____ cm²

Show that the area of each shape is 100 cm².



Convert metric units of volume H

Notes and guidance

Volume has not yet been explicitly covered in KS3, but students following the Higher strand should be familiar with units of volume from KS2 and should also be confident in finding the volume of a cuboid. The large numbers created by volume conversion are a useful context on which to revisit numbers expressed in standard form.

Key vocabulary

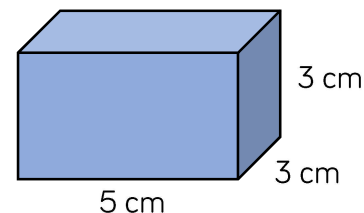
Area	Perpendicular	Units
Cubic units	Dimensions	

Key questions

How do you calculate the volume of a cuboid/cube?
 What happens to all the dimensions if we change them from (e.g.) m to cm?
 Is there a connection between volume and cube numbers?

Exemplar Questions

Work out the volume of the cuboid, giving your answer in cm^3 .



Now change the lengths to mm and work out the volume of the cuboid, giving your answer in mm^3 .

Use your answers to complete the statement.

$$1 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ mm}^3$$

Which of these measures are equivalent to 1 m^3 ?

Justify your answer.

- ☒ 100 cm^3
☒ $10\,000 \text{ cm}^3$
☒ $1\,000\,000 \text{ cm}^3$
☒ 1000 cm^3
☒ $100\,000 \text{ cm}^3$

Complete the statements using $<$, $>$ or $=$

$$8 \text{ m}^3 \underline{\hspace{1cm}} 800\,000 \text{ cm}^3$$

$$8 \text{ cm}^3 \underline{\hspace{1cm}} 8000 \text{ mm}^3$$

$$8\,000 \text{ cm}^3 \underline{\hspace{1cm}} 0.08 \text{ m}^3$$

$$800 \text{ mm}^3 \underline{\hspace{1cm}} 0.08 \text{ cm}^3$$

$$8 \times 10^7 \text{ cm}^3 \underline{\hspace{1cm}} 80 \text{ m}^3$$

$$8 \times 10^{-2} \text{ cm}^3 \underline{\hspace{1cm}} 80 \text{ mm}^3$$

Time and the calendar

Notes and guidance

This topic is often regarded as 'common knowledge' but without explicit teaching/reminding, students are often prone to errors. Revisiting conversions in starters, or in other contexts, may be necessary to help students remember them. The use of an 'empty number line' to model calculating time differences is very helpful, emphasising that time is not a decimal quantity.

Key vocabulary

12-hour clock	24-hour clock	Week
Month	Year	Leap year

Key questions

To find the amount of time between (e.g.) 9:40 and 11:25, why can't you just do $11.25 - 9.40$ on a calculator?

Which months have 30 days? How can you remember these?

How can you tell if a time is given in 12 or 24 hour clock?

Exemplar Questions

Three consecutive months in 2020 have a total of 91 days.

What might the months be?

How many possibilities can you find?

What are least and greatest possible totals of four consecutive months in 2020? Why might 2021 be different?

Feb 1st 2022 is a Tuesday.

What dates in January 2022 will be Tuesdays?

How many hours are there in one week?

It is believed that to become an expert at a skill takes around 10 000 hours of practice.

To the nearest week, how many weeks is 10 000 hours?

If you practised a skill for 3 hours a day every single day, how many years and months would it take to become an expert?

Investigate Dora's claim, assuming she counts one number per second.

How long would it take to count to one billion at the same rate?

It would take me about 10 weeks to count to one million.



21 : 50

A film starts 9:50 pm and lasts $2\frac{3}{4}$ hours.

? : ?

What time will the film end?

How could a number line solve the problem?