Summer Term

Year 9

#MathsEveryoneCan

White Rose Maths

WRM – Year 9 Scheme of Learning

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Reasoning with Algebra**						**Constructing in 2 and 3 Dimensions**					
	Straight line graphs		Forming and solving equations		Testing conjectures		Three-dimensional shapes			Constructions and congruency		
Spring	**Reasoning with Number**						**Reasoning with Geometry**					
	Numbers		Using percentages		Maths and money		Deduction		Rotation and translation		Pythagoras' Theorem	
Summer	**Reasoning with Proportion**						**Representations and Revision**					
	Enlargement and similarity		Solving ratio & proportion problems		Rates		Probability		Algebraic representation	Revision		

Summer 1 : Reasoning with Proportion

Weeks 1 and 2: Enlargement and Similarity

Students develop their knowledge of transformations to include enlargement, learning the mathematical meaning of the word similar. You can link back to other transformations as necessary. If appropriate students can move on to negative scales factors. All students should experience finding unknown sides in similar shapes and this can be extended to formal similar triangles problems and trigonometry in the 30/60/90 triangle. General trigonometry is introduced at the start of Year 10.

National Curriculum content covered includes:

- construct similar shapes by enlargement, with and without coordinate grids
- use scale factors, scale diagrams and maps
- apply angle facts, triangle congruence, similarity and properties of quadrilaterals to derive results about angles and sides
- understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction
- use Pythagoras' Theorem and trigonometric ratios in similar triangles to solve problems involving right-angled triangles

Weeks 3 and 4: Ratio and Proportion

Building on students' experience in previous years, here they solve all types of ratio problems and make the links with direct proportion and graphs. Students formally study inverse proportion for the first time, and if following the Higher strand they also look at graphs of inverse relationships. If appropriate, students could also look at more complex problems involving algebra. Students also revisit 'best buys' comparing unit pricing from earlier in the year with alternative methods such as using scaling.

National Curriculum content covered includes:

- divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio
- understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction
- solve problems involving direct and inverse proportion, including graphical and algebraic representations
- use compound units such as speed, unit pricing and density to solve problems

Weeks 5 and 6: Rates

Students develop their knowledge of inverse relationships to explore speed, distance and time in detail. They also look at graphs and the link between the speed/distance/time formulae and density/mass/volume. Students go on to explore other compound units including exploring flow problems such as how long it will take to fill/empty tanks of different shapes at different rates. Students following the Higher strand will also look at converting compound units such as m/s to km/h. You could also include metric and imperial conversions here if desired.

National Curriculum content covered includes:

- use compound units such as speed, unit pricing and density to solve problems
- understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction
- change freely between related standard units [for example time, length, area, volume/capacity, mass]

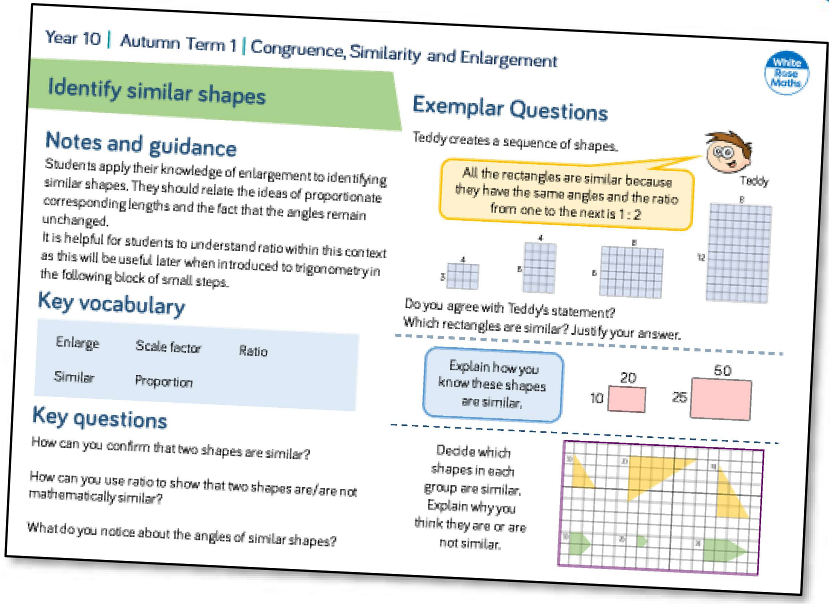
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a “small steps” approach.

