Summer Term

# Year 8

#MathsEveryoneCan

White Rose Maths

| | Week 1 | Week 2 | Week 3 | Week 4 | Week 5 | Week 6 | Week 7 | Week 8 | Week 9 | Week 10 | Week 11 | Week 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Autumn | **Proportional Reasoning** | | | | | | **Representations** | | | | | |
| | Ratio and scale | | Multiplicative change | | Multiplying and dividing fractions | | Working in the Cartesian plane | | | Representing data | | Tables & Probability |
| Spring | **Algebraic techniques** | | | | | | **Developing Number** | | | | | |
| | Brackets, equations and inequalities | | | | Sequences | Indices | Fractions and percentages | | | Standard index form | | Number sense |
| Summer | **Developing Geometry** | | | | | | **Reasoning with Data** | | | | | |
| | Angles in parallel lines and polygons | | | Area of trapezia and circles | | Line symmetry and reflection | The data handling cycle | | | | Measures of location | |

# Summer 1: Developing Geometry

## Weeks 1 and 2: Angles in parallel lines and polygons

This block builds on KS2 and Year 7 understanding of angle notation and relationships, extending all students to explore angles in parallel lines and thus solve increasingly complex missing angle problems. Links are then made to the closely connected properties of polygons and quadrilaterals. The use of dynamic geometry software to illustrate results is highly recommended, and students following the Higher strand will also develop their understanding of the idea of proof. They will also look start to explore constructions with rulers and pairs of compasses. This key block may take slightly longer than two weeks and the following blocks may need to be adjusted accordingly.

National Curriculum content covered includes:

- apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles
- understand and use the relationship between parallel lines and alternate and corresponding angles
- derive and use the sum of angles in a triangle and use it to deduce the angle sum in any polygon, and to derive properties of regular polygons
- use the standard conventions for labelling the sides and angles of triangle ABC
- derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language and technologies
- derive and use the standard ruler and compass constructions (H only)

## Weeks 3 and 4: Area of trapezia and circles

Students following the Higher strand will have met the formulae for the area of a trapezium in Year 7; this knowledge is now extended to all students, along with the formula for the area of a circle.

A key aspect of the unit is choosing and using the correct formula for the correct shape, reinforcing recognising the shapes, their properties and names and looking explicitly at compound shapes.

National Curriculum content covered includes:

- derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, trapezia
- calculate and solve problems involving: perimeters of 2-D shapes (including circles), areas of circles and composite shapes

## Weeks 5 and 6: Line symmetry and reflection

The teaching of reflection is split from that of rotation and translation to try and ensure students attain a deeper understanding and avoid mixing up the different concepts. Although there is comparatively little content in this block, it is worth investing time to build confidence with shapes and lines in different orientations. Students can revisit and enhance their knowledge of special triangles and quadrilaterals and focus on key vocabulary such as object, image, congruent etc.

Rotation and translations will be explored in Year 9

National Curriculum content covered includes:

- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons that are reflectively and rotationally symmetric
- identify properties of, and describe the results of reflections applied to given figures

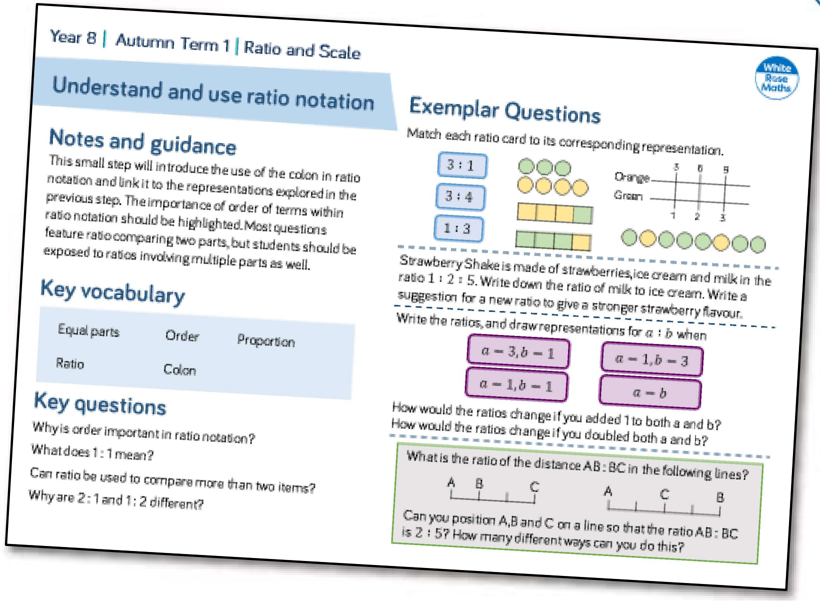
# Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

# What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



Year 8 | Autumn Term 1 | Ratio and Scale

## Understand and use ratio notation

### Notes and guidance

This small step will introduce the use of the colon in ratio notation and link it to the representations explored in the previous step. The importance of order of terms within ratio notation should be highlighted. Most questions feature ratio comparing two parts, but students should be exposed to ratios involving multiple parts as well.

### Key vocabulary

| | | |
|---|---|---|
| Equal parts | Order | Proportion |
| Ratio | Colon | |

### Key questions

Why is order important in ratio notation?

What does 1 : 1 mean?

Can ratio be used to compare more than two items?

Why are 2 : 1 and 1 : 2 different?

### Exemplar Questions

Match each ratio card to its corresponding representation.

3 : 1

3 : 4

1 : 3



Strawberry Shake is made of strawberries, ice cream and milk in the ratio 1 : 2 : 5. Write down the ratio of milk to ice cream. Write a suggestion for a new ratio to give a stronger strawberry flavour.

Write the ratios, and draw representations for $a : b$ when

$a = 3, b = 1$ $a = 1, b = 3$

$a = 1, b = 1$ $a = b$

How would the ratios change if you added 1 to both a and b?

How would the ratios change if you doubled both a and b?

What is the ratio of the distance AB : BC in the following lines?



Can you position A, B and C on a line so that the ratio AB : BC is 2 : 5? How many different ways can you do this?

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

# Angles in parallel lines & polygons

## Small Steps

- Understand and use basic angles rules and notation (R)
- Investigate angles between parallel lines and the transversal
- Identify and calculate with alternate and corresponding angles
- Identify and calculate with co-interior, alternate and corresponding angles
- Solve complex problems with parallel line angles
- Constructions triangles and special quadrilaterals (R)
- Investigate the properties of special quadrilaterals
- Identify and calculate with sides and angles in special quadrilaterals

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

# Angles in parallel lines & polygons

## Small Steps

- **Understand and use the properties of diagonals of quadrilaterals** (H)
- Understand and use the sum of exterior angles of any polygon
- Calculate and use the sum of the interior angles in any polygon
- Calculate missing interior angles in regular polygons
- **Prove simple geometric facts** (H)
- **Construct an angle bisector** (H)
- **Construct a perpendicular bisector of a line segment** (H)

(H) denotes Higher Tier GCSE content

(R) denotes 'review step' – content should have been covered at KS3

# Basic angle rules and notation R

## Notes and guidance

This step revisits key angle facts learnt in Year 7, and reminds students of three-letter angle notation. Students often find this notation difficult, so plenty of practice is helpful here. When finding missing angles, students should justify their answers using fully correct mathematical reasons e.g. “the angles on a straight line add up to 180°,” rather than, “it’s a straight line”.

## Key vocabulary

| | | |
|---|---|---|
| Adjacent | Angles at a point | Vertically Opposite |
| Straight | Acute/Obtuse/Reflex/Right angle | |

## Key questions

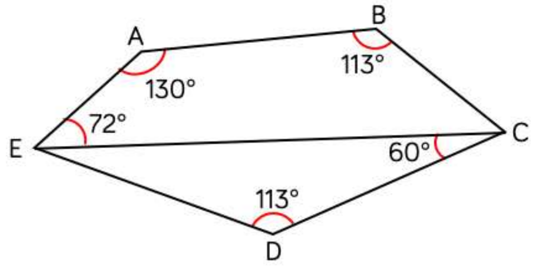
How is a right angle shown on diagrams?

How do you draw an angle of 180°?

What’s the difference between an acute angle and an obtuse angle?

What angle rules do you know? How could they be applied to this diagram?

## Exemplar Questions




- Angle ABC is the same size as angle ______.
- Angle _____ is 70° greater than angle ______.
- Angle _____ and _____ are examples of acute angles.
- Angle ECB = __________°

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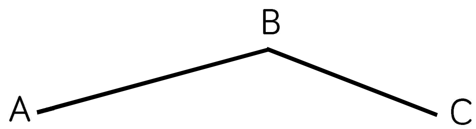
Amir says that angle ABC is a reflex angle.




Is Amir correct? How could you be sure?

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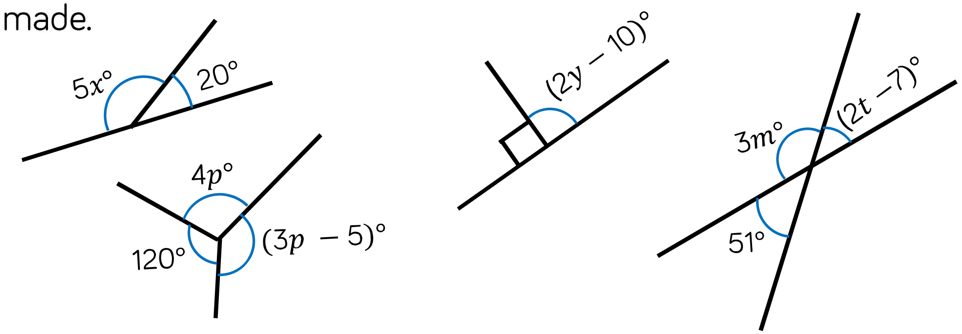
Form and solve equations to find the values of the unknown letters. Give reasons for your answers and state any assumptions you have made.



# Angles between parallel lines

## Notes and guidance

In this step, students will investigate angles between parallel lines using vocabulary such as alternate angles, corresponding angles and transversal. It is helpful to include examples and non-examples of parallel lines and find where the relationships hold. Parallel lines should be varied to include horizontal and vertical sets (including more than two lines) as well as other orientations. Dynamic geometry software is extremely useful!

## Key vocabulary

| | | |
|---|---|---|
| Parallel | Transversal | Alternate |
| Corresponding | Vertically Opposite | |

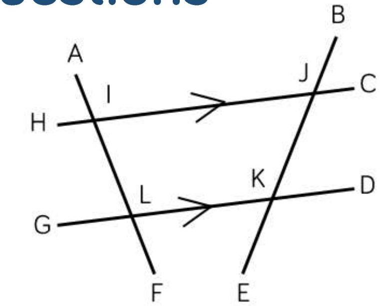
## Key questions

How do you know when two or more lines are parallel?

Name a pair of alternate/corresponding angles on the diagram. Which line(s) is/are transversal?

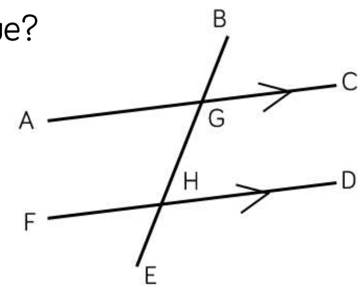
What relationships can you see between the angles? Will this work if you move the transversal line?

## Exemplar Questions

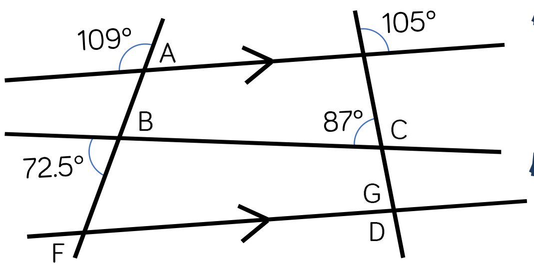



- Lines _____ and _____ are transversal lines.
- Angle GLF is vertically opposite angle _____.
- Measure angle CJK and DKE. What do you notice?
- Measure and label all angles on the diagram.
- Lines HC and GD are parallel because they do not_________.

Which of the following statements are true?




- Lines AC and FD are parallel.
- Angle BGC is equal to BGA.
- Line BE is a transversal line.
- There are four pairs of equal angles.




- Which pairs of angles are alternate and/or corresponding on the diagram?
- When are they equal or not equal?

# Alternate & corresponding angles

## Notes and guidance

Students will now look more formally at calculating alternate and corresponding angles between parallel lines. Students should also know how to recognise that a pair of lines are parallel because corresponding or alternate angles are equal. As with all angles rules, correct language needs emphasising e.g. "because alternate angles are equal," rather than, "because they are alternate".

## Key vocabulary

| | | |
|---|---|---|
| Angle | Parallel | Transversal |
| Line | Supplementary | Points |

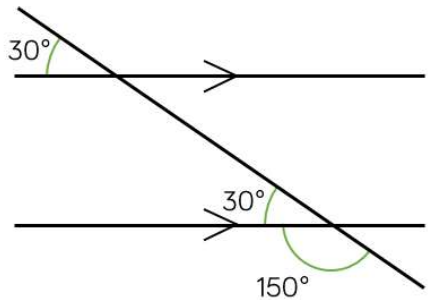
## Key questions

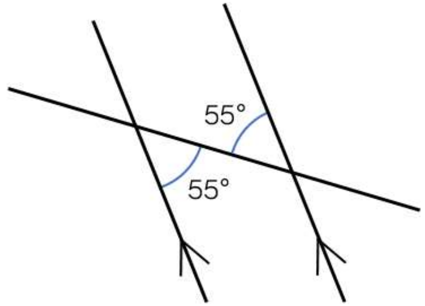
How do you identify a pair of corresponding angles or a pair of alternate angles?

Which angle(s) can you work out directly from the information given on the diagram? What other angle(s) can you then work out?

