Spring Term

Year 9

#MathsEveryoneCan

White Rose Maths

WRM – Year 9 Scheme of Learning

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Spring 1: Reasoning with Number

Weeks 1 and 2: Numbers

Students will develop their knowledge of the number system to include rational and real numbers, with the higher strand also looking at simple surds. The block provides plenty of opportunity for students to revisit and practise their number skills both with and without a calculator as necessary. Standard form and HCF/LCM are also revisited.

National Curriculum content covered includes:

- use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative
- use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation property
- interpret and compare numbers in standard form $A \times 10^n, 1 \leq n < 10$ where n is a positive or negative integer or zero
- appreciate the infinite nature of the sets of integers, real and rational numbers.

Weeks 3 and 4: Using Percentages

Building on their revision of fractions in the last block, students relate these to fractions and decimals, extending their learning in Year 8. All students will look at ‘reverse’ percentage problems with higher attainers stretched by looking at repeated percentage change. Both calculator and non-calculator methods are encouraged, with the use of decimal multipliers again key.

National Curriculum content covered includes:

- define percentage as ‘number of parts per hundred’, interpret percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%
- interpret fractions and percentages as operators
- solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics

Weeks 5 and 6: Maths and Money

Students practise their number skills in various financial contexts in this block. The language of financial mathematics, already introduced in Years 7 and 8 is further developed. Simple ideas of tax and wages are introduced, and the percentages studied in the last block are applied in various contexts including simple and compound interest.

National Curriculum content covered includes:

- solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics
- select and use appropriate calculation strategies to solve increasingly complex problems
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics

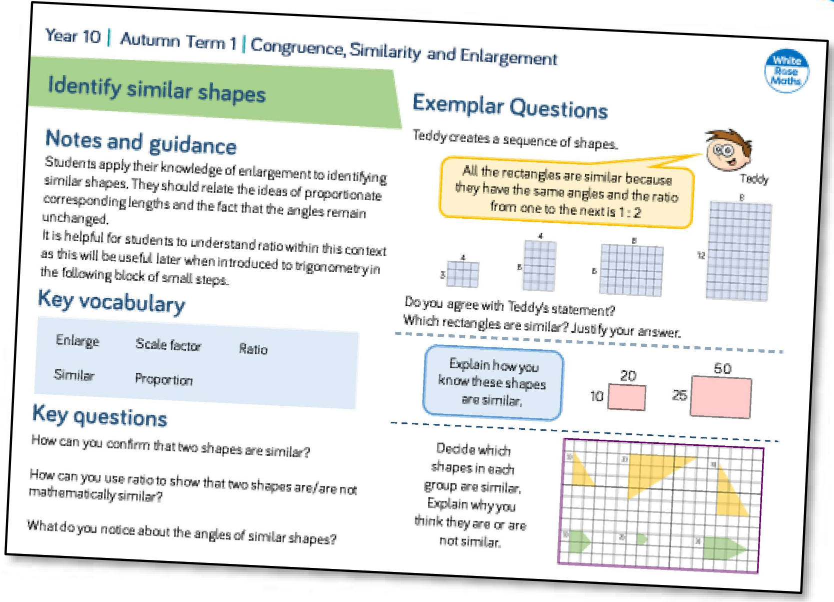
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.

It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge Scale factor Ratio

Similar Proportion

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

All the rectangles are similar because they have the same angles and the ratio from one to the next is 1 : 2

Do you agree with Teddy's statement?
Which rectangles are similar? Justify your answer.

Explain how you know these shapes are similar.

Decide which shapes in each group are similar. Explain why you think they are or are not similar.

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol 💡.
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered earlier in Key Stage 3 are labelled R.

Numbers

Small Steps

Integers, real and rational numbers	
Understand and use surds	H
Work with directed number	R
Solve problems with integers	
Solve problems with decimals	
HCF and LCM	R
Adding and subtracting fractions	R
Multiplying and dividing fractions	R

H denotes higher strand and not necessarily content for Higher Tier GCSE

R denotes ‘review step’ – content should have been covered earlier in KS3

Numbers

Small Steps

- Solve problems with fractions
- Numbers in standard form R

H denotes higher strand and not necessarily content for Higher Tier GCSE

R denotes 'review step' – content should have been covered earlier in KS3

Integers, real and rational numbers

Notes and guidance

Students will already be familiar with the word integer, and here this knowledge is built on by looking at rational and real numbers. It is useful to link rational to the root word ratio by noting that e.g. $\frac{2}{3}$ can result from the ratio 2 : 1 ($\frac{2}{3}$ of the whole) as well as comparing the parts in the ratio 2 : 3. Students sometimes think recurring decimals are irrational, so be prepared to address this misconception. Formal poof of irrational numbers is not needed at KS3

Key vocabulary

Integer
Real
Rational
Irrational
Root

Key questions

Why are all fractions rational numbers?

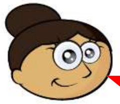
Can you find the square root of a negative number?

Are recurring decimals rational?

Exemplar Questions

Which of these numbers are integers?

18 1.8 −1.8 −1 −18 0



$\frac{1}{4}$ isn't an integer but $\frac{8}{4}$ is.

Do you agree with Dora?

Which of the equations have rational solutions?

$2x = 4$ $4x = 2$ $x^2 = 4$ $x^2 = 2$

$2x = 5$ $2x = 4.5$ $2x = \pi$ $2x^2 = 4$

Investigate whether the statements are always, sometimes or never true. Give examples and/or counterexamples to justify your answers.

- Integers are rational numbers
- Rational numbers are integers
- Rational numbers are real numbers
- Terminating decimals are rational numbers
- Recurring decimals are rational numbers
- The square roots of negative numbers are real, but not rational

Understand and use surds

H

Notes and guidance

Students are introduced to the idea of a surd as a number that cannot be simplified to remove a square root or cube root etc. This topic is explored in much more depth at KS4, with the focus here being on identifying simple surds (e.g. $\sqrt{7}$ is a surd but $\sqrt{81}$ is not) and manipulation of the form $\sqrt{8} = 2\sqrt{2}$ You may also wish to include examples of the form $\sqrt{\frac{9}{16}}$

Key vocabulary

Square root

Simplify

Cube root

Rational

Surd

Irrational

Key questions

Are surds rational or irrational?

What types of factors do you look for when simplifying a surd? Why?

Are all square roots surds? Why or why not?

Exemplar Questions

Which of these cards have integer values?

$\sqrt{25}$	$\sqrt{50}$	$\sqrt{125}$	$\sqrt[3]{125}$
$\sqrt{9 \times 4}$	$\sqrt{2 \times 32}$	$\sqrt{12 \times 4}$	$\sqrt[3]{9 \times 3}$

Find the value of each card. Give your answers to 3 significant figures.

$\sqrt{8}$	$3\sqrt{3}$	$2\sqrt{2}$	$\sqrt{32}$
$4\sqrt{2}$	$\sqrt{80}$	$4\sqrt{5}$	$\sqrt{27}$

Which cards are equal in value?

Mo says that $\sqrt{50} = 2\sqrt{5}$
Show that Mo is correct.

Copy and complete the workings to write $\sqrt{75}$ in simplest surd form.

$$\sqrt{75} = \sqrt{25 \times 3} = \sqrt{\square} \times \sqrt{\square} = \sqrt{\square}$$

Use the same method to write these in simplest surd form.

🟩 $\sqrt{18}$ 🟩 $\sqrt{12}$ 🟩 $\sqrt{500}$ 🟩 $\sqrt{72}$

Work with directed number R

Exemplar Questions

$-4-8=+12$ because two negatives make a positive.

Explain why Annie is incorrect.

Work out the calculations shown on the cards

$2+-5$	$2--5$	2×-5	-2×-5
$-2+-5$	$-2--5$	$2\div-5$	$-2\div-5$

$a=6$ $b=4$ $c=-2$

- How many expressions can you find using a, b and c that have a value of 12?
- How many expressions can you find using a, b and c that have a value of −20?
- What expression using a, b and c has the greatest value?
- What expression using a, b and c has the least value?

$x<y$

Are the statements always, sometimes or never true?
Give examples and/or counterexamples to explain why.

- $x+5<y+5$
- $x^2<y^2$
- $-x<-y$

Notes and guidance

This review step is intended to provide practice and keep skills sharp rather than introduce new content. As well as looking at calculations, you could also revisit the use of directed number in the contexts of equations and inequalities, particularly in the more challenging form when the coefficient(s) of the unknown(s) are negative. The third exemplar question is a good opportunity for exploration, and the numbers can be varied as suits the students in your class.

Key vocabulary

Positive	Negative	Directed
Inverse	Square	Cube

Key questions

What's the same and what's different about adding a negative number to another number and subtracting a negative number from another number?

Explain why x^2 is never negative.

Solve problems with integers

Notes and guidance

This is an opportunity for students to revise working with integers in any context you choose. You may need to refresh students' memories about the written methods for the four operations of arithmetic, comparing these with mental approaches or strategies such as dividing by 24 by dividing by 4 and then dividing the result by 6. As well as procedural fluency, students need to practise interpreting questions and choosing the correct operation.

Key vocabulary

Operation	Integer	Quotient
Product	Sum	Difference

Key questions

What mental methods of calculation do you know? How/why do they work?

How do you know whether to (e.g.) multiply or divide numbers to solve a problem?

Exemplar Questions

Which of these calculations can be done mentally, and which would you do using a written method?
Explain your choices and the methods you would use.

187 + 564	187 + 599	800 – 243	512 – 189
26 × 2000	124 × 73	3336 ÷ 8	4272 ÷ 24

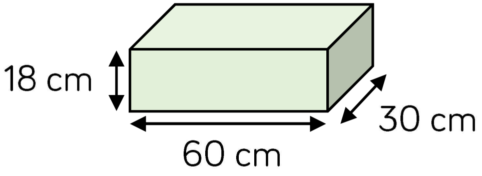
A cinema has 26 rows with 23 seats and 14 rows with 16 seats.
The seats cost £12 each.
What is the total amount of money from seat sales when the cinema is full?

360 students are going on a school trip.
For every 15 students, there must be 1 adult.
How many 52-seat buses are needed for the trip?

Rosie has to fill the cuboid with oil.

A bottle of oil contains 3 litres.
Each bottle costs £7

Rosie has £75



Does Rosie have enough money to fill the cuboid with oil?

Solve problems with decimals

Notes and guidance

Building from the last step, students revisit the formal methods of calculation for decimals, comparing these to mental methods where appropriate. Likewise, when the focus is on the correct choice of operation, students should be encouraged to use calculators to ease cognitive load. Dividing by a decimal can be compared to dividing by a fraction later in this block.

Key vocabulary

- Integer
- Adjust
- Decimal
- Compensate
- Remainder
- Operation

Key questions

What's the same and what's different about multiplying and dividing by decimals?

Why do 292 ÷ 0.4 and 2920 ÷ 4 have the same answer?

Why are multiplying and dividing by powers of 10 easy to do without a calculator?

Exemplar Questions

Which of these calculations can be done mentally, and which would you do using a written method?
Explain your choices and the methods you would use.

56.7 + 94	3.99 + 4.63	20 – 6.57	61.4 – 9.53
3.6 × 100	5.7 × 8.2	62 ÷ 2000	292 ÷ 0.4

Huda is going on holiday to Turkey.
The exchange rate is £1 = 8.56 lira
Huda changes £350 to lira.
Work out how many lira she should get.

Use the information that 8.7 × 54 = 469.8 to find the value of

- 8.7 × 540
- 8.7 × 0.54
- 469.8 ÷ 5.4

Filip travels to work by train. Here are some ticket prices options.

Monthly saver
£112.50

Day return
£7.65

He works 18 days a month and buys a day return ticket every day he works.
How much cheaper is it to buy a monthly saver ticket?.

The perimeter of a square is 26 cm.
Work out the area of the square.