As a result, for each block of content in the scheme of learning we will provide a “small step” breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students’ attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.

It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge, Scale factor, Ratio, Similar, Proportion

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

All the rectangles are similar because they have the same angles and the ratio from one to the next is 1 : 2

Teddy

Do you agree with Teddy’s statement? Which rectangles are similar? Justify your answer.

Explain how you know these shapes are similar.

Decide which shapes in each group are similar. Explain why you think they are or are not similar.

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered earlier in Key Stage 3 are labelled R.

Enlargement and Similarity

Small Steps

- Recognise enlargement and similarity
- Enlarge a shape by a positive integer scale factor
- Enlarge a shape by a positive integer scale factor from a point
- Enlarge a shape by a positive fractional scale factor
- **Enlarge a shape by a negative scale factor** H
- Work out missing sides and angles in a pair of given similar shapes
- **Solve problems with similar triangles** H
- **Explore ratios in right-angled triangles** H

H denotes higher strand and not necessarily content for Higher Tier GCSE

R denotes 'review step' – content should have been covered earlier in KS3

Recognise enlargement and similarity

Notes and guidance

Students already know that shapes are similar if all pairs of corresponding sides are in the same ratio, and this can be used as the definition of enlargement. Photographs and posters are useful references. They need to realise that if shape A is an enlargement of shape B then shape B is also an enlargement of shape A i.e. that scale factors can be both greater than 1 and between 0 and 1. Conservation of angle is also worth discussing to prevent misconceptions.

Key vocabulary

Similar	Ratio	Enlargement
Scale factor	Corresponding	Object/Image

Key questions

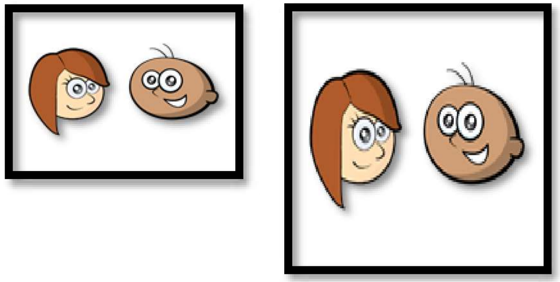
How can you show if one shape is an enlargement of another or not? What information do you need?

What do you notice about the angles of similar shapes?

Are all squares similar? Are all rectangles similar?

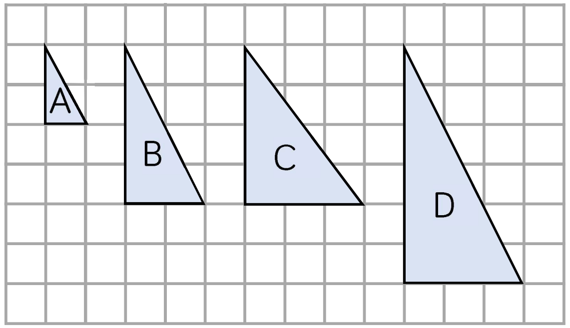
Exemplar Questions

Rosie has enlarged a photograph of her and Tommy.



Is her enlargement correct?
How do you know?
Can you prove your answer?

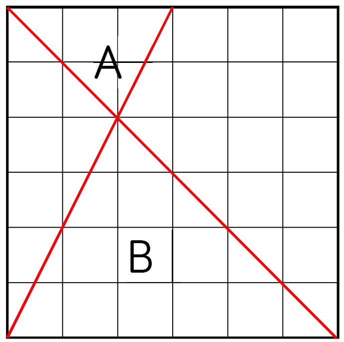
Which triangles are similar?
How can you tell?



Create another triangle similar to triangle A with side lengths

- Greater than 10
- Less than 1

Whitney says that triangles A and B are similar.



Do you agree? Explain why?