# Exemplar Questions

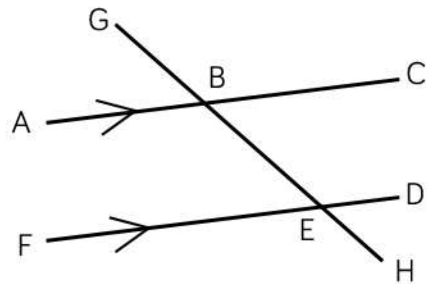
Work out all of the missing angles. Give reasons for your answers.



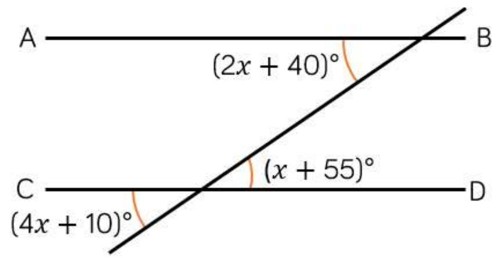

$\angle DEH = 53°$

- How can you use this fact to work out $\angle GBC$?
- What other angle(s) do you need to work out first?
- What other angles can you work out?
  Justify your answers.




Form and solve an equation to work out the value of $x$.

Are line segments AB and CD parallel?



# Calculating with co-interior angles

## Notes and guidance

Once familiar with working with both corresponding and alternate angles, students can then move on to calculate with co-interior (also known as "allied") angles. Again it is useful to explore examples and non-examples using parallel lines and non-parallel lines to establish whether a given pair will add to give 180 degrees.

## Key vocabulary

| | | |
|---|---|---|
| Parallel | Transversal | Co-interior |
| Alternate | Corresponding | |

## Key questions

Why are co-interior angles different to corresponding and alternate angles?

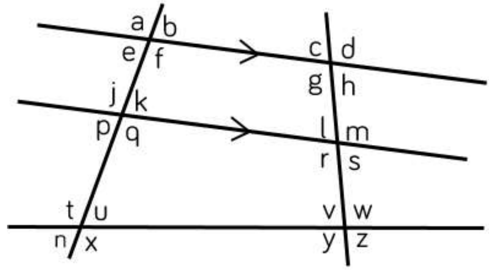
Explain, using understanding of alternate/corresponding angles, why the sum of co-interior angles equal 180°

Can you have co-interior angles in a pair of lines which are not parallel?

## Exemplar Questions

Which pairs of angles sum to 180°?

Which pairs of angles are co-interior?

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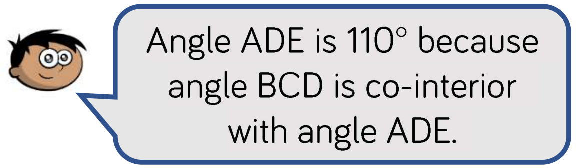
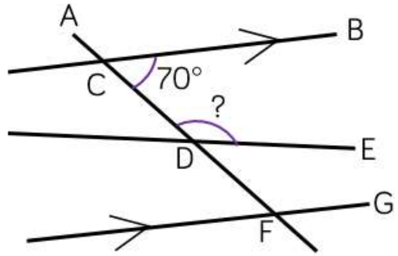
Amir is working out the size of angle ADE.




Rosie says that Amir is wrong as there is not enough information.
Who do you agree with? Explain why.

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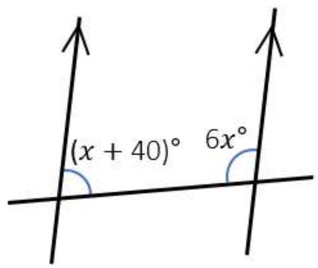
Which equations represent the information on the diagram?




$x + 40 = 180$

$x + 40 = 180 - 6x$

$x + 40 + 6x = 180$

$140 - x = 6x$

Work out the size of the labelled angles.

# Parallel lines problems

## Notes and guidance

In this step, students are exposed to all variations of the angle facts they have learned in the previous steps, including those from previous years. This is an excellent opportunity to develop mathematical talk around the problems, and scaffold their approach through careful questioning. Misconceptions could also be drawn out through 'spot the mistake' examples.

## Key vocabulary

| | | |
|---|---|---|
| Parallel | Transversal | Co-interior |
| Alternate | Corresponding | |

## Key questions

What other information do we know that we can add to the diagram?

What tells us if the lines are parallel?

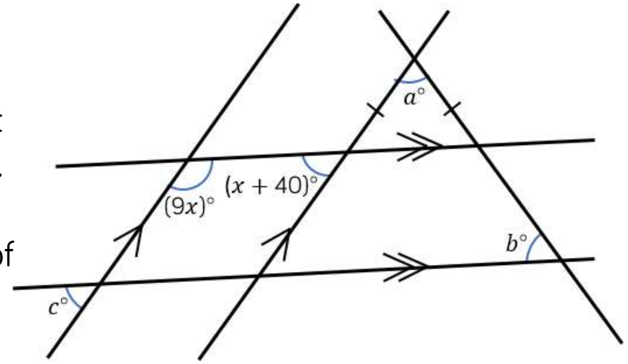
What angle facts do we need to use for this question?

## Exemplar Questions

Work out the value of $x$.

Use your answer to work out the size of angles $a$, $b$ and $c$.

Give reasons for each step of your working.

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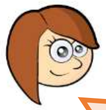
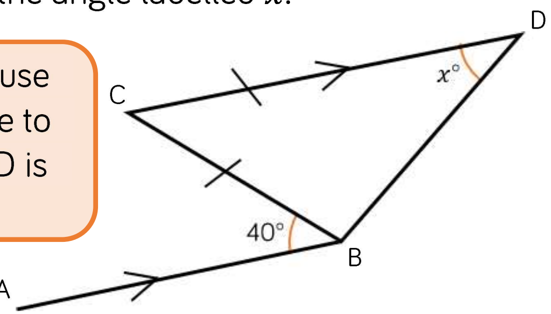
Rosie is calculating the value of the angle labelled $x$.



$x$ is equal to 40 because angle BCD is alternate to ABC and triangle BCD is isosceles.




What mistake has she made?

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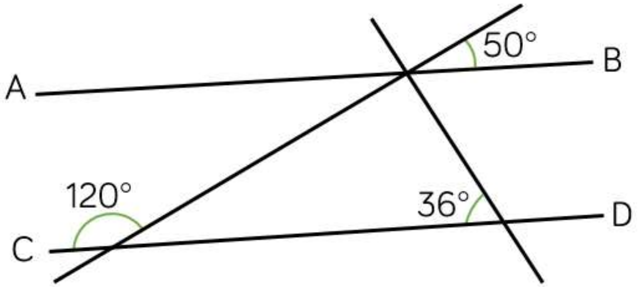
Is AB parallel to CD?
Use the given angles to justify your answer.



# Triangles and quadrilaterals R

## Notes and guidance

In this step, students will revisit constructing triangles given SSS, SAS or ASA, and special quadrilaterals. Students should consider the information given and what mathematical equipment is needed. They can also practise measuring angles by checking each other's work.

## Key vocabulary

| | | |
|---|---|---|
| Isosceles | Equilateral | Scalene |
| Right-angled | Rhombus | Parallelogram |

## Key questions

Why don't you need a protractor to draw an equilateral triangle?

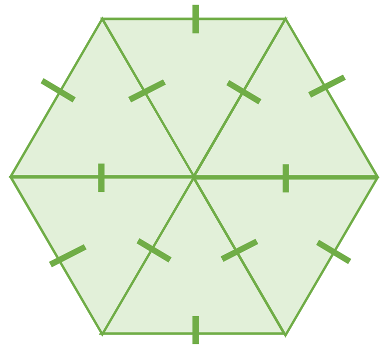
How much information do you need to draw an isosceles triangle?

How is a rhombus different from a parallelogram?

## Exemplar Questions

Construct a hexagon of side length 5 cm using only a ruler, a pair of compasses and a pencil.

Use the diagram to help you.



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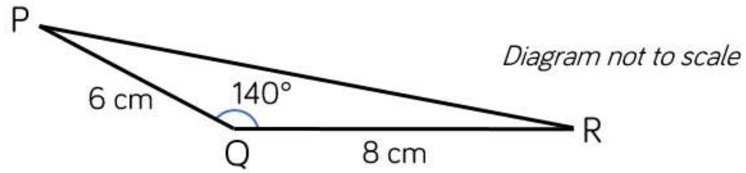
Construct triangle PQR using the information on the diagram.




How may different parallelograms can you make with P, Q and R as three of the vertices?

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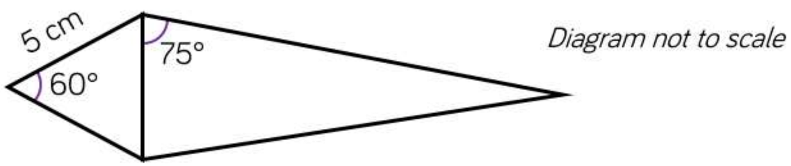
Construct the quadrilaterals:

- a square of side length 12 cm
- a rhombus ABCD with side lengths 10 cm and ∠ABC = 50°
- a kite as shown in the diagram



# Properties of quadrilaterals

## Notes and guidance

In this step, students will focus on investigating special quadrilaterals such as squares, rectangles, trapeziums, rhombi and parallelograms. Symmetry will be covered in a later unit, so students should focus on side lengths and angles only. Again, the use of dynamic geometry can help bring the properties to life.

## Key vocabulary

| | | |
|---|---|---|
| Square | Rhombus | Parallelogram |
| Trapezium | Rectangle | Kite |

## Key questions

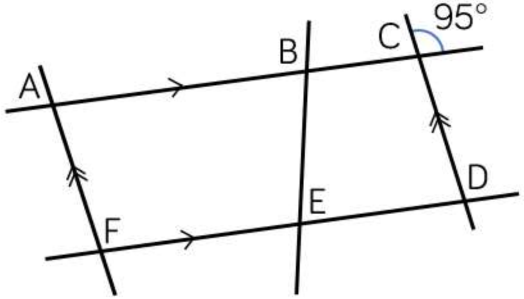
I am a four-sided shape with two pairs of parallel lines, what might I be?

Draw a standard example and a peculiar example of a quadrilateral. Compare your shapes with a partner's.

Which quadrilaterals are regular and which are not?

## Exemplar Questions

AF is parallel to CD and AC is parallel to FD.




Explain whether the following statements are true or false.

- ACDF is a rectangle
- ABEF and BCDE are trapezia
- CDFA is a parallelogram

Which quadrilaterals satisfy each of these statements?

| | |
|---|---|
| Two pairs of parallel sides | Only one pair of parallel sides |
| Four right angles | Two pairs of equal sides |
| Two pairs of equal angles | One pair of equal angles |

How many quadrilaterals satisfy 2, 3 or more of the statements.

Which of this statements are true and which are false?

- All squares are rectangles
- All rectangles are squares
- No rhombuses are kites

# Calculate sides and angles

## Notes and guidance

This step could be taught in conjunction with the previous step, with students focusing on applying their knowledge of angle facts and properties of parallel lines to investigate special quadrilaterals and deduce unknown information. Students should be encouraged to discuss and label what information they know or can work out on their diagrams.

## Key vocabulary

| | | | |
|---|---|---|---|
| Rhombus | Parallelogram | Trapezium | Kite |
| Rectangle | Isosceles | Equilateral | |

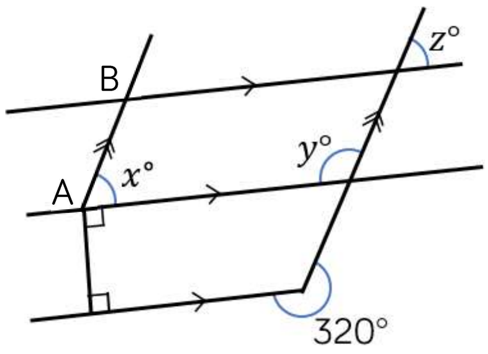
## Key questions

What properties does a rhombus have that a parallelogram does not? What similar properties do they have?

Give me an example of a quadrilateral which only has one obtuse angle/two obtuse angles.

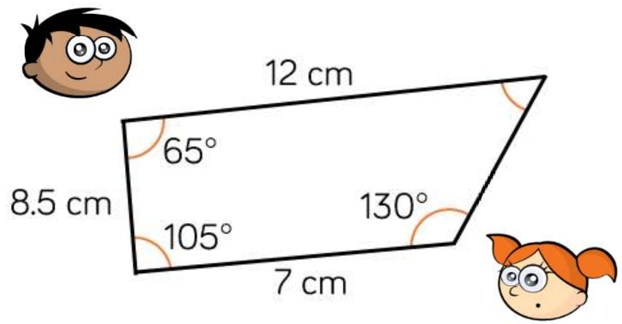
What makes a trapezium an isosceles trapezium?

## Exemplar Questions

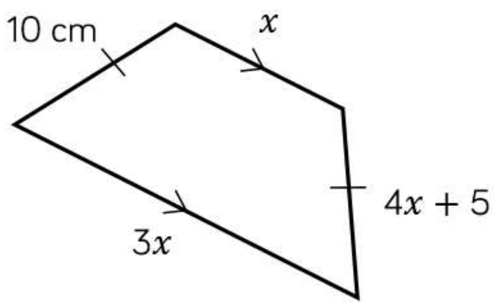



Using angle facts, find the unknown angles $x$, $y$ and $z$.
What quadrilaterals can you see in the diagram?
How did you decide?
If you knew the length of AB, what else could you work out?

Amir says the shape is a trapezium. Rosie thinks there's not enough information to be able to tell.
Who do you agree with?
Why?




Work out the perimeter of the isosceles trapezium.



