Autumn Term

Year 9

#MathsEveryoneCan

White Rose Maths

WRM – Year 9 Scheme of Learning

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Reasoning with Algebra**						**Constructing in 2 and 3 Dimensions**					
	Straight line graphs		Forming and solving equations		Testing conjectures		Three-dimensional shapes			Constructions and congruency		
Spring	**Reasoning with Number**						**Reasoning with Geometry**					
	Numbers		Using percentages		Maths and money		Deduction		Rotation and translation		Pythagoras' Theorem	
Summer	**Reasoning with Proportion**						**Representations and Revision**					
	Enlargement and similarity		Solving ratio & proportion problems		Rates		Probability		Algebraic representation	Revision		

Autumn 1: Reasoning with Algebra

Weeks 1 and 2: Straight line graphs

This block builds on Year 8 content where students plotted simple straight line graphs. They now study $y = mx + c$ as the general form of the equation of a straight line, interpreting m and c in abstract and real-life contexts, and reducing to this form in simple cases. This will be explored further in the next block when students rearrange formulae. Higher strand students will also consider inverse relationships and perpendicular lines.

National Curriculum content covered includes:

- develop algebraic and graphical fluency, including understanding linear and simple quadratic functions
- recognise, sketch and produce graphs of linear and quadratic functions of one variable with appropriate scaling, using equations in x and y and the Cartesian plane
- interpret mathematical relationships both algebraically and graphically
- reduce a given linear equation in two variables to the standard form $y = mx + c$; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically and algebraically
- use linear and quadratic graphs to estimate values of y for given values of x and vice versa and to find approximate solutions of simultaneous linear equations
- solve problems involving direct and inverse proportion, including graphical and algebraic representations

Weeks 3 and 4: Equations and inequalities

Students revisit and extend their knowledge of forming and solving linear equations and inequalities, including those related to different parts of the mathematics curriculum. They also explore rearranging formulae, seeing how this links to solving equations and reinforcing their understanding of the difference between equations, formulae, identities and expressions. This is a good opportunity to practise non-calculator skills if appropriate.

National Curriculum content covered includes:

- move freely between different numerical, algebraic, graphical and diagrammatic representations [for example…equations and graphs]
- use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement)
- understand and use standard mathematical formulae; rearrange formulae to change the subject
- model situations or procedures by translating them into algebraic expressions or formulae, and by using graphs

Weeks 5 and 6: Testing conjectures

Reasoning is encouraged throughout the White Rose Maths scheme of learning, and this block allows time for direct teaching of this. The opportunity is taken to revisit primes, factors and multiples, which provides a wealth of opportunity to make and test simple conjectures. As well as testing given conjectures, students should be encouraged to create and test their own. An example given in the block is through looking at relationships in a 100 square; another great source of patterns is Pascal's triangle. Students also develop their algebraic skills through developing chains of reasoning and learning how to expand a pair of binomials, which Higher strand students met in Y8

National Curriculum content covered includes:

- make and test conjectures about patterns and relationships; look for proofs or counterexamples
- begin to reason deductively in geometry, number and algebra
- use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation
- simplify and manipulate algebraic expressions to maintain equivalence by expanding products of two or more binomials

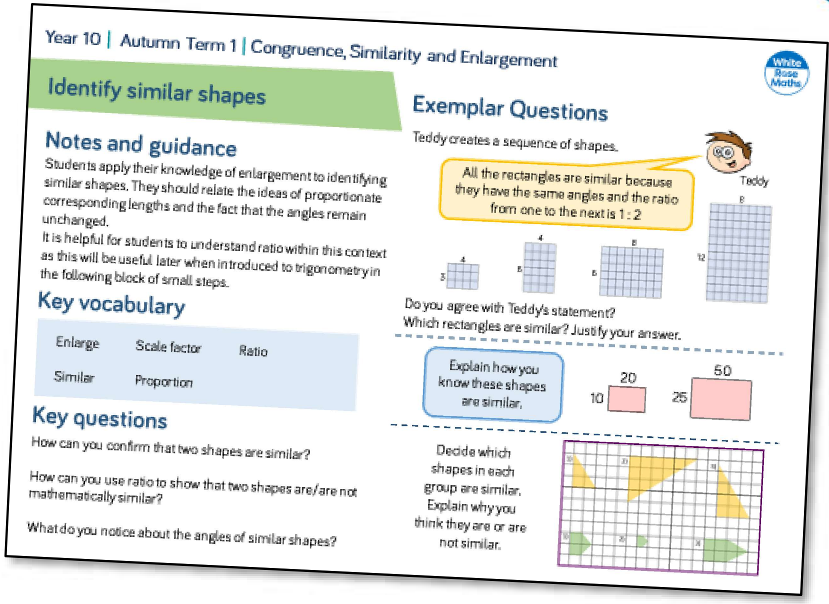
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.

It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge, Scale factor, Ratio, Similar, Proportion

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

All the rectangles are similar because they have the same angles and the ratio from one to the next is 1 : 2

Teddy

Do you agree with Teddy's statement?

Which rectangles are similar? Justify your answer.

Explain how you know these shapes are similar.

Decide which shapes in each group are similar. Explain why you think they are or are not similar.

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered earlier in Key Stage 3 are labelled R .

Key Representations

Input → $\times 2$ → -1 → Output

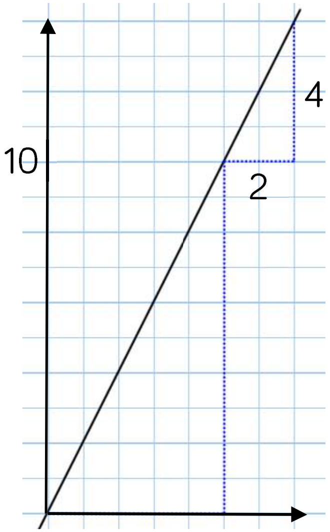
$y = mx + c$

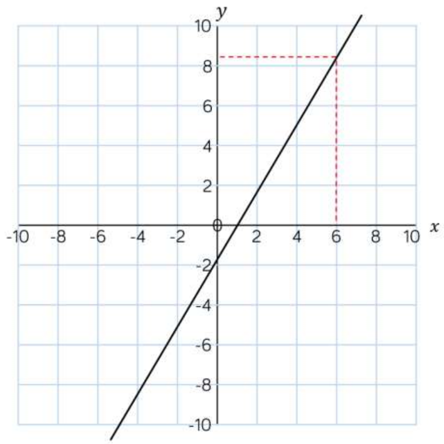
$y = 2x - 1$

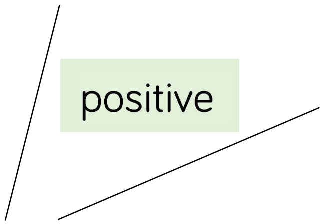
x	1	2	3	4
y	1	3	5	7

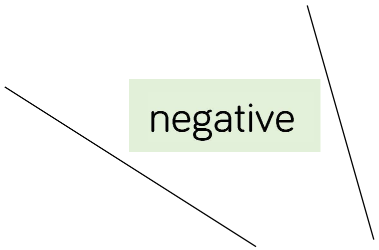
$\frac{\text{Increase in } y}{\text{Increase in } x}$

(1, 1)
(2, 3)
(3,5)
(4, 7)









In this block, axes that allow the use of all four quadrants should be used. It is important that children see a wide range of axes, for example, those with different scales. If appropriate, counters and small cubes can be used to demonstrate a coordinate before it is marked onto the grid.

Students should make links between the different representations of the same relationship – the equation of the line, the table of values, the coordinate pairs and the graph itself. The key focus of the unit is to understand the meaning of m and c in the standard format of the equation of a straight line given by $y = mx + c$. They should appreciate when and why m is positive or negative and relate the formula for gradient to the graphical representation.

Straight line graphs

Small Steps

- Lines parallel to the axes, $y = x$ and $y = -x$ (R)
- Using tables of values (R)
- Compare gradients
- Compare intercepts
- Understand and use $y = mx + c$
- **Write an equation in the form $y = mx + c$** (H)
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs

(H) denotes Higher Tier GCSE content

(R) denotes 'review step' – content should have been covered earlier in KS3

Straight line graphs

Small Steps

- Model real-life graphs involving inverse proportion (H)
- Explore perpendicular lines (H)

(H) denotes Higher Tier GCSE content

(R) denotes 'review step' – content should have been covered earlier in KS3

Parallel to the axes, $y = \pm x$ R

Notes and guidance

This small step revises content covered earlier in KS3. Students need to be able to plot and recognise lines of the form $x = a, y = b, y = x$ and $y = -x$. Students should understand that the equation of a line describes a relationship between any pair of coordinates on that line and so at any point on the line $y = 3$, the y coordinate is equal to 3 Similarly, at any point on the line $y = x$, the y coordinate is equal to the x coordinate.

Key vocabulary

Parallel	Horizontal	Vertical	Straight line
Axis	Equation	Graph	Intercept

Key questions

Which axis is $y = 4$ parallel to? How do you know?

All of the points on the line $y = x$ have something in common. What is it?

What is the equation of the x-axis?

What is the equation of the y-axis?

Exemplar Questions

Which of these coordinates lie on the line $x = 4$?

$(0, 4)$ $(4, 0)$ $(0, 0)$ $(-4, 0)$ $(0, -4)$

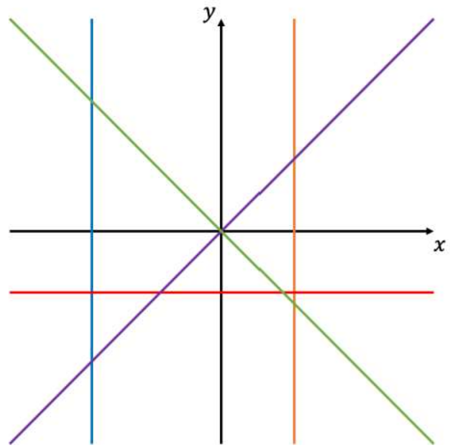
$(2^2, 5)$ $(4, 6.7)$ $\left(\frac{8}{2}, \frac{9}{3}\right)$ $\left(\frac{1}{4}, 4\right)$ $(15, \sqrt{16})$

The point $(-3, 3)$ doesn't lie on the line $y = -x$ because the x coordinate is the negative of the y coordinate, not the other way round.

Do you agree with Ron? Why or why not?

Here are the equations of 6 lines.

- $y = x$
- $y = -2$
- $y = -x$
- $x = 3$
- $y = 5$
- $x = -5$



Five of the lines have been drawn on the grid.
Label each line and explain how you know.
Sketch the graph of the sixth line.

Using tables of values

R

Notes and guidance

Students need to be able to complete and use a table of values to plot a straight line graph. The use of function machines can enable students to understand how the y coordinate is generated. Students should start to look for patterns in their tables of values and it is useful to include tables where the value of x increases by varying amounts to compare with those where the values of x only increases by 1

Key vocabulary

Linear	Equation	Graph
Straight line	Table of values	Function

Key questions

How does the function machine link to the equation?
If the x coordinate is ___, then what is the y coordinate?
If the y coordinate is ___, then what is the x coordinate?
If you know the equation of a line, how can you work out the value of y for a given value of x? How can you work out a value of x for a given value of y?

Exemplar Questions

Match each equation to its correct function machine.

$y = 3x + 2$ — $x \rightarrow \times 2 \rightarrow +3 \rightarrow y$

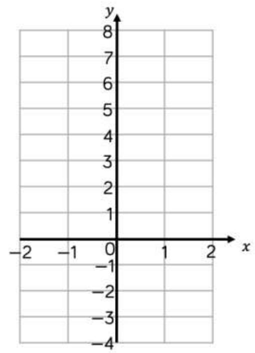
$y = 2(x + 3)$ — $x \rightarrow \times 3 \rightarrow +2 \rightarrow y$

$y = 2x + 3$ — $x \rightarrow +3 \rightarrow \times 2 \rightarrow y$

Complete the table of values for $y = 2x + 3$

x	−2	−1	0	1	2
y					

On the grid, draw the graph of $y = 2x + 3$ for values of x from −2 to 2



Complete the table of values for each graph.

$y = 3x + 1$ $y = 4x + 1$

$y = 1 - 3x$ $y = 1 - 5x$

x	−2	−1	0	1	2
y					

- Look at the sequences formed by the y values. What do you notice? How do the sequences relate to the equations of the lines?
- All of the graphs have one point in common. State the coordinates of this point and explain why this is the case

Compare gradients

Notes and guidance

Students need to recognise that the coefficient of x in the equation of a line in the form $y = mx + c$ tells us the gradient of the line. They should first look at lines of the form $y = mx$ for both positive and negative values of m so that they focus solely on the effect this has on the line, before moving on to exploring different gradients in lines of the form $y = mx + c$. Students should be aware that the greater the gradient of the line, the steeper the line is.

Key vocabulary

Equation	Gradient	Slope	Steep
Positive	Negative	Parallel	Straight line

Key questions

How does changing the coefficient of x in the equation of a line affect the line?

How can you tell from a graph if a line has a positive or negative gradient? How can you tell from the equation of the line?

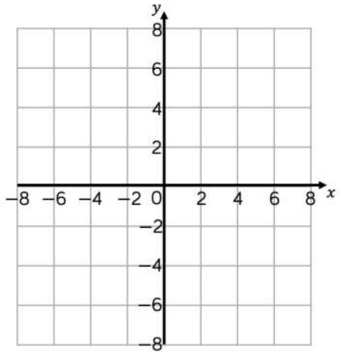
What do you know if two lines have the same gradient?

Exemplar Questions

Draw each of the lines on the grid.

$y = x$ $\quad y = 2x$ $\quad y = 5x$

$y = \frac{1}{2}x$ $\quad y = -3x$ $\quad y = -\frac{x}{2}$

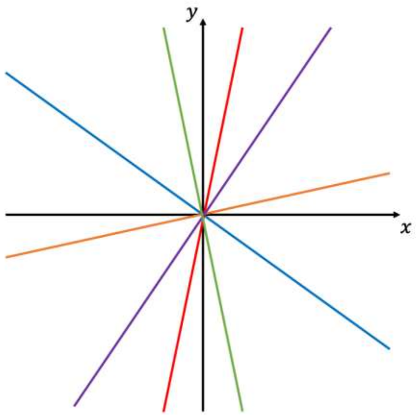


What do you notice?

Five lines have been drawn on the grid.
Here are the equations of the lines.

$y = 3x$ $\quad y = \frac{1}{3}x$ $\quad y = 7x$

$y = -\frac{3}{5}x$ $\quad y = -4x$



Which equation belongs to which line?
How do you know?

Draw each of the lines on the same set of axis.

- $y = 2x$
- $y = 2x + 3$
- $y = 2x - 5$
- $y = 2x + \frac{1}{2}$

What do you notice?

Compare intercepts

Notes and guidance

In this small step students will focus on how the value of c affects a line. They should first look at lines of the form $y = x + c$ to recognise that the value of c is the point at which a line intercepts the y-axis. Once they are secure in this they should then also look at lines of the form $y = mx + c$ to explore it further. Students need to be familiar with the term y-intercept to describe the point at which a graph intersects with the y-axis.

Key vocabulary

Equation	Intercept	Axis
Coordinate	y-intercept	

Key questions

What does it mean if the y-intercept of a straight line is positive/negative?
What does it mean if two lines have the same y-intercept?
What are the coordinates of the y-intercept? What always stays the same? What changes?
Is it possible to have an x-intercept?

Exemplar Questions

Complete the coordinates for a point that lies on each line.

- $y = x$ (0, __)
- $y = x + 1$ (0, __)
- $y = x + 2$ (0, __)
- $y = x - 5$ (0, __)
- $y = x - 11$ (0, __)
- $y = \frac{27}{4} - x$ (0, __)

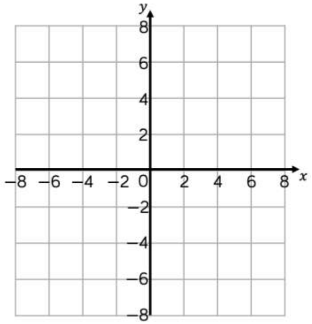
What do you notice?

Plot each of the lines on the grid.

$y = x + 4$ $y = 2x + 4$

$y = 4 + 3x$ $y = 4 - x$

What do you notice? Why does this happen?



Which of these lines intercept the y-axis at the same point?

$y = 5x - 3$ $y = 3 - 5x$ $y = -3 + 8x$ $y = -5$

What are the coordinates of the point where they intercept the axis?