Positive integer scale factors

Notes and guidance

Here students practise drawing accurate enlargements. They should explore shapes where all lines are vertical and horizontal first and then consider shapes with some diagonal lines, noting e.g. that a line that joins two points that are, "3 squares to the left and 2 squares up," when enlarged by scale factor 2 is, "6 squares to the left and 4 squares up". This helps when none of the lines on the object are vertical nor horizontal.

Key vocabulary

Object	Image	Scale factor
Integer	Positive	Enlargement

Key questions

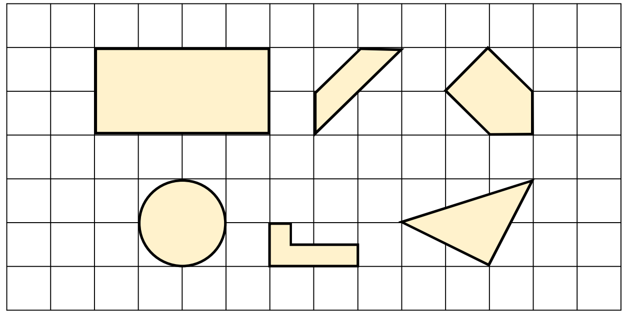
How to the lengths of the image compare to the lengths of the object? How do their perimeters compare?

Does it matter where on a grid an enlargement is drawn?

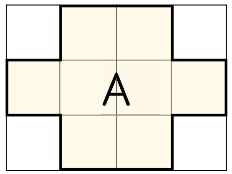
How do you enlarge a shape that has no horizontal or vertical sides?

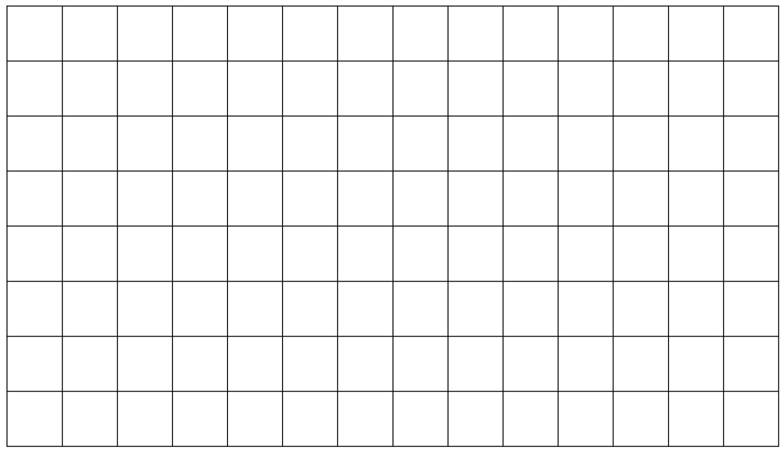
Exemplar Questions

Enlarge each shape by scale factor 3



What is the greatest integer scale factor that shape A can be enlarged by so that it fits on the grid?





A regular hexagon has been enlarged by scale factor 4
The image has a perimeter of 36 cm.
What is the length of each side of the object?

Enlarge a shape from a point

Notes and guidance

In this small step, students need to see the effect an enlargement from a point has on distances between the centre of enlargement and the shape's vertices as well as the side lengths. Using vectors is usually more accurate than rays, though it is useful for students to explore both methods. Dynamic geometry software can help show the effect the centre of enlargement and scale factor has on the position of the image.

Key vocabulary

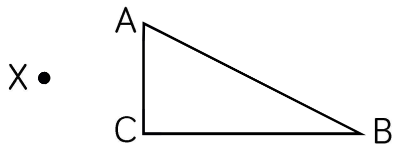
Centre	Scale Factor	Image
Object	Distance	Position