# Diagonals of quadrilaterals H

## Notes and guidance

Students should be aware that a diagonal joins the opposite vertices of a quadrilateral, and that they don't necessarily "go diagonally"! Geoboards and/or squared paper are very useful for discovering properties, as is dynamic geometry. It is also useful to use straws of various sizes to reverse the process, starting with diagonals with certain properties and deducing what the quadrilateral must be.

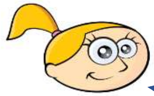
## Key vocabulary

| | | | |
|---|---|---|---|
| Rhombus | Parallelogram | Trapezium | Kite |
| Rectangle | Perpendicular | Bisect | Delta |

## Key questions

Is it possible for the diagonals of a quadrilateral to be horizontal or vertical?
What types of quadrilateral have diagonals that are equal in length? Why can't this be the case for the other special quadrilaterals?
Is it possible for a diagonal to be outside the shape?

## Exemplar Questions



I'm thinking of a quadrilateral.
The diagonals of my quadrilateral are perpendicular.

- What are the three types of quadrilateral Eva could be thinking of?
- Is it possible for the diagonals of these quadrilaterals to also be equal in length?
- Is it possible for the diagonals of these quadrilaterals to also bisect each other?

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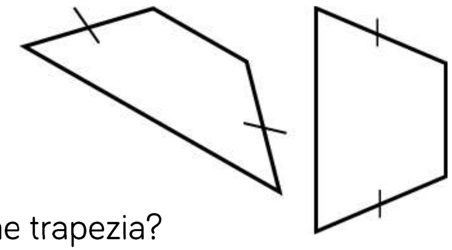
Draw some different isosceles trapezia and investigate their diagonals.



- Do they bisect each other?
- Do they bisect the angles of the trapezia?
- Is it possible for them to be equal in length?

Use dynamic geometry to check your findings.

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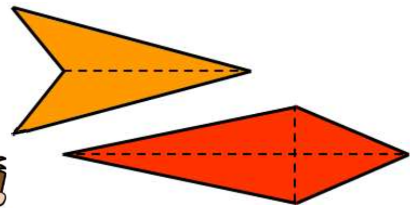
A delta only has one diagonal, but a kite has two.



Do you agree with Teddy?

# Sum of exterior angles

## Notes and guidance

In this step, students should explore the meaning of external angles and how to find them by extending the lines of a polygon. Using a pen or pencil to go around the outside of a polygon from one side to the next demonstrates that there is one full turn needed, no matter how many sides, and so the exterior angle sum is always 360°. Students need to know that the exterior angles will only be equal if the polygon is regular.

## Key vocabulary

| | | | |
|---|---|---|---|
| Exterior | Interior | Regular | Polygon |
| Sum | Total | Pentagon/Hexagon etc. | |

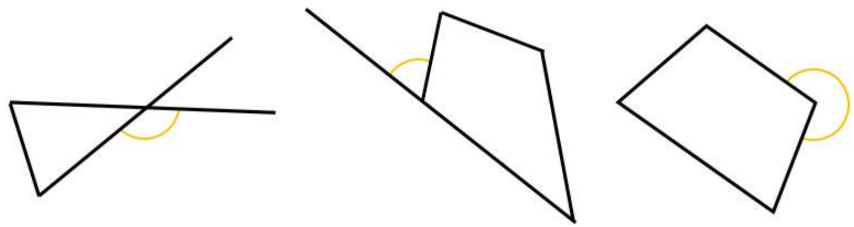
## Key questions

What are the two conditions that make a polygon regular?

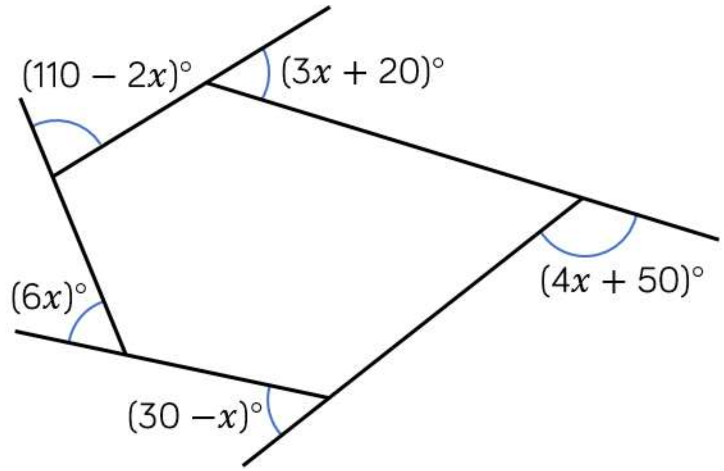
What is the sum of the external angles of a polygon? If the polygon is regular, what is the size of each external angle?

## Exemplar Questions

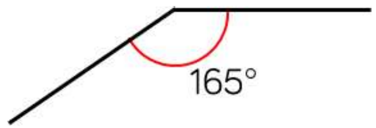
Which of the following diagrams shows an exterior angle of the shape?



Form and solve an equation to find the size of the largest external angle of the pentagon.




The diagram shows 2 sides of a regular polygon.
How many sides does the polygon have?



# Sum of interior angles in polygons

## Notes and guidance

In this step, students should explore the sum of the interior angles in different polygons – students following the Higher strand may have covered this last year. Students should explore the links between the number of sides a polygon has and the number of internal triangles a polygon has, and so deduce the interior angle sum is given by $(n - 2) \times 180°$ They explore a regular polygon's angles in the next step.

## Key vocabulary

| | | | |
|---|---|---|---|
| Exterior | Interior | Regular | Polygon |
| Sum | Total | Pentagon/Hexagon etc. | |

## Key questions

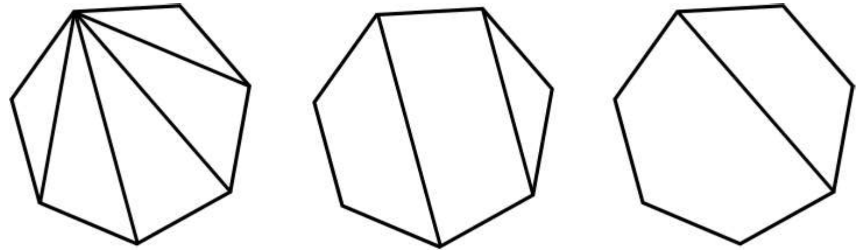
If a polygon is regular, what do we know about its angles?

Will the interior angles of a 20-sided shape be greater than or less than those of a 19-sided shape? What about the exterior angles?

Is it possible to have a reflex interior angle in a polygon? Give me an example.

## Exemplar Questions

Which of the following diagrams would be helpful in finding the sum of the interior angles of regular heptagon?



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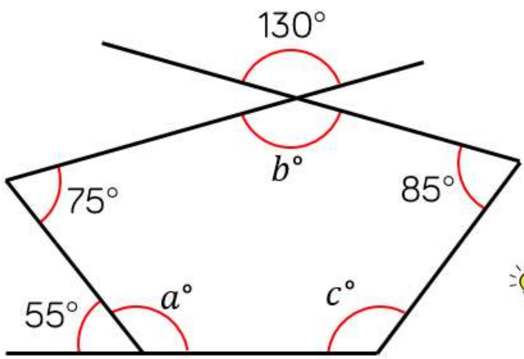
Annie is calculating the sum of interior angles in a 12-sided shape.

> The angles in a quadrilateral sum to 360°
> A 12-sided shape has 3 times as many sides, so its angle sum will be $3 \times 360° = 1080°$

Prove that Annie is wrong.

---




Calculate the unknown angles in this polygon. Give mathematical reasons for all your answers.

Does the order in which you find the angles matter?

# Missing angles in regular polygons

## Notes and guidance

Students sometimes misunderstand ‘regular’ as only meaning equal sides, or even rectilinear, so this is a good opportunity to discuss regularity whilst using the recently learnt rules of interior and exterior angle sums. It is also useful to compare different methods to find the size of one interior angle. Students could take this further, exploring which regular polygons tessellate and why.

## Key vocabulary

| | | | |
|---|---|---|---|
| Exterior | Interior | Regular | Polygon |
| Sum | Total | Pentagon/Hexagon etc. | |

## Key questions

Will the interior angles of a regular polygon be different from those of an irregular polygon?

Explain why neither a rectangle nor a rhombus are regular.

What’s the connection between the interior and the exterior angles of a polygon?

## Exemplar Questions

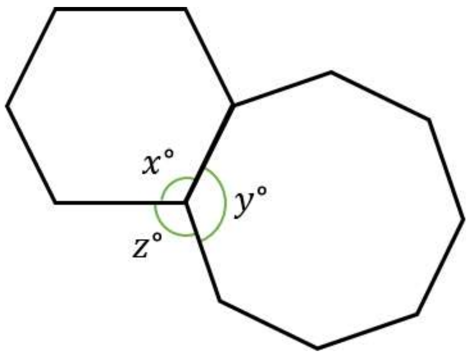
Which of the following calculations would be correct for working out the size of one interior angle of a regular decagon?

| | | |
|---|---|---|
| $10 - 2 = 8$<br>$360° \div 8 = 45°$ | $10 - 2 = 8$<br>$8 \times 180° = 1440°$<br>$1440 \div 10 = 144°$ | $10 + 2 = 12$<br>$12 \times 180° = 2160°$<br>$2160 \div 10 = 216°$ |

Compare Whitney and Tommy’s methods for working out the size of an interior angle of a regular nonagon.

The interior angle sum is
$(9 - 2) \times 180° = 1260°$
So one angle is
$1260° \div 9 = 140°$

Each exterior angle is $360° \div 9 = 40°$
So each interior angle is $180 - 40° = 140°$




The diagram shows a regular hexagon and a regular octagon that meet at a common point.

Work out the values of $x, y$ and $z$.

# Prove geometric facts

H

## Notes and guidance

The formal proof that the sum of the angles in a triangle is 180° is a good introduction to this step as it builds on previous work. It is also useful to compare this to tearing corners off a triangle, illustrating the difference between proof and demonstration. Students could then do small "show that" activities (e.g. the value of a missing angle) before moving on to short formal proofs like those illustrated in the exemplars.

## Key vocabulary

Demonstration  Justify  Proof

Alternate/Corresponding/Parallel etc.

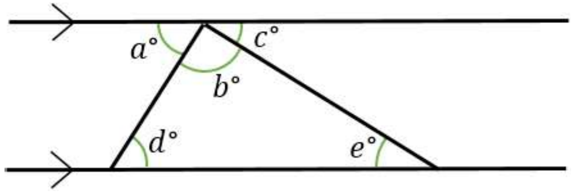
## Key questions

What's the difference between a proof and a demonstration?

How do we know the result will always be true?

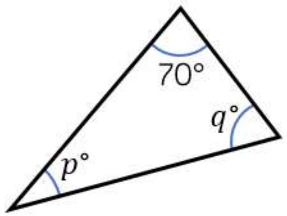
What can we find out first?

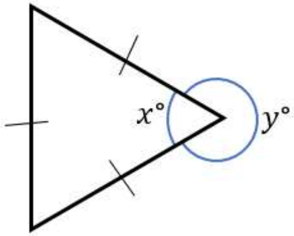
## Exemplar Questions

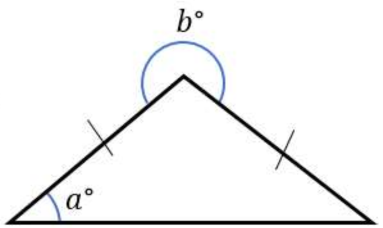



Complete the proof that angles in a triangle add to 180°

$a + b + c = 180°$ (Angles on a ________add up to 180°)
$a = d$ (Alternate angles are _____)
$c =$ __ (_____ angles are _____)
So $a + b + c = d + b +$ __
So $d + b + e = 180°$



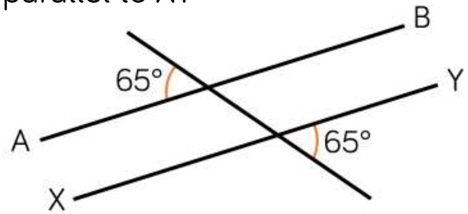




Prove $p + q = 110°$

Show that $y = 300°$

Prove $b = 180 + 2a$

Prove that AB is parallel to XY



# Angle bisectors

H

## Notes and guidance

It is useful to use a visualiser to demonstrate the method of bisecting an angle using only a pencil and a pair of compasses. Students should practise with angles of different sizes and in different orientations. This topic does not link easily to other areas of maths, so it might be worthwhile revisiting occasionally as a starter (comparing with the next step) to help students to remember the technique.

## Key vocabulary

| | | |
|---|---|---|
| Bisect | Bisector | Acute |
| Obtuse | Reflex | Compasses |

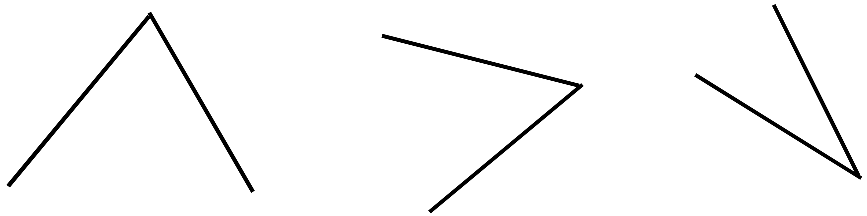
## Key questions

What does bisect mean? What does the stem "bi" tell us?

Describe the steps to construct the bisector of an angle without using a protractor.

## Exemplar Questions

Bisect the acute angles.



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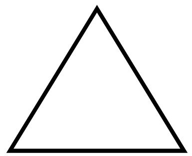
Use a protractor to draw angles of these sizes.

- 140°
- 170°
- 90°
- 10°

Now use a pair of compasses to help bisect the angles, and check how accurate you were with a protractor.

---

Thinking about how you would construct an equilateral triangle, construct an angle of 60° without using a protractor.
Now construct the angles on the cards.



30° | 120° | 150° | 210°

---

Draw a large acute-angled triangle.
Bisect each angle.
Using the point where the three angle bisectors meet as the centre, draw a circle that just touches each of the three sides of the triangle.
(This is called the 'incircle' of the triangle).

# Perpendicular line bisectors H

## Notes and guidance

As with angle bisectors, it is useful to use a visualiser to demonstrate the method of bisecting an angle using only a pencil and a pair of compasses. Students should again practise with lines of different size and in different orientations. This topic does not link easily to other areas of maths, so it might be worthwhile revisiting occasionally as a starter (comparing with the last step) to help students to remember the technique.

## Key vocabulary

| | |
|---|---|
| Line | Line segment |
| Perpendicular | Bisector |

## Key questions

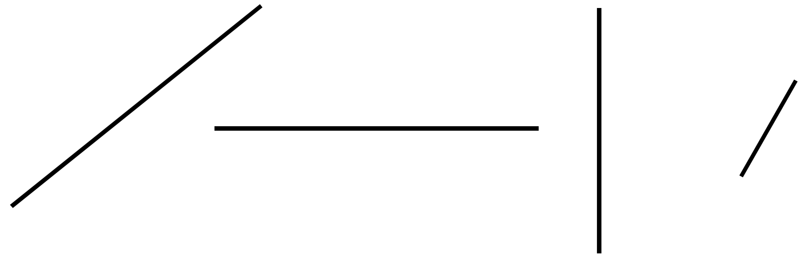
Tell me what perpendicular means?

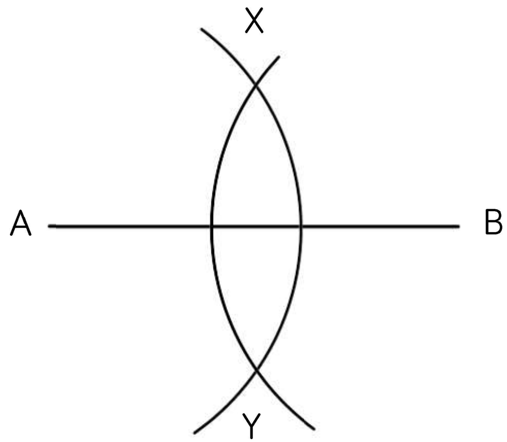
What does bisect mean? What does the stem "bi" tell us?

What's the connection between the method for constructing a perpendicular bisector and what we know about the diagonals of a rhombus?

## Exemplar Questions

Construct the perpendicular bisectors of these line segments.






Construct the perpendicular bisector, XY, of a line segment AB as shown, leaving on all the construction lines.
Join the points to form the quadrilateral AXBY.
What type of quadrilateral is AXBY?
What so you notice about its diagonals?

Draw a large acute-angled triangle.
Bisect each side.
Using the point where the three angle bisectors meet as the centre, draw a circle that just touches each of the vertices of the triangle.
(This is called the 'circumcircle' of the triangle)

# Area of Trapezia and Circles

## Small Steps

- Calculate the area of triangles, rectangles and parallelograms R
- Calculate the area of a trapezium
- Calculate the perimeter and area of compound shapes (1)
- Investigate the area of a circle
- Calculate the area of a circle and parts of a circle without a calculator
- Calculate the area of a circle and parts of a circle with a calculator
- Calculate the perimeter and area of compound shapes (2)

H denotes Higher Tier GCSE content

R denotes ‘review step’ – content should have been covered earlier at KS3

# Reviewing area

R

## Notes and guidance

This small step revises and extends KS2 and KS3 work. Teachers might first check that students understand the links between the formulae for area of rectangle, triangle and parallelogram. A possible difficulty can be finding the perpendicular height when triangles are not in 'standard' orientations. Ensure students have exposure to these, and include questions that revisit unit conversions.

## Key vocabulary

| | | |
|---|---|---|
| Formula | Area | Triangle |
| Square | Parallelogram | Rhombus |

## Key questions

Why is the formula to find the area of a rectangle the same as the formula to find the area of a parallelogram?

Why do we use the perpendicular height when finding the area of a triangle and not the sloping height?

How can you find the area of a rhombus? How do you know?

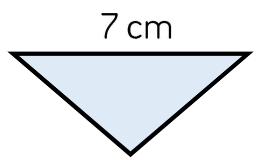
## Exemplar Questions

The large rectangle has been split into four smaller rectangles. Calculate the perimeter of the large rectangle.

| | |
|---|---|
| 15 cm² | 21 cm² |
| 30 cm² | 42 cm² |

---

The area of this triangle is 21 $cm^2$




True or False?
The perpendicular height of this triangle = 3 cm
Explain your answer.

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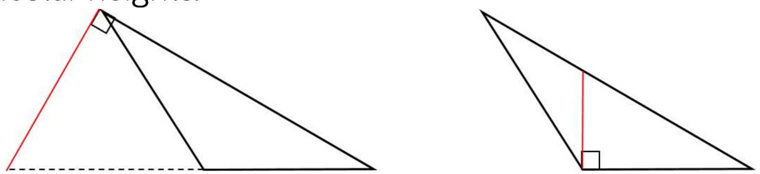
The red lines do not show the perpendicular heights of the triangles. Re-draw the diagrams, keeping the same base, to show the correct perpendicular heights.



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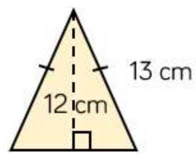
The perimeter of this isosceles triangle is 0.36 m.

Show that the area of the triangle is 60 $cm^2$.

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Calculate the area of the parallelogram.

# Calculate the area of a trapezium

## Notes and guidance

Teachers might start by ensuring students can identify trapezia using different standard (e.g. isosceles trapezium) and non-standard (e.g. with two right angles) examples. Students can then explore the formula for the area of a trapezium by using congruent trapezia to form a parallelogram. They can then compare the formula with those for the area of other quadrilaterals.

## Key vocabulary

| | |
|---|---|
| Trapezium/Trapezia | Parallel |
| Perpendicular height | Formula |

## Key questions

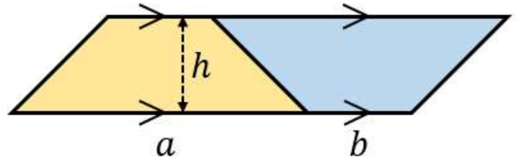
Compare a rectangle, parallelogram and trapezium. What's the same and what's different?

Why does the formula for the area of a trapezium also work if it is applied to parallelograms, rectangles and squares?

Are the parallel sides of a trapezium always horizontal?

## Exemplar Questions

Dora places two congruent trapezia next to each other:




What shape has she made?

Copy and complete the following

Length of base $= a + \_\_$

Area of parallelogram $= (a + \_\_) \times \_\_$

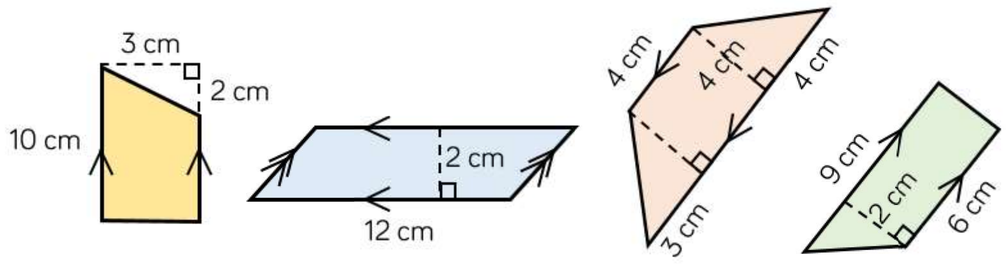
This means that:

Area of trapezium $= \frac{(a+b)\times h}{2}$

Explain why Dora is correct.

---

Give the mathematical name of each shape.




Find the areas of the shapes.

Did you use the same method for each one?

---

Draw a trapezium which has area of 24 cm$^2$ and

- a base of 10 cm and a height of 3 cm
- parallel sides of lengths 4 cm and 8 cm
- a height of 12 cm

# Compound shapes (1)

## Notes and guidance

Students sometimes simply multiply all of the dimensions marked in an attempt to find the area. Model splitting up different compound shapes before introducing students to compound shapes with dimensions labelled. Ensure students are aware that there are different methods of splitting compound shapes and that they should aim to be efficient.

## Key vocabulary

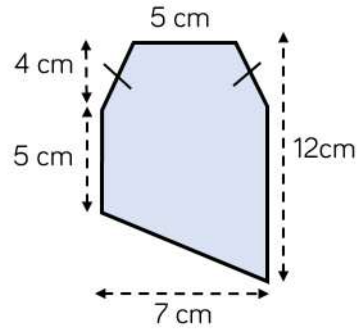
| | | |
|---|---|---|
| Compound | Component shapes | |
| Parallelogram | Trapezium | Perpendicular |

## Key questions

How can you divide this compound shape up into shapes we know how to find the area of? Name each of these shapes.

What length(s) do you need to substitute into your formula? Is this length given, or do you need to calculate it first? What is your strategy for find the missing length(s)?

## Exemplar Questions




Mo is finding the area of this shape. Draw lines on the shape to show how he could split it into easier shapes. Show that there is more than one way of doing this.

Mo decides to split the shape into two trapezia, A and B. Here are his workings:

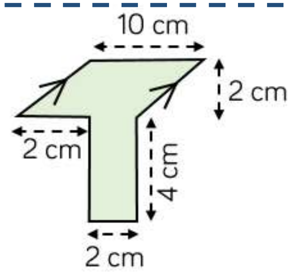
Area A $= \frac{7+5}{2} \times 4$

Area B $= \frac{12+5}{2} \times 7$

Identify and correct any mistakes that he has made and calculate the total area of the shape.

---

Show that the area of the shape is 28 cm$^2$.
What smaller shapes did you split the shape into?
Compare the methods in your class.
Which method was the most efficient?

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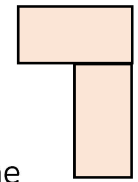
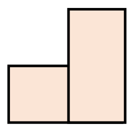
A square of perimeter of 48 cm is cut in half and the two halves are put together to make this shape.
Calculate the perimeter and area of this shape.





A new shape is made using the same two rectangles as shown.
Calculate the area and perimeter of this shape.

# Investigate the area of a circle

## Notes and guidance

Students explore the area of a circle by cutting up circles into sectors and placing them in an arrangement to resemble a parallelogram or (with more sectors) a rectangle. They might need teacher guidance to notice that as the number of sectors increases, the shape that they can make becomes more rectangular. They then use a known area formula to deduce the area of a circle.

## Key vocabulary

| | | | |
|---|---|---|---|
| Sector | Parallelogram | Rectangle | |
| Estimate | Infinity | Radius | $\pi$ |

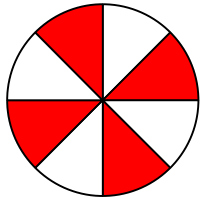
## Key questions

Where is the radius of the circle?
How do we find the circumference of a circle?
How do we find the area of a parallelogram?

As the number of sectors increases, is our estimate for the area more or less accurate? Explain why.

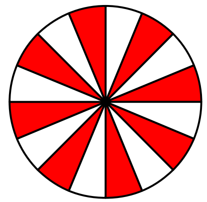
What does this tell you about the area of a circle?

## Exemplar Questions



Cut up the circle so that you have 8 separate sectors. Can you place the sectors adjacent to each other to make a shape that it roughly like a parallelogram?

Repeat this using a circle divided into 16 sectors. Compare the two shapes that you have made. What would happen if you used a circle split into 32 sectors, or 3 200 sectors?



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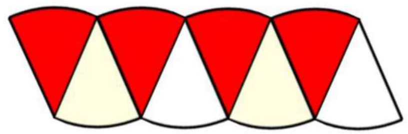
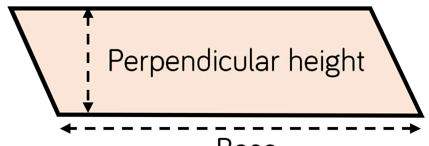





Ron compares the diagram he made using sectors of a circle to a parallelogram. He deduces:

Perpendicular height = radius = $r$

Base = half of the circumference of the circle = $\frac{\pi d}{2} = \pi r$

Explain why Ron is correct.

Ron says,

Area of parallelogram $= r \times \pi \times r$
$= \pi r^2$

What does this tell you about the area of a circle? Explain why.

# Area of a circle without a calculator

## Notes and guidance

This small step focusses on how to estimate the area of a circle using the approximations $\pi = 3$ and $\pi = \frac{22}{7}$. It then builds to calculating the exact area of a circle, leaving answers in terms of $\pi$. Starting with a recap on squaring and the order of operations can avoid later issues in calculations, so that students can concentrate on new learning.

## Key vocabulary

| | | |
|---|---|---|
| Approximately | Estimate | Diameter |
| Radius | In terms of $\pi$ | |

## Key questions

How do you round a number to 1 significant figure?

Use a calculator to change $\frac{22}{7}$ into a decimal. What do you notice when you compare this to $\pi$?

How do I know whether to substitute the radius or the diameter? What mistake do you think people often make?

## Exemplar Questions

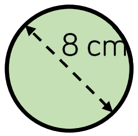
$4 \times 3^2$

$4 \times 3 = 12$, $12^2 = 144$

$4 \times 9 = 36$

$4 \times 6 = 36$

Which calculation is correct? What mistakes have been made in the other calculations?



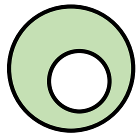

Whitney rounds $\pi$ to 1 significant figure to estimate the area of the circle.