HCF and LCM

R

Notes and guidance

Students may need to revise the language of factors, multiples and primes to support this step. You may also need to go over expressing a number as a product of its prime factors. Students could be encouraged to look for links between the HCF and LCM of different sets of numbers e.g. comparing 8 and 12 with 80 and 120. Sets with more than two numbers should also be explored.

Key vocabulary

Factor	Multiple	Common Factor/Multiple
Prime	HCF/LCM	Product of primes

Key questions

How do you express a number as a product of its prime factors?

How do you find the highest common factor of a set of three numbers?

Explain why the lowest common multiple of a set of numbers isn't always the product of the numbers?

Exemplar Questions

5	8	12	30	4
24	3	50	17	9

From the numbers in the rectangle choose

- A factor of 10
- A multiple of 15
- A common factor of 12 and 20
- A common multiple of 6 and 8

Explain why each the statements are incorrect.

- 7 and 13 are both prime so they don't have a common factor.
- The highest common factor of 60 and 80 is 10
- The lowest common multiple of 6 and 18 is 6

$840 = 2^3 \times 3 \times 5 \times 7$

$900 = 2^2 \times 3^2 \times 5^2$

- Work out the highest common factor of 840 and 900
- Write, in exponent form, the lowest common multiple of

840 and 900 840 and 1800 8400 and 900

Buses to Newtown leave a bus station every 24 minutes.
Buses to Oldtown leave the same bus station every 40 minutes.
A bus to Newtown and a bus to Oldtown both leave the bus station at 8 am.
When will a bus to Newtown and a bus to Oldtown next leave the bus station at the same time?

Add and subtract fractions R

Notes and guidance

Students may need concrete or pictorial support to remind them how the processes for adding and subtracting fractions work. Encourage them to use estimation to get a sense of the size of their answers and to compare their estimates with their calculated answers. They should also be familiar with using the fraction key on their calculators and could use this to check their answers to questions in this step.

Key vocabulary

Fraction	Numerator	Denominator
Mixed number	Common denominator	

Key questions

Why do you need to find a common denominator in order to add or subtract fractions?

How do you convert an improper fraction to a mixed number?

When can you check the answer to a fraction calculation by converting to decimals?

Exemplar Questions

Is the statement true or false?

> "To find the sum of two fractions, you add their numerators and add their denominators"

Give an example or counterexample to support your answer, illustrating with a diagram why it is true or false.

For each card, find some possible pairs of values for a and b.

$a + b = \frac{3}{4}$ $a - b = \frac{3}{5}$ $a + b = 2\frac{1}{4}$

Can you find fractions a and b that have the same denominator?

Can you find fractions a and b that have the different denominators?

Can you find fractions a and b that are both improper?

Can you find integer values for a or b or both?

Dexter thinks the answer to $0.3 + \frac{1}{3}$ is 0.6

What mistake do you think Dexter has made?

Explain how to find the answer to $0.3 + \frac{1}{3}$ without using a calculator.

Compare these methods for finding the sum of $5\frac{3}{4}$ and $9\frac{2}{5}$

- Change to improper fractions
- Add the integer and fractional parts separately
- Convert the numbers to decimals
- Use the fraction key on a calculator

Find the difference between $5\frac{3}{4}$ and $9\frac{2}{5}$

Multiply and divide fractions R

Notes and guidance

Students can often mix up the procedures for multiplication and division of fractions both with each other and with those for addition and subtraction. Refer back to the diagrams used in their earlier study to support students' understanding. Estimation is again a useful tool for checking whether an answer is sensible or not. Including examples that feature all four operations will challenge students to think about each step rather than leap to a procedure.

Key vocabulary

Fraction	Numerator	Denominator
Mixed number	Improper fraction	

Key questions

What does the word reciprocal mean?

Draw a diagram to show why the method of multiplying two fractions together works.

When does multiplication/division by a fraction result in an answer greater than/less than the original number?

Exemplar Questions

Is the statement true or false?

> "To find the product of two fractions, you multiply their numerators and multiply their denominators"

Give an example or counterexample to support your answer, illustrating with a diagram why it is true or false.

Work out the calculations.

$7 \times \frac{1}{3}$	$7 \times \frac{2}{3}$	$7 \times 2\frac{1}{3}$	$7\frac{1}{2} \times 2\frac{1}{3}$
$7 \div \frac{1}{3}$	$7 \div \frac{2}{3}$	$7 \div 2\frac{1}{3}$	$7\frac{1}{2} \div 2\frac{1}{3}$

What's the same and what's different about how you approach them?

What's the same and what's different about finding the numbers in these two questions?

- Three-quarters of a number is 60. What is the number?
- What number is three-quarters of 60?

- Write down the reciprocal of $\frac{7}{12}$
- Show that $\frac{3}{8} \div \frac{7}{12} = \frac{9}{14}$

Solving problems with fractions

Notes and guidance

This step puts together the skills of the last two steps in context questions, where students have to choose, as well as perform, the correct operation(s). It can be helpful to think what approach would be taken if the numbers were integers and then use this approach with the given fractions. Estimation and checking with a calculator are still to be encouraged. You can choose examples from other areas of the curriculum as appropriate for further practice.

Key vocabulary

- Operation
- Denominator
- Fraction
- Mixed number
- Numerator
- Improper

Key questions

What's the first step you need to take to solve this problem?

What's the same and what's different about solving this problem if the numbers were integers rather than fractions?

How can you check you answer?

Exemplar Questions

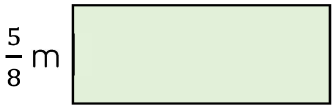
The area of an A4 sheet of card is $\frac{1}{16}$ m^2.
The card has a mass of 240 g per m^2.
Work out the total mass of 20 sheets of A4 card.

Lunchtime is $\frac{3}{4}$ hour.

Eva spends $\frac{2}{3}$ of this playing football.

What fraction of an hour does Eva spend playing football?

The perimeter of the rectangle is $3\frac{1}{2}$ m.



Find the area of the rectangle.
Give your answer as a mixed number.

A piece of wood is $4\frac{1}{2}$ m long.

How many lengths of $\frac{3}{8}$ m can be cut from the piece of wood?

How much wood is left over?

What fraction of the original length of wood is left over?

Nijah makes purple paint by mixing $2\frac{1}{3}$ litres of blue paint with $1\frac{3}{4}$ litres of red paint.
How much purple paint does she make?
She uses $\frac{4}{5}$ of the purple paint. How much does she have left over?

Numbers in standard form

R

Notes and guidance

Students looked at standard form in depth in Year 8. This small step provides opportunity to revisit both conversion to/from standard form as well as solving problems both with and without a calculator. When working without a calculator, students need to be careful when answers such as 12×10^6 or 0.5×10^6 arise, and may need reminding of how to convert these into standard form.

Key vocabulary

Standard form
Exponent
Power
Million
Index
Billion

Key questions

When is a number in standard form?
How can you convert a number to/from standard form?
What keys do you need to press on your calculator to enter a number in standard form?
What's the difference between a million and a billion?

Exemplar Questions

Explain why $3 \times 10^4 > 4 \times 10^3$

Without using a calculator, find the values of the cards in standard form.

$(6 \times 10^5) \times (8 \times 10^4)$

$(6 \times 10^5) - (8 \times 10^4)$

$\frac{4.5 \times 10^6}{5000}$

$(5 \times 10^3)^2$

$\frac{9 \times 10^{-2}}{3000}$

$(8 \times 10^{-5}) \times (2 \times 10^3)$

$(7 \times 10^{-3}) + (8.5 \times 10^{-4})$

A sheet of paper is 8×10^{-3} cm thick.
Amir wants to put 500 sheets of paper into the paper tray of his printer.
The paper tray is 3.5 cm deep.
Is the paper tray deep enough for 500 sheets of paper?
Explain your answer.

The distance from Earth to Mars is about 66.3 million km.

- Write 66.3 million km in standard form.

The distance from Earth to Pluto is about 5.12 billion km.

- Write 5.12 billion km in standard form.

How many times further away from Earth than Mars is Pluto?
Give your answer in standard from correct to 2 significant figures.

Using percentages

Small Steps

- Use the equivalence of fractions, decimals and percentages R
- Calculate percentage increase and decrease R
- Express a change as a percentage R
- Solve 'reverse' percentage problems
- Recognise and solve percentage problems (non-calculator)
- Recognise and solve percentage problems (calculator) R
- **Solve problems with repeated percentage change** H

H denotes higher strand and not necessarily content for Higher Tier GCSE

R denotes 'review step' – content should have been covered earlier in KS3

FDP equivalence

R

Notes and guidance

Students could begin this review step by exploring the fraction, decimal and percentage conversions that they know and use these to derive new ones e.g. using $\frac{1}{4} = 25\% = 0.25$ to find $\frac{1}{8} = 12.5\% = 0.125$

They could then revise formal methods of conversion through multiplication and division by 100 and simplifying fractions. Number lines and bar models are useful supports.

Key vocabulary

Fraction	Decimal	Percentage
Convert	Equivalent	

Key questions

Can any percentage under 100% be changed to a decimal or a fraction? What about percentages greater than 100%?

Describe the steps you need to convert a percentage to a fraction.

Why do you divide by 100 to find the decimal equivalent of a percentage?

Exemplar Questions

How many of these fractions, decimals and percentages can you convert into another form because they are “known facts”?

50%	$\frac{1}{3}$	0.2	$\frac{3}{4}$
$\frac{2}{5}$	30%	$\frac{1}{8}$	0.25

What strategies might you use to convert the others?

0.34 is the same as 34%
So 0.2 is the same as 2%
and 0.164 is the same as 164%

- Explain why Dora is wrong.
- Describe how to convert from a decimal to a percentage.

$$85\% = \frac{85}{100} \overset{\div 5}{\underset{\div 5}{=}} \frac{17}{20}$$

Convert the percentages to fractions in their simplest form.

- 35%
- 62%
- 84%
- 19%
- $3\frac{1}{2}\%$

Percentage increase/decrease R

Notes and guidance

This step is a reminder of the multiplier method used in Year 8 Students may still prefer to find a percentage and then add/subtract from the original, but familiarity with multipliers will make work with reverse percentages much easier. Drawing bar models to show the relationship with the original is useful here. It is worth discussing the different multipliers needed between increasing or decreasing by e.g. 40%, 4% and 0.4%

Key vocabulary

Increase
Reduce
Decrease
Multiplier

Key questions

When finding a percentage increase, will a multiplier be more or less than 1? Why?

Can a number be increased by more than 100%?

Why is the multiplier to decrease by e.g. 35% not 0.35?

Exemplar Questions

Which is the correct multiplier to increase a number by 4%?

1.4	0.4	1.04	0.04

Which is the correct multiplier to decrease a number by 4%?

96	9.6	0.96	0.096

Miss Chang's salary is £28 000 a year.
She get's a 12% pay rise.
Work out her new salary.

A television set is priced at £1200
In a sale the price is reduced by 7.5%.
Work out the sale price of the television set.

Mo sees a special offer in a shop.

Phone	£940
Headphones	£180

Buy together and get a 5% discount

Mo buys both items.
How much does he pay?

I think of a number. I increase my number by 20% and then decrease my answer by 20%. Is my final answer smaller than or greater than my starting number? By what percentage?

Change as a percentage R

Notes and guidance

You may need to revise writing one number as a percentage of another as a precursor to this step. Students may also need reminding that percentage change should be found as a comparison to the original rather than final amount. Problems can be tackled on a case-by-case basis, but the general result $\frac{\text{Change}}{\text{Original}} \times 100\%$ may also be useful.

Key vocabulary

- Profit
- Original
- Loss
- Change

Key questions

What is the difference between profit and loss?

How can you express the change as a fraction? How can you then convert the fraction to a percentage?

What degree of accuracy is appropriate for expressing the percentage change?

Exemplar Questions

Last year Seb paid £568 for his car insurance.
This year he has to pay £715 for his car insurance.
Work out the percentage increase in his car insurance.
Give your answer to 1 decimal place.

Dani buys 3 kg of sweets for £12
She puts the sweets into bags, with 120 g of sweets in each bag.
She sells each bag of sweets for 60 p.
Work out Dani's percentage profit.

The table shows the estimated world tiger population since 1970

Year	Population
1970	37 500
1980	28 000
1990	12 500
2000	6300
2010	3200
2020	3900

- Work out the percentage decrease in the number of tigers between 1970 and 2010
- Work out the percentage increase in the number of tigers between 2010 and 2020
- In which decade was the greatest percentage change?

Reverse percentage problems

Notes and guidance

It is worth approaching this step through discussion of related facts building from e.g. “Given 100% what other percentages can be found?” to, “Given 40%...” and then, “Given 120%...” Bar models are then useful to illustrate what percentage there is after a change and what approach to take forward. Including examples which can be solved without a calculator and relating these to the multiplier method can then lead into problems with more complex arithmetic.

Key vocabulary

Original

Change

Increase/Decrease

Reverse

Related facts

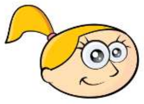
Key questions

What’s different about finding an original amount and finding a percentage increase or decrease?

How could a bar model be used to represent this problem?

Is the answer to the problem going be greater than or less than the given value? How do you know?

Exemplar Questions

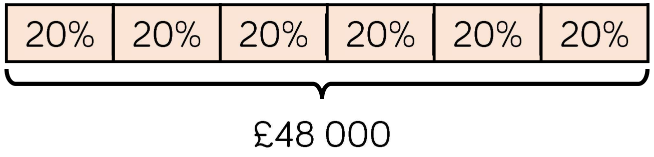


40% of my number is 12

150% of my number is 45

Show that Eva and Amir are thinking of the same number.

After a 20% pay rise, Jez earns £48 000 a year.



Explain how the bar model can be used to help work out how much Jez earned before the pay rise.

May earns £40 000 a year after a 20% pay cut.
How much did May earn before the pay cut?

What’s the same and what’s different about solving these problems?

The price of a train season ticket increased by 6%.
The price of the ticket increased by £156
Work out the price of the ticket before the increase.

The price of a train season ticket increased by 6%.
The price of the ticket after the increase is £3021
Work out the price of the ticket before the increase.

Non-calculator percentage problems

Notes and guidance

There is no “new maths” in this step, but students have the opportunity to develop their reasoning skills by recognising the type of problem and selecting the correct approach. Include problems with relatively simple numbers first advancing to those that require more complex working with division and/or fractions. Although this is also a good point to practise non-calculator skills, the key point is interpreting questions, so do not let arithmetic be a barrier.

Key vocabulary

Increase
Change
Decrease
Bar model
Original

Key questions

How can you tell what approach to take to a percentage problem? What are the clues in the questions?

What would a bar model representing this problem look like? Does this help you to choose the first step in finding the solution?

Exemplar Questions

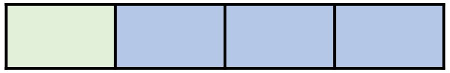
Which is greater? Justify your answer.

35% of 60

41% of 50

In a sale, normal prices are reduced by 15%.
The normal price of a book is reduced by £2.40
Work out the normal price of the book.

There are some teachers and students at a school football match.
There are 120 students at the football match.
25% of the people at the football match are teachers.
Work out the total number of people at the football match.
You may use the bar model to help you.



Rosie buys a pack of 20 bottles of water.
The pack costs £10
Rosie sells 12 of the bottles for 80 p each and the rest of the bottles for 20 p each.
What percentage profit or loss does Rosie make?

In the last 10 years, the population of a village has decreased by 30%.
The population of a village is now 560
What was the population of a village 10 years ago?

Calculator percentage problems

Notes and guidance

Building from the last step, students now tackle problems with greater arithmetical demand. The focus remains on recognising whether they need to find an increase, a decrease, an original value or to express a quantity as a percentage. Bar models can again be used, and the earlier focus on the multiplier method should pay dividends. A supporting strategy is to consider how to solve a problem with easier numbers and then apply the same approach.

Key vocabulary

Increase

Change

Decrease

Bar model

Original

Key questions

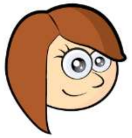
How can you tell what approach to take to a percentage problem? What are the clues in the questions?

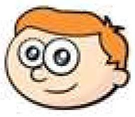
What would a bar model representing this problem look like? Does this help you to choose the first step in finding the solution?

What multiplier do you need to use?

Exemplar Questions

Todd buys a house for £170 000
He sells the house for £185 000
What percentage profit does Todd make?





I scored 80% in the test.

I got 50 of the 65 marks.

Did Rosie or Ron do better in the test?

The cost of a litre of petrol increased by 12%.
A litre of petrol then cost 145.6 p.
Work out the price of a litre of petrol before the increase.

Amina buys a television for £675
She pays a 20% deposit and then the rest of the money is paid in monthly payments of £45
How many monthly payments will Amina need to pay?

A school only has students in Years 7, 8 and 9. The tables shows the number of students at school one day.
Did the school meet it's 96% attendance target?
Did any of the year groups meet the target?

Year	Present	Absent
7	172	12
8	175	7
9	168	8

Repeated percentage change H

Notes and guidance

Students can explore the effect of repeated change and may need to be guided to notice that the effect is not linear. Comparing repeated increases with the much quicker multiplication by e.g. 1.25^n is useful to highlight the efficiency of the latter method. This will be revisited when we study compound interest in the next block. Students could also use trial and improvement to find how long e.g. a repeated increase of 10% would lead to doubling.

Key vocabulary

Repeated
Depreciate
Change
Power/Index/Exponent
Multiplier

Key questions

Explain why increasing a number by a percentage and then decreasing the answer by the same percentage doesn't get you back to the original number.

What's a quick way of multiplying by the same number twice? Three times? Four times?

Exemplar Questions

A shop decreases all its prices by 25% and then by a further 25%.

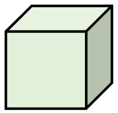
Show that Annie is wrong.

There are 5000 bacteria in a test tube.
The number of bacteria increase by 10% each hour.
How many bacteria will there be in the test tube after

- 1 hour
- 2 hours
- 3 hours?

Find the percentage increase in the number of bacteria after 3 hours.

A cube's length is increased by 15%.
Find, to 3 significant figures, the increase in the cube's volume.



Another cube's volume is increased by 15%.
Find, to 3 significant figures, the increase in the cube's length.

A car's value is decreasing by 10% each year.
Which of the statements are true and which are false?

The car will be worth nothing in 10 years' time.

It will take 7 years for the car to lose half its value.

After 3 years, the car is worth 72.9% of its current price.

Maths and Money

Small Steps

- Solve problems with bills and bank statements
- Calculate simple interest
- Calculate compound interest
- Solve problems with Value Added Tax
- Calculate wages and taxes
- Solve problems with exchange rates
- Solve unit pricing problems

H denotes higher strand and not necessarily content for Higher Tier GCSE

R denotes ‘review step’ – content should have been covered earlier in KS3

Bills and bank statements

Notes and guidance

Students should be familiar with the language of financial mathematics from their learning in previous years, but they may need a reminder. They could practise using calculator and non-calculator skills as appropriate, and there is plenty of opportunity to encourage estimation. Using real bills and statements is possible e.g. examples provided by banks can be found online.

Key vocabulary

Total
Balance

Debit
Expense

Credit
Bill

Key questions

What's the difference between credit and debit?

How is the balance of an account calculated? What does it mean if the balance is negative?

Why does a calculator display £12.50 as 12.5?

Exemplar Questions

Ron buys 6 bonbons, 8 strawberry laces and a fizzy drink.
Ali buys one of each item.

- Who spends more money, Ron or Ali?
- Could you spend exactly £5? £10? Why or why not?

Menu	Price
Bonbons	5 p
Strawberry Laces	10 p
Chocolate Bar	85 p
Fizzy Drink	£1.19

Fill in the blank entries on the bank statement.

Date	Description	Credit (£)	Debit (£)	Balance (£)
Oct 1	Opening balance			42.17
Oct 3	Phone bill		36.98	
Oct 4	Birthday money	65.00		
Oct 6	Shopping			20.21

Scott's employer covers the cost of his travel expenses.
He receives 45 p for every mile he travels for work.
He notes the mileage readings from his car at the start and end of a work day.

Start	0	3	4	0	8	8	**End**	0	3	4	1	3	3

Work out how much money Scott receives for his travel expenses.

Calculate simple interest

Notes and guidance

This small steps gives students opportunity to practise calculating percentages using a multiplier. Although most interest paid in real life is compound, simple interest over 1 year is often used in credit agreements. Students can look at both calculator and non-calculator methods and will need to read questions carefully to determine whether they are finding the interest or the total value of an investment. The initial amount invested is known as the 'principal amount'.

Key vocabulary

Percentage	Interest	Annual
Deposit	Principal	Rate

Key questions

How do you work out a percentage of an amount without a calculator? With a calculator?

How do you find a multiplier to calculate percentages ?

What is a credit agreement?

Exemplar Questions

Which of the following calculations find 5% of 200

0.05×200

$200 \div 5$

$200 \div 20$

$200 \div 10 \div 2$

$\frac{1}{5}$ of 200

Alex deposits £250 into an account paying simple interest at 3% a year. How much money is in the account after

- 1 year
- 4 years
- 10 years?

Alex thinks that simple interest earned, I, can be calculated using the formula $I = \frac{PTR}{100}$ where P is the principal amount, T is the time in years and R is the rate of interest. Is Alex correct?

John pays for an £800 guitar with a credit agreement.
He pays a 10% deposit. The outstanding balance is to be paid over 12 months at 9% interest. How much will John pay for the guitar in total?

Amir deposits £12 000 into a savings account with a bank that pays 5.7% simple interest annually. He wishes to withdraw his money once the amount exceeds £16 000
How many years will Amir need to wait?

Esther deposits the same amount at a different bank but only has to wait half the time of Amir for the balance to exceed £16 000
Does this mean the interest rate was double 5.7%? Why or why not?

Calculate compound interest

Notes and guidance

In this small step students should see that the interest is added to the current value of the investment at the end of each year and so next year's interest is greater. Students who did not cover the small step on repeated percentage change in the last block will need more time to appreciate the quicker way of finding the total value after a number of years.
Graphing future amount using simple and compound interest helps to compare linear and exponential growth.

Key vocabulary

Compound	Interest	Multiplier
Principal	Rate	Per annum

Key questions

How do you find a percentage increase using a multiplier?
Why is the amount of interest earned different each year?
In the calculation 300 × 1.04^3, what is the principal amount? The interest rate? The time period?
What is the difference between simple and compound interest?

Exemplar Questions

Match the multiplier with the correct percentage statement.

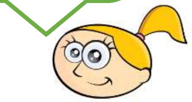
Increase by 20%
Increase by 100%
Increase by 12%
Increase by 2%
Increase by 1.2%

1.012
1.02
2
1.2
1.12

Jack invests £300 into a savings account at 5% compound interest for 2 years. He works out how much he will have at the of the 2 years.

5% of 300 = £15
Balance after 1 year = £315
5% of 315 = £15.75
Balance after 2 years = £330.75

I used a multiplier method.
300 × 1.05^2 = £330.75

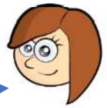


Compare Jack and Eva's methods.

Rosie also invests £300 into a savings account at 2% compound interest for 5 years.

Do you agree with Rosie?
Explain why.

I think I will receive the same amount as Jack



TETRA-BANK
3% compound interest for the first year
1% for each additional year.

BANK OF CLARITY
1.7% compound interest per year.

Which bank will pay more interest on £3500 invested for 3 years?

Value Added Tax

Notes and guidance

Students may not be aware of taxation of goods, so it may be beneficial to start with a discussion on the different rates of VAT (e.g. gas and electricity bills currently include 5% VAT as opposed to the standard 20%) and which goods are exempt. Students can then practise both increasing by a percentage and finding the original amount within this context. You could also compare different rates of VAT in different countries.

Key vocabulary

Tax
VAT
Rate
Original
Value Added

Key questions

What is VAT? How is VAT calculated?

How do you find the cost of an item before VAT is added?

How could you represent a VAT problem using a bar model?

Exemplar Questions

In the UK, VAT is set at 20%.
The prices of the goods in the table shown do not include VAT.

Item	Price
Headphones	£40
Smart Watch	£225
Television	£582.50

Calculate the price of the goods including VAT.

Chocolate without VAT is traded at 70 p per 100 g.
VAT in Switzerland is 2.5% and in the UK it is 20%.

- What is the difference in cost between a 200 g bar of chocolate bought in the UK and the same bar bought in Switzerland?
- How much more chocolate can be bought in Switzerland for the same cost in the UK?

A brand new car in the UK is £19 995 including VAT.

- Work out the price of the car before VAT is added.
- Explain why your answer is not the same as finding 80% of £19 995

Mo has used a rate of 20% to work out his total gas bill is £165, but the rate of VAT for gas is 5%.
What should his total bill be?

Calculate wages and taxes

Notes and guidance

In this small step, students should be shown the similarities and differences between wages and salary. Overtime terminology such as 'time and a half' will introduce students to the complexities of budgeting. Likewise, the meaning and workings of tax thresholds will need to be explained and modelled. You could also include examples of piecework where employees are paid per item produced.

Key vocabulary

Income
Annual
Salary
Exemption
Wage
Overtime

Key questions

What is meant by 'time and a half'? What steps do you need to take to work out a wage that includes overtime?

How much of a £20 000 salary is taxed?

Exemplar Questions

Dora works 8 hours a day Monday to Friday and 6 hours on a Sunday.
She is paid £9.50 per hour.
How much does Dora earn in one week?

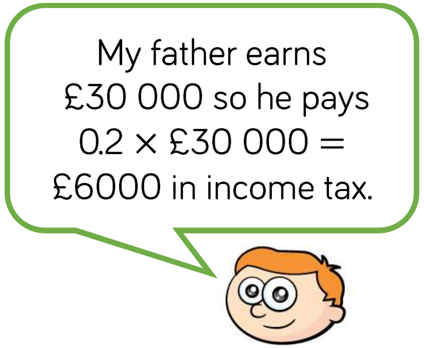
Dexter's rate of pay is £8.72 per hour.
When he works for more than 8 hours he is paid overtime at 'time and a half'.

- Dexter works for 10 hours one day. How much does he earn?

- How much overtime will Dexter need to work to achieve his goal?

In the UK, income above £12 500 is taxable.
The table below denotes the tax rate.

Taxable Income	Tax Rate
£12 501 to £50 000	20%
£50 001 to £150 000	40%
over £150 000	45%



- Explain why Ron is wrong and work out how much income tax does Ron's father pays.
- Will the amount of income tax paid on a salary of £60 000 per year be double that paid on £30 000? Explain why or why not?

Exchange rates

Notes and guidance

Students have seen exchange rates in previous learning in Year 8 so this small step gives plenty of chance to build upon those skills. Use a variety of representations to develop students' understanding of the multiplicative nature in currency conversion. Encourage students to estimate their answers before calculating to ensure they have a sensible result from using a calculator.

Key vocabulary

Currency

Convert

Rate

Exchange

Key questions

How is the conversion between pounds and euros different from euros to pounds?

Will the number of (e.g.) euros be greater than or less than the number of (e.g.) pounds?

How can you find an estimate to the calculation?

Exemplar Questions

Annie is going on holiday to Spain.
The exchange rate is £1 = €1.15
She changes £200 into euros (€)
How many euros does she receive?
Explain the connections shown in the diagram.

$$\begin{tikzcd} \pounds 1 \arrow[r, "\times 1.15"] \arrow[d, "\times 200"'] & \text{€}1.15 \arrow[d, "\times 200"] \\ \pounds 200 \arrow[r, "\times 1.15"'] & ? \end{tikzcd}$$

Annie comes back from holiday with €60 and changes these back into pounds.
The exchange rate is now £1 = €1.20
Work out how many pounds Annie receives.

$$\begin{tikzcd} \pounds 1 \arrow[r, "\times 1.2"] \arrow[d, "\times ?"'] & \text{€}1.20 \arrow[d, "\times ?"] \\ \pounds ? \arrow[r, "\times 1.2"'] & \text{€}60 \end{tikzcd}$$

A camera costs $400 in the United States.
The same camera costs £325 in the UK.
The exchange rate is £1 = $1.25
Is the camera cheaper in the United States or in the UK?

I converted £325 into $ by doing 325 × 1.25

I converted $400 into £ by doing 400 ÷ 1.25

Are both approaches correct? Explain why or why not.

The exchange rate for British Pounds (£) to Japanese Yen (¥) is shown.

Today's Rate
£1 = ¥140

What is the exchange rate for Yen to pounds?

Unit pricing

Notes and guidance

This small step focuses on the use of the unitary method, where students find the cost of one item or the amount of an item that can be bought for £1 or 1 p as appropriate. The use of double number lines and proportion diagrams offer students a chance to develop their understanding of multiplication embedded within proportion. You could also explore alternative approaches, which will be studied in more depth in the "Best buys" small step later in the year.

Key vocabulary

Value
Cost
Proportion
Unit
Unitary

Key questions

If 4 lemons cost £1, how much does one lemon cost?

What is meant by "unitary"?

What's the same and what's different about 'cost per unit' and 'unit per cost'?

Exemplar Questions

Supermarket
4 lemons for £1

Greengrocer
A bag of 5 lemons
£1.20 per bag.

How much is one lemon from the supermarket?
How much is one lemon from the greengrocer?
Which offers better value for money?

A brand of sugar is available in two sizes.

1 kg for 40 p   1.6 kg for 60 p

Work out which bag is better value for money.

You need the cost per kg, so divide the price of the larger bag by 1.6

Whitney

Tommy

You need to find the amount of sugar you get per penny in each bag.

Compare the methods. Which do you prefer? Why?

A school is planning a summer barbecue.
Two shops have deals on burgers.

Fresh Foods
Burgers 64 p each
Buy two get one free

Farm Best
24 burgers plus 25% extra
£10

Which deal offers the best value for money?

Spring 2: Reasoning with Geometry

Weeks 1 and 2: Deduction

In this block students revise and extend their knowledge of angles rules and properties of shapes, applying them to increasingly complex problems. The block also builds on the ideas of the earlier Testing Conjectures block looking at deduction in a geometric rather than algebraic and numerical contexts. Students also revise the constructions covered in Year 8 and look more deeply at how and why these work.

National Curriculum content covered includes:

- derive and use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); recognise and use the perpendicular distance from a point to a line as the shortest distance to the line
- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons that are reflectively and rotationally symmetric
- apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles
- understand and use the relationship between parallel lines and alternate and corresponding angles

Weeks 3 and 4: Rotation and Translation

Building on their study of line symmetry and refection in Year 8, students now look at rotational symmetry and rotation. They then move on to study translations, which are described in vector form They compare the different effects of the transformations studied so far, noticing that the objects and images are congruent.

National Curriculum content covered includes:

- identify properties of, and describe the results of, translations, rotations and reflections applied to given figures
- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons that are reflectively and rotationally symmetric
- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems

Weeks 5 and 6: Pythagoras' Theorem

Students revise squares and square roots before moving on to investigate the relationship between the sides of a right-angled triangle. The converse of the theorem is emphasised so that students are aware that if the sides of a triangle satisfy the rule $a^2 + b^2 = c^2$ then the triangle must be right-angled. Students explore using the theorem in a variety of context, including on coordinate axes, and a higher step is included using 3-D shapes. There is an opportunity to revisit the learning in the next block when students explore similarity in right-angled triangles as an introduction to trigonometry.

National Curriculum content covered includes:

- use Pythagoras' Theorem to solve problems involving right-angled triangles
- apply angle facts, triangle congruence, similarity and properties of quadrilaterals to derive results about angles and sides, including Pythagoras' Theorem, and use known results to obtain simple proofs
- interpret mathematical relationships both algebraically and geometrically
- begin to reason deductively in geometry, number and algebra, including using geometrical constructions
- begin to model situations mathematically and express the results using a range of formal mathematical representations

Deductions

Small Steps

- Angles in parallel lines R
- Solving angles problems (using chains of reasoning)
- Angles problems with algebra
- Conjectures with angles
- Conjectures with shapes
- **Link constructions and geometrical reasoning** H

H denotes higher strand and not necessarily content for Higher Tier GCSE

R denotes 'review step' – content should have been covered earlier in KS3

Angles in parallel lines

R

Notes and guidance

This review step provides students with a reminder of the rules connecting the angles formed by a pair of parallel lines and a transversal. Knowledge of basic angle rules such as angles at a point, angles on a straight line and vertically opposite angles is also needed. Encourage students to verbalise their thinking and their answers. Three-letter notation is useful to describe which angle they are referring to in each step of working.

Key vocabulary

Alternate	Corresponding	Co-interior
Transversal	Parallel	

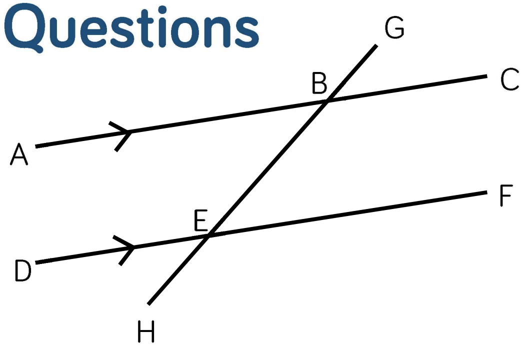
Key questions

What do co-interior angles sum to?

Name a pair of corresponding/alternate angles shown in the diagram.

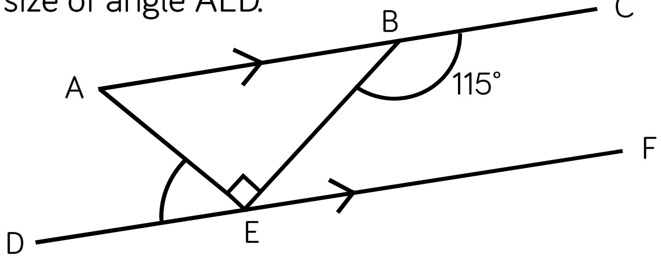
How can you prove if a pair of lines are parallel or not?

Exemplar Questions



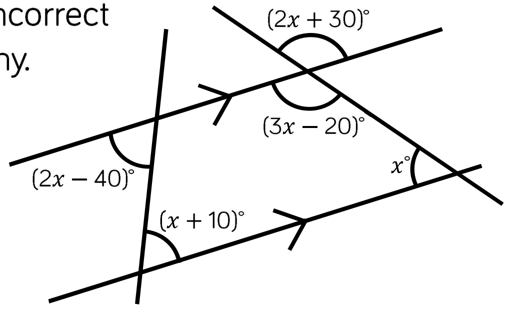
- Lines AC and DF are ____________ lines.
- Line HG is a ____________ line.
- Angles _____ and ______ are corresponding angles.
- Angles _____ and ______ are alternate angles.

Work out the size of angle AED.



Which of the following equations is incorrect for finding the value of x? Discuss why.

$x + x + 10 = 180$	$2x + 30 = 3x - 20$
$3x - 20 + x = 180$	$2x - 40 = x + 10$



Solving angles problems

Notes and guidance

Building on the previous step, students now look at more complex problems with more steps of working. They could be challenged to create their own "angle chasing" problem like the first exemplar question and to work out the minimum amount of information needed to evaluate all the unknown angles. Again the focus is the use of correct mathematical language as students begin to use longer chains of reasoning.

Key vocabulary

Angles at a point
Parallel
Alternate
Co-interior
Corresponding
Isosceles

Key questions

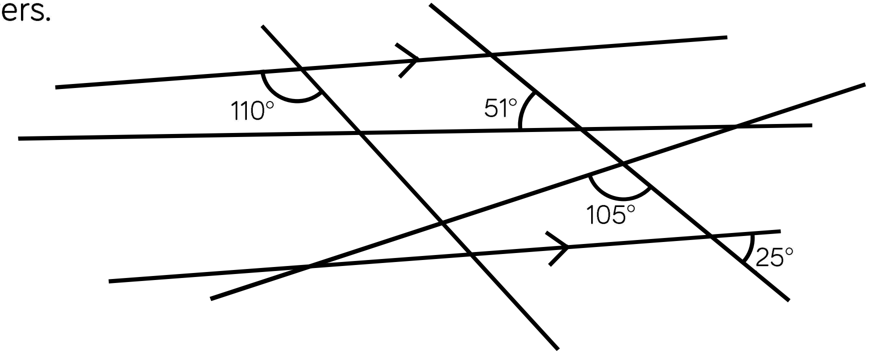
What angle facts can we use to solve this problem?

Which pair of angles are corresponding/ alternate/ co-interior?

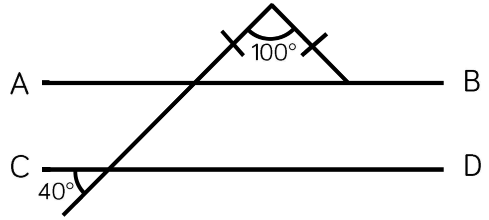
Which other angles can we label?

Exemplar Questions

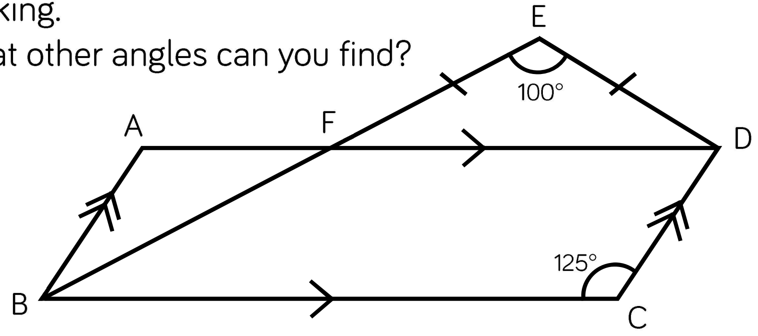
How many angles can you work out? Give a reason for each of your answers.



Show that lines AB and CD are parallel giving a reason for each step.



- Show that ∠FBC is 40°. Give a reason for each step of your working.
- What other angles can you find?



Angles problems with algebra

Notes and guidance

In this step, students interleave the forming and solving of equations with their use of the angles rules. You could also include inequalities if you wish. Depending on students' attainment, you may wish to focus on simpler equations or include some with unknowns on both sides e.g. a pair of vertically opposite angles labelled as $ax \pm b$. This is a good point to bring in problems involving the interior and exterior angle sums of polygons that students covered in Y8.

Key vocabulary

Interior	Exterior	Regular	Equation
Polygon	Sum	Total	

Key questions

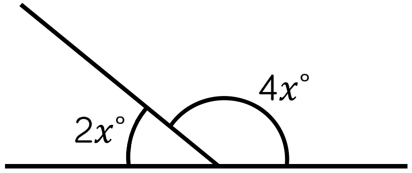
What makes a polygon regular?

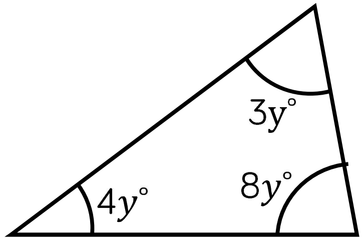
How do you simplify an algebraic expression?

What's the same and what's different about summing the interior and exterior angles of a polygon?

Exemplar Questions

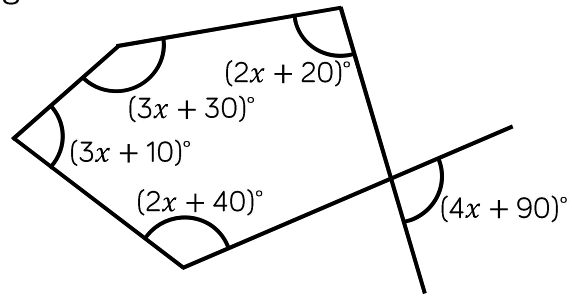
Form and solve an equation to find the value of x and y.



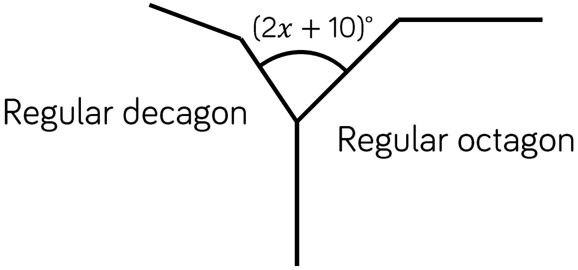


What is the same about these questions?

Form and solve and equation to find the size of the greatest interior angle in the polygon.



Calculate the value of x.



Conjectures with angles

Notes and guidance

Here students revisit the ideas of “True or False” and “Always, Sometimes, Never true?” in the context of angles. They could be encouraged to come up with their own conjectures and test these, appreciating that a single counterexample is enough to disprove a conjecture. Some of the circle theorems could be used as a vehicle for exploration here as a useful preliminary to formal study of these at Key Stage 4.

Key vocabulary

Conjecture	Prove	Justify
Example	Counterexample	

Key questions

What is a conjecture?

Is one example enough to prove a conjecture? How many examples do you need?

What is meant by ‘counterexample’?

Exemplar Questions

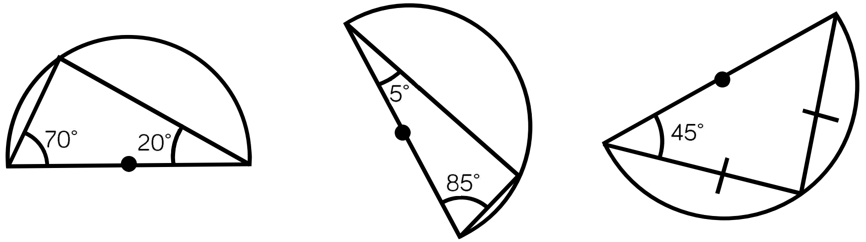
Decide whether the following statements are always, sometimes or never true.

‘The sum of two obtuse angles is a reflex angle’

‘When three angles meet at a point on a straight line they are all acute angles’

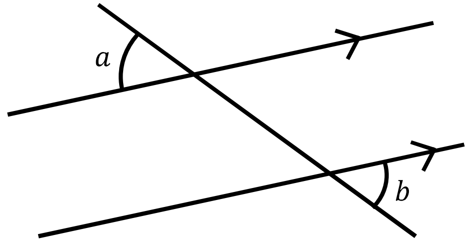
‘The sum of the exterior angles of a regular pentagon is less than the sum of the exterior angles of a irregular pentagon’

Investigate the missing angles in the triangles within a semi circle.



Can you make your own conjecture about angles in semi-circles?

Mo conjects that angles a and b will always be equal.
Give reasons to support Mo’s conjecture.



Conjectures with shapes

Notes and guidance

Again students could be encouraged to come up with their own conjectures in this step. This is an excellent opportunity to revisit the properties of shapes and remind students about the differences between the various types of special quadrilaterals, and symmetry. They could also look at diagonals of quadrilaterals, exploring lengths and angles and be asked to complete and test conjectures starting, “If the diagonals of a quadrilateral are equal in length then…”

Key vocabulary

Parallelogram	Rhombus	Kite
Diagonal	Bisect	Regular

Key questions

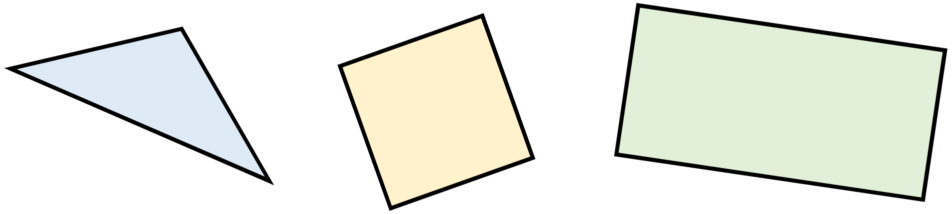
Make a conjecture about …

Give me an example that supports the conjecture…

Give me a counterexample to disprove the conjecture…

Exemplar Questions

Ron comes up with a conjecture about the area and perimeter of shapes. He conjects that the greater the area, the greater the perimeter. Investigate Ron's conjecture using triangles, squares and rectangles.



Investigate the following conjectures about pentagons. Give examples to support them or counterexamples to disprove them.

- A pentagon with equal sides is regular
- A regular pentagon can be constructed using only equilateral triangles
- You can split any pentagon into a quadrilateral and a triangle

Which of the following conjectures are true?
Justify your answers.

The number of sides that a regular shape has is the same as the number of lines of symmetry it has.

The number of sides that a regular shape has is the same as the order of rotational symmetry it has.

Constructions and reasoning H

Notes and guidance

All students could have the opportunity to revisit the standard constructions with ruler and compasses in this step and higher attainers should be challenged to see the links between the properties of shapes formed and the bisectors. You could also compare and contrast the methods of construction of a perpendicular to and from a point with that for a perpendicular bisector and investigate how to construct parallel lines.

Key vocabulary

Locus	Equidistant	Point
Construct	Bisector	Perpendicular

Key questions

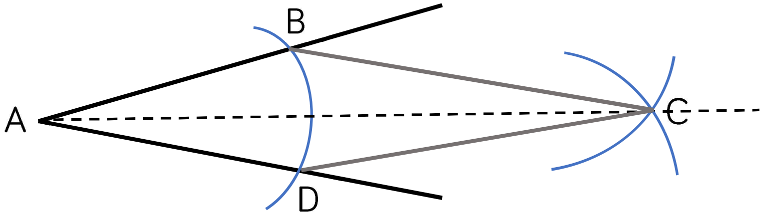
What's the same and what's different about 'drawing' and 'constructing'?

What angles can be constructed without using a protractor?

Why does the construction work?

Exemplar Questions

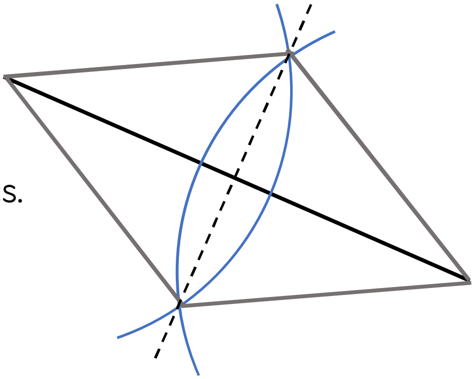
Construct the angle bisector of any angle. Join up the arc lines with two straight lines. What shape is ABCD? Will this always work?



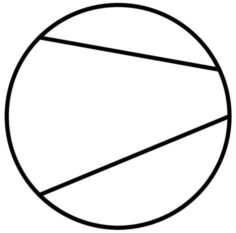
Construct the perpendicular line bisector of a line.

Join up the ends of the lines with the intersection point of the two arcs.

What do you notice about the shape you have made?



Construct the perpendicular bisector of two chords of a circle. What do you notice about where they intersect?



Rotation and translation

Small Steps

- Identify the order of rotational symmetry of a shape
- Compare and contrast rotational symmetry with lines of symmetry
- Rotate a shape about a point on a shape
- Rotate a shape about a point not on a shape
- Translate points and shapes by a given vector
- Compare rotation and reflection of shapes
- **Find the result of a series of transformations** H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Rotational symmetry

Notes and guidance

This small steps introduces students to the idea of rotational symmetry and finding the order of rotational symmetry of a shape. A common misconception when identifying rotational symmetry is that a shape can have order or rotational symmetry 0; as a shape lands exactly on itself is after a full turn then it actually rotation symmetry of order 1. Tracing paper is very useful here, but students need to be aware to use the exact centre of the shape.

Key vocabulary

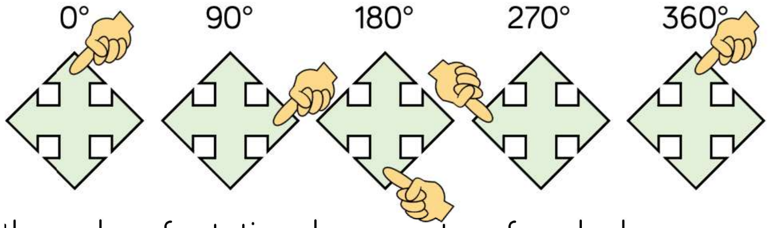
Shape	Rotational	Symmetry
Order	Regular	Irregular

Key questions

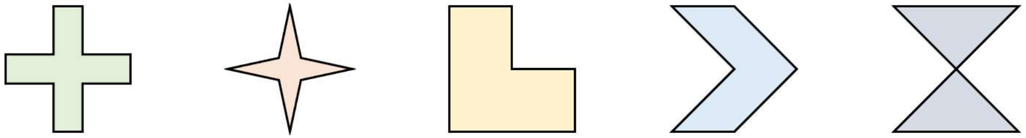
How do you find the order of rotational symmetry of a shape?
What is the order of rotational symmetry of a circle?
What is the centre of rotation?
How does a pattern on a shape affect the order of rotational symmetry?

Exemplar Questions

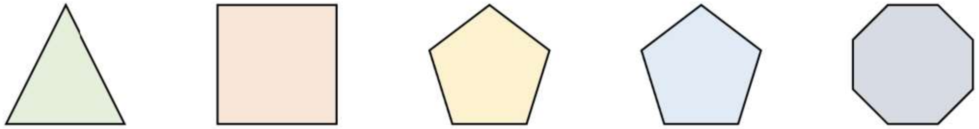
This shape has rotational symmetry of order 4 because it lands exactly on itself 4 times in a 360° turn.



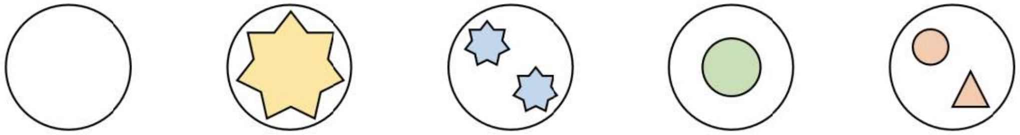
Identify the order of rotational symmetry of each shape.



Each of the shapes is regular.



Identify the order of rotational symmetry of each shape.
What do you notice? Explore this with other regular polygons.



How does the pattern inside the shapes affect the rotational symmetry?

Symmetry – rotational and lines

Notes and guidance

This step builds on the previous step to also revises line symmetry as explored in Year 8. . It's easy for students to encompass both of these skills under one umbrella of symmetry, and ultimately confuse the two. This small steps provides opportunities to see both things alongside of each other and compare and contrast. It's important that students understand that the number of lines of symmetry a shape has is not necessarily equal to its order of rotational symmetry.

Key vocabulary

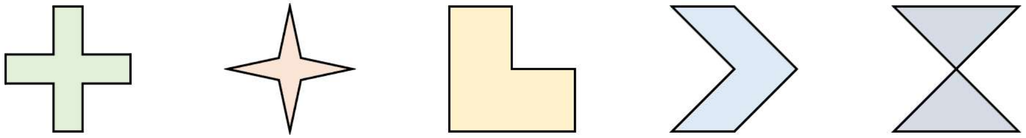
Rotational	Line	Symmetry
Order	Mirror	Shape

Key questions

How many lines of symmetry does a ________ have?
What is the order of rotational symmetry of a ________?
Can a shape have order of rotational symmetry 2/3/4… but no lines of symmetry?
Can a shape have more lines of symmetry than its order of rotational symmetry?

Exemplar Questions

How many lines of symmetry does each shape have?



Compare the number of lines of symmetry with the order of rotational symmetry. What do you notice?

Decide whether each statement is always, sometimes or never true. Explain your reasoning, including examples where appropriate.

- The more lines of symmetry a shape has, the greater its order of rotational symmetry.
- The order of rotational symmetry of a shape is equal to the number of lines of symmetry.
- If a shape has line symmetry then it has rotational symmetry of order greater than 1

The interior angles of a regular polygon is 156°

- How many sides does the polygon have?
- How many lines of symmetry does the polygon have?
- Identify the order of rotational symmetry of the polygon.

Rotation (1)

Notes and guidance

In this small step students rotate shapes by multiples of 90° about a point that is on the shape. They should see examples both on a squared grid and on a set of coordinate axes. Consider first using a vertex of the shape as a centre of rotation, and then extended this by using centres of rotation both on the edge of the shape or within a shape. Students may struggle if the image overlaps the object, and should be exposed to examples like these to help them build confidence.

Key vocabulary

Rotation
Object
Direction
Image
Invariant
Centre
Clockwise
Anti-clockwise

Key questions

How does rotation affect the orientation of an object?
Where is the centre of rotation?
Why does the image overlap the object?
Which vertex has not moved/is invariant? Why?
Why do you not need a direction when rotating by 180 degrees?

Exemplar Questions

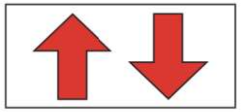
Match each card to the turn shown.

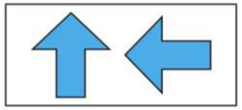
90° anti-clockwise

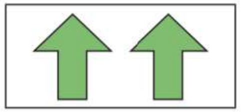
180°

270° clockwise

360°



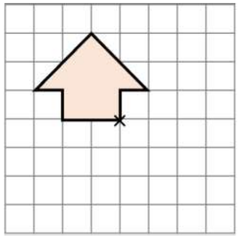




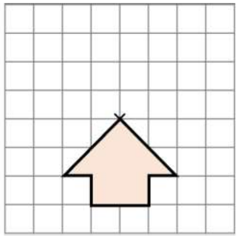
Why do some turns not have a direction?

Rotate each shape as instructed using × as the centre of rotation.

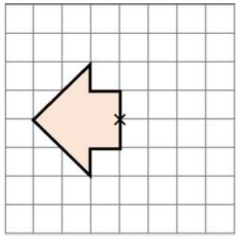
180°

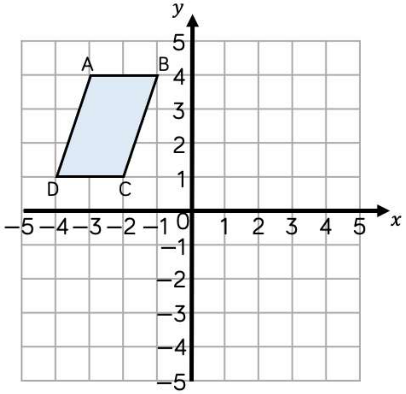


90° anti-clockwise



270° clockwise





- Rotate ABCD 90° clockwise using (−2, 1) as the centre of rotation.
- Rotate ABCD 180° using (−3, 1) as the centre of rotation.
- Rotate ABCD 270° clockwise using (−2, 3) as the centre of rotation.

Rotation (2)

Notes and guidance

Students build on their learning from the previous small step and now rotate shapes using a centre of rotation that is not on the shape. Again, they should see this on a squared grid as well as on a set of coordinate axes. Students should now be confident in determining the orientation of their image and need to play close attention to how the position of this image is affected by the centre of rotation. They should explore how the position of the centre affects the position of the object.

Key vocabulary

Rotation	Direction	Invariant	Clockwise
Object	Image	Centre	Anti-clockwise

Key questions

Why does a 90 degree rotation clockwise produce the same image as a 270 degree rotation anti-clockwise?

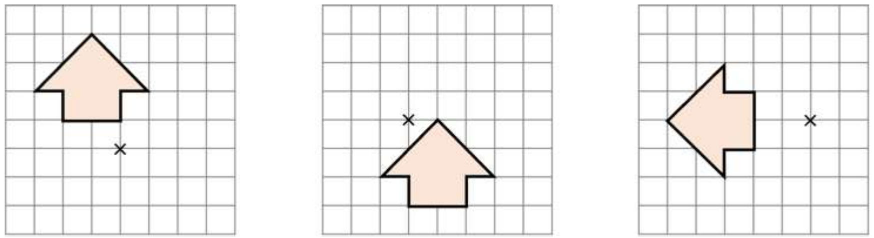
Is the orientation of your rotated image correct? What about the position? How do you know?

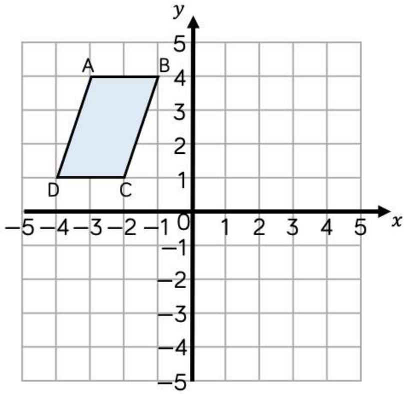
Why is you image not touching the centre of rotation?

Exemplar Questions

Rotate each shape as instructed using × as the centre of rotation.

180° 90° anti-clockwise 270° clockwise





- Rotate ABCD 90° clockwise using (−1, 1) as the centre of rotation.
- Rotate ABCD 180° using the origin as the centre of rotation.
- Rotate ABCD 270° clockwise using (−2, −1) as the centre of rotation.

Shape A is an isosceles trapezium with vertices at (0, 3), (0, 9), (4, 4) and (4, 8).
Shape A is rotated 90 degrees anti-clockwise about the origin to give shape B.

- Find the area of shape B. Do you need to perform the rotation?
- Find the equation of the line of symmetry of shape B.

Translation

Notes and guidance

In this small steps students translate both points and shapes by a given vector. Translation is on the Key Stage 2 National Curriculum and so students may have seen this before, however the idea of describing the translation as a vector is new learning. Students should be confident in interpreting a vector before performing translations. Whilst they should use their understanding of coordinates to support knowledge of vectors, they should also understand the differences between the two. Consider starting by translating individual points before moving on to translate shapes.

Key vocabulary

- Translate
- Vertical
- Vector
- Move
- Horizontal
- Vertex

Key questions

Why does a translation vector have two components?
What does the top/bottom number represent?
What does it mean if a component is negative?
What's the difference between coordinates and vectors?

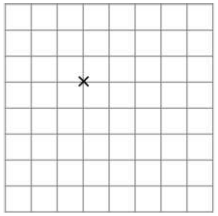
Exemplar Questions

Describe each vector in words.

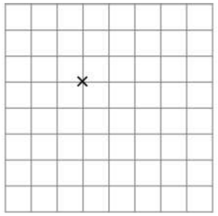
$\begin{pmatrix}1\\2\end{pmatrix}$ $\begin{pmatrix}1\\-2\end{pmatrix}$ $\begin{pmatrix}-1\\2\end{pmatrix}$ $\begin{pmatrix}-1\\-2\end{pmatrix}$ $\begin{pmatrix}2\\-1\end{pmatrix}$

Translate each point by the vector given.

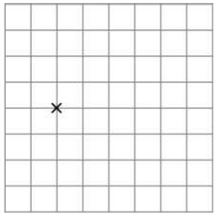
$\begin{pmatrix}2\\-3\end{pmatrix}$



$\begin{pmatrix}-2\\2\end{pmatrix}$

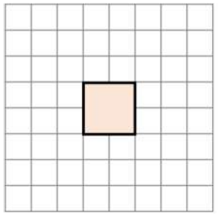


$\begin{pmatrix}3\\0\end{pmatrix}$

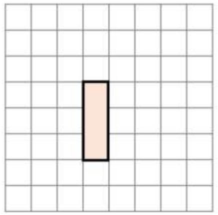


Translate each shape by the vector given.

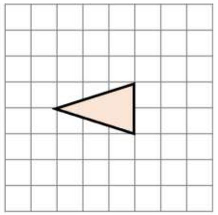
$\begin{pmatrix}2\\-3\end{pmatrix}$



$\begin{pmatrix}-2\\2\end{pmatrix}$



$\begin{pmatrix}3\\0\end{pmatrix}$



A triangle has vertices at (23, 17), (20, −19) and (−3, 1)

The triangle is translated by the vector $\begin{pmatrix}17\\-2\end{pmatrix}$

Find the coordinates of each vertex after the translation.

Compare rotation and reflection

Notes and guidance

This step builds on the previous steps to also revises line symmetry as explored in Year 8. Students should be able to perform both rotations and reflections and compare the two. They could be presented with multiple objects and images and be able to identify whether the transformation was a rotation or a reflection. Students could look at examples which give the same image and be extended to consider variant and invariant points and lines within this.

Key vocabulary

Rotate
Centre
Direction
Reflect
Line
Variant
Invariant

Key questions

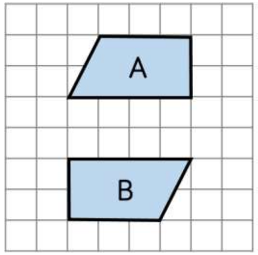
What information do you need to perform/describe a rotation/reflection?

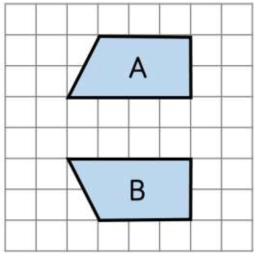
What's the same and what's different about rotations and reflections?

What does it mean for a point or a line to be invariant?

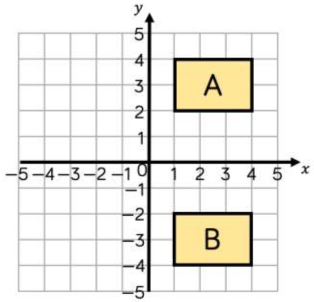
Exemplar Questions

For each diagram decide whether B is a reflection or a rotation of A.





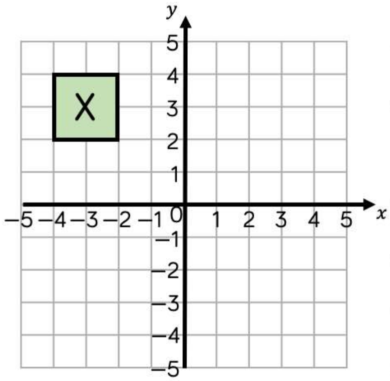
A transformation has been performed on shape A to give shape B.



Ron says "Shape B is a reflection of shape A"

Dora says "Shape B is a rotation of shape A"

Who do you agree with? Explain why.



Reflect X in the line $x = -2$
Write down the coordinates of each vertex of the image.

Rotate X 180° about $(-2, 2)$
Write down the coordinates of each vertex of the image.

Which vertices remain invariant under each transformation? How do you know?

Series of transformations

H

Notes and guidance

Students have so far performed single transformations and looked at them in detail. This small step provides students opportunity to perform a series of transformations. They should explore how changing the order affects the final image to realise that the order is usually important. Whilst focus should mainly be placed on performing these transformations, students could be stretched to consider if/how the same image could be produced using a single transformation.

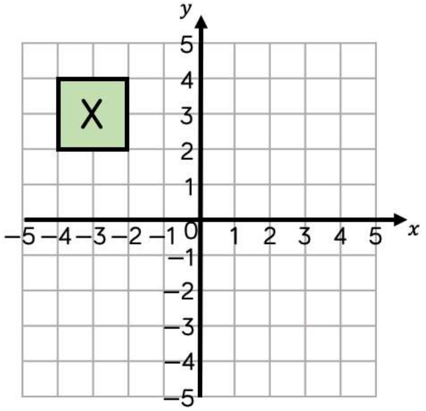
Key vocabulary

Reflect	Line	Rotate	Centre
Direction	Translate	Vector	Single

Key questions

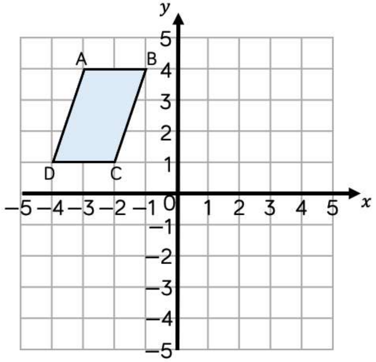
Which transformation should you do first? Why?
Why do you not start with the original shape when performing the second transformation?
Could you perform a single transformation on shape A such that the image is shape C?

Exemplar Questions



- Rotate X 90° clockwise using the origin as the centre of rotation.
  Label this shape Y.
- Reflect shape Y in the x-axis.
  Label this shape Z.
- Investigate what would happen if the transformations were performed in a different order.

Translate ABCD by the vector $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$ and then reflect in the y-axis.



Shape P has vertices at (−3, 5), (−1, 7) and (1, 5)
It is rotated 180 degrees about the origin and the translated by the vector $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ to give shape Q.
Describe fully the single transformation that maps shape P onto shape Q.

Pythagoras' Theorem

Small Steps

- Squares and square roots R
- Identify the hypotenuse of a right-angled triangle
- Determine whether a triangle is right-angled
- Calculate the hypotenuse of a right-angled triangle
- Calculate missing sides in right-angled triangles
- Use Pythagoras theorem on coordinate axes
- Explore proofs of Pythagoras' theorem
- **Use Pythagoras' theorem in 3-D shapes** H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered earlier in KS3

Squares and Square Roots

R

Notes and guidance

This small step provides an opportunity for retrieval on squaring and square roots from earlier in KS3. Concrete and pictorial representations can be useful to address misconceptions of squaring and square rooting as doubling and halving such as $4^2 = 8$ and $\sqrt{16} = 8$ Discuss with students that whilst two numbers will square to give the answer e.g. 25, the square root is the positive value. You may need to revise approximation including significant figures.

Key vocabulary

Square	Square root	Integer
Significant figures	Decimal places	

Key questions

What's the difference between the square of a number and the square root of a number?

What is the first step when calculating e.g. $\sqrt{12 + 9^2}$?

What two numbers square to give the answer 25? If a square has an area of 25 cm^2 what might the side length be? How are these questions different?

Exemplar Questions

Teddy has 30 counters.
How many different square numbers can he make?

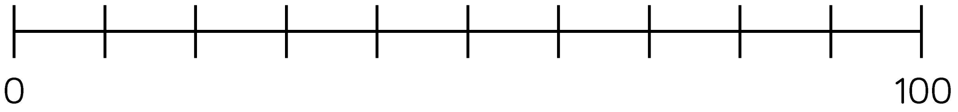
Complete the following using <, > or =

$\sqrt{16}$ ○ 2^2

3^2 ○ $\sqrt{49}$

$\sqrt{64}$ ○ 4^2

Evaluate each card and place them on the number line as accurately as you can.



$\sqrt{121}$	8^2	$3^2 + \sqrt{4}$
$(3^2 - 1^2)^2$	$2 \times \sqrt{25}$	$\sqrt{12 + 9^2}$

"The sum of two different square numbers is equal to another square number."

Is this always, sometimes or never true? Justify your answer.

Identify the hypotenuse

Notes and guidance

This is a key prerequisite for students before they meet Pythagoras' theorem. Students should be given plenty of exposure to examples/non-examples with triangles in various orientations. You can extend this to look at the hypotenuses within a polygon made up of right-angled triangles. Discuss the relationship between the size of angles in a triangle and the length of the sides opposite them, noting the hypotenuse must be the longest side of a right-angled triangle.

Key vocabulary

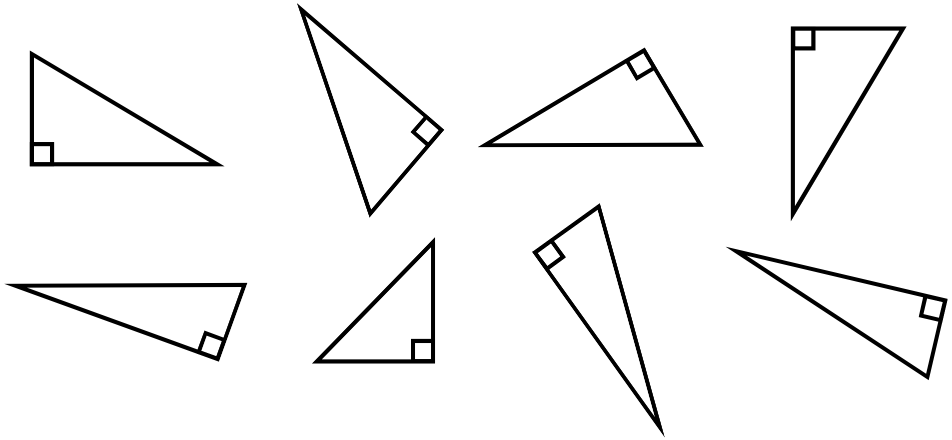
- Hypotenuse
- Right-angled triangle
- Opposite
- Adjacent

Key questions

What is the hypotenuse of a right-angled triangle?
How can you identify that a side is the hypotenuse?
What is the difference between the adjacent and opposite sides?
Why will the hypotenuse always be the longest side?

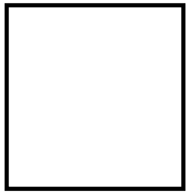
Exemplar Questions

Identify the hypotenuse in each right-angled triangle.



Amir says, "It is not possible to have a hypotenuse in a square as it is a quadrilateral."

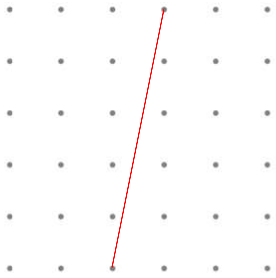
Do you agree?



The line shown is the hypotenuse of a right-angled triangle.

Complete the triangle.

Can you find all 6 possibilities?



Determine whether a triangle is right-angled

Notes and guidance

This small step provides the opportunity for students to use Pythagoras' Theorem without moving straight to calculating unknown sides. Students could investigate the areas of squares on each side of a right-angled triangle and discover or be led to the connection $a^2 + b^2 = c^2$. You could use "hyp" for c if preferred. Students can then use the rule to determine if other triangles are right-angled or not. You could extend to consider whether triangles contain an obtuse angle.

Key vocabulary

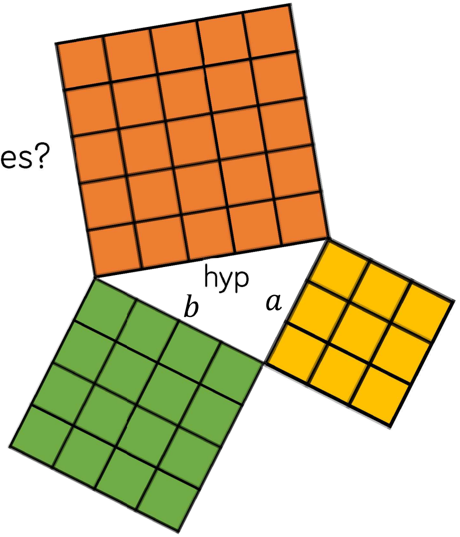
Hypotenuse	Right-angled triangle	
Opposite	Adjacent	Sum

Key questions

What is the sum of the areas of the two smaller squares?
How does this compare to the area of the larger square?
How can you use the lengths of the sides of a triangle to determine whether the triangle is right-angled?
If $a^2 + b^2 = c^2$, is it also true that $a + b = c$?

Exemplar Questions

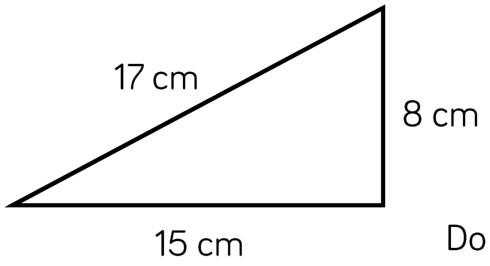
A triangle is enclosed by three squares.
Calculate the area of each square.
What is the sum of the two smaller squares?
What do you notice?

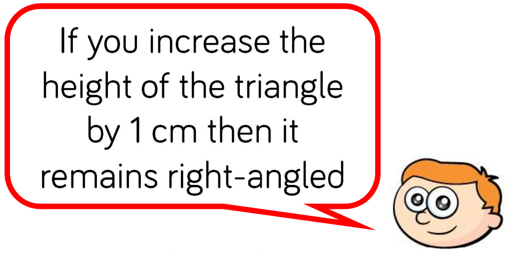


Fill in the table and discuss your results.

a	b	hyp	a^2	b^2	hyp^2
3	4	5			
4	5	6			
6	8	10			

Show that this triangle contains a right angle.





Do you agree with Ron?

Ron

A football pitch has a length of 100 m and a width of 70 m.
The diagonal length is 125 m.
Justify whether the pitch is rectangular or not.

Calculate the hypotenuse

Notes and guidance

Students now move on to calculating the length of a hypotenuse meeting both integer and non-integer answers. Knowing the hypotenuse is the longest side provides a check. Using non-standard right-angled triangle examples in various orientations provides opportunity for students to correctly identify the hypotenuse. Exploring the length of diagonals within 2D shapes widens the network of application of Pythagoras' theorem.

Key vocabulary

- Hypotenuse
- Right-angled triangle
- Opposite
- Adjacent
- Square root

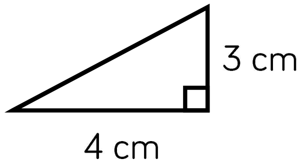
Key questions

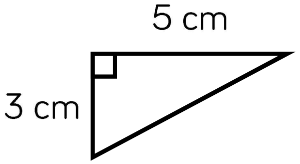
Which term in Pythagoras' theorem represents the hypotenuse? Does it matter which of the shorter two sides is a and which is b?

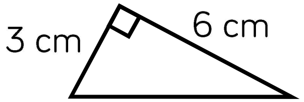
Why is the diagonal length of a square not equal to its side length? How can Pythagoras' theorem be used to find the diagonal of rectangles and squares?

Exemplar Questions

Calculate the length of the hypotenuse in each triangle.

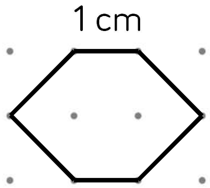




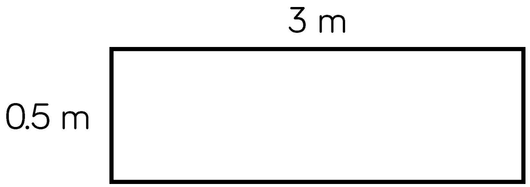


What is the same? What is different?

A hexagon is drawn on a centimetre square grid. Annie says that the perimeter of the hexagon is 6 cm. Explain why Annie is wrong.



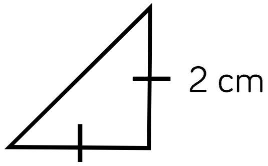
Find the length of the diagonal of the rectangle. Give your answer to two decimal places.



Calculate the length of the hypotenuse of the triangle.

Mo

In an isosceles right-angled triangle, the length of the hypotenuse will always be at least 40% greater than the length of the equal sides.



Is Mo correct? Explain why.

Calculate missing sides

Notes and guidance

In this small step it is worthwhile including examples where the unknown sides are and are not the hypotenuse. This gives purposeful decision making practice. $a^2 + b^2 = c^2$ is widely used, but using $a^2 + b^2 = \text{hyp}^2$ may help students correctly identify which term is the hypotenuse. Model the setting out of solutions and encourage students to show clear workings. They should also check their answers using the knowledge that the hypotenuse must be the longest side.

Key vocabulary

- Hypotenuse
- Opposite
- Right-angled triangle
- Adjacent
- Square root

Key questions

Why does it not matter when labelling the shorter sides a and b?

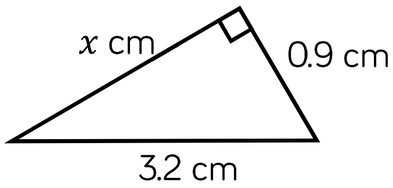
Why can the hypotenuse never be the same as another length in a right-angled triangle?

How does comparing the length you have calculated with the given lengths provide a check of your answer?

Exemplar Questions

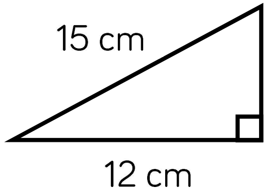
Alex is calculating the missing length of the right-angled triangle

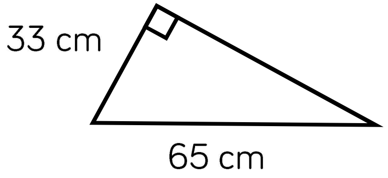
$$\boxed{\begin{aligned} a^2 + b^2 &= \text{hyp}^2 \\ x^2 + 0.9^2 &= 3.2^2 \\ x^2 &= 3.2^2 - 0.9^2 \end{aligned}}$$



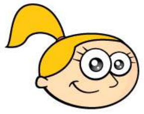
Complete Alex's working to find the missing side.

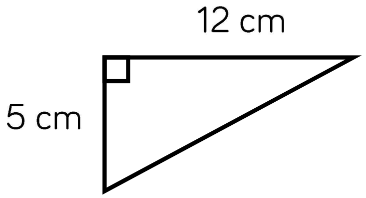
Calculate the areas of the right-angled triangles.

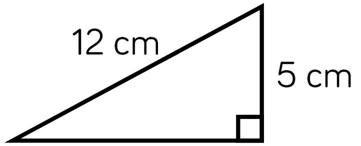




Eva says that the two triangles have the same perimeter.







Show that Eva is wrong.

Pythagoras' theorem on coordinate axes

Notes and guidance

This small step interleaves learning on the Cartesian plane working in all four quadrants. Students should be encouraged to discover that vertical and horizontal lengths are sufficient enough for calculating the length of a line segment. Be aware of students confusing line length with gradient. Line segments that cross one or more of the axes may also cause concern for students. Counting squares is useful to check any directed number arithmetic.

Key vocabulary

Origin	Quadrant	Negative
Line segment	Hypotenuse	Gradient

Key questions

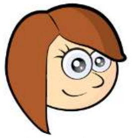
What's the same and what's different about the points with coordinates (2, 1) and (1, 2)?

What is the difference between finding the length of a line segment and the gradient of a line segment?

Why do you not always need a right angle triangle to find the length of a line segment?

Exemplar Questions

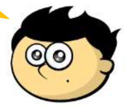
Who do you agree with? Why?



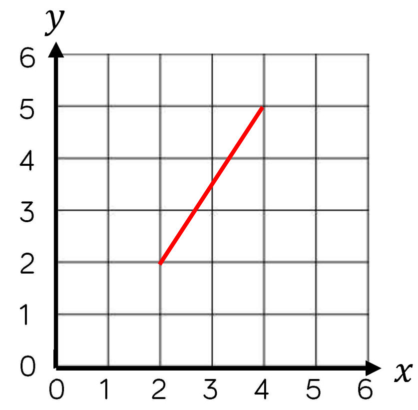
Rosie

I can use Pythagoras' theorem to find the length of the line segment.

You can't use Pythagoras' theorem as there is no right-angled triangle.

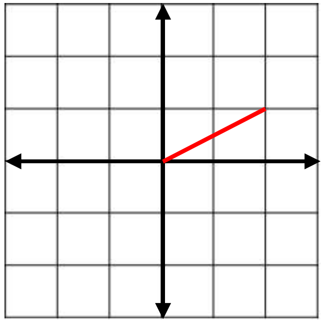


Jack



Find the distance between the origin and the point (2, 1).

Find all seven points with integer coordinates that are the same distance away from the origin as this point.



The points A (0, 0), B (2, 4) and C (−2, 1) form a triangle.
Calculate the lengths

- AB
- BC
- AC

Show that triangle ABC is a right-angled triangle.

Proofs of Pythagoras' theorem

Notes and guidance

This small step is not asking students to proof Pythagoras Theorem independently, but to explore some of the hundreds of proofs that exist. An algebraic and a geometric example are included here as examples you might wish to build on. You could start be demonstrating the result using dynamic geometry software and move on to formal proofs using resources such as videos available online. Students could practise their construction skills exploring Perigal's dissection.

Key vocabulary

Hypotenuse	Right-angled triangle
Opposite	Adjacent

Key questions

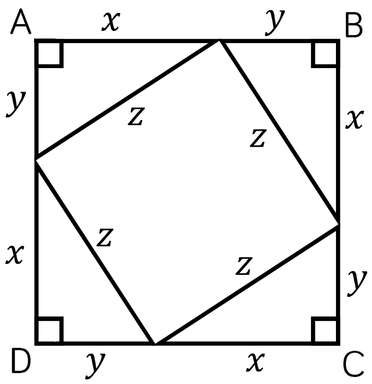
What's the difference between a demonstration and a proof?

Does Pythagoras' theorem hold for all right-angled triangles?

Does Pythagoras' theorem hold for any triangles that are not right-angled? Why or why not?

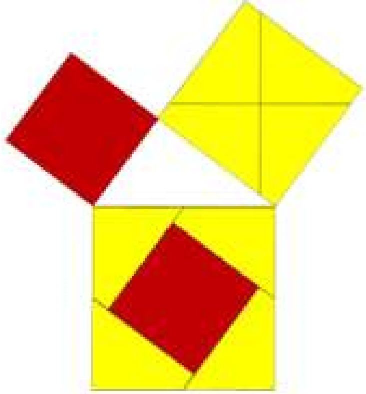
Exemplar Questions

Four congruent right-angled triangles are arranged to make a square ABCD.



- Find an expression for the area of ABCD as the sum of areas of the four triangles and the smaller square.
- Find another expression for the area of ABCD by expanding $(x+y)^2$
- Equate your two expressions to prove Pythagoras' theorem.

British mathematician Henry Perigal provided a proof of Pythagoras' theorem known as 'Perigal's dissection'. Investigate his proof!



Pythagoras' theorem in 3-D

H

Notes and guidance

You might introduce this with asking students to find the length of the diagonal of the classroom, as this will support students who find 3-D visualisation difficult. Annotating diagrams may lead to congestion in their workings therefore encouraging separated triangles may be useful when modelling worked solutions. Prioritising the use of the theorem twice over $\sqrt{l^2 + w^2 + h^2}$ will enable students to apply their skills to other 3-D shapes.

Key vocabulary

2-D	3-D	Cuboid
Diagonal	Hypotenuse	

Key questions

What is the same and what is different about using Pythagoras theorem with 2-D and 3-D shapes?

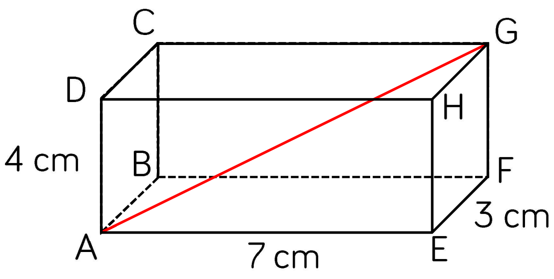
What is the greatest distance between the vertices of a cube?

Why might drawing right-angled triangles be useful when finding the length of a diagonal in a 3-D shape?

Exemplar Questions

Annie and Dora have been asked to calculate the length of AG.

$$\sqrt{7^2 + 3^2 + 4^2}$$
$$= 8.6 \text{ cm}$$



I found AF first

$$\sqrt{7^2 + 3^2} = 7.615 \ldots$$

and then I did

$$\sqrt{7.615 \ldots^2 + 4^2} = 8.6 \text{ cm}$$

Compare their methods.
Investigate finding lengths of diagonals in other cuboids.

Find the length of the longest diagonal in a cube of side length 10 cm.

Brett has a 15 cm × 7 cm × 7 cm pencil case.
He purchases a 18 cm pencil.

Will the pencil fit in Brett's pencil case? Justify your answer.