A straight line has a gradient of -4
It intercepts the y-axis at the point $(0, -2.5)$
What is the equation of the line?

Understand and use $y = mx + c$

Notes and guidance

In this small step, students bring together what they have covered in the previous two small steps. They need to be able to interpret equations of a line given in the form $y = mx + c$, identifying both the gradient and the y-intercept. They should be exposed to examples such as $y = 5 - 2x$ where they may wrongly think the gradient is 5. They need to know that when two lines have the same gradient, they are parallel. Students should know that the coordinates of the y-intercept are (0, c).

Key vocabulary

Gradient	y-intercept	Equation
Parallel	Linear	Straight line

Key questions

In $y = mx + c$, what do m and c represent?

In $y = mx + c$, what do x and y represent?

What does it tell us about two lines if they have the same gradient? What does it tell us about two lines if they have the same y-intercept?

Exemplar Questions

Write down the gradient of each line.

- $y = 3x + 5$
- $y = 7 + 11x$
- $y = \frac{1}{2}x - 9$
- $2x + 0.5 = y$
- $y = \frac{x}{2} + \frac{1}{4}$
- $y = 5 - x$

Which two lines are parallel? How do you know?

Write down the coordinates of the y-intercept of each line.

- $y = 3x + 5$
- $y = 7 + 11x$
- $y = \frac{1}{2}x - 9$
- $2x + 0.5 = y$
- $y = \frac{x}{2} + \frac{1}{4}$
- $y = 5 - x$

Which two lines cross the y-axis at the same point? How do you know?

A straight line l_1 has a gradient of 19 and y-intercept of 11.2
What is the equation of l_1?

Another line, l_2, is parallel to l_1 and passes through the point (0, −17)
What is the equation of l_2?

A straight line is given by the equation $y = px + b$
What is the gradient of the line?
State the coordinates of the y-intercept of the line.

Write in the form $y = mx + c$ H

Notes and guidance

Students will formally study rearranging formulae in the next block, so teachers may wish to leave this step until then and study simple and complex examples together. At this stage, students could deduce an equation of the form $y = mx + c$ from e.g. $2y = \cdots$ or simple equations that require one step of rearrangement/deduction. For example, given $y + 3 = x$ students could consider coordinates of the points on the line and realise that $y = x + 3$ is equivalent.

Key vocabulary

Gradient	y-intercept	Equation	Parallel
Linear	Straight line	Rearrange	

Key questions

Is the equation of the line given in the form $y = mx + c$?
From the given equation, can you find an expression for y?
Why is the form $y = mx + c$ more useful than other forms?
Are the lines $2y = 4x$ and $y = 2x$ parallel? How do you know?

Exemplar Questions

Write each equation in the form $y = mx + c$.

- $2y = 16x + 12$
- $y - 1 = x$
- $y - x = 2$
- $3y = 3(5 - 2x)$
- $4y = 20 + x$
- $x + 15 = 3y$

Match the equations that represent the same straight lines.
Check by testing with coordinates of some points on the lines.

$y = \frac{x}{4}$ | $y = x - 4$ | $4x = y$

$y - 4 = x$ | $y - x = 4$ | $y = 4 - x$ | $4y = x$

$y = x + 4$ | $y = 4x$ | $x + y = 4$

Work out the gradient and y-intercept of each line.

- $3y = 18x + 12$
- $y - 2 = x$
- $y - x = 0$
- $7y = 14(5 - 2x)$
- $3 - 12x = 6y$
- $5(y - 8) = x$

$y = 9 + 2x$ | $y = 8 - 2x$

$2y = 2x + 16$ | $y = 4\left(\frac{1}{2}x + 9\right)$

Which lines are parallel?

Which lines have the same y-intercept?

How do you know?

Equation of a line from a graph

Notes and guidance

Students may need to revise finding the gradient of a line before they find its equation. They sometimes find it conceptually difficult to 'work backwards' to find the values of m and c and then deduce the equation of the line. It is helpful to consider what information can be seen immediately from the graph (usually the y-intercept) before calculating the gradient. Students should be encouraged to look carefully at the scales of the graphs before calculating the gradients.

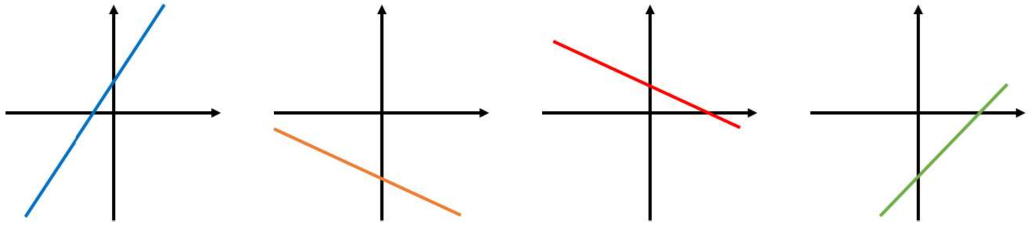
Key vocabulary

Gradient	y-intercept	Equation
Parallel	Linear	Straight line

Key questions

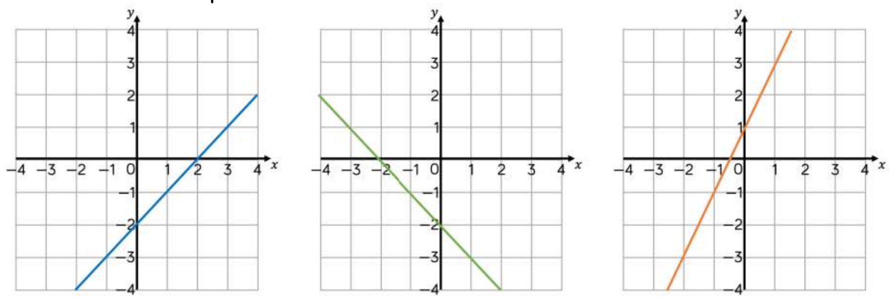
How do you know from its graph if a line has a positive/negative gradient?
How do you know from its graph if a line has a positive/negative y-intercept?
How do you work out the gradient of a line? How can you then find its equation?

Exemplar Questions

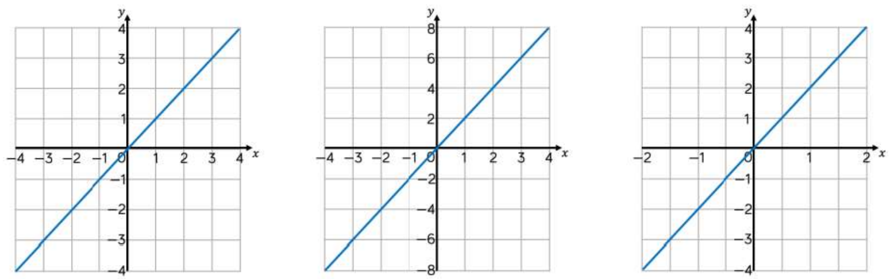


Is the gradient of each line positive or negative? How do you know?
Is the y-intercept of each line positive or negative? How do you know?

What is the equation of each line?



What is the equation of each line?



What is the same and what is different?

Gradient/intercept real-life graphs

Notes and guidance

Students will continue to find the gradient and y-intercept of a line, now interpreting it in a given context. They should see examples where the graph exemplifies direct proportion, and examples where it does not. For example, if a straight line graph is representing the cost of a taxi for a given number of miles, then the gradient of the line represents the cost per mile, and the y-intercept is the minimum fare for that journey. They need to be aware that graphs that do not start at (0,0) do not represent direct proportion.

Key vocabulary

Gradient	y-intercept	Interpret
Direct proportion	Table of values	Real-life

Key questions

Why does the table of values only have positive values?

What does the y-intercept represent?

What does the gradient represent?

Exemplar Questions

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

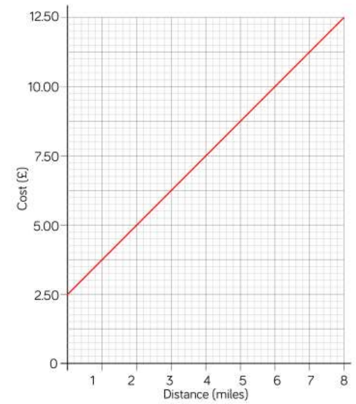
Time (h)	0	1	2	3	8
Cost (£)	£25				£125

Draw a graph to represent this information. What do you notice?
Is the cost of the plumber directly proportional to the time?

A box of pens costs £2.30
Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

Draw a graph to represent this information. What do you notice?
Is the cost of the pens directly proportional to the number of boxes bought?



The graph shows the cost of a taxi for a given number of miles.

- What is the y-intercept of the graph? What does this mean abut the taxi?
- What is the gradient of the graph? What does this mean about the taxi?
- What is the equation of the line?

Model inverse real-life graphs H

Notes and guidance

This small step continues to look at real life graphs but this time where the graph shows inverse proportion. As the graph will not be a straight line, students should know that they can't find the gradient nor the y-intercept. Instead, they can look at the curve of the graph and explore why the graph forms an asymptote at each axis. For example, if there are no builders the house will never get built, and if there an infinite number of builders it will still require some time to build the house.

Key vocabulary

Real-life	Graphs	Inverse Proportion
Curve	Asymptote	Interpret

Key questions

Why does the graph not form a straight line?

Why does the graph never meet the axis?

Is it possible to build a house in 0 days?

Is it possible for a house to be built if there are 0 builders?

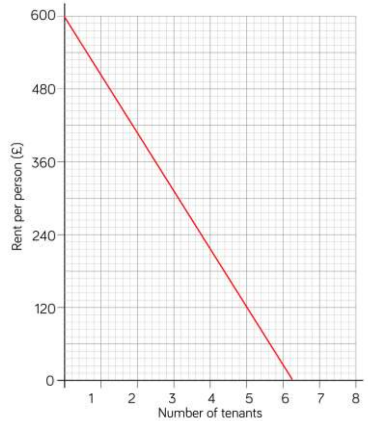
Exemplar Questions

It takes 1 builder 60 days to build a house.
Complete the table of values to show how long it will take if there are more builders who are working at the same rate.

Builders	1	2	3	4	10
Days	60				

Why is there no column in the table for 0 builders?
Draw a graph to represent the information. What do you notice?

The rent for a house is £600 per month.
5 people live in the house each pay £120 per month.
Dexter has drawn a graph to show how the rent changes if there are different numbers of tenants living in the house.
Explain why Dexter is incorrect.
Draw a correct graph to represent the information.



The time taken to decorate a house is inversely proportional to the number of decorators.
It takes 4 decorators 16 hours to decorate the house.

- Complete a table of values for this information.
- Draw a graph to represent the information.
- Describe the key features of the graph.

Explore perpendicular lines H

Notes and guidance

Students should already be familiar with the concept of perpendicular lines. They now need to be able to recognise perpendicular lines on a graph. They should look at lines e.g. $y = 2x$ and $y = -\frac{1}{2}x$ and recognise that the product of the gradients of a pair of perpendicular lines will always be -1
Students need to know that when two lines are perpendicular, one gradient is the negative reciprocal of the other.

Key vocabulary

Parallel	Perpendicular	Gradient
Product	Reciprocal	Negative reciprocal

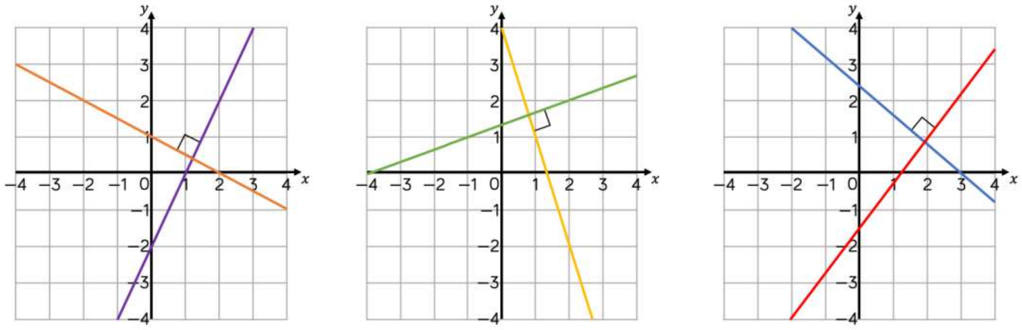
Key questions

When two lines are perpendicular, why must one gradient be positive and one be negative?

What is the product of the gradients of a pair of perpendicular lines?

Exemplar Questions

Each graph shows a pair of perpendicular lines.



Find the gradients of each pair of lines.
Find the products of the gradients of each pair of lines.
What do you notice?

Fill in the missing numbers.

$8 \times \square = -1$ $\quad -\frac{1}{3} \times \square = -1$ $\quad \frac{12}{15} \times \square = -1$

Write down the negative reciprocal of each number.

- 4
- -6
- $\frac{5}{4}$
- $-\frac{3}{11}$
- 4.5

Line A is given by the equation $y = 3x - 15$
Line B is given by the equation $15y = 17 - 5x$

Are lines A and B perpendicular? Explain your reasoning.

Forming and solving equations

Small Steps

- Solve one- and two-step equations and inequalities R
- Solve one- and two-step equations and inequalities with brackets R
- Inequalities with negative numbers
- Solve equations with unknowns on both sides
- Solve inequalities with unknowns on both sides
- Solving equations and inequalities in context
- Substituting into formulae and equations
- Rearranging formulae (one-step)

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered earlier in KS3

Forming and Solving Equations

Small Steps

- Rearrange formulae (two-step)
- **Rearrange complex formulae including brackets and squares** H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered earlier in KS3

1/2-step equations and inequalities R

Notes and guidance

Students will be familiar with equations and inequalities from previous learning. This step will provide an opportunity for students to revisit key ideas before looking at more complex examples. Students could have access to calculators throughout this step if appropriate and examples should include decimals to avoid "spotting" answers. Look out for the common misconceptions of changing an inequality sign for an equals sign.

Key vocabulary

Equation	Inequality	Greater/ less than	
Solution	Unknown	Inverse	Solve

Key questions

What is the difference between an equation and an inequality?

How many solutions does an inequality have?

How many solutions does an equation have?

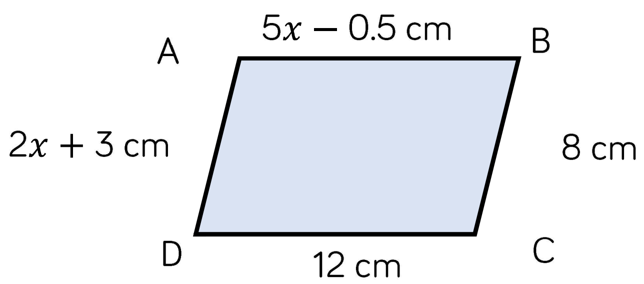
Exemplar Questions

Amir is solving some equations.
What mistakes has he made?

$$36 = 10x + 2$$
$$3.6 = x + 2$$
$$1.6 = x$$

$$\frac{x-3}{2} = 10$$
$$\frac{x}{2} = 13$$
$$x = 26$$

Prove that ABCD is a parallelogram.



Which of the following have the solution set $x < 7.5$?

$15 > 2x$	$x > 15 - x$
$5x - 17 < 23.5$	$23 > 3x - \frac{1}{2}$
$3.75 > \frac{x}{2}$	$\frac{x}{4} < 1.875$

Equations/inequalities with brackets R

Notes and guidance

Students should now be secure in solving one- and two-step equations. In this step questions that do not have integer solutions should be encouraged. Students need to be clear that they can leave solutions in fractional form. Calculators could be used to support this. Students should be exposed to varying methods to solve the questions and these should be discussed in depth asking 'what is the same? what is different?'

Key vocabulary

Equation	Inequality	Greater/ less than	
Solution	Unknown	Inverse	Expand

Key questions

Do you have to expand the brackets first to be able to solve the equation?

Can we check the solution is correct? How?

Can we have a solution that is not a whole number? Give me an example.

Exemplar Questions

Match the cards that have the same solution or set of solutions.

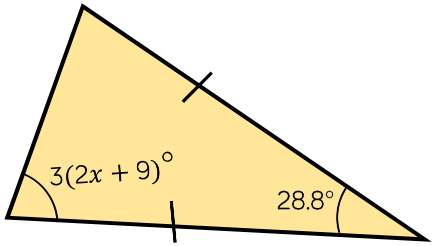
$\frac{3}{2} = 3(x + 5)$	$2(x - 5) < 1.4$
$2.85 < \frac{x}{2}$	$3x + 9 = 22.5$
$0.7 > x - 5$	$2.8 < 8(\frac{1}{2}x - 2.5)$
$22.5 = 3(3 + x)$	$30 + 6x = 3$

Compare the two methods to solve $5(x - 2) \geq 17.7$
What is the same? What is different?

$$5(x - 2) \geq 17.7$$
$$x - 2 \geq 3.54$$
$$x \geq 5.54$$

$$5(x - 2) \geq 17.7$$
$$5x - 10 \geq 17.7$$
$$5x \geq 27.7$$
$$x \geq 5.5.4$$

Mo is working out value of x. Comment on the mistake he has made, and work out the value of x.



$$3(2x + 9) = 28.8$$
$$6x + 27 = 28.8$$
$$6x = 1.8$$
$$x = 0.3$$

Inequalities with negative numbers

Notes and guidance

In this step students will explore and therefore understand the need to reverse the inequality when multiplying and dividing by a negative number. Students could compare and contrast with solving equations and check their solution sets by testing values either side of the boundary values found. Number lines are useful to support this. It is useful to include examples with non-integer solutions. Calculators should be used to support fluency with the "change sign" key.

Key vocabulary

Inequality	Satisfy	Reverse
Solve	Greater/less than (or equal)	

Key questions

What is the same and what is different about solving an inequality where the variable has a negative coefficient?

Explain why the direction of inequality sign has changed.

What is the first step you are going to take?

Exemplar Questions

Fill in the blanks.

- $20 \geq 6 - 5x$
  - $+5x$ $+5x$
  - $5x + 20 \geq 6$
  - -20 ☐
  - $5x \geq -14$
  - ☐ ☐
  - $x \geq$ ☐

- $-3x < 16$
  - ☐ ☐
  - $0 < 16 + 3x$
  - ☐ ☐
  - ☐ $< 3x$
  - ☐ ☐
  - ☐ $< x$

Here is an inequality. $-10 < -8$

Is the inequality still true if:

- 2 is added to both sides?
- Both sides are multiplied by 2?
- 2 is subtracted from both sides?
- Both sides are multiplied by -2?
- Both sides are divided by -2?

Which inequality is the same as $x > 5$?

$-x > -5$ $\quad$ $-x < -5$

Solve the inequalities.

- $-8y + 6 > 10$
- $8y - 6 < -10$
- $8 \geq 7 - 10y$
- $-10 - 8y \leq 20$
- $-\frac{1}{3}y > 2$
- $7 > 5 - \frac{2}{5}y$

Unknowns on both sides - equations

Notes and guidance

Students should now be fully confident using the 'balance' method to solve equations and inequalities and we now focus on solving equations where we have unknowns on both sides. Bar models should be used alongside, rather than instead of, the abstract calculation. Students should be exposed to examples with the larger coefficient on the right as well as on the left. Again, non-integer solutions and checking by substitution should be encouraged.

Key vocabulary

Equation	Balance	Coefficient
Solve	Unknown	Check

Key questions

Why do we do the same operation to both sides of an equation?

When solving a four-term equation should we deal with the variables or constants first? Why?

When solving an equation do we always start by subtracting something? Why or why not?

Exemplar Questions

Which of the equations **does** the bar model represent?

x	x	x	x	18.7
x	x	x	23.1	

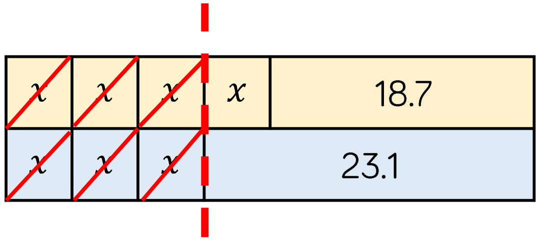
$23.1 + 3x = 18.7 + 4x$

$3x + 23.1 = 4x + 18.7$

$18.7 + 4x = 3x + 23.1$

$18.7 + 3x = 4x + 23.1$

Use the bar model and fill in the blanks to solve



$4x + 18.7 = 3x + 23.1$

$-3x$ ☐

$x + 18.7 = 23.1$

☐ ☐

$x =$ ☐

Match the equations that have the same solutions.

$1 + y = 3y + 5$

$7y + 4 = 5 + 5y$

$7y - 7 = 3 + 2y$

$1.5 - y = 2y$

$3y - 5 = -13 - y$

$20 - \frac{y}{2} = 4y + 11$

Unknowns on both sides - inequalities

Notes and guidance

In this step students will extend their learning and understanding of the balancing method for solving equations and inequalities with unknowns on both sides.
Students should only move onto this step once they are fully secure with solving equations and inequalities with unknowns.
Throughout this step students should be encouraged to fully check their solutions through substitution.

Key vocabulary

Equation	Inequality	Substitute
Solve	Unknown	Check

Key questions

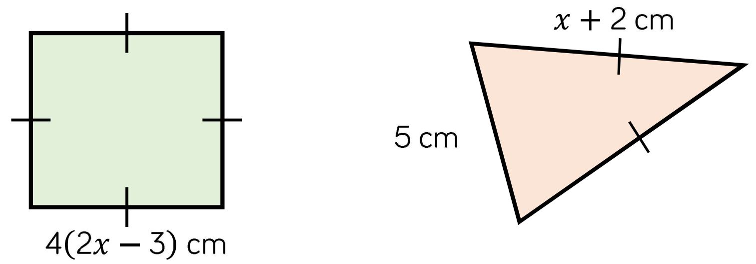
What would be the first step you would take to solve…?

"An equation only has one solution." Is this True or False? Give an example.

How can we check that the solution to an inequality is correct?

Exemplar Questions

The perimeter of the square is greater than the perimeter of the isosceles triangle.
Form and solve the inequality to find the possible values of x.



Rosie is solving $16 - 4x \leq 5.5 + x$.

Find the mistakes in her solution.

Solve the inequality.

$$16 - 4x \leq 5.5 + x$$
$$10.5 - 4x \leq x$$
$$10.5 \leq -3x$$
$$-3.5 \geq x$$

Solve the inequalities. What is the same? What is different?

- $4x + 9 \leq 9x$
- $9 + 4x \geq 9x - 4$
- $-4x + 9 > -9x + 4$
- $-4x - 9 < -9x$
- $4x - 9 \geq 9x + 4$
- $-4x + 9 > 9x$

Equations and inequalities in context

Notes and guidance

Here students look at forming and solving equations in mathematical contexts. This gives them the opportunity to revisit e.g. angles rules, types of triangles and quadrilaterals, probability, the mean and range, and a host of other areas. Teachers can choose the topics their classes need to revise the most and choose/create equation or inequality-based questions accordingly.

Key vocabulary

Form	Solve	Equation
Inequality	Check	

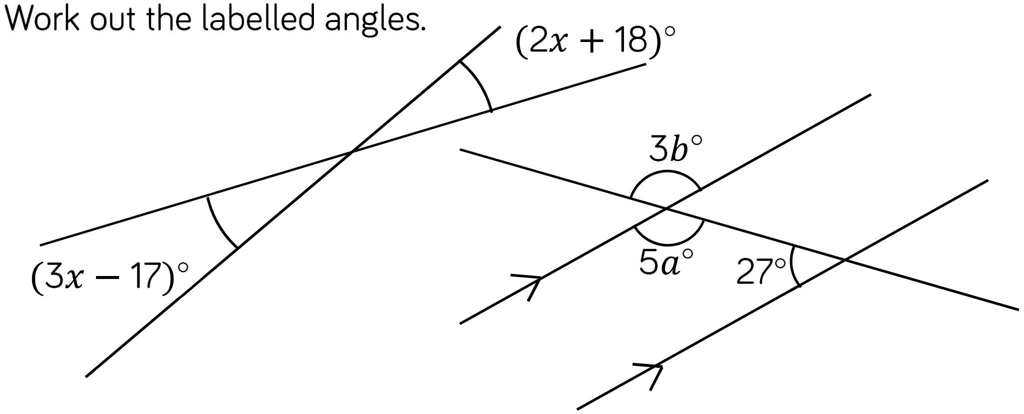
Key questions

Is your answer realistic given the context of the question?
How can you check your answer?

What facts do we know that will hep us to form an equation/inequality in this question?

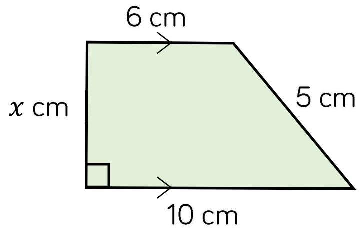
Exemplar Questions

Work out the labelled angles.



Annie has some coins.
Dora has three more coins than Annie.
Mo has three times as many coins as Dora.
Altogether they have more than 70 coins.
What is the smallest number of coins Dora could have?

The area of the trapezium is 32 cm^2.
Work out the perimeter of the trapezium.



The mean of the numbers on the cards is 9
Find the range.

x	$x - 2$	6	$2x - 1$	12

Formulae and equations

Notes and guidance

Here students explore the difference between formulae and equations and substitute numbers into formulae to produce equations to solve. The concept of the subject of the formula can be introduced here, as this step leads into the next few steps which focus on rearrangement. Teachers may wish to interleave familiar formulae that students should know, rounding and limits of accuracy, use of calculators, and/or using numbers expressed in standard form.

Key vocabulary

Formula	Equation	Solve
Variable	Substitute	Subject

Key questions

What is the difference between a formula and an equation?
How do you know which letter represents what quantity in a formula?
Can you substitute into a formula/equation?
Can you solve a formula/equation?

Exemplar Questions

A plumber charges a £35 call out fee, plus £15 per hour worked. Which is correct formula for the total cost, £C, to hire the plumber for t hours?

$C = (35 + 15)t$ | $C = 35 + 15t$ | $C = 35t + 15$

Form and solve equations to find:

- The cost of a job that takes the plumber 9 hours.
- The length of a job that costs £125

The perimeter of a rectangle is given by the formula $P = 2(l + w)$.
What do each of P, l and w represent?
Find the perimeter of a rectangle of length 11 cm and width 9.3 cm.
Find the width of a rectangle of length 9.2 cm and perimeter 27 cm.

The formula for Ohm's law, which links current, voltage and resistance is $V = IR$.

- Work out V when $I = 14$ and $R = 18$
- Work out I when $V = 14$ and $R = 18$
- Work out R when $I = 14$ and $V = 18$

Pressure (p) is found by dividing force (F) by area (A).

- Write down the formulae connecting p, F and A
- Work out p when $F = 20$ and $A = 0.4$
- Work out F when $p = 1\,000$ and $A = 0.1$
- Work out A when $F = 140$ and $p = 7$

Rearrange formulae (one-step)

Notes and guidance

Here students explore the link between solving one-step equations and rearranging one-step formulae. They could begin with simple "think of a number puzzles" and move from particular solutions to general ones, and then repeat the process with symbols. Bar models are useful tools from which to see both the original and rearranged formulae. Substitution is a useful strategy to check the new formula.

Key vocabulary

Formula	Subject	Rearrange
Make the subject of		Inverse operation

Key questions

Which variable is the subject of the formula? How do you know?

What is the inverse of ____?

Does it make a difference if a formula reads $x =$… or … $= x$? Would it be different if it was an equation?

Exemplar Questions

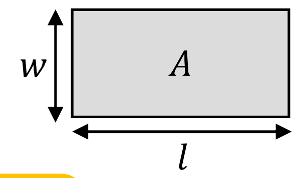
Which of these formulae have A as the subject?

$p = \frac{F}{A}$ $A = bh$ $A = \frac{1}{2}bh$ $V = Ax$

The area of a rectangle is given by the formula $A = lw$.

How can you find the area of a rectangle of width 10 cm and height 3.7 cm?

How do you find the width of a rectangle of area 864 cm^2 and length 32 cm?



Complete the formula for the width of a rectangle given its length and its area.

$A = \frac{\square}{\square}$

Match the formulae on the left with rearrangements on the right. Substitute values of x, y and z to check your answers.

$x = y + z$	$y = x - z$
$x = yz$	$y = xz$
$x = \frac{y}{z}$	$y = \frac{z}{x}$
$x = y - z$	$y = z + x$

Rearrange the formulae to make z the subject.

Rearrange formulae (two-step)

Notes and guidance

This builds from the last step, and students need to be confident with the more basic formulae before proceeding to these. Again "I think of a number, double it and add 7" type puzzles are a good introduction to the idea. This is a very good opportunity to build on the last block of work on straight line graphs, rearranging into the form $y = mx + c$ and revising the meaning of/how to find the gradient and y-intercept.

Key vocabulary

Formula	Subject	Rearrange
Make the subject of		Inverse operation

Key questions

What is the first step you need to take to rearrange the formula?

If you are multiplying or dividing, why is it important to do this to every term?

How do you choose good values to check your answers by substitution?

Exemplar Questions

Compare solving the equation $3x - 5 = 46$ to making x the subject of the formula in $p = 3x - 5$. What is the same and what is different?

$$3x - 5 = 46$$
$+5$ $\quad$ $+5$
$$3x = 51$$
$\div 3$ $\quad$ $\div 3$
$$x = 17$$

$$p = 3x - 5$$
$+5$ $\quad$ $+5$
$$p + 5 = 3x$$
$\div 3$ $\quad$ $\div 3$
$$\frac{p + 5}{3} = x$$

Rearrange the equations of straight lines to the form $y = mx + c$.
State the gradient and y-intercept of each line.

- $x = 4y + 3$
- $2y + 8x = 10$
- $5(y - 8) = x$
- $3 = 6y - 12x$
- $3x - 2y = 0$
- $14(5 - 2x) = 7y$

Make the letter in bold the subject of each formula.
What is the same and what is different?

- $v = \boldsymbol{u} + t$
- $x = \boldsymbol{u} + t$
- $x = \boldsymbol{y} + t$
- $v = 2\boldsymbol{u} + t$
- $x = 2\boldsymbol{u} + t$
- $x = 2\boldsymbol{y} + t$
- $v = t + 2\boldsymbol{u}$
- $x = t + 2\boldsymbol{u}$
- $x = t + 2\boldsymbol{y}$
- $v = \frac{\boldsymbol{u}}{2} + t$
- $x = \frac{\boldsymbol{u}}{2} + t$
- $x = \frac{\boldsymbol{y}}{2} + t$

Rearrange complex formulae H

Notes and guidance

This final step looks at slightly more complex rearrangement that involve more steps. In particular, students explore formulae that include squaring or square rooting and that have terms in brackets. This is a Higher strand step and should only be covered when students are fully confident with the previous two steps. Note that rearrangement where the subject occurs more than once is not covered here, as this is left until KS4

Key vocabulary

Formula	Subject	Rearrange
Make the subject of	Inverse	Square/Root

Key questions

What is the first step you need to take to rearrange the formula?

What is the inverse of squaring/cubing/square rooting/cube rooting?

Do you need to multiply out the brackets or not in this case?

Exemplar Questions

Dexter is rearranging the formula for the area of a circle to make r the subject.

What mistake has he made?

$$A = \pi r^2$$

Divide by π

$$\frac{A}{\pi} = r^2$$

Square root

$$\frac{\sqrt{A}}{\pi} = r$$

Dora and Teddy are rearranging $P = 2(l + w)$ to make w the subject.
Who is correct? How can you verify your answer?

$$P = 2(l + w)$$
$$P = 2l + 2w$$
$$P - 2l = 2w$$
$$\frac{P - 2l}{2} = w$$

$$P = 2(l + w)$$
$$\frac{P}{2} = l + w$$
$$\frac{P}{2} - l = w$$

Make u the subject of each formula.

- $v = u + at$
- $v^2 = u^2 + 2as$
- $s = ut + \frac{1}{2}at^2$

Starting with the same formulae, make a the subject of each.

Make b the subject of each formula.

- $T = \frac{1}{2}ab^2$
- $T = \frac{1}{2}a\sqrt{b + c}$
- $T = \frac{1}{2}a\sqrt{b}$
- $T = \frac{1}{2}a\sqrt{b^2 + c}$

Testing conjectures

Small Steps

- Factors, Multiples and Primes (R)
- True or False?
- Always, Sometimes, Never true
- Show that
- Conjectures about number
- Expand a pair of binomials
- Conjectures with algebra
- Explore the 100 grid

(H) denotes Higher Tier GCSE content

(R) denotes 'review step' – content should have been covered earlier in KS3

Factors, Multiples and Primes R

Notes and guidance

This review step serves both as a reminder of useful content and as a good context on which to base forthcoming work on conjectures. Confusion between the concepts of factors and multiples may need addressing, as might misconceptions about 1 being prime. Students may need to be reminded how to express a number as a product of primes and the use of correct language is important here. HCF/LCM will be revisited next term.

Key vocabulary

Factor	Multiple	Prime	Common
Odd	Even	Express	

Key questions

What's the difference between a factor and multiple? Can one number be both a factor and a multiple?

What does 'express' mean? How do you express a number as a product of prime factors?

What does 'common' mean in the context of multiples and factors?

Exemplar Questions

Which of the numbers are factors of 12 and which are multiples of 12?
Are any of the numbers both factors and multiples of 12?

1	6	36	12	4	48	30

List the first 10 prime numbers.
Which numbers between 40 and 60 are prime?
Explain how you can tell by looking at some numbers whether or not they are prime.

A fair die numbered 1 to 9 is rolled once.
Find the probability that it shows:

- a prime number
- a multiple of 4
- a factor of 36
- a common factor of 9 and 18
- a common multiple of 2 and 4

By drawing a factor tree or otherwise, express 72 as a product of prime factors, giving your answer in index form.
Use your answer to express these numbers as products of prime factors.
Explain your reasoning.

- 144
- 216
- 720
- 360

The HCF of two numbers is 6 and the LCM of the numbers is 24
What might the numbers be? How many solutions can you find?

True or False?

Notes and guidance

As an introduction to forming and testing conjectures, this step focuses on identifying whether given statements are true or false. Many of the exemplars are just factual recall or involve a small element of reasoning, before becoming more complex. Students may have met the idea of counterexamples in Year 7, but this may need revisiting. Teachers could choose statements from topics that need revision and/or common misconceptions.

Key vocabulary

Conjecture	True/False	Verify
Counterexample	Demonstrate	Prove

Key questions

How many examples do you need to prove that a statement or a conjecture is false? What do we call this type of example?

How can you show the statement is true of false? How many cases do you need to look at? Is this a demonstration or a proof?

Exemplar Questions

Which of these statements are true and which are false.
Explain how you know.

- $\frac{1}{3}$ is the same as 30%
- $\pi = 3.14$
- $\frac{x}{x}$ is the same as 1
- $y - 3x = 2$ has a negative gradient
- There is only one prime number between 90 and 100
- $86.3 + 104.7 > 104.6 + 86.2$
- A quadrilateral can have 0, 1, 2, 3 or 4 right angles
- These numbers cannot be probabilities: 1.5, 0.00000147, -0.2, $\frac{7}{5}$
- You cannot find the square root of a negative number
- $3x + 1 = 3x + 2$ has no solutions
- Numbers have at least 2 factors

$8 = 2^3$ and has 4 factors.
$16 = 2^4$ and has 5 factors.
I think numbers of the form 2^n always have $n + 1$ factors.

Is Whitney's conjecture correct? Can you explain why?

Investigate whether 3^n and 4^n follow similar rules or not.

Always, Sometimes, Never true

Notes and guidance

The statements met in this step require a little more reasoning because e.g. to establish a statement is sometimes true needs examples to show both when it is true and when it is false. Students will need to consider negative numbers and fractions for many of the statements that appear to be true just by considering positive integers. They could be encouraged to come up with 'always, sometimes, never true' ideas of their own as a starting point for forming conjectures.

Key vocabulary

Conjecture	True	False
Verify	Demonstrate	Prove

Key questions

What values could you substitute to test the statement?

Is the result true for 0 and 1? What about fractions? What about negative numbers?

Could you draw a picture to prove your result?

Exemplar Questions

Are the statements on the cards always, sometimes or never true?
If always true or never true, can you explain or prove why?
If sometimes true, give examples of when and when not.

Multiples of 3 are also multiples of 6	Multiples of 6 are also multiples of 3	Factors of 6 are also factors of 60
Square numbers have an odd number of factors	Cube numbers have an even number of factors	
The sum of two odd numbers is odd	Multiples of 5 are odd	The square of a negative number is positive

By testing different values of the letters, decide whether these algebraic statements are always, sometimes or never true.
Explain or prove your answers.

$3m + 5 > 3m + 1$	$3m + 5 > 2m + 1$	$3m < 2m + 1$	
$a - b = b - a$	$2a > a$	$x^2 > x$	$x^2 < 0$

Are these conjectures always, sometimes or never true?

- The LCM of two numbers is equal to the product of the numbers
- The squares of prime numbers have exactly three factors
- If n is odd, then $n + 3$ is even
- If p is prime, then $p - 1$ is even
- $(n + 1)$ multiplied by $(n - 1)$ is one less than n^2

Show that

Notes and guidance

This step develops more formal demonstrations that a statement is true or not. It is good to start with simple numerical verification and then proceed to verifying algebraic identities before, if appropriate, moving to simple proofs. Teachers will choose the level of complexity suitable for their classes. It is good to use concrete and pictorial support e.g. proving the sum of two odd/even numbers is even using multilink or pictures as shown in the second exemplar.

Key vocabulary

Conjecture	True	False
Verify	Demonstrate	Prove

Key questions

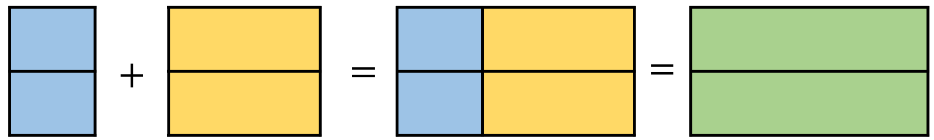
How can you work out the left hand side of the statement? Can you then compare it to the right hand side? Are they the same or different? Can either expression be written in a different way?
What can you work out from the given information? What could you find out next?

Exemplar Questions

Show that the following statements are true.

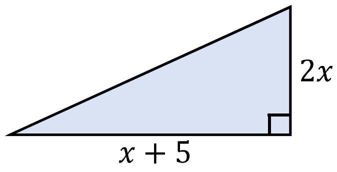
- $\frac{1}{3}$ of $60 = 50\%$ of 40
- $\frac{1}{3} + \frac{1}{6} = 0.5$
- $3(4x + 2) + 4x \equiv 2(3 + 8x)$
- $2^4 = 4^2$
- $2(3a - 3) + 4(4 + a) \equiv 10(a + 1)$
- $3^4 \neq 4^3$

Explain how the diagram below show that the sum of two even numbers is also an even number.



Show that the perimeter of the triangle is $6(x + 1)$

Show that the area of the triangle is $x^2 + 5x$



Multiples of 3 can be written as $3k$, where k is an integer.
How might we write multiples of 6? Use algebra to show that

- a multiple of 6 is also a multiple of 3
- a multiple of 6 is also a multiple of 2
- the square of an even number is always a multiple of 4

Conjectures with number

Notes and guidance

Conjectures about the sums and products of even and odd numbers are relatively easy to see and can be verified using diagrams e.g. two equal rows for even with an extra 1 for odd. This makes sense of the later algebraic approach of $2n$ and $2n + 1$. Students could also look at factors of numbers and related numbers, building on the activities of the earlier steps in this block.

Key vocabulary

Conjecture	True	False
Verify	Demonstrate	Prove

Key questions

How do we know if a number is even or odd?

If a number is even and we multiply it by an integer, what can we say about the result?

Can you draw a diagram or use manipulatives to show this?

Exemplar Questions

Add together different combinations of odd and even numbers.

Even + Even = ?

Odd + Even = ?

Odd + Odd = ?

What do you notice? Can you explain your result using words, pictures or symbols?
What is the result when you add 2, 3, 4…odd numbers? What conjectures can you make? Can you explain/prove they are true?

Multiply together different combinations of odd and even numbers.

Even × Even = ?

Odd × Even = ?

Odd × Odd = ?

What conjectures can you make? Can you explain/prove they are true?
Investigate further with more than two numbers.

$x\%$ of $y = y\%$ of x

Test Amir’s conjecture by comparing 30% of 40 and 40% of 30 and other numbers of your choice.
Is the result, always, sometimes or never true?
Use fractions and multiplication to prove your findings.

The Goldbach conjecture is a famous unproved problem in mathematics. Investigate!

“Every even number greater than 2 is the sum of two primes.”

Expand a pair of binomials

Notes and guidance

Students following the Higher strand will have met this briefly in Year 8. This step focuses on expansion where all the terms are positive, but teachers could bring in negative terms as an extension if appropriate. The expansions produced will be used to form and test conjectures in this and future steps. Students need to be familiar with the language of binomial (i.e. having two terms) and quadratic (i.e. having four terms, although these will simplify to three or fewer).

Key vocabulary

Expand	Factorise	Binomial
Term	Expression	Quadratic

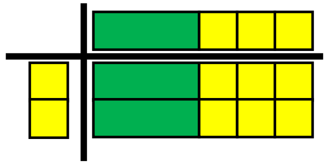
Key questions

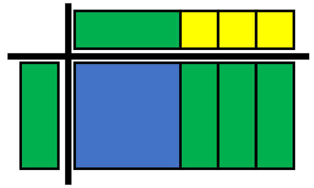
What is the difference between a numerical and algebraic factor? Why is an expression like $x + 3$ called a binomial? What other words use the prefix 'bi-'?

Why is the expansion of a pair of binomials called a quadratic expression? What other words use the prefix 'quad-'?

Exemplar Questions

Annie uses algebra tiles to expand $2(x + 3)$ and $x(x + 3)$





$2(x + 3) \equiv 2x + 6$ $\quad$ $x(x + 3) \equiv x^2 + 3x$

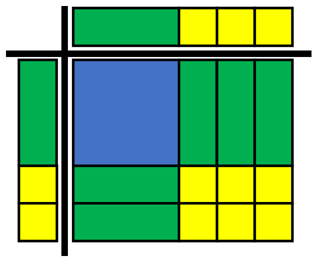
Mo uses a written method:

$2(x + 3) \equiv 2x + 6$ $\quad$ $x(x + 3) \equiv x^2 + 3x$

Verify the results are correct by substituting some values for x

"Expanding $x(x + 5)$ will give the same result as expanding $x(5 + x)$"

Use your preferred method to show that Dexter is correct.



- Using the picture of the algebra tiles or otherwise, expand $(x + 2)(x + 3)$
- How can your answer be simplified?
- Ron conjectures that expansions of the type $(x + a)(x + b)$ will always have four terms that can be simplified to three terms. Do you agree?

$(a + 4)^2 \equiv a^2 + 16$

Use a counterexample to show the statement is wrong. Find the correct expansion of $(a + 4)^2$

Conjectures with algebra

Notes and guidance

The level of difficulty of the conjectures used in this step can be moderated by the teacher using their knowledge of the class' current attainment in algebra. A key aspect would be to understand that expressions of the form $2n$ are even and those of the form $2n+1$ are odd. Where appropriate, students can formally prove some of the conjectures with numbers met in earlier steps, and all students can explore terms in sequences.

Key vocabulary

Conjecture	True	False
Verify	Demonstrate	Prove

Key questions

What values would be useful to test the conjecture?

How many values do we need to show a conjecture is false?

Why is it harder to show that a conjecture is true than it is to show a conjecture is false?

Exemplar Questions

Tommy is investigating the sequence given by the rule $5n+1$
He makes two conjectures.

The last digit of every term is either a 1 or a 6

None of the terms are multiples of 3

Test Tommy's conjectures.

---

m is an even number, n is an odd number and p is a prime number.
Test the conjectures.

n^2 is odd | $m+p$ is odd | mn is odd | n^m is odd

Eva and Ron both make Fibonacci sequences with 5 terms.

3, 4, 7, 11, 18

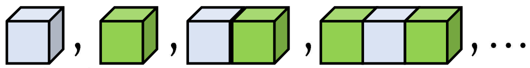
1, 9, 10, 19, 29

They conjecture that if you add the first and last terms of a 5-term Fibonacci sequences, the result is three times the middle number.

$3+18=21=3\times 7$

$1+29=30=3\times 10$

Eva uses cubes to try and prove the conjecture



Ron uses algebra:

$a, b, a+b, \ldots$

Complete their working to show that their conjecture is correct.

Explore the 100 grid

Notes and guidance

This step provides an introduction to more formal proof with students using the hundred square to form expressions and then practising the skills of simplification and expanding two binomials. The tasks here are very open ended and could be extended if appropriate. For example, students could explore generalising for different sizes of grid, different shapes etc., bringing in more variables if appropriate.

Key vocabulary

Conjecture	Verify	Demonstrate
Prove	In terms of n	Simplify

Key questions

What does "in terms of n" mean?
What is the expression for the number to the right of n on a grid?
On a 10 by 10 gird, what is the expression for the number one row below n? What is the number two rows below n?
How would the expressions change for an 8 by 8 grid?

Exemplar Questions

Whitney is exploring the products of the opposite corners of 2 by 2 squares on a hundred square.

1	2	3	4	5	6	7	8	9	10
11	**12**	**13**	14	15	16	17	18	19	20
21	**22**	**23**	24	25	26	27	28	29	30
31	32	33	34	35	36	**37**	**38**	39	40
41	42	43	44	45	46	**47**	**48**	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

$12 \times 23 = 276$
$22 \times 13 = 286$

She conjectures the difference between the products will always be 10.
Check her result using other squares.

Whitney then generalises her findings.
She calls the top left number of the square n.
What will the other numbers be in terms of n?

n	?
?	?

Find the products of the algebraic expressions and prove whether Whitney's conjecture is correct.

Investigate the difference between the products of different sizes of square and rectangle on the hundred grid.
Test and prove your conjectures.

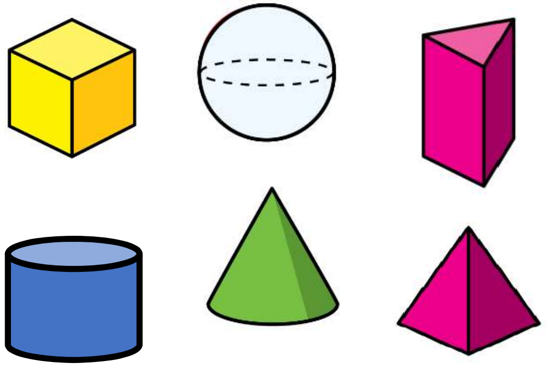
The shapes highlighted on the grid are T_{13} and T_{28}
The total of the numbers in T_{13} is 100
Find the total of the numbers T_{28}
Generalise your result.
What if the grid were 8 by 8?

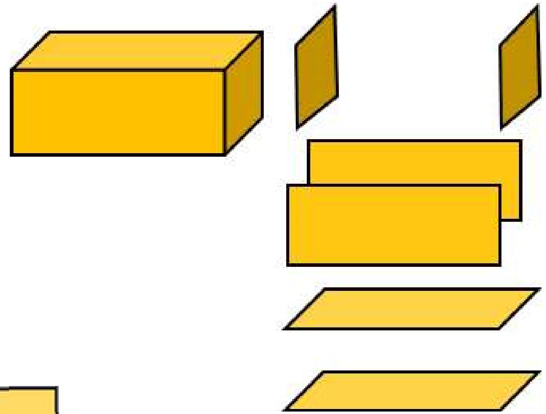
1	2	3	4	5	6	7	8	9	10
11	12	**13**	**14**	**15**	16	17	18	19	20
21	22	23	**24**	25	26	27	**28**	**29**	**30**
31	32	33	**34**	35	36	37	38	**39**	40
41	42	43	44	45	46	47	48	**49**	50

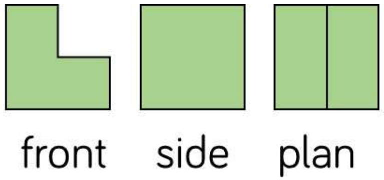
Autumn 2: Constructions in 2 and 3 Dimensions

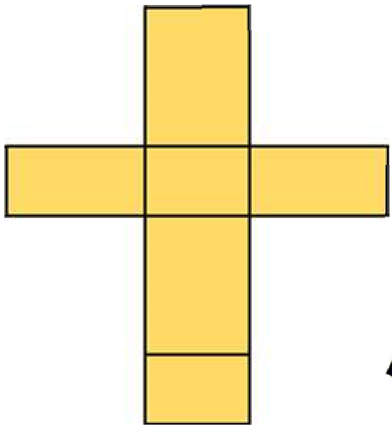
Weeks 1 to 3: Three-dimensional shapes	Weeks 4 to 6: Constructions and Congruency
This is the first time students have studied 3-D shapes formally at KS3, so they will need reminding about the associated vocabulary. Students could be supported by the use of practical equipment such as cubes, squared and isometric paper. As well as surface area and volume, students will also explore plans and elevations. There is a wide variety of software available to support this, and again practical work is very useful to develop visualisation and understanding. For students following the Higher strand, there is a step on investigating volumes of other 3-D shapes; as this is KS4 content this could be omitted if time is short. National Curriculum content covered includes: • use language and properties precisely to analyse numbers, algebraic expressions, 2-D and 3-D shapes • use the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres to solve problems in 3-D • derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, trapezia, volume of cuboids (including cubes) and other prisms (including cylinders)	This block builds on the constructions studied during Years 7 and 8 to formally look at the idea of a locus and the standard constructions using a straight edge and a pair of compasses. This is a very practical unit and it is useful to explore the loci using objects and rulers as well as the paper-based approach. Indeed 'human geometry' is a very engaging way of promoting understanding through e.g. asking students to all line up 2 m from a point or 2 m from a wall to explore the different loci formed. Congruency is also explored, again taking a practical approach to compare congruent figures of all kinds before looking at the formal aspect of identifying congruent triangles. National Curriculum content covered includes: • draw and measure line segments and angles in geometric figures, including interpreting scale drawings • derive and use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); recognise and use the perpendicular distance from a point to a line as the shortest distance to the line • describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons that are reflectively and rotationally symmetric • use the standard conventions for labelling the sides and angles of triangle ABC, and know and use the criteria for congruence of triangles

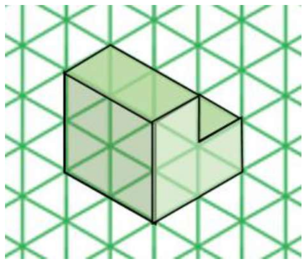
Key Representations

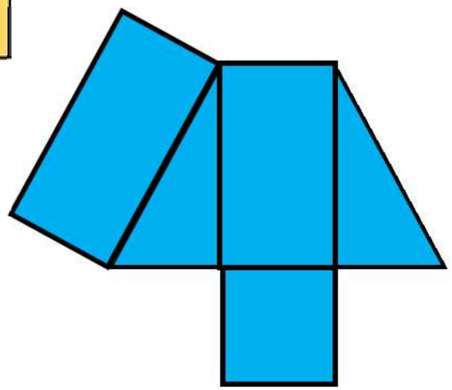












Drawing 3-D shapes can be a challenge for some students, so it is worth spending some time practising drawing cubes step by step. Isometric paper should be used after students have shown they can draw sketches. Nets of cubes and cuboids could be drawn on Excel for precision.

To distinguish a sphere from a circle, students could use a dashed ellipse as shown on the left.

An exploded diagram can be a good way of ensuring all sides are accounted for in surface area calculations.

Nets of triangular prisms may prove difficult to draw at this stage, given that students have not yet covered Pythagoras' theorem or constructions. This could be revisited in those blocks.

Three-dimensional shapes

Small Steps

- Know names of 2-D and 3-D shapes
- Recognise prisms (including language of edges/vertices)
- Accurate nets of cuboids and other 3-D shapes
- Sketch and recognise nets of cuboids and other 3-D shapes
- Plans and elevations
- Find area of 2-D shapes R
- Surface area of cubes and cuboids
- Surface area of triangular prisms

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered earlier in KS3

Three-dimensional shapes

Small Steps

- Surface area of a cylinder
- Volume of cubes and cuboids
- Volume of other 3-D shapes – prisms and cylinders
- Explore volumes of cones, pyramids and spheres H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered earlier in KS3

Names of 2-D and 3-D shapes

Notes and guidance

Students should be familiar with most names from earlier study, so this is a good opportunity to check their understanding of the vocabulary used for describing shapes. The key vocabulary of vertices, edges and faces should all be revisited when making descriptions. Discussion could focus on similarities and differences e.g. between a tetrahedron and square-based pyramid.

Key vocabulary

Dimensions		Cube/Cuboid	Cylinder
Cone	Sphere	Pyramid	Tetrahedron

Key questions

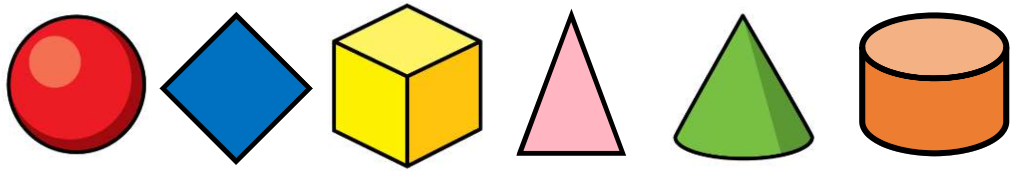
What is a dimension?

What are the main differences between 2-D and a 3-D shapes?

What 2-D shapes are the faces of (e.g.) a pyramid?

Exemplar Questions

Match each shape to its name.



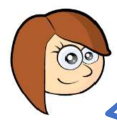
Circle Triangle Cuboid Cube
Cone Pyramid Square Cylinder

Some of the names do not have a matching picture.
Draw a sketch of each shape that does not have a match.

What is the same and what is different about the following pairs?
Think of at least two similarities and two differences for each pair.

- a circle and a sphere
- a cube and a cuboid
- a cylinder and a cone
- a triangular-based pyramid and a tetrahedron

Do you agree or disagree with Ron and Rosie?



Recognise prisms

Notes and guidance

Allowing students the opportunity to sort concrete 3-D shapes into prisms and non-prisms will help to explore and secure the language and properties of prisms.
The uniform constant cross-sectional face on a prism is defined to be a polygon. A cylinder does not have a polygonal face, but is considered to have similar properties and is therefore included in this set of 3-D shapes.

Key vocabulary

Face	Edge	Vertex
Polygon	Prism	Cross-section

Key questions

How do we know if a solid shape is a prism?

Do prisms have symmetry?

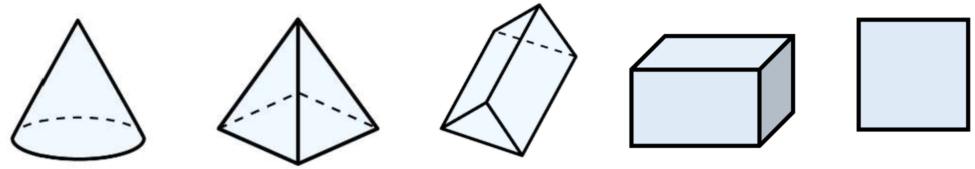
Is a cylinder a prism?

Which faces contain the constant cross-section?

How do you know that a shape is *not* a prism?

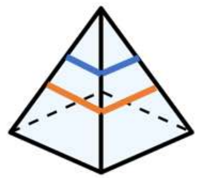
Exemplar Questions

Which of the following shapes are prisms?



What are the common features of the shapes that are prisms?
You could consider the faces, the cross section, the dimensions…

Brett slices the shape below across the blue and orange lines.

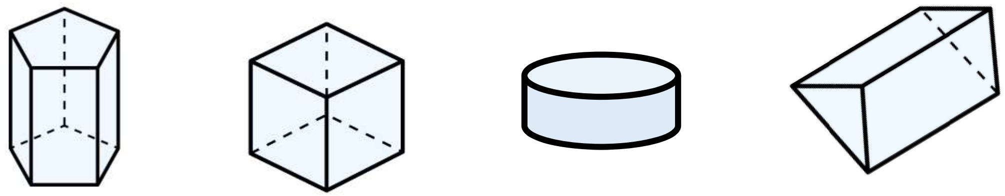


He notices that the cross sections are both squares and decides the shape must be a prism.
Explain why Brett is mistaken.

Write down the names for each of the shapes below.
Put them in order of

- number of faces (f)
- number of edges (e)
- number of vertices (v)

Does the formula $f + e - v = 2$ work for all the shapes?
What do you notice about the shape(s) it doesn't work for?



Accurate nets of cuboids

Notes and guidance

In this step, centimetre squared paper will help students to draw nets of simple cubes and cuboids. Cutting out pieces with the same dimensions as the faces of the 3-D shape then rearranging the pieces to make up the net gives them a good idea of which combinations work/don't work.
One of the simplest nets is a tetrahedron, which is easily folded from paper and drawn using isometric paper.

Key vocabulary

Face	Edge	Vertex
Net	Dimensions	

Key questions

How many faces does the 3-D shape have?

What are the dimensions of the faces?

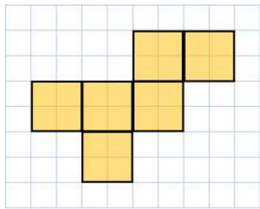
Is it a cube or a cuboid?

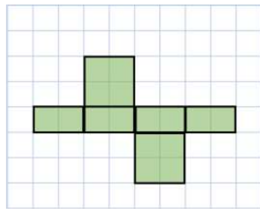
How many sides does each edge connect to?

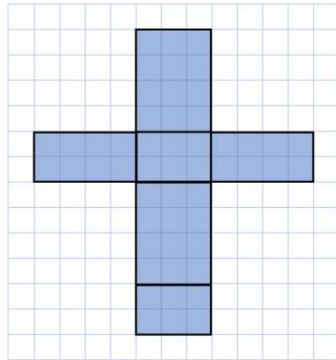
Is there more than one way of drawing the net?

Exemplar Questions

Here are the nets of some cuboids.
What are the dimensions of shapes?
What is the same/different about them?







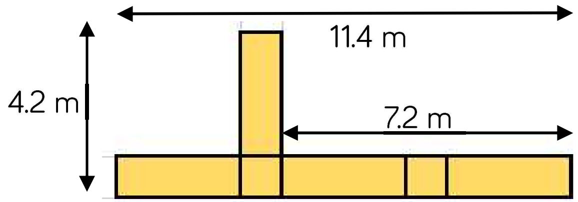
Draw accurate nets for each of the shapes.

- A 3 cm cube
- A cuboid with sides 3 cm, 3 cm and 4 cm
- A cuboid with sides 3 cm, 4 cm, 5 cm

Compare your nets with a partner's, did you draw the same net?
How many different nets are there for each shape?
For the nets that don't work, can you explain why they don't?

The opposite faces of a dice have a sum of 7
Draw an accurate net of a dice, labelling the faces.
Is there more than one way of doing this?

Here is the net of a cuboid. What are the dimensions of the cuboid?



Sketch and recognise nets

Notes and guidance

Encouraging students to deconstruct simple boxes will secure the connection between 3-D shapes and their nets. They can then move on to recognising and sketching nets, labelling the key lengths represented. They should be able to identify which edge joins with which when folding up a net. Practical exploration will again be useful here. The next block, 'Constructions and Congruency', could revisit this topic to draw truly accurate nets of shapes with triangular faces.

Key vocabulary

Face	Edge	Vertex
Net	Dimensions	Area

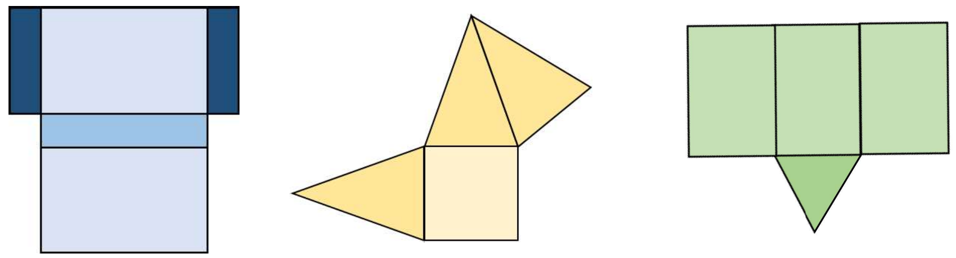
Key questions

What is the net of a shape?
How many faces does the 3-D shape have?
Which faces are 'connected'?
How many different nets are there for the shape?
If one of the lengths was increased, which parts of the net would change?

Exemplar Questions

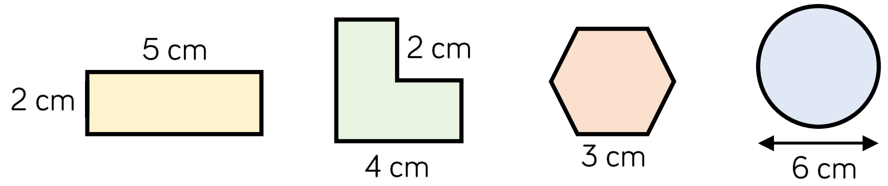
Draw the net of a 3 × 3 cm cube.
Compare it with a partner, did you both draw the same net?
How many different nets of a cube are possible?
Compare the number of edges of a cube to the number of joined faces on your net. What do you notice? Will this be the same for all nets?

One of the faces from each of the nets below has been forgotten. Copy them and draw the missing face. Is there more than one place the missing face could go?



Cuboid   Square-based pyramid   Triangular prism

Here are the cross-sections of 4 prisms, all of which are 5 cm long. Draw a net for each prism.



Plans and Elevations

Notes and guidance

As with most of the steps in this block, having the shapes in the classroom is an invaluable route to understanding this topic. Get students to move around the object instead of moving the object around to avoid confusion. Dynamic geometry software can also be very helpful here. Some students may not be familiar with isometric paper and will need help in determining how to orient and how to use this.

Key vocabulary

Plan	Front/Side Elevation	Face
Perspective	Isometric	Solid

Key questions

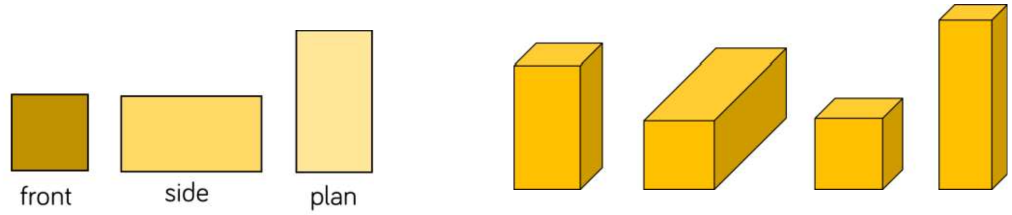
What can you see looking at the shape from the front/side/above?

Are there any parts of the shape you cannot see?

Why do you need to have three different perspectives to be able to construct the shape?

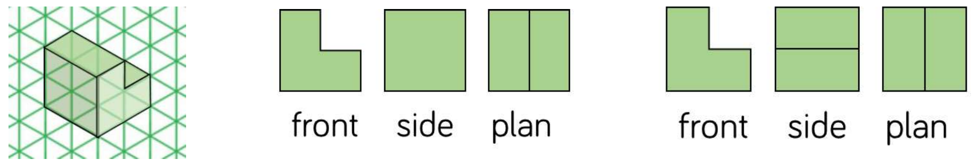
Exemplar Questions

Here are the front elevation, side elevation and plan of a cuboid.
To which of the cuboids on the right could they belong?



Whitney says the cuboid could be either of the first two.
Do you agree?

A 3-D shape has been drawn on isometric paper.
Give reasons why both of the sets of plans and elevations could represent the shape.



Build the shape and add one extra cube to it.
Draw the plans and elevations of your new shape and draw the new shape on isometric paper.

Take an object in your classroom. Draw it from above, this is the plan. Draw it from the side, this is the side elevation. Draw it from the front. See if someone else in the room can work out what the object is from just one of your drawings.

Find area of 2-D shapes

R

Notes and guidance

This review step is to support students with the forthcoming step on surface area; hopefully most will only need a quick reminder of these familiar formulae. Squares, rectangles, triangles and circles are the most common in nets, but you could remind students about trapezia as well. It might be worth doing some compound shapes, which could be the cross section of a prism. Remind students of the units, linking the power of 2 to the 2 dimensions.

Key vocabulary

Area	Perpendicular height	Units
Formulae	Compound	Dimensions

Key questions

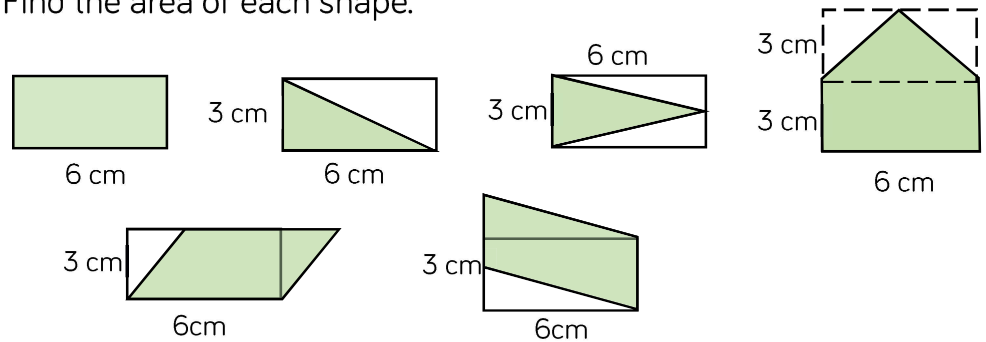
Which dimensions do you need in order to find the area of the shape?

What is the formula for the area of a circle?

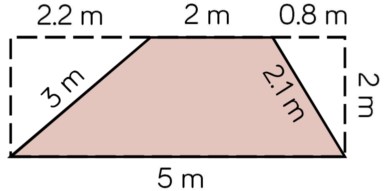
Is there more than one way of finding the area? Which method is most efficient?

Exemplar Questions

Find the area of each shape.

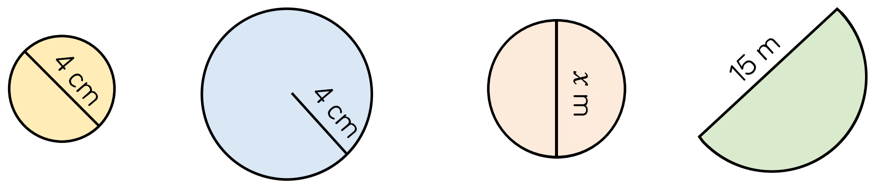


How many different methods for working out the area of the trapezium can you find?
Which lengths did you use for each method?



A piece of paper measuring 20 cm by 30 cm has a 5 cm square cut out from each corner. What is the area of the remaining piece of paper? Draw a diagram to show your working.

Find the area of the shapes below.
Write your answers in terms of π and to 3 significant figures.



Surface area of cubes and cuboids

Notes and guidance

Drawing nets in the first instance will ensure understanding of how to find the total surface of a cube or cuboid. They should also consider what's the same and what's different about finding the surface area of open and closed shapes.
3-D graphing software helps with the exploration of nets of cubes and cuboids and is well worth spending time to develop students' proficiency.

Key vocabulary

Faces	Area	Surface
Units	Dimensions	Open/Closed

Key questions

How many faces does a cuboid have?

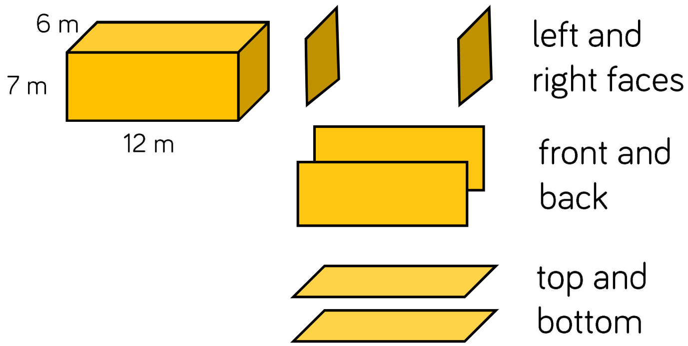
What is the same/different about the faces of cubes and cuboids?

What is an 'open' cube?

Exemplar Questions

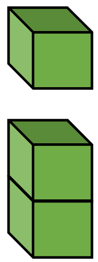
Draw an accurate net of a cube with sides 4 cm.
How many faces does your cube have? Are any of the faces the same?
What is the total surface area of the cube?

Draw an accurate net of a cuboid with sides 3 cm, 4 cm and 6 cm.
How many faces does your cuboid have? Are any of the faces the same?
What is the total surface area of the cuboid?



Use the exploded diagram to help you find the surface area of the cuboid.

What other methods could you use?



Find the surface area of a cube with sides 20 cm.

If you stack two of the cubes on top of each other, what will the new surface area be?

What if you stacked three cubes? Twenty cubes?

Surface area of triangular prisms

Notes and guidance

As with the previous small step, sketching nets will help students to decide which numbers to use in the required calculations. It is worth discussing why a sketch is sufficient rather than a full net. They should also be encouraged to set their solutions out in an organised fashion to help them to identify any errors. Providing both perpendicular heights and slanted lengths will avoid the use of Pythagoras' theorem, which will be covered in the Spring term.

Key vocabulary

Perpendicular height		Prism
Net	Faces	Surface Area

Key questions

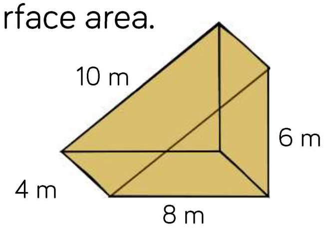
How many faces do all triangular prisms have?

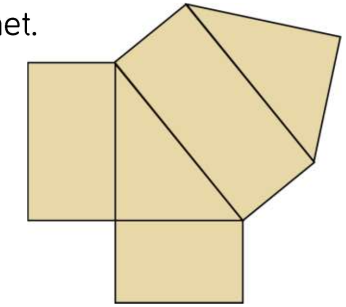
Are any of the faces the same?

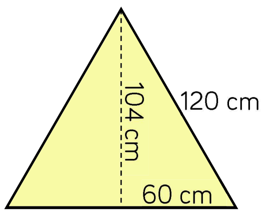
Which dimensions of a triangle are used to find its area?

Exemplar Questions

Here is a right-angled triangular prism and its net.
Label the lengths of all the sides on the net.
Are any of the faces the same?
Find the total surface area.



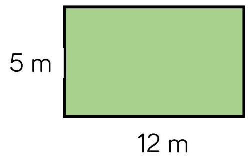


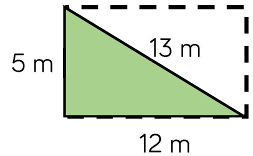


Here is the cross section of an equilateral triangular prism 50 cm long.
Draw a sketch of the net of the prism.
Find the area of the cross section.
Find the surface area.

Does doubling the length of the prism double its surface area?

Here is one face of a cuboid and one face of a right-angled triangular prism.





Both solids are 15 cm long.
Sketch a net of each prism and find their surface areas.
What is the same and what is different about the calculations?

Surface area of a cylinder

Notes and guidance

There is potential for error in this step as both the formulae for the circumference and area of a circle are needed to establish the surface area of a cylinder. Students could be challenged to explain which they need when. Practical wrapping or unwrapping of cylinders (e.g. labels of tin cans) will help them to understand the links. There are excellent animations of nets of cylinders available online.

Key vocabulary

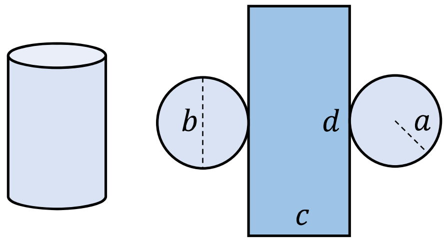
Circumference	Area	π
Curved surface area	Cylinder	

Key questions

How many faces does a cylinder have?
How many of the faces are the same?
How do you find the area of the curved face?
How do you find the circumference of a circle?
What does "in terms of π" mean? Is this form of answer exact or approximate?

Exemplar Questions

Here is a cylinder and its net.
Which of the statements are true?

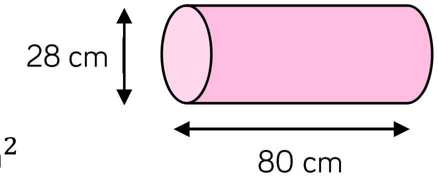


- a is the radius of the cross-section
- b is the width of the prism
- c is the height of the prism
- d is the circumference of the circular cross-section

Which part of the net is the curved surface of the cylinder?
How do you find the area of the curved surface?
Make a model of a cylinder to show how the circumference of the base is used to find the area of the curved surface area.

Explain each of the steps used to calculate the surface area of the cylinder shown.

$$28 \div 2 = 14$$
$$\pi \times 14^2 = 616$$
$$2 \times \pi \times 14 \times 80 = 7\,037$$
$$616 + 616 + 7\,037 = 8\,720 \text{ cm}^2$$



Find the surface area of a cylinder that is 40 cm wide and 40 cm high.
Give your answer to 3 significant figures and in terms of π.
If you double the radius of the cylinder, does the surface area also double? Justify your answer.

Volume of cubes and cuboids

Notes and guidance

Students have not revisited volume in a mathematical sense since KS2. Using a Frayer model will help students to develop a definition and better understanding of volume. Building with unit blocks will help students to understand the formula, especially since this step deals exclusively with cubes and cuboids. There are links to cube numbers and to the commutativity of multiplication. The volumes of compound shapes could also be explored.

Key vocabulary

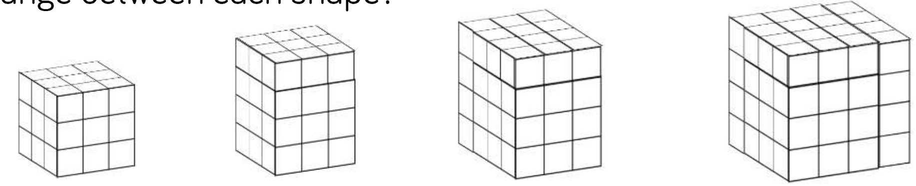
Cube	Cuboid	Height	cm^3
Width	Length	Commutative	

Key questions

What units are used to measure volume?
Can you build the shape using blocks?
What is the difference between a cube and a cuboid?
What is the difference between finding the volume and finding the surface area of a cube or cuboid?

Exemplar Questions

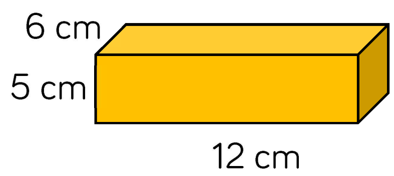
Build each of the shapes and find its volume. How does the volume change between each shape?



Which are cubes, and which are cuboids?
Use your models or the diagrams above to explain how each of these could be used to find the volume of a cuboid.

- length × width × height
- area of face × depth

Here is a block of butter. Explain why each of the following could be used to work out the volume.



- 30 × 12
- 60 × 6
- 72 × 5

Explain why the volume is not 360 cm^2.

Find the volume of a cuboid with sides 8 m, 10 m and 3 m.
One of the lengths of the cuboid is increased by 1 m. Which of the following could be the volume of the new shape?
Justify your answers.

- 270 m^3
- 264 m^3
- 248 m^3
- 320 m^3
- 241 m^3

Volume of other 3-D shapes

Notes and guidance

Students can establish that the volume of a 3-D shape is the product of the area of its cross-section and its length by comparing the volume of a right-angled triangular prism to that of a cuboid. They then generalise to all prisms and cylinders. It is worth exploring prisms in various orientations to discuss where the cross-section can be seen and whether we should multiply by height, length or depth depending on the context.

Key vocabulary

Constant	Cross-section	Height
Prism	Area of face	Length

Key questions

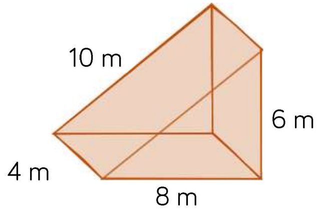
Which face is the constant cross-section?

What is the area of the constant cross-section?

What is the height of the prism? Could this also be called the depth of the prism?

Exemplar Questions

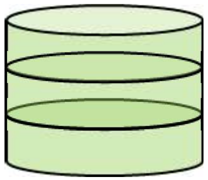
Here is a triangular prism. Explain the two methods used to find its volume.



Volume of cuboid $= 8 \times 6 \times 4 = 192$ m^3
Volume of prism $= 192 \div 2 = 96$ m^3

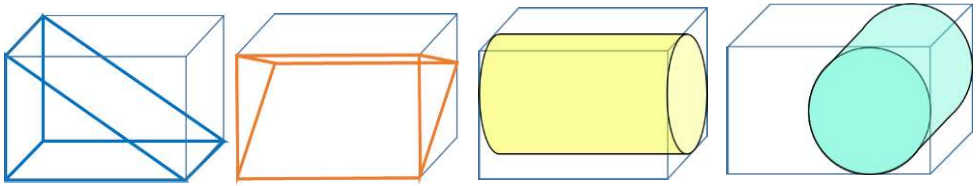
Area of cross-section $= \frac{6 \times 8}{2} = 24$ m^2
Volume of prism $= 24 \times 4 = 96$ m^3

Eva works out the volume of the cylinder.
Area of face $= \pi \times 5^2 = 78.5$ cm^2
Volume = area of face × height
$= 78.5 \times 1 = 78.5$ cm^2
Eva thinks this can't be right because the volume is the same as the area. Do you agree?



Three of the cylinders are stacked to make a new cylinder. What is the volume of this new shape?
Another stack has a volume of 1 099 cm^3.
How high is this stack?

The prisms are all enclosed in the same cuboid, sides 6.2 cm, 6.2 cm and 10 cm. Find the volume of each prism.



Cones, pyramids, spheres

H

Notes and guidance

This step will introduce volume formulae that will be studied in more depth at KS4, so it could be omitted if time is short. Students should be aware that the volume of a pyramid is $\frac{1}{3}$ × base area × height and note the similarity between the formula for the volume of a cone and that of cylinder. An interesting extension is to identify volume/area/length formulae by considering their dimensions.

Key vocabulary

Pyramid	Cone	Vertex
Sphere	Base	Perpendicular height

Key questions

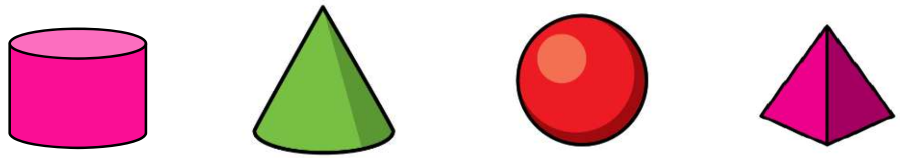
What is the perpendicular height of the pyramid/cone?

How do you identify a pyramid?

How many cones would fit into a cylinder of the same dimensions?

Exemplar Questions

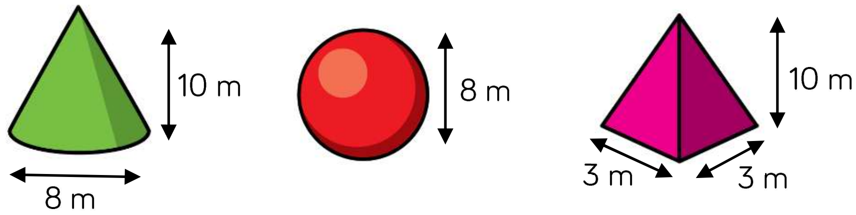
Match the volume formulae to the shapes.



$\frac{1}{3}b^2h$ $\frac{4}{3}\pi r^3$ $\frac{1}{3}\pi r^2h$ πr^2h

What does h stand for where it appears in the formulae?
Why does it not appear in the formulae for all of the shapes?

Sort these shapes in ascending order of volume.



Sketch a triangular-based prism that would be second in your list.
What dimensions did you give?
Sketch another shape with a volume that would rank third in the list.

A sphere has the same volume as a cube.
If the cube has sides 4 m, what is the radius of the sphere?
If the sphere has a radius of 4 m, what is the side of the cube?
What was the same/different about how you answered both of these questions?

Constructions & congruency

Small Steps

- Draw and measure angles R
- Construct and interpret scale drawings R
- Locus of distance from a point
- Locus of distance from a straight line/shape
- Locus equidistant from two points
- Construct a perpendicular bisector
- Construct a perpendicular from a point
- Construct a perpendicular to a point

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered earlier in KS3

Constructions & congruency

Small Steps

- Locus of distance from two lines
- Construct an angle bisector
- Construct triangles from given information (R)
- Identify congruent figures
- Explore congruent triangles
- Identify congruent triangles

H denotes Higher Tier GCSE content

R denotes ‘review step’ – content should have been covered earlier in KS3

Draw and measure angles R

Notes and guidance

Although this is familiar material, students need to practise both drawing and measuring angles in order to ensure accuracy. This step could be combined with other steps if students are confident, or included as a brief "skills check" activity in another lesson. The vital skill of estimation is important to regularly revisit, particularly if students are prone to using the wrong scale on a protractor.

Key vocabulary

Acute	Obtuse	Reflex
Right angle	Estimate	Protractor

Key questions

How do you line up the arms of an angle with a protractor? Where does the 0 go?

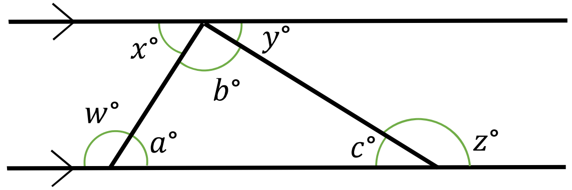
How do you know which scale to use on a protractor?

How can you classify an angle just by looking at it?

Exemplar Questions

Draw a line segment AB 5 cm long.
Add a line segment BC, 5cm long to your diagram so that ∠ABC = 110°

Now draw a line segment XY 5 cm long.
Add a line segment XZ 5 cm long to your diagram so that ∠YXZ = 110°



Which of the angles are acute and which are obtuse?
Which angle is most difficult to classify? Why?
Measure the angles with a protractor and compare your answers.

What's the same and what's different about drawing an angle of 100° and drawing an angle of 80°?
Describe how you can draw an angle of 310° using a 180° protractor.

Without using a protractor, try to draw angles of the given sizes.

- 60°
- 110°
- 200°
- 45°
- 5°
- 305°

Measure your angles and see how close you were.
Which were the most difficult to draw "by eye"? Why?

Scale drawings

R

Notes and guidance

Here we again review familiar material. Links can be made to ratio and enlargement, both of which are revisited later in the year. Students need to consider carefully whether to multiply or divide when performing calculations and a revision of unit conversions may also be needed. Using scale drawings to make estimates (see third exemplar question) is also a useful skill to practise.

Key vocabulary

Scale	Ratio	Multiplier
Estimate	Conversion	mm/cm/m/km

Key questions

How do the angles in a scale drawing relate to the angles in the real-life object?

How do you work with a scale such as 1 : 100 000 to work out lengths in different metric units?

Exemplar Questions

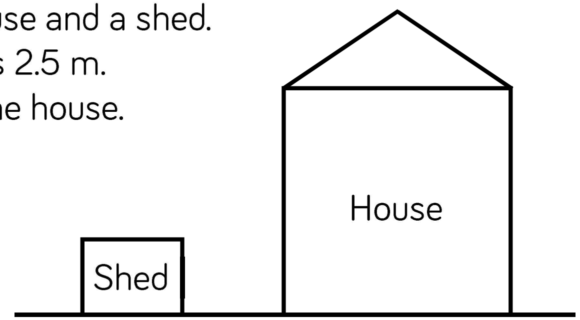
A plan of a classroom has a scale of 1 : 50
Find the measurements on the plan that would represent

- a student's desk of 50 cm by 100 cm
- a teacher's desk of 75 cm by 200 cm
- a storage cupboard 1 m by 2.5 m

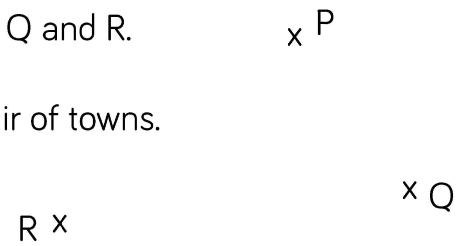
The plan measures 20 cm by 16 cm.
Find the area of the classroom, giving your answer in m^2.

A model boat has length 7 cm.
The scale of the model is 1 : 80
Work out the length of the real boat, giving your answer in metres.

The scale drawing shows a house and a shed.
The actual height of the shed is 2.5 m.
Estimate the actual height of the house.



The scale drawing shows three towns P, Q and R.
The scale is 1 : 150 000
Calculate the distance between each pair of towns.
Give your answers in kilometres.



Locus of a distance from a point

Notes and guidance

Students sometimes struggle with the concept of a locus as a set of points with a common property, so it is used consistently throughout the block to develop familiarity. Many of the loci in this block can be introduced through 'human geometry' activities such as asking students to stand 3 m from a central point, or by using counters as shown in the exemplar. As well as full circles, students should consider the locus of points at the same distance from e.g. a vertex.

Key vocabulary

Locus	Path	Equidistant
Construction lines		Point

Key questions

What do all the radii of a circle have in common?

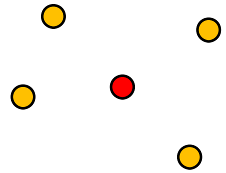
What's the difference between the shape of the locus of points 2 m away from the centre of the room and the shape of the locus 2m away from the corner of a room?

Exemplar Questions

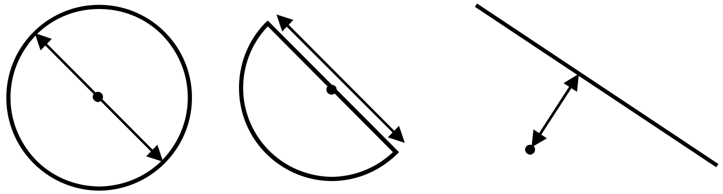
Put a red counter at the centre of your desk.

- Arrange a set of yellow counters so that each is 5 cm from the red counter.
- Arrange a set of yellow counters so that each is 10 cm from the red counter.

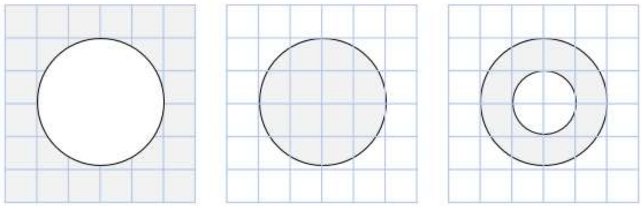
What's the same and what's different?



A point, P, moves so that it is always 6 cm from the point O
Which diagram(s) shows the locus formed by point P?



Match the description to the diagram.



Locus of points less than 2 cm from A	Locus of points more than 1 cm and less than 2 cm from A	Locus of points more than 2 cm from A

Locus of a distance from a straight line/shape

Notes and guidance

A sports stadium is a familiar context that helps students identify the locus of points at the same distance from a straight line, and the resulting shape is known as a stadium or a discorectangle. With a shape, careful consideration needs to be made about the vertices, and whether or not the points inside the shape should be included as well as those outside. Practice using compasses is important here.

Key vocabulary

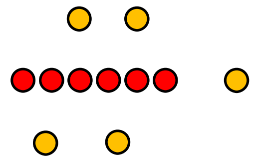
Locus	Equidistant	Path
Stadium	Vertex	Discorectangle

Key questions

What's the difference between the locus of the points 2 m away from a line and the locus of the points 2 m away from a point? How does this affect the point at the end of a line?

Exemplar Questions

Put a straight line of red counters at the centre of your desk.
Arrange a set of yellow counters so that each is 5 cm from the line of red counters.

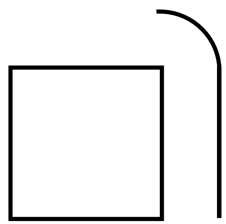


Describe the shape created by the yellow counters.

Tommy draws the set of points 1 cm from the line segment AB.

Which parts of Tommy's diagram are correct, and which are incorrect?
Draw a line segment AB 4 cm long and draw the correct locus of all the points 1 cm from the line AB.

Use a ruler, pencil and pair of compasses to draw the locus of the points 2 cm away from a square with sides 4 cm.
Part of the diagram is shown in the sketch.



Locus equidistant from two points

Notes and guidance

This step could be taught in conjunction with the next step, as when considering this locus, students can be guided to see that it forms the perpendicular bisector of the line segment that joins the two points. A useful introduction to this, as modelled in the first exemplar question, is to consider a series of equidistant points before deducing the general path and showing that is can be formed by considering two points.

Key vocabulary

Locus	Equidistant	Arc	Perpendicular
Bisector	Construction lines		Line segment

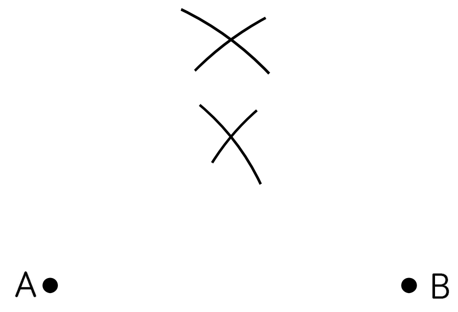
Key questions

What does equidistant mean?

If you draw circles of the same radius from the two points what do you notice about the points where the circles intersect?

Exemplar Questions

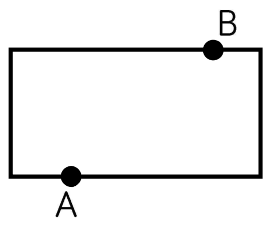
Mark two points A and B, 5 cm apart.
Use a pair of compasses to find a point 3 cm from A and 3 cm from B.
Then use a pair of compasses to find a point 4 cm from A and 4 cm from B.
Repeat for other distances.
What do you notice?



Draw a line XY 7 cm long.
Construct the locus of all the points 5 cm from X.
Construct the locus of all the points 5 cm from Y.
How many points on your loci are 5 cm from both X and Y?
Construct the locus of all the points that are 5 cm from both X and Y.

The rectangle is 6 cm by 4 cm.
A and B are both 1 cm away from the vertices of the rectangle.

- Construct the locus of the points equidistant from points A and B.
- Shade the points that are closer to A than B.
- Draw more rectangles and vary the positions of A and B. What do you notice? Can you predict where the region will be?



Construct a perpendicular bisector

Notes and guidance

This builds from the previous step. It is useful to use a visualiser to demonstrate the method of bisecting a line at right angles using only a pencil and a pair of compasses. Students should again practise with lines of different sizes and in different orientations. It might be worthwhile revisiting occasionally as a starter to help students to remember the technique. Higher strand students may have met this in Year 8

Key vocabulary

Perpendicular	Bisector	Arc
Construction lines	Line segment	

Key questions

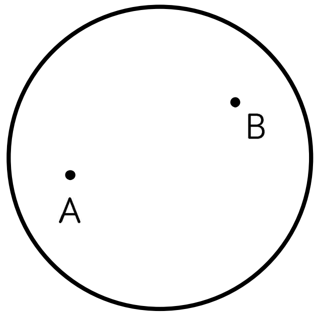
What does the prefix bi- mean? What does a bisector do?

What does perpendicular mean? How can we check whether a line is a perpendicular bisector?

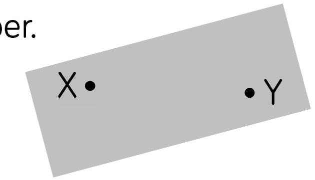
How is a perpendicular bisector connected to the diagonals of a rhombus?

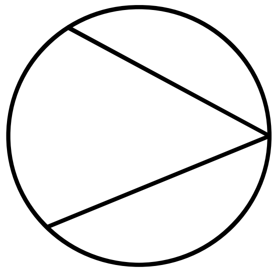
Exemplar Questions

Draw a circle and mark two points A and B inside it.
Find two points P and Q on the circle such that P and Q are equidistant from A and B.
Draw the line segments AB and PQ.
Mo says PQ is the perpendicular bisector of AB. Is he correct?



Mark two points X and Y on a piece of tracing paper.
Fold the paper so that X and Y coincide.
Open the paper.
What do you notice about the fold line?
On another piece of tracing paper mark three points X, Y and Z.
Fold the paper so that X and Y coincide then open out the paper.
Repeat with Y and Z and then X and Z.
What can you say about all three folds?





Mark three points on the circumference of a circle and join them to form two chords.
Draw the perpendicular bisectors of both chords.
Where do they meet?
Repeat with another circle and compare results as a class.

Construct a perpendicular from a point

Notes and guidance

Connections can be made between this construction and the previous and next steps, considering how they are the same and how they are different. You can also consider the shapes formed by joining the points involved in the construction e.g. linking the fact that PXQY in the first exemplar is a kite, and the diagonals of a kite meet at right angles. This is a good opportunity it revisit the area of a triangle, particularly if a vertex is beyond the end of the base.

Key vocabulary

Locus	Equidistant	Point	Path
Arc	Perpendicular	Construction lines	

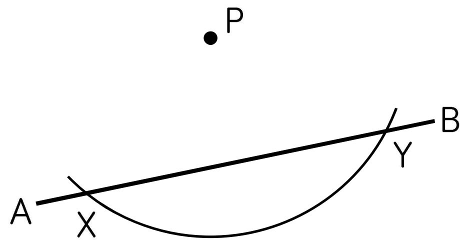
Key questions

Does it make a difference if the line segment is horizontal or vertical or neither?

What do you need to do to the line segment if the point you want to draw a perpendicular from is past the end point of the line segment?

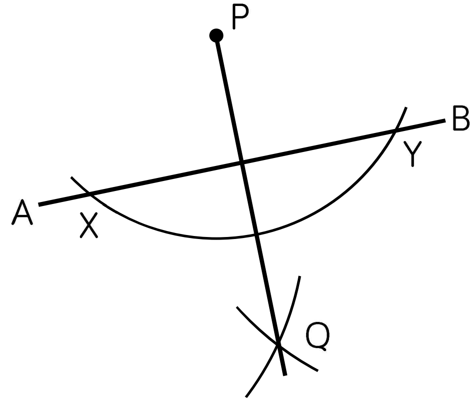
Exemplar Questions

Draw a line segment AB with a point P above it as shown.



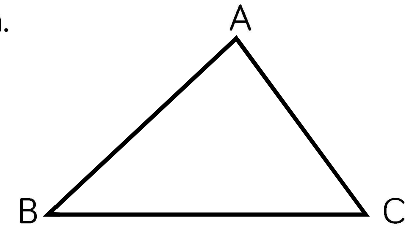
Use centre P, to draw an arc to cut AB at points X and Y.

Use centres X and Y to draw two arcs of equal radius that intersect at point Q.



Join the points P and Q and verify that PQ is perpendicular to AB.
Which radii need to be equal and which do not?

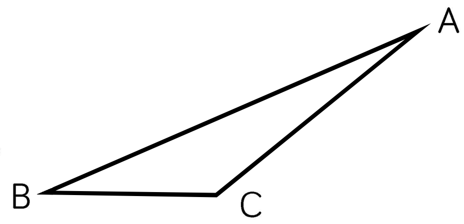
Draw a triangle ABC like the one shown.



Construct a perpendicular height A to the side BC using a ruler and pair of compasses.

Find the area of your triangle.

Repeat with a triangle like this one.
What's the same and what's different?



Construct a perpendicular to a point

Notes and guidance

As with the previous step, the first exemplar provided here demonstrates the method of construction. Students can be guided to observe that the shape XQY is an isosceles triangle, so PQ is perpendicular as it is the line of symmetry of the triangle. As with the previous construction, it may be necessary to extend the line, in this case to provide X and Y that are sufficiently far apart to make it easier to be accurate.

Key vocabulary

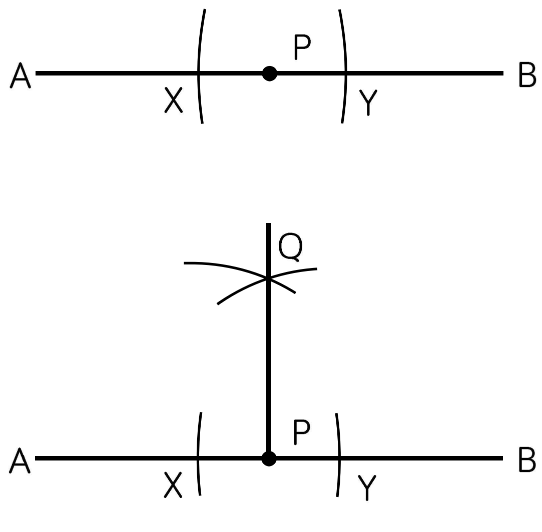
Locus	Equidistant	Point	Path
Construction lines		Perpendicular	

Key questions

When do you need to keep the radii of the arcs equal and when can they change?

What shapes can you make by joining the points from the construction arcs?

Exemplar Questions

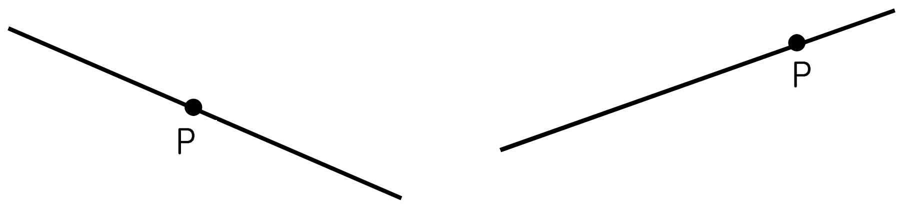


Mark a point P on a line segment AB.
Use centre P to draw two arcs of the same radius cutting AB at points X and Y.

Use X and Y as centres to draw two arcs of the same radius meeting at point Q.
Verify PQ is perpendicular to AB.

Do XP and XQ need to be the same length?

Use a ruler and compasses to construct the perpendicular to each line segment that passes through the point P.
You must show all construction lines.



Use diagrams with construction lines to explain the difference between constructing a perpendicular from a point and constructing a perpendicular to a point.

Locus of a distance from two lines

Notes and guidance

This step provides an introduction to the construction of an angle bisector and as such can be combined with the next step. Again it is modelled to be introduced through investigation so students can appreciate the similarities and differences between this and the perpendicular bisector. If students appreciate the structure of the constructions they are more likely to remember them and less likely to mix them up.

Key vocabulary

Locus	Equidistant	Point
Path	Construction lines	

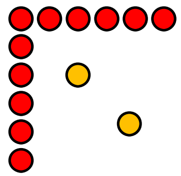
Key questions

What's the same and what's different about the locus of points equidistant from two points and the locus of points equidistant from two lines?

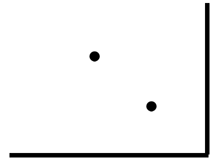
What does the locus do to the angle between the lines?

Exemplar Questions

Set up two rows of red counters perpendicular to each other.
Add some yellow counters that are the same distance from each set of red counters.



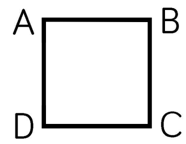
Repeat using points the same distance from a corner of a page from your book.



Investigate what happens if the two lines are not perpendicular to each other.

Draw a square ABCD of side 4 cm.
Draw the loci of the points that are
- equidistant from AB and AC.
- equidistant from BC and CD.

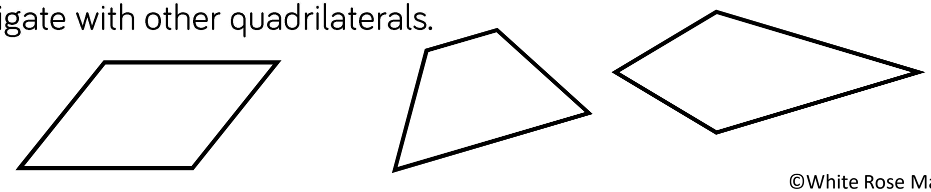
What can you say about the point the two loci meet?



Repeat with rectangle PQRS, where PQ = 6 cm and QR = 4 cm.
Draw the loci of the points that are
- equidistant from PQ and PS.
- equidistant from QR and RS.

Do you get the same result?

Investigate with other quadrilaterals.



Construct an angle bisector

Notes and guidance

This builds from the previous step to now formally bisect an angle. It is again useful to use a visualiser to demonstrate the method of bisecting an angle using only a pencil and a pair of compasses. Students should again practise with angles of different size in different orientations. As with the other constructions, revisiting in starters may help retention. Higher strand students may have met this in Year 8

Key vocabulary

Locus	Equidistant	Point	Path
Construction lines		Perpendicular	

Key questions

What's the difference between an angle bisector and a perpendicular bisector?

What's the difference between how you construct an angle bisector and how you construct a perpendicular bisector?

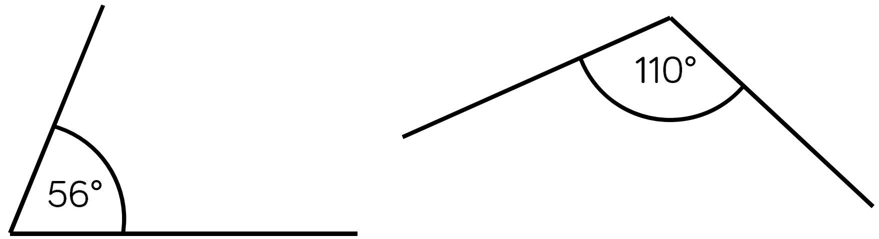
Exemplar Questions

Investigate Annie's claim.

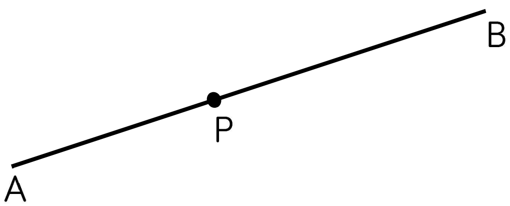
The locus of points which are equidistant from two fixed lines bisects the angle between the lines

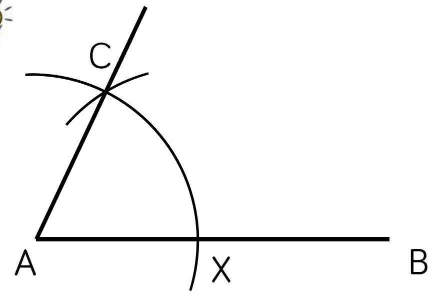
---

Use a protractor to draw the angles.
Use a ruler and compasses to bisect the angles, and check your accuracy with a protractor.



Construct a perpendicular to line segment AB passing through the point P.
Bisect the right angle formed to find a point Q so that $\angle APQ = 45°$





Draw an arc from A that cuts line segment AB at X.
Use the same radius to draw an arc from X to cut the first arc at C.
Bisect angle ACX and measure the result. Explain the answer.

Construct triangles

R

Notes and guidance

Students will be familiar with constructing triangles given SSS, SAS and ASA from Years 7 and 8. They can use their knowledge to construct accurate nets of triangular prisms and shapes made up of triangles, such as the rhombi described here. This can also be combined with previous steps such as using the equilateral triangle construction lines to create an angle of 60° and bisecting this for 30° etc., and combinations of these key values.

Key vocabulary

SSS	SAS	ASA	Net
Prism	Equilateral/Scalene/Isosceles		

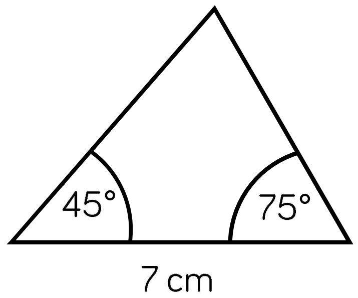
Key questions

What information do you need in order to complete the construction of a triangle?

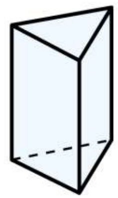
How many faces will the net of a triangular prism have? How many of them will be triangles? In drawing the net, which face should you draw first?

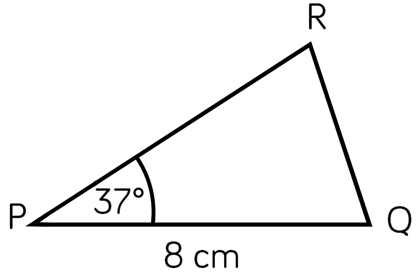
Exemplar Questions

Here is a sketch of a triangle.
Use a ruler and protractor to construct this triangle accurately.



Construct an accurate net for a prism 8 cm long with constant cross-section of an equilateral triangle side length 4 cm.





PQR is a triangle.
PQ = 8 cm
PR = 3 cm
$\angle QPR = 37°$
Make an accurate drawing of triangle PQR.

In each case, do you have enough information to construct a rhombus given the information shown?

- Side length 7 cm and length of diagonal is 9 cm
- Side length 7 cm and one angle is 110°

Construct the rhombi or explain why it cannot be done.

Identify congruent figures

Notes and guidance

Much work in congruency focuses on the conditions for congruent triangles, but students should also be able to recognise other congruent figures. Students sometimes think that reflection does not produce a congruent figure because of the change in orientation and this misconception is directly addressed here. This could be revisited when translation and rotation are covered later in the year, as again shapes remain invariant under these transformations.

Key vocabulary

Congruent	Identical
Invariant	Reflection

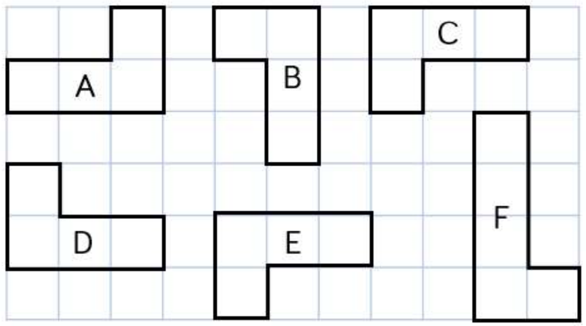
Key questions

Will a pair of congruent shapes have the same area? Will they have the same perimeter?

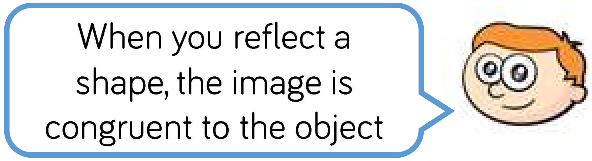
Does a pair of shapes need to be in the same orientation in order to be congruent?

Exemplar Questions

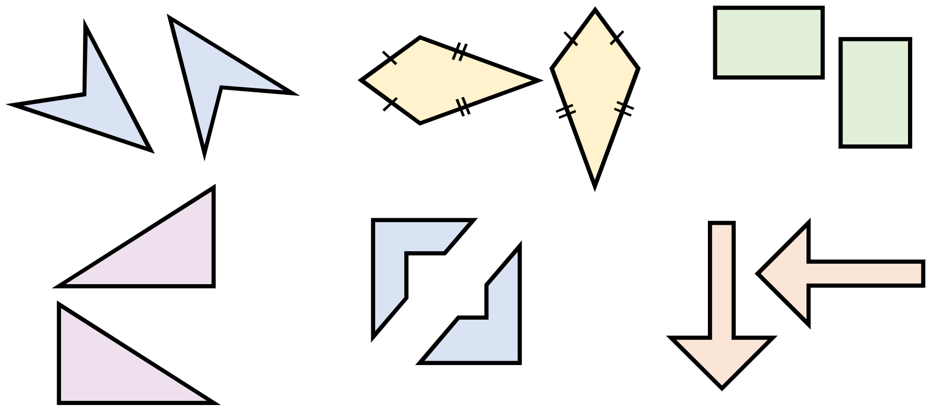
Which shapes are congruent to shape A?
How do you know?



Is Ron's claim always, sometimes or never true?
Give examples to justify your answer.



Which pairs of shapes are congruent? Explain your answers.



Explore congruent triangles

Notes and guidance

In this step we practise constructing triangles and observe which sets of information produce a unique triangle and which do not. The difference between “two sides and an angle” and “two sides and an included angle” is vital, and so the meaning of ‘included’ in this context angle needs careful consideration. The condition of right angle–hypotenuse–side can be revisited and justified when Pythagoras’ theorem is studied later in the year.

Key vocabulary

Congruent	Included Angle	Unique
Reflection	Corresponding side	

Key questions

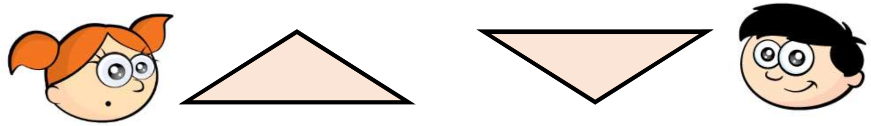
What does ‘included angle’ mean?

Will a side and two angles from that side always produce the same triangle?

Why does three given sides produce a unique triangle even though you could swap the point from which you draw the second and third sides?

Exemplar Questions

Alex and Dexter both construct an isosceles triangle with sides 8 cm, 6 cm and 6 cm.

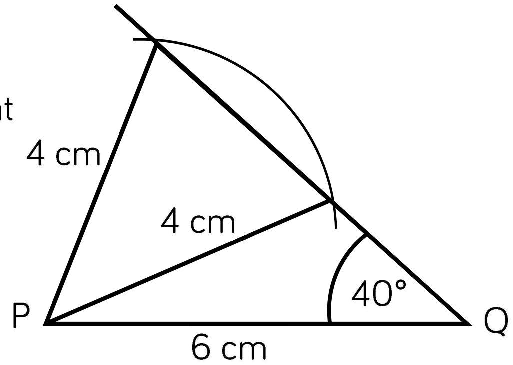


Are the triangles congruent? Explain why or why not.

Construct triangle ABC with AB = 5 cm, AC = 4 cm and $\angle ABC = 37°$.
Compare your triangle with a partner’s. Are they congruent?

Construct triangle PQR with PQ = 6 cm, PR = 4 cm and $\angle PQR = 40°$.
Compare your triangle with a partner’s. Are they congruent?

Use the diagram to explain why there is more than one possible triangle that can be constructed from the given information.



The angles in triangle ABC are equal to the angles in triangle XYZ.
Which statement do you agree with?

- ABC is definitely congruent to XYZ
- ABC could be congruent to XYZ
- ABC cannot be congruent to XYZ

Identify congruent triangles

Notes and guidance

The main purpose of this step is to recognise pairs of congruent triangles using the four sets of conditions, with more formal proof covered in Year 10. It is again worth exploring and reinforcing shape properties here, linking to symmetry and constructions. Students do not always recognise AAS when the third angle needs to be calculated, so this is explicitly looked at in the second exemplar question.

Key vocabulary

Side-side-side	Side-angle-side
Angle-side-angle	Right angle-hypotenuse-side

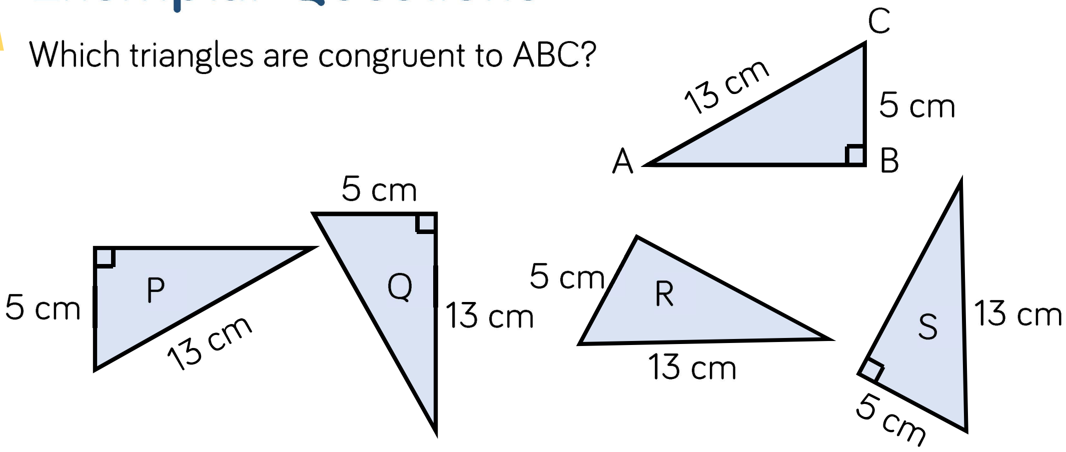
Key questions

How can you tell from markings on a pair of triangles which sides and which angles are equal?

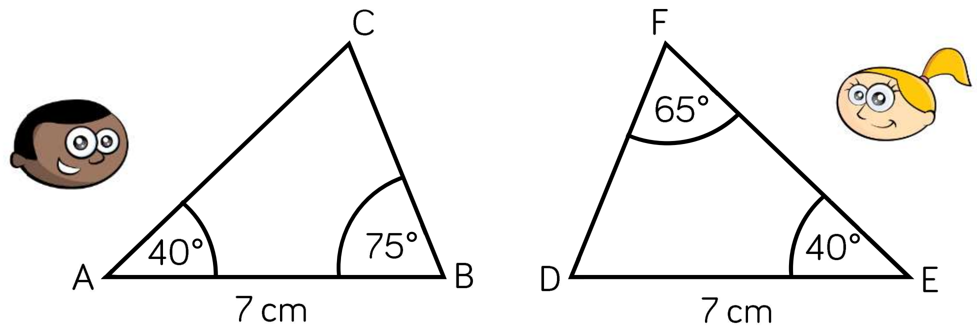
If you know two angles in a triangle, what else do you know?

Exemplar Questions

Which triangles are congruent to ABC?



Mo thinks that triangles ABC and DEF are congruent, but Eva thinks they're not. Who do you agree with? Show working to justify your answer.



Investigate the statements.

- Any diagonal of a rectangle splits it into two congruent triangles.
- Any diagonal of a kite splits it into two congruent triangles.
- Any diagonal of a parallelogram splits it into two congruent triangles.
- The diagonals of a rhombus split it into four congruent triangles.