$\pi = 3.1415...$

Area $\approx 3 \times 8^2 = 192$ cm$^2$

What mistake has Whitney made?

Whitney then cuts a circular hole out of the circle. The radius of the circular hole is 3 cm. Estimate the area of the remaining shape.



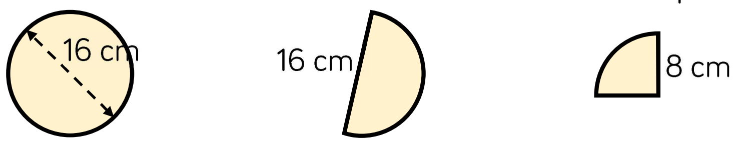
Use $\pi \approx \frac{22}{7}$ to estimate the area of a circle of radius 7 cm.

Jack

I can find the exact area of this circle without using a calculator by leaving my answer in terms of $\pi$

Area of this circle $= \pi \times 7^2$
$= \pi \times 49$
$= 49\pi$ cm$^2$

Use Jack's method to work out the exact areas of the shapes.



# Area of a circle with a calculator

## Notes and guidance

Before considering the area of a circle, students might need to practise entering squares of numbers and $\pi$ into their calculators. Students need to confidently identify the diameter and radius of a circle and know which to substitute into the formula for area. They also may need a reminder of rounding to an appropriate number of decimal places or significant figures.

## Key vocabulary

| | | |
|---|---|---|
| Decimal place | Estimate | Calculate |
| Substitute | Significant figures | |

## Key questions

Where is the $\pi$ key on your calculator? How do you enter e.g. $3^2$ into your calculator? Is there more than one way of doing this?

Why is it useful to firstly calculate an estimate of the area?

How many decimal places or significant figures should you round your answer to? Why?

## Exemplar Questions

Use your calculator to work out these values.

- $11^2$
- $\pi \times 121$
- $\pi \times 11^2$
- $\pi \times r^2$ when $r = 4$

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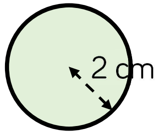



The area of the circle is about 12 cm$^2$

Explain why Dora's estimate is a good one.

Choose the calculation which will work out the exact area.

Explain your choice.

- $\pi \times 4^2$
- $\pi \times 4$
- $\pi \times 2^2$
- $(\pi \times 2)^2$

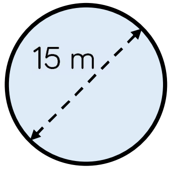
Use your calculator to work out the area of the circle.

Compare your answer to the estimate.

Is your answer likely to be correct?

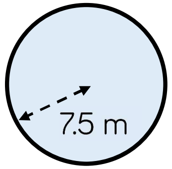
---

Calculate the four areas, rounding your answers to 1 decimal place.




A circle of diameter 12.8 cm

A circle of radius 6.4 cm




What do you notice about your answers?

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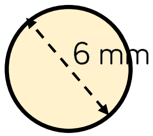



The area of half the circle is $\pi \times 3^2$

Is Tommy correct? Justify your answer.

# Compound shapes (2)

## Notes and guidance

In this small step, students are encouraged to identify standard shapes within compound shapes. When two semi-circles are involved, they might discuss whether they can use the formulae for a whole circle, rather than performing separate calculations for each. Students should identify the dimensions required in a formula, and then think about how to calculate these if they are not given.

## Key vocabulary

| | | |
|---|---|---|
| Compound | Component shapes | |
| Parallelogram | Trapezium | Perpendicular |

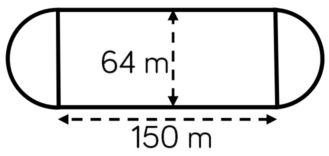
## Key questions

Do we need to work out the area/arc length of each semi-circle separately? Why or why not?

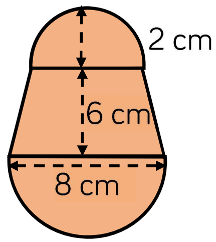
Which standard shapes can you identify in the compound shape?

Identify the dimensions you need to be able to calculate the area. How can you work out the missing ones?

## Exemplar Questions

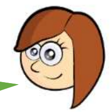



The diagram shows a running track.
To be able to use this for regional school competitions, the perimeter of the track needs to be at least 400 m.
Determine if the track can be used for regional competitions, justifying your answer.



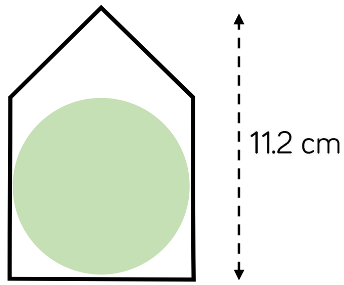

Here is the outline of a logo.
The logo consists of two semi-circles and a trapezium.

The length of the top of the trapezium must be 4 cm



The radius of the bottom semi-circle must be 4 cm

Explain why both Rosie and Jack are correct.
Calculate the total area of the logo.




The shape is made up of a square and triangle.
The circle touches the sides of the square and has radius 3.8 cm.

What percentage of total area of the shape is shaded?

# Line symmetry and reflection

## Small Steps

- Recognise line symmetry
- Reflect a shape in a horizontal or vertical line 1 (shapes touching the line)
- Reflect a shape in a horizontal or vertical line 2 (shapes not touching the line)
- Reflect a shape in a diagonal line 1 (shapes touching the line)
- Reflect a shape in a diagonal line 2 (shapes not touching the line)

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered earlier in KS3

# Recognise line symmetry

## Notes and guidance

Students will be familiar with the concept of line symmetry from Key Stage 2. As well as looking at conventional shapes and counting lines, students can explore the structure of shapes and how this affects the number of lines e.g. considering why a quadrilateral cannot have 3 lines of symmetry and/or how designs for shapes with 3 lines are based around an equilateral triangle.

## Key vocabulary

| | | |
|---|---|---|
| Line symmetry | Regular | Polygon |
| Isosceles | Equilateral | Rhombus etc. |

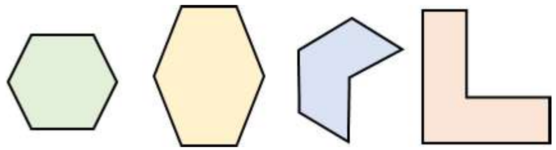
## Key questions

Do all regular polygons have lines of symmetry?

Why does a rhombus have two lines of symmetry but a parallelogram none? What do you notice about the other special quadrilaterals?

## Exemplar Questions

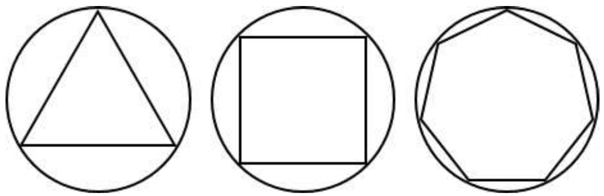
How many lines of symmetry do these hexagons have?



- What assumptions have you made?
- What types of triangle or quadrilateral can you make with 0, 1, 2, 3 or 4 lines of symmetry?

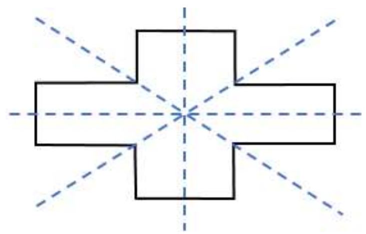
Dora says a circle has an infinite number of lines of symmetry. Do you agree? Explain why or why not.

A company is looking at designs for a new logo. These logos are circles that contain regular polygons.



How many lines of symmetry do the logos have?

Dexter has drawn four lines of symmetry on the shape. Do you agree with Dexter's choices? Why or why not?



# Reflect vertically/horizontally (1)

## Notes and guidance

In this step, students can make links with the previous step, noticing that reflecting when a shape is "on the line" automatically produces a line of symmetry. As before, they could use paper folding and mirrors to check their results. Students could also be challenged to find the areas of the shape either by counting squares or recalling and revisiting formulae as appropriate.

## Key vocabulary

| | | |
|---|---|---|
| Reflect | Line symmetry | Congruent |
| Object | Image | Vertical/Horizontal |

## Key questions

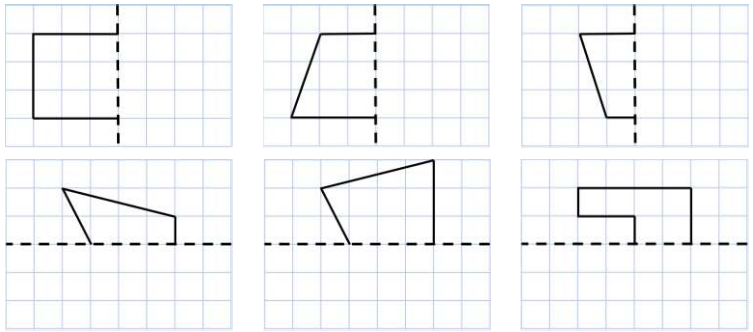
After a reflection, does the resulting shape always have a line of symmetry? Why or why not?

What's the same and what's different about the two parts of a shape following a reflection?

What's the area of the original shape? What's the area of the resulting shape?

## Exemplar Questions

Reflect the shapes in the given lines.



- Give the mathematical name of each resulting shape.
- Draw on any additional lines of symmetry the resulting shapes may have. Check by folding.

---

On a coordinate grid with axes from −5 to 5 in both directions, plot and join the points given to form a trapezium.

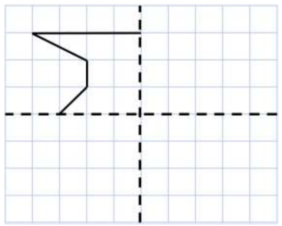
(1, 0) (3, 0) (3, 2) (2, 2)

Reflect the trapezium:

- in the $x$-axis
- in the line $x = 3$
- in the line $x = 2$
- in the line $x = 1$
- in the line $y = 2$

---

Complete the shape so both dotted lines are lines of symmetry. Create your own design with two lines of symmetry on a squared grid.



# Reflect vertically/horizontally (2)

## Notes and guidance

This step now moves students on to shapes that are not touching the line. Students can again use folding and mirrors to check their results and will need to be encouraged to take care that their images are the same distance away from the mirror line as the object. This step provides a good opportunity to revisit equations of lines parallel to the axes which were met in the Autumn term.

## Key vocabulary

| | | |
|---|---|---|
| Reflect | Line symmetry | Congruent |
| Vertical/Horizontal | Object | Image |

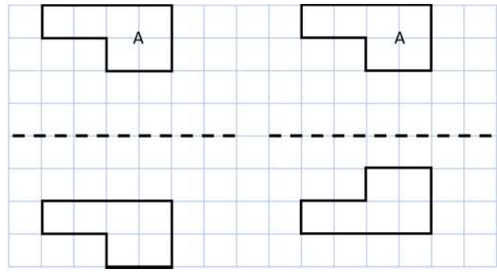
## Key questions

How far is each vertex of the object from the mirror line? What does this tell us about the position of the image?

How do we know whether the equation of a line parallel to an axis is of the form $x = \ldots$ or $y = \ldots$?

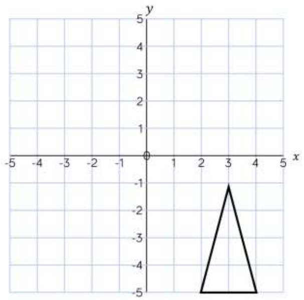
## Exemplar Questions

Explain why both attempts to reflect shape A in the given line are incorrect.
Copy shape A and the line on squared paper and draw the correct reflection.



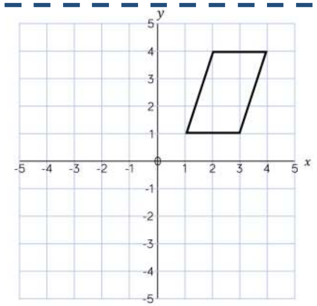

Reflect the triangle:

- in the $x$-axis
- in the $y$-axis
- in the line $x = 1$
- in the $y = -1$
- in the $y = -3$



Write down the coordinates of the vertices of the parallelogram. Compare these with the coordinates of the vertices of the parallelogram formed by

- reflecting in the $x$-axis
- reflecting in the $y$-axis



Draw a pair of axes and draw a square whose opposite corners are at the points $(-3, 3)$ and $(5, -1)$.
What are the equations of the vertical and horizontal lines of symmetry of the square? Can you generalise?

# Reflect in a diagonal line (1)

## Notes and guidance

Using mirrors or tracing paper and folding to support and check answers is even more important for the more challenging diagonal lines. It is also helpful to model drawing a perpendicular line from the vertices of the object to the mirror line and then extend this to find the position of the corresponding vertices of the image.

## Key vocabulary

Reflect     Object     Image     Vertex

Perpendicular distance

## Key questions

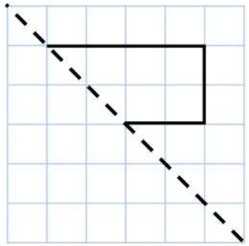
Why does it help to rotate your exercise book when reflecting in diagonal lines?

Why don't we have to worry about points/vertices that are on the line?

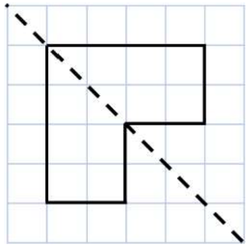
How do we know how far the vertices of the image are from the mirror line?

## Exemplar Questions

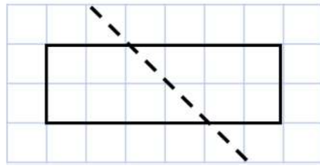
Amir and Mo are both trying to reflect this shape in the line shown.
Compare their answers.
Who do you agree with? Why?
Use a mirror to check.



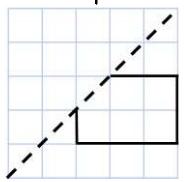
Amir

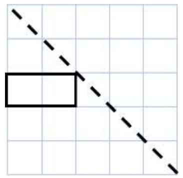


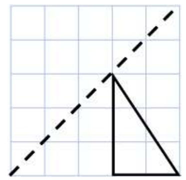
Mo

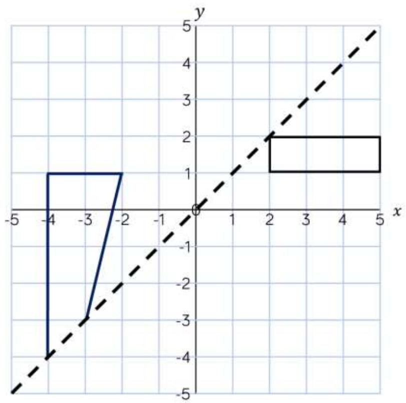


Reflect the shapes in the lines shown.










What is the equation of the diagonal line going through (0, 0), (1, 1) etc.?

Reflect both shapes in this line.

**Hint** – draw perpendicular lines from each vertex to the line to help.

# Reflect in a diagonal line (2)

## Notes and guidance

We now consider shapes that are not touching a diagonal line. Students can again use folding and mirrors to check their results, and will again need to be encouraged to take care that their images are the same perpendicular distance away from the mirror line as the object. Practising on cm- or even 2 cm-square paper can makes this (and earlier steps) more accessible than using very small squares.

## Key vocabulary

| | | |
|---|---|---|
| Reflect | Object | Image |
| Vertex | Perpendicular distance | |

## Key questions

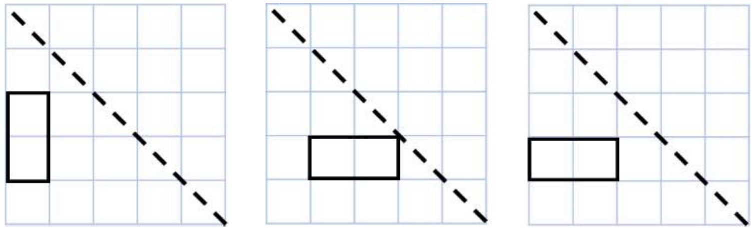
What is the equation of the line that goes through (0,0), (1,1) etc.?

How can we tell the lines $y = x$ and $y = -x$ apart?

Why do we count the distance to the mirror line diagonally rather than horizontally?

## Exemplar Questions

Reflect the rectangles in the lines.



What's the same and what's different?

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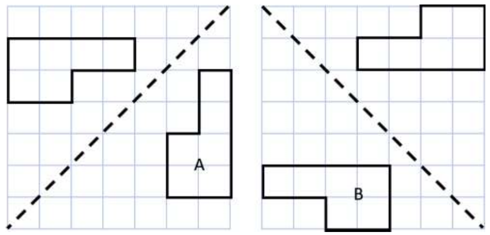
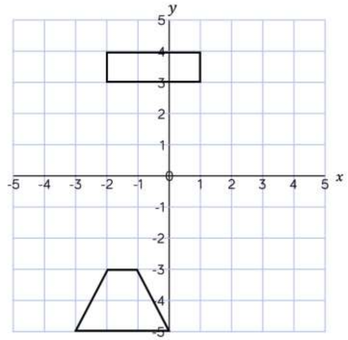



Dora has reflected both shape A and shape B incorrectly.
Draw the shapes and the correct reflections.

---

Reflect the rectangle and the isosceles trapezium in:

- the line $y = x$
- the line $y = -x$

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Plot the points (1, 5), $(-2, 3)$ and $(1, -2)$ and join them to form a triangle.
Reflect the triangle in the line $y = x$
Investigate reflecting other shapes that "cross" $y = x$ and $y = -x$

# Summer 2: Reasoning with Data

| Weeks 1 to 4: The data handling cycle | Weeks 5 and 6: Measures of Location |
|---|---|
| Much of the statistics content in Key Stage 3 is a continuation of that studied at primary school, and many of the charts and graphs in this block have been used in Year 7 and earlier in Year 8. A particular focus is using charts to compare different distributions. We also explore when graphs may be misleading, an important real-life consideration. Collection of data is also covered, including designing and criticising questionnaires. As we are covering the elements of the data handling cycle, it may well be worth delivering these steps (and some of those in the next block) through an extended project so students become aware of the pitfalls and difficulties of data collection and interpretation as well as the procedural production of graphs and charts.<br><br>National Curriculum content covered includes:<br>• describe, interpret and compare observed distributions of a single variable through: appropriate graphical representation involving discrete, continuous and grouped data; and appropriate measures of central tendency (mean, mode, median) and spread (range, consideration of outliers)<br>• construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data | Students have already met the median and the mean earlier in KS3. This block introduces the mode and also looks at when and why each average should be used. Students following the Higher strand will look at the mean from grouped and ungrouped frequency tables, and these steps may well also be accessible to the vast majority of students following the Core strand. The previous block is built on as students have the opportunity to compare distributions, use these averages and the range. We also consider outliers, considering what effect these have on all the measures studied, and whether they should be included or excluded in our calculations. Again, much of the material in the block is suitable for exploring through project work.<br><br>National Curriculum content covered includes:<br>• describe, interpret and compare observed distributions of a single variable through appropriate measures of central tendency (mean, mode, median) and spread (range, consideration of outliers) |

# The Data Handling Cycle

## Small Steps

- Set up a statistical enquiry
- Design and criticise questionnaires
- Draw and interpret pictograms, bar charts and vertical line charts R
- Draw and interpret multiple bar charts
- Draw and interpret pie charts R
- Draw and interpret line graphs
- Choose the most appropriate diagram for given set of data
- Represent and interpret grouped quantitative data

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered earlier in KS3

# The Data Handling Cycle

## Small Steps

- Find and interpret the range
- Compare distributions using charts
- Identify misleading graphs

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered earlier in KS3

# Set up a statistical enquiry

## Notes and guidance

The focus of this step is to consider the steps involved in a statistical enquiry. In particular, students will consider how to write a suitable hypothesis and design an appropriate data collection sheet. Students should also discuss the pros and cons of sampling, and the advantages and disadvantages of using primary and secondary data. They may need to be reminded about the difference between discrete and continuous data.

## Key vocabulary

| | | |
|---|---|---|
| Hypothesis | Investigation | Enquiry |
| Primary/secondary data | | Sample |

## Key questions

What is a hypothesis? Why do you need a hypothesis?
What is the difference between discrete and continuous data?
What are the advantages/disadvantages of using primary/secondary data?
What features do you need on a data collection sheet?

## Exemplar Questions

Put these steps for a statistical investigation into a suitable order.

- Collect, interpret and present your data
- Design a questionnaire/experiment to test your hypothesis
- **Confirm, deny or inconclusive?** Have you found enough information to support or disprove your hypothesis?
- **Ask a question**: what do you want to find out?
- **Create a hypothesis**: what do you expect to be the outcome of your statistical investigation?

---

Eva investigates how far students in her class can run in 1 minute.
All 30 students run between 80 m and 250 m.
Here is the start of Eva's data collection sheet.
What could be a problem with her data collection sheet?
Draw a more suitable data collection sheet.

| Distance | Tally | Frequency |
|---|---|---|
| 80 m | | |
| 81 m | II | 2 |
| 82 m | | |
| 83 m | | |
| … | | |

---

Jack wants to test the hypothesis: Boys can run further than girls.

Design a data collection sheet that would help him collect the data he needs to test his hypothesis.

# Design and criticise questionnaires

## Notes and guidance

Real life examples of questionnaires may be useful when exploring this step. Students could discuss the language used in questions – leading language or judgemental language may influence a reader's answers or make them less likely to be honest in their response. Students should also be able to identify issues with e.g. missing/overlapping response boxes, missing time frames. They should also consider when/whether open questions are appropriate.

## Key vocabulary

| | | |
|---|---|---|
| Questionnaire | Questions | Design |
| Multiple choice | Response box | Biased |

## Key questions

Imagine you are completing this questionnaire, which questions would you find difficult to answer? Why?
Why could having multiple choice answers/ranges make a questionnaire easier to answer?
Do you think a name should be included on a questionnaire? What influence might this have?

## Exemplar Questions

Dexter has created a questionnaire about students' pocket money.

1) How old are you?
☐ 10 – 12 ☐ 12 – 14 ☐ 14 - 16

2) How much pocket money do you get each week?
☐ £1 – £2 ☐ £3 – £4 ☐ £5 – £10

What is wrong with the response boxes Dexter has chosen?
Rewrite the questions with more suitable response boxes.
What other questions could be useful to ask in this questionnaire about students' pocket money?

---

Dora wants to find out about students' physical activity.
She includes the following questions.

1) Doing exercise is good for you. How many hours do you spend doing exercise?
☐ 0 – 2 ☐ 3 – 4 ☐ 5 – 6 ☐ 7 – 8 ☐ 8 +

2) Why is running better than other exercise?
☐ Burn more calories ☐ Feel good ☐ Lose weight

How could you improve these questions?

---

With a partner, choose a topic that you would like to investigate.
Write a hypothesis, and design a questionnaire that would help you test your hypothesis.

# Pictograms, bar and line charts R

## Notes and guidance

Students will be very familiar with these basic charts from KS2 and Year 7, so the focus here will be on interpreting and criticising charts more than drawing them. They should consider issues around scale, readability and consistency of the symbols used in pictograms. It is useful to include both vertical and horizontal examples of line and bar charts. They will review diagrams for continuous data that have been organised into equal groups in a later step.

## Key vocabulary

| | | |
|---|---|---|
| Pictogram | Bar chart | Line chart |
| Tally | Frequency | |

## Key questions

Why is it important to include e.g. a key, labels on the axes etc.?

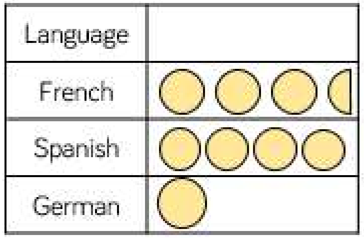
Is this discrete or continuous data? Is the data qualitative or quantitative?

How are a line chart and bar chart the same? How are they different?

## Exemplar Questions

The tally chart shows the number of people who study French, Spanish and German in a class. Complete the tally chart.

| | Tally | Frequency |
|---|---|---|
| French | 卌 卌 \|\|\|\| | |
| Spanish | 卌 卌 卌 \| | |
| German | \|\|\|\| | |



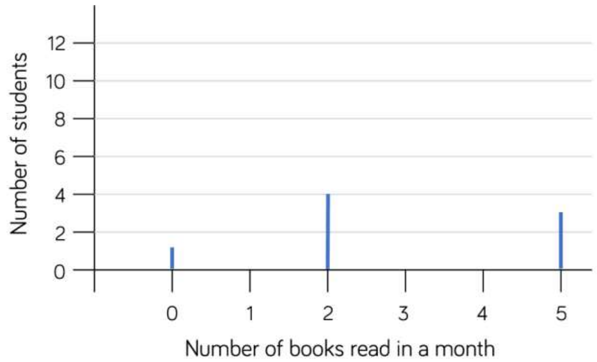

Dora puts the information into a pictogram.
What mistakes has Dora made?
Why might this pictogram be difficult to interpret without the frequency table?

Draw a bar chart to represent this data.

---

The table shows the number of books 28 students read in a month. Complete the line chart.

| Books | Frequency |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 12 |
| 4 | 6 |
| 5 | 3 |




- How many students read 4 books in a month?
- How many students read 3 or more books in a month?

# Multiple bar charts

## Notes and guidance

Students may be less familiar with multiple bar charts, so it would be useful to construct as well as interpret here.
Students should be aware of the need for a clear key. They may need support in choosing suitable scales and deciding where gaps should be placed to support reading of the charts.
Students could be asked to find examples of multiple bar charts from newspapers, magazines, and online, and compare their features.

## Key vocabulary

| | | |
|---|---|---|
| Multiple bar chart | Scale | Axes |
| Comparison | Key | |

## Key questions

When might it be useful to create a multiple bar chart?

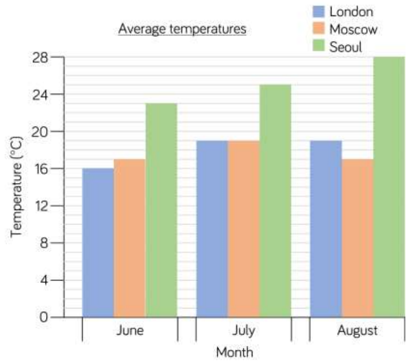
Why do multiple bar charts need a key?

What other questions could you ask about the bar chart?

What would you put on each of the axes? How can you decide your scale for the vertical axes?

# Exemplar Questions

The bar chart shows the average temperatures in London, Moscow and Seoul over the summer months.




- Which city is warmest in the summer?
- Which city has a decrease in temperatures between July and August?
- Which city is warmer in August, London or Moscow? What is the temperature difference?
- What other questions could you ask?

---

The information shows the scores of four students in their most recent science and geography exams. Draw a bar chart to represent the data.

| | Deacon | Sophie | Callum | Zulaika |
|---|---|---|---|---|
| Science | 8 | 17 | 14 | 10 |
| Geography | 10 | 16 | 8 | 15 |

---


Over the week, shop A has 10 more customers than shop B. Complete the bar chart to show how many customers went to shop A on Friday.

Describe the differences between the number of customers in each shop over the week. Is there a pattern?

# Draw and interpret pie charts R

## Notes and guidance

It would be a good idea to refresh students' memory on using a protractor. Exemplar question 1 offers visual support to students to think about the proportion of the pie chart taken up as a fraction, which is a useful stepping stone to help students find the proportion of 360° that is required for each part. Students should also be encouraged to think about more efficient methods when the total number is a factor of 360

## Key vocabulary

| | |
|---|---|
| Pie chart | Fraction |
| Full turn | Proportion |

## Key questions

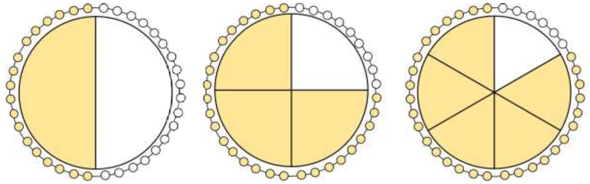
What are the factors of 360?

If you had e.g. 36 people in total, would you use the fraction of 360 or a multiplier to get to 360 in order to find the number of degrees? What about e.g. 35 people?

What type of data would you represent in a pie chart?

## Exemplar Questions

In each of the diagrams, 36 counters have been placed around the outside of a circle.



In each diagram, what fraction of the circle is made of yellow counters?

Since a full turn is 360°, how many degrees are:

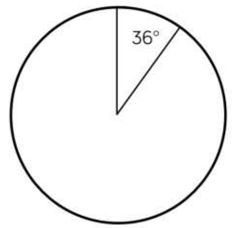
- 18 counters?
- 9 counters?
- 6 counters?
- 1 counter?

---

240 people were asked their favourite colours.

| Favourite colour | Red | Green | Blue | Orange |
|---|---|---|---|---|
| Number of people | 72 | 48 | 96 | 24 |

For each colour, write the fraction of people who chose it. What fraction of the pie chart is labelled? Which colour does this represent?




How many degrees would the others be? Complete the pie chart.

---

60 people are asked what type of pet they have.

| Type of pet | Dog | Cat | Hamster |
|---|---|---|---|
| Frequency | 32 | 25 | 3 |

$\frac{32}{60} \times 360 = 192°$

$32 \times 6 = 192°$

Rosie and Amir have worked out how many degrees would represent the number of dogs in the pie chart. Explain their methods.

# Draw and interpret line graphs

## Notes and guidance

Line graphs are most commonly used to show change over time for one or more sets of data. Researching real-life examples is again useful. Many line graphs show the points joined with solid lines which can imply it is possible to 'read off' information from between data points. It is useful to discuss with students whether this is meaningful e.g. estimate temperature between two recorded points or read off values between two years.

## Key vocabulary

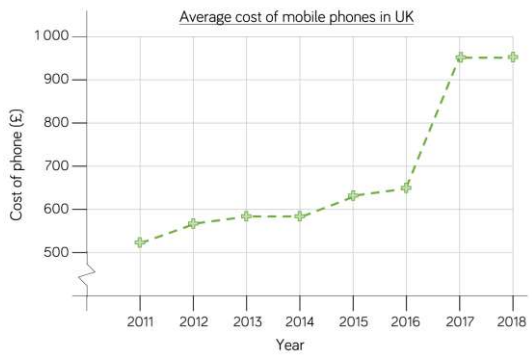
| | | |
|---|---|---|
| Line graph | Axes | Scale |
| Change | Read off/read from | |

## Key questions

Does the line graph have to start at 0? How can you show that your axis has not started from 0?
Is it possible to read off points between those given?
Would it be better to use a solid or a dotted line here?
What other information/comparisons can you make from the line graph? What other questions could you ask?

## Exemplar Questions

The line graph shows the increase in the cost of a mobile phone from 2011 to 2018.



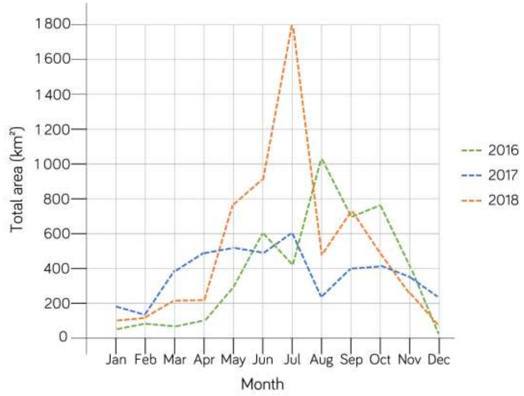

- How much did a mobile phone cost in 2011?
- How much more did it cost in 2017?
- Between which years was there very little change in price?
- Between which years was there a significant price rise?

---

The data shows the number of books sold in a book shop over one week. Draw a line graph to show the data.

| Days | Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|---|---|---|---|---|---|---|---|
| No. of customers | 72 | 68 | 54 | 75 | 82 | 112 | 64 |

---




The line graph shows deforestation of land in a country over a three year period. In which year and month was the highest area of deforestation recorded? How much more than 2016 is this? In which year was 600 km$^2$ of the rainforest eared in June?

# Choose appropriate diagram

## Notes and guidance

Given a free choice, students may struggle to decide which is the best choice of diagram to use. It is useful to discuss what they can/cannot ascertain from the diagrams such as those in the first exemplar. Students can then draw generalisations as to what advantages/disadvantages each of the diagrams have e.g. proportions are more easily seen from a pie chart, but the actual values cannot be seen. It is also a good opportunity to revisit the idea of scatter graphs for bivariate data.

## Key vocabulary

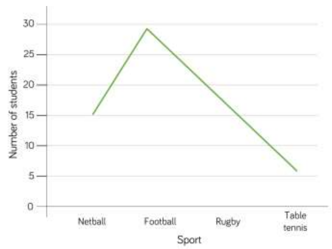
| | | |
|---|---|---|
| Pie/Bar charts | Proportion | Comparison |
| Scatter graph | Line graph | Bivariate |

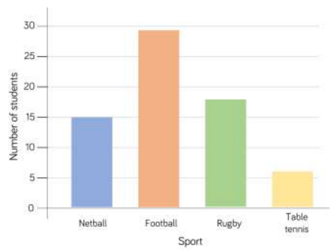
## Key questions

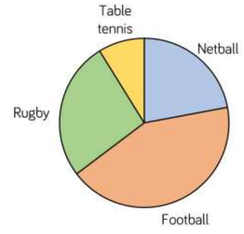
In which situation is a pie chart/bar chart/line graph the most useful? Why?
Which chart best shows changes over time/proportion/ comparison?
When would you/wouldn't you use a scatter graph to represent a set of data?

## Exemplar Questions

Dora wants to use a diagram to represent the number of students that attended each after school sports club.




Which diagram best represents the information? Why?

---

Would you use a pie chart, line graph, bar chart or scatter graph to represent the information shown on each card.
Explain your choices.

| | |
|---|---|
| Proportion of ingredients used in a cake | Profits made by company over the last 5 years |
| The heights and weights of 10 people | Number of TVs per household in a class |

---

| Year group | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|
| Number of students | 160 | 153 | 181 | 175 | 159 |

Eva is going to use a pictogram to represent the information shown in the table. Explain why this is not a good idea and suggest an alternative representation.

# Grouped quantitative data

## Notes and guidance

In this step, students will practise tabulating data into tables, interpreting tables with both discrete and continuous data, and drawing/interpreting grouped frequency diagrams (equal class width only). Comparisons should be made between bar charts and frequency diagrams to help students distinguish the use of each and choosing the appropriate representation. Students could also consider which distributions could be shown on a pie chart.

## Key vocabulary

| | | |
|---|---|---|
| Grouped data | Frequency diagram | |
| Discrete | Continuous | Intervals |

## Key questions

Why do we leave a space between the bars on a bar chart, but we don't on a frequency diagram?

How do we know which group/class a data item belongs to?

Why is it helpful to tally data to find the frequencies?

## Exemplar Questions

25 batteries were tested to see how long they lasted. The results were:

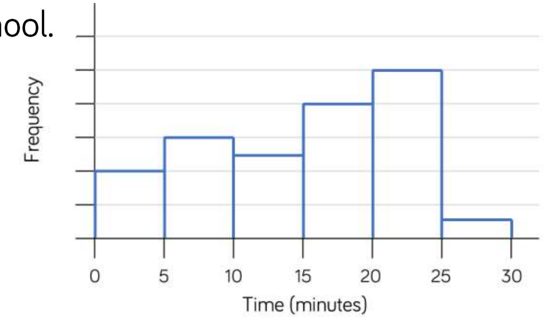
| | | | | |
|---|---|---|---|---|
| 23 | 15 | 5 | 9 | 17 |
| 18 | 27 | 19 | 12 | 18 |
| 15 | 20 | 27 | 22 | 13 |
| 23 | 12 | 18 | 11 | 26 |
| 14 | 23 | 22 | 19 | 16 |

| Duration (hours) | Tally | Frequency |
|---|---|---|
| $5 \le t < 10$ | | |
| $10 \le t < 15$ | | |
| $15 \le t < 20$ | | |
| $20 \le t < 25$ | | |
| $25 \le t < 30$ | | |

Which class does the battery that lasted 15 hours belong to?
Complete the table to show the number of batteries in each class.
Draw a grouped frequency diagram to represent the information.

---

The grouped frequency diagram and table show information about the amount of time it takes for students in one class to travel to school.




| Time (minutes) | Frequency |
|---|---|
| $0 \le t < 5$ | |
| $5 \le t < 10$ | 6 |
| $10 \le t < 15$ | |
| $15 \le t < 20$ | |
| $20 \le t < 25$ | |
| $25 \le t < 30$ | |

The scale is missing from the axis showing frequency. Use the table to work out the scale and complete the rest of the table.
How many students are in the class altogether?

# Find and interpret the range

## Notes and guidance

Students have met the concept of the range in Year 7 and this step extends their learning to consider interpreting the range as a measure of spread as well as finding the range from the a diagram and a list. A common error is to give the range as e.g. “20 to 75” rather than calculating the range as 75 – 20 = 55. The effect of outliers on the range is discussed in the next block.

## Key vocabulary

| | |
|---|---|
| Range | Spread |
| Consistent | Average |

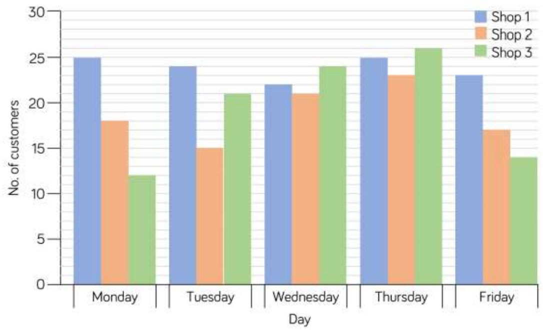
## Key questions

How can you work out the range? What does the range tell you about a set of data? Is it an average?

Does a large range mean the data is more spread out or less spread out?

If the data has a range of 0, what does this tell you about the data?

## Exemplar Questions




The bar chart shows the number of customers who came into three shops over a week.

On which day did Shop 1 have the most customers?

On which day did Shop 1 have the least customers?

What is the range in the number of customers that came into each shop? Which has the highest/lowest range? What does that tell us about the shops?

---

Dexter and Annie throw 20 rounds of 3 darts each.
They both have the same average score.
Dexter’s scores have a range of 23
Annie’s scores have a range of 8
Who is the more consistent player? Why?

---

The length of 50 caterpillars had a range of 10 cm

30 students sat a test and their scores had a range of 10%

The height of trees in a wood had a range of 10 cm

Each of the data sets has a range of 10

In which of the contexts would this be considered a small range?

In which of the contexts would this be considered a large range?

Explain your answers.

# Compare distributions using charts

## Notes and guidance

This step again develops students' interpretation skills as they compare distributions using two or more charts. It may be useful to provide students with stem sentences to support them to make comparisons. They could consider the range, the totals and whether the data is spread evenly or more towards one end of a distribution. Real-life charts could again be very useful here.

## Key vocabulary

| | |
|---|---|
| Compare | Distribution |
| Proportion | Range |

## Key questions

What is the same and what is different the charts?

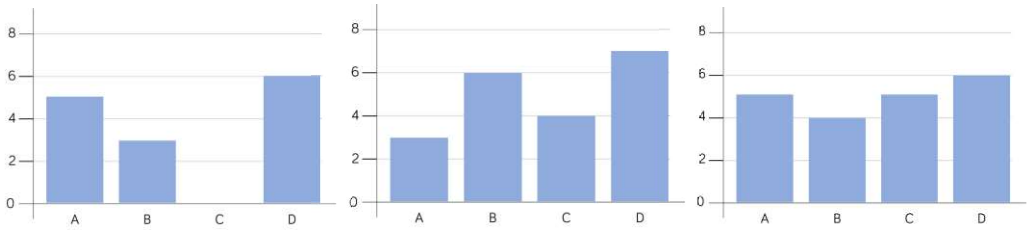
Is the data symmetrical or not? How does this compare to the other distribution?

What can you, and what can't you, tell about each distribution from the charts?

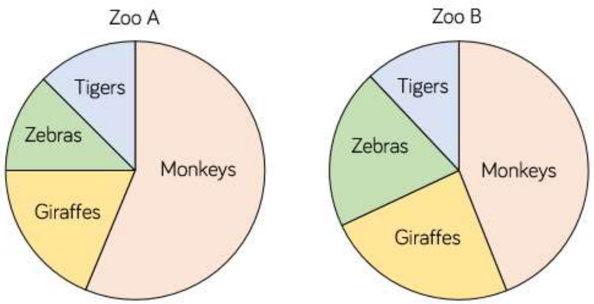
## Exemplar Questions

Compare the bar charts.

Bar chart 1 Bar chart 2 Bar chart 3




- What is the same and what is different about the data sets?
- Which data set has the highest range? Which has the lowest range?




There are fewer monkeys in Zoo B than Zoo A

There are the same number of tigers in Zoo A and Zoo B

The proportion of zebras is greater in Zoo B than in Zoo A

- Is there enough information in the charts so you can tell if the statements are true or false?
- What can you find out from the charts?

# Identify misleading graphs

## Notes and guidance

This small step draws students' attention to how graphs can be used to mislead an audience; they may have already come across this in earlier steps. There are many useful examples online of how graphs have been used by advertisers, media, politicians etc. to represent information to try to support dubious claims. It could be useful to discuss these examples and relate to real-life contexts for students.

## Key vocabulary

| | | |
|---|---|---|
| Scale | Broken axis | Mislead |
| Difference | Proportion | |

## Key questions

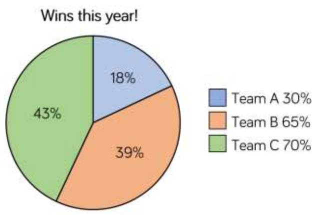
Who has made the chart/graph? Why might the data/representation of data be biased?

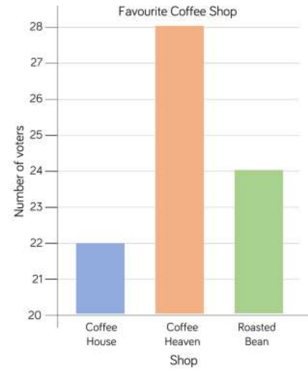
What information should you check on a graph to ensure the data is not misleading?

How could the information be represented more clearly/more fairly?

# Exemplar Questions

Why are these graphs hard to interpret, and how could they be improved?



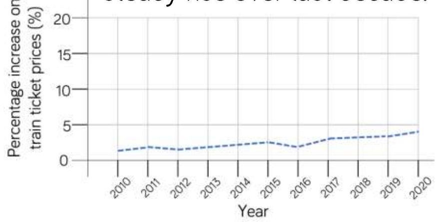

The cards show claims made by the owner of Coffee Heaven. Are these claims justified?

Coffee Heaven is 4 times as popular as other coffee shops!

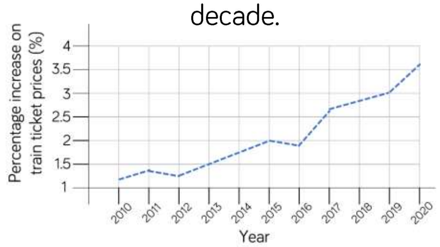
Coffee Heaven is the most popular coffee shop in town!

Each pair of line graphs shows the same data.
Do you agree with the statements on each graph? Why or why not?

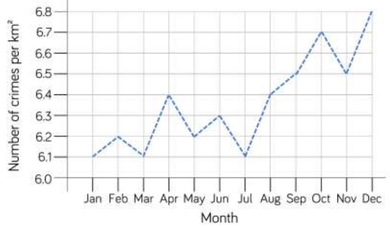
Train price increases show slight, but steady rise over last decade.



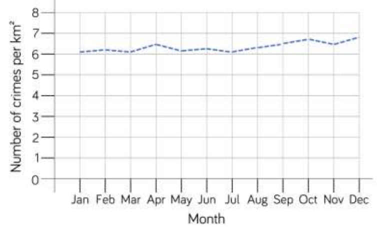

Train price increases have tripled over last decade.




Dramatic increase in crimes in Yorkshire.




Crime in Yorkshire has remained consistently low over the last year.



# Measures of Location

## Small Steps

- Understand and use the mean, median and mode
- Choose the most appropriate average
- **Find the mean from an ungrouped frequency table** H
- **Find the mean from an grouped frequency table** H
- Identify outliers
- Compare distributions using averages and the range

H denotes Higher Tier GCSE content

R denotes ‘review step’ – content should have been covered earlier in KS3

# Mean, median and mode

## Notes and guidance

Students will be familiar with both the mean and the median from Year 7 content, but this is the first time they have met the mode at KS3. It is worth introducing the term “modal value” as well as “the mode” as this will help with the later “modal class” in grouped frequency tables. Understanding is deepened by working backwards e.g. given the median, what might the data set be?

## Key vocabulary

| | | |
|---|---|---|
| Average | Mean | Median |
| Mode | Modal value | Total |

## Key questions

What’s the same and what’s different about finding the median of four numbers and the median of five numbers?

Why is it helpful to order data when finding averages? Which one is it most helpful for?

If you know the mean of a set of numbers, how can you find the total?

## Exemplar Questions

Find the mode of each of these data sets.

- 1 5 5 7 9 11 11 11 17
- A A E I I I O O U
- 6 8 8 7 6 5 8 2 6 6
- R R G G B R G B G B R

Which sets are easier to find the mode? Why?

---

What’s the same and what’s different about finding the modal value of these sets of data?

| | |
|---|---|
| 1 5 7 8 8 9 9 9 10 | 1 2 3 4 5 6 7 8 9 |
| 1 1 2 2 2 3 3 3 4 4 | 1 1 2 2 6 6 7 7 8 8 |

---

The mean of five numbers is 7
What is the total of the numbers?

? ? ? ? ?

- If the median of the numbers is 6, what might the numbers be?
- If the mode of the numbers is also 6 find a different set of possible numbers.
- Compare your solutions with others in your class.

---

Find a set of four numbers so that their

Mean value < Median value < Modal value

Investigate for other orderings of the mean, median and mode. Can you make all the numbers/none of the numbers integers?

# Choose an appropriate average

## Notes and guidance

Although students may be familiar with the different types of averages, they sometimes lack confidence in deciding which to use when, and why. It is important to emphasise that the average is meant to be representative of the data set and should be compared with the set as a whole to see whether it is or isn't appropriate e.g. non-integer means may be more useful than the mode in some cases.

## Key vocabulary

| | | |
|---|---|---|
| Average | Mean | Median |
| Mode | Frequency | Represent |

## Key questions

Is it possible (e.g.) to have 3.9 people in family? What would be a better average to use?

How does the ____ compare to the actual numbers in the data set? It is roughly the same as all, some, or none of them?

## Exemplar Questions

Which averages can and can't be found for these sets of data?

- Heights of students
- Scores in a test
- Eye colour
- Number of siblings
- Favourite app
- Ages in a class

---

Find the means of these sets of numbers.

2 5 8 10 12 16 19 20

5 35 35 37 38 39 40

Does the mean give a fair representation of the data sets?
Would any of the other averages be better?

---

The table shows how many books some students read last month.

| Number of books | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Frequency | 37 | 24 | 21 | 10 | 6 | 2 |

Dexter says the average number of books read is 0
Which average has Dexter found? Is this a good average to use?

---

Here are the weekly wages of the employees of a small firm.

| | | | | |
|---|---|---|---|---|
| £440 | £440 | £440 | £440 | £440 |
| £460 | £460 | £500 | £550 | £900 |

Find the mean, median and mode of the weekly wages.
Which average do you think represents the wages best?
Why might someone use one of the other averages to represent the wages?

# Mean from a frequency table H

## Notes and guidance

This step explores tabulating data to more quickly find subtotals and so the overall total when calculating a mean of large amounts of data. Students need to be careful not to divide by the number of rows rather than the total frequency; estimating roughly what the mean will be can help here. As well as using tables, students could read frequencies from bar/line charts and then find the mean.

## Key vocabulary

| | | |
|---|---|---|
| Average | Mean | Subtotal |
| Mode | Frequency | Estimate |

## Key questions

How could you estimate the mean from a table before doing any calculations?

How do you decide if the answer is reasonable?

What other average can you see immediately from a table?

## Exemplar Questions

The scores of 30 students in a test were

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 7 | 8 | 7 | 8 | 7 | 9 | 6 | 8 | 7 | 8 |
| 6 | 8 | 7 | 7 | 8 | 6 | 9 | 7 | 8 | 6 |
| 7 | 8 | 9 | 7 | 9 | 7 | 8 | 9 | 7 | 6 |

Show the results in a table.

How might the table help us find the total mark of the students?

Given the total, what would we need to divide by to find the mean?

---

Mo finds the mean number of goals scored from the table.

| Number of goals | Frequency | Subtotal |
|---|---|---|
| 0 | 9 | 0 |
| 1 | 6 | 6 |
| 2 | 5 | 10 |
| 3 | 2 | 6 |
| 4 | 3 | 12 |
| | Total | 34 |

$$\text{Mean} = \frac{\text{Total}}{\text{Number of items}}$$

$$= \frac{34}{5}$$

$$= 6.8$$ ✗

What mistake has he made? Find the actual mean.

---

| Number of pets | Frequency | Subtotal |
|---|---|---|
| 0 | 6 | |
| 1 | 11 | 11 |
| 2 | 5 | |
| 3 | 5 | 15 |
| 4 | | |
| Total | 30 | |

The mean number of pets is 1.6

Find the missing values.

Teddy thinks the mode is 5 as it occurs twice, but Rosie thinks the mode is 11 as it is highest.

Explain why they are both wrong.

# Mean of grouped data

H

## Notes and guidance

To practise estimating the mean from a grouped frequency table, it would be useful to look at both discrete and continuous data. Students may need reminding of the inequality notation and again take care to choose the correct values when dividing. This is a good opportunity to also revise grouped frequency diagrams (if the classes are equal) studied in the Autumn term.

## Key vocabulary

Midpoint

Estimate

Mean

Modal Class

Frequency

## Key questions

How do we find the midpoint of a class interval?

Why is our value an estimate of the mean rather than the exact mean?

Would the estimate be more or less accurate if you had more/fewer classes?

## Exemplar Questions

The scores of 30 students in a test were

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 28 | 31 | 40 | 39 | 42 | 54 | 31 | 57 | 45 | 29 |
| 44 | 52 | 58 | 45 | 37 | 45 | 52 | 41 | 22 | 41 |
| 34 | 47 | 56 | 21 | 56 | 43 | 17 | 24 | 38 | 22 |

Group the results into classes 10 – 19, 20 – 29 etc. in a table.
Which is the modal class?
Explain why it is impossible to find the exact mean just using the table.

---

Ron uses mid-points to represent each class and to find an estimate of the weights in the table.

| Weight ($w$) | Frequency | Midpoint | Midpoint × Frequency |
|---|---|---|---|
| $30 < w \leq 40$ | 9 | 35 | 315 |
| $40 < w \leq 50$ | 12 | | |
| $50 < w \leq 60$ | 15 | | |
| $60 < w \leq 70$ | 4 | | |
| Total | | | |

Complete Ron's workings.
Which column do you **not** need to find the total of?
Which values should you divide to find an estimate of the mean?

---

| Time | $40 < w \leq 50$ | $50 < w \leq 60$ | $60 < w \leq 70$ | $70 < w \leq 90$ |
|---|---|---|---|---|
| Frequency | 6 | 12 | 18 | 4 |

The table shows how long, in minutes, 40 students took to complete a test. Find an estimate of the mean time taken to complete the test.

# Identify outliers

## Notes and guidance

There is no need to look at a formal definition of an outlier through calculation/formulae here. Good discussion points include whether values are genuine outliers (e.g. just an unusually tall person) or errors in recording the data. Whether these values should be included in calculations is another interesting discussion. Students could also identify outliers graphically e.g. in a scatter diagram.

## Key vocabulary

Outlier

Median

Range

Mean

## Key questions

How do you decide which values are outliers?

Are any of the values impossible/unreasonable? Should these values be included in any calculations we might do?

Which averages are most affected by outliers?

Will outliers always affect the range? Why or why not?

## Exemplar Questions

Here is some data collected from a group of 12 students.

Number of siblings
0 1 0 1 3 20 4 6 2 4 1 2

Shoe size
4 6 5 6 -2 10 4 42 7 4 5 6

Height in cm
152 150 142 158 182 151 153 149 156 160 151 144

Do any of the data sets contain a value, or values, very different from the others? Do you think these values are errors or outliers?

---

Alex's practice lengths, in m, for a long jump event are shown.

1.64 1.71 1.68 1.66 0.67 1.79 1.58 1.69 1.75

- Find the range of the lengths.
- Which one of the lengths is an outlier?
- Find the range of the lengths without the outlier.
- Compare your two answers for the range.
- Do you think it is a good idea not to include the outlier? Why or why not?

---

Tommy checks the weights, in grams, of 10 packets of crisps.

25.7 25.9 26.1 25.2 24.8 25.6 51.2 24.3 25.9 25.8

- Find median and mean weights of the packets of crisps both with and without the outlier value.
- What effect does removing the outlier have on the mean?
- What effect does removing the outlier have on the median?

# Compare distributions

## Notes and guidance

In the last block, students compared distributions by looking at diagrams. We now extend this to compare distributions using an average or the range. The idea that the range represents consistency is sometimes difficult to grasp and may need reinforcing. The choice of an appropriate average could again be considered, as could whether the range is reliable given the presence of any outliers.

## Key vocabulary

Range

Average

Consistent

Outlier

## Key questions

Is it better to have a low or high range?

Why does a high range mean the (e.g.) scores are less consistent?

Which averages are affected by outliers?

Which average is most useful for comparing these groups of data?

## Exemplar Questions

Show that the means of these sets of numbers are equal.

3 3 3 3 3 3 3 3

1 2 3 3 3 3 4 5

1 1 1 3 3 5 5 5

-1 0 1 2 4 5 6 7

What's the same and what's different about the sets of numbers?

Dora and Jack do a spelling test every week. The table summarises their performances over a term.

| | Dora | Jack |
|---|---|---|
| Mean | 7.5 | 7.4 |
| Range | 6 | 2 |

I'm better than Jack at spelling, as both my mean and range are higher.

Do you agree with Dora? Why or why not?

Here are the numbers of runs scored by Brett and Amir in their last seven games of cricket.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Brett | 42 | 35 | 47 | 32 | 51 | 45 | 48 |
| Amir | 41 | 60 | 90 | 23 | 47 | 14 | 31 |

Complete the sentences, using calculations to justify your answers.

______ is less consistent than ______ because his scores have a greater range.

______ performed better on average because his scores have a greater ______ .