Key questions

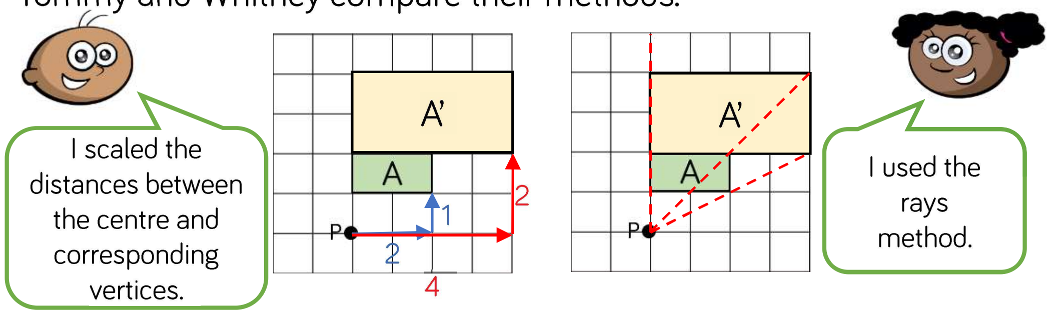
What is meant by the "centre" of an enlargement?
Can the image be anywhere on the grid?
How does the scale factor affect the position of the image?
What happens when the centre of enlargement is on the edge of a shape? Inside the shape?

Exemplar Questions

X is 2 cm away from the midpoint of AC. Enlarge triangle ABC by scale factor 2 about the point X.

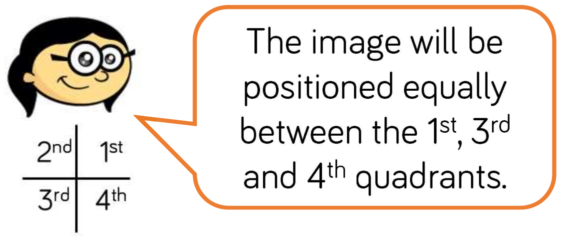


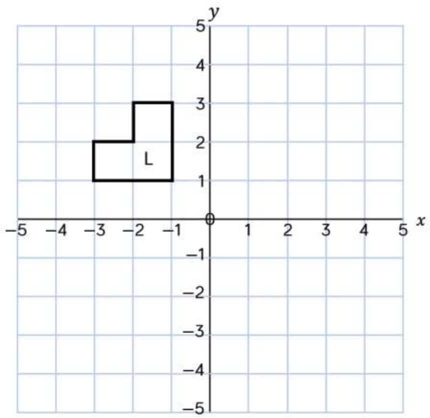
Rectangle A has been enlarged by scale factor 2 from P. Tommy and Whitney compare their methods.



Which method do you prefer?

Annie enlarges shape L from (−3, 3) with scale factor 2





Show that Annie is wrong.
Find a centre of enlargement that would make Annie's claim true.

Positive fractional scale factor

Notes and guidance

Building on the previous steps, students practise drawing enlargements with scale factors between 0 and 1. They should experience enlarging by fractional scale factors both with and without a given centre. Encourage students to generalise that enlargements by a scale factor greater than 1 makes a shape bigger and by scale factors between 0 and 1 makes a shape smaller. They should link multiplication by $\frac{1}{a}$ with division by a.

Key vocabulary

Scale Factor

Fraction

Enlargement

Greater than 1

Between 0 and 1

Key questions

Does enlargement always mean make a shape larger?

What's the same and what's different about enlarging by scale factor 4 and enlarging by scale factor $\frac{1}{4}$?

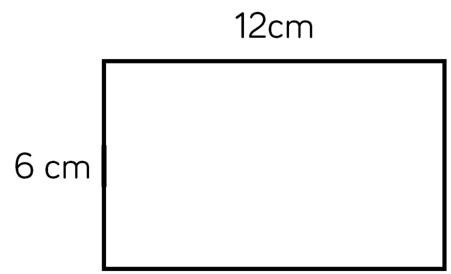
How do you enlarge a shape by scale factor $\frac{2}{3}$?

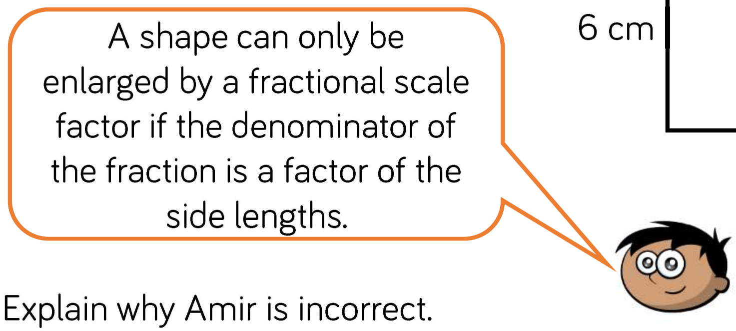
Exemplar Questions

Match up the cards of equal value.

1.5×8	$\frac{3}{4}$ of 12	$\frac{4}{5}$ of 10	One third of 9
$\frac{4}{3} \times 9$	$\frac{1}{2}$ of 8	$\frac{3}{2}$ of 2	$6 \times \frac{2}{3}$

Draw an enlargement of the rectangle by scale factor $\frac{1}{3}$





Explain why Amir is incorrect.

A triangle has vertices (2, 4), (10, 2) and (8, 0).

Enlarge the triangle by scale factor $\frac{1}{2}$

- about the origin
- about the point (8, 2)
- about the point (−2, −2)

Negative scale factor

H

Notes and guidance

In this small step, graphical software can help students see the journey so far with scale factors, transitioning from greater than 1 to between 0 and 1 and then to less than 0. This can be linked back to the structure of the Cartesian plane. Starting with the centre of enlargement as the origin can be helpful for students to see the difference and similarities between enlarging by e.g. 2 and −2. You could also compare an enlargement by scale factor −1 and a rotation by 180°

Key vocabulary

Inverted	Negative	Rotation
Scale factor	Orientation	Centre

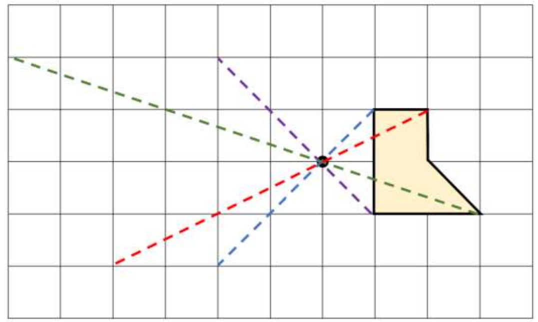
Key questions

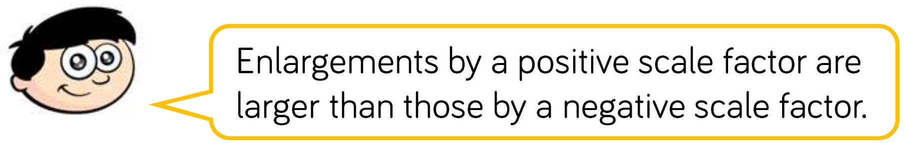
What's the same and what's different about the relative positions of the image and object after a negative enlargement?

Which scale factor produces a larger shape −2 or $\frac{1}{2}$?

Exemplar Questions

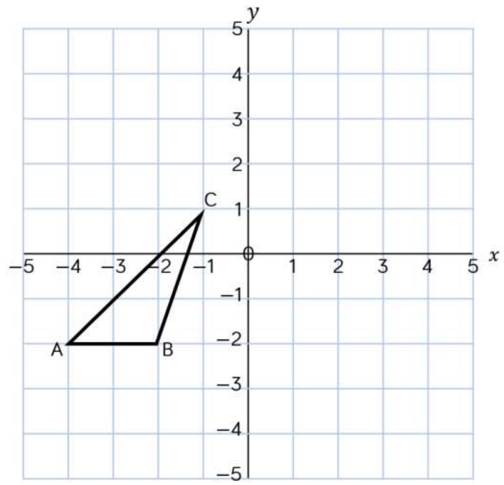
Use the rays to enlarge the pentagon by scale factor −2 from the point.





Is Dexter's claim always, sometimes or never true? Justify your answer.

Enlarge triangle ABC with scale factor −1 from each of the centres of enlargement



- (0, 0)
- (−1, 1)
- (−2, −1)

With which of the centres would an enlargement of scale factor −2 still fit on the grid?

Calculations in similar shapes

Notes and guidance

Students build on their previous learning in Year 8 to calculate unknown sides and angles in pairs, or larger sets, of similar shapes. Provide students with plenty of opportunity to explain what is meant by 'similar' using ratios and identifying scale factors. Students may spot some answers intuitively, so consider using less obvious scale factors such as 3.5 to encourage a slightly more formal approach. Pairs of shapes in different orientations will need modelling and support.

Key vocabulary

Similar	Corresponding	Ratio
Scale factor	Enlargement	

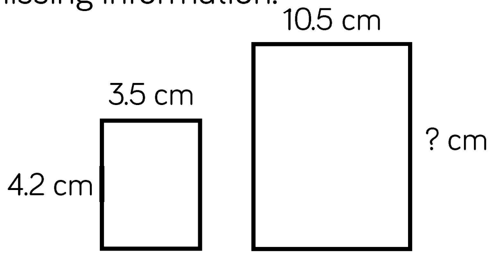
Key questions

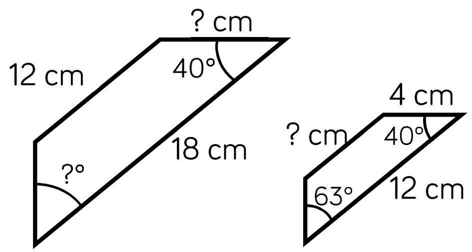
How do you identify the corresponding sides and angles in a pair of similar shapes?

What's the connection between similarity and scale factors of enlargement?

Exemplar Questions

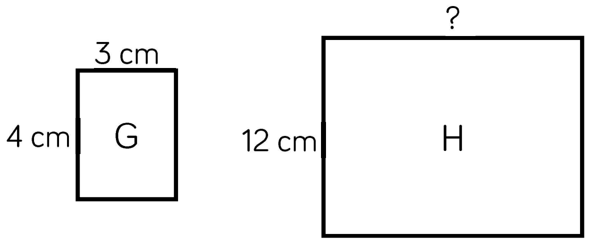
Identify the scale factor between each pair of shapes and find any missing information.





Rectangles G and H are similar.

Eva thinks the missing length in H is 9

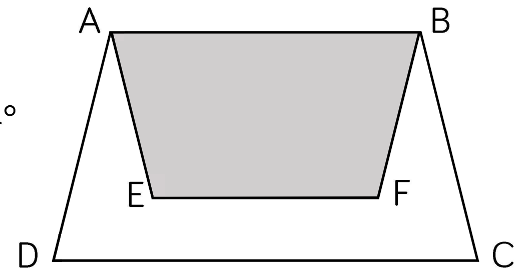


Do you agree? Justify your answer.

ABCD and ABEF are similar isosceles trapezia.

$\angle DAE = 28°$ and $\angle BCD = 104°$
Work out $\angle AEF$

The ratio of AB:DC is 2 : 3
Work out the ratio of EF: DC



Similar triangles

H

Notes and guidance

Students will have found missing sides in pairs of similar triangles in the previous step. This Higher step formalises the process to look at typical similar triangles problems that might be seen at GCSE. Students need to be careful identifying corresponding sides and angles. Labelling the vertices of the triangles is helpful with communicating their thinking both verbally and in writing. Students should explore ratios within triangles as well as between them.

Key vocabulary

Similar	Corresponding	Ratio
Scale factor	Enlargement	

Key questions

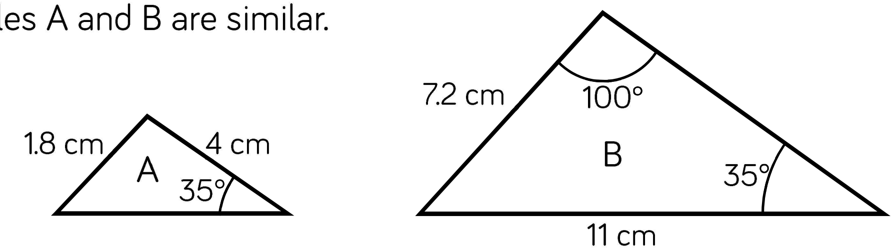
What do you know about angles that correspond to each other in similar triangles?

How do you establish a pair of triangles are similar given information about their sides/about their angles?

Are similar triangles always in the same orientation?

Exemplar Questions

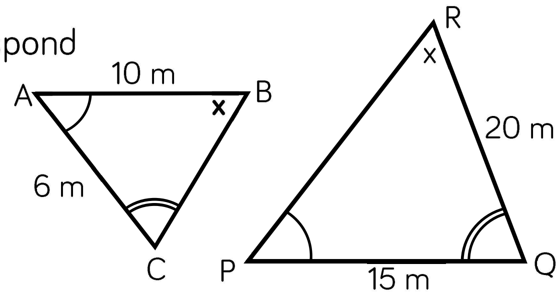
Triangles A and B are similar.



- What's the scale factor of enlargement from triangle A to triangle B?
- Work out the missing angles.
- Work out the missing lengths.

The two triangles are similar.
Which sides in triangle ABC correspond to which sides in triangle PQR?

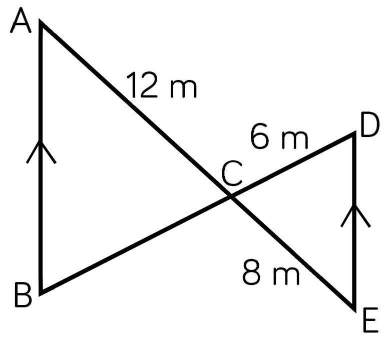
Which sides and angles can you Work out?



Show that triangles ABC and CDE are similar.

The perimeter of ABC is 36 m.

Find the missing lengths in ABC and CDE.



Explore ratios in right-angled triangles H

Notes and guidance

Here, prior to the formal study of trigonometry in Y10, students explore right-angled triangle with 30° and 60°, building on the ratios within triangles looked at in the last step. They look at the ratio of the lengths of the hypotenuse, adjacent and opposite sides, and these names will need defining and explaining. Students can use their findings to work out missing sides. Depending on time, you could explore ratios in other right-angled triangles as well.

Key vocabulary

Opposite	Adjacent	Hypotenuse
Angle	Right-angle	Ratio

Key questions

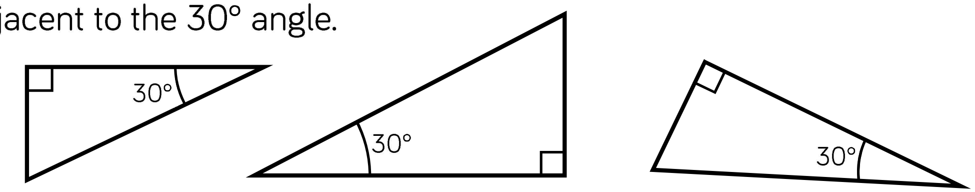
There are two sides that touch the 30° angle, so how do you know which one to label as 'adjacent'?

Explain why all right-angled triangles with an angle of 30° are similar.

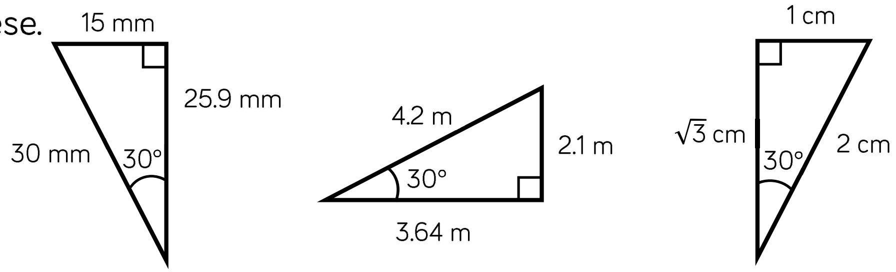
If you know the length of the side opposite a 30° angle, how can you find the length of the hypotenuse?

Exemplar Questions

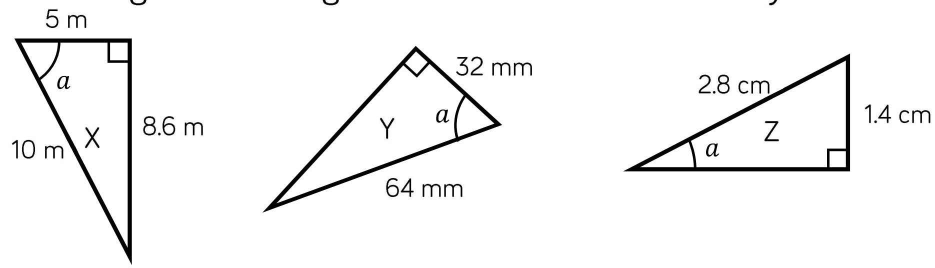
In each triangle, label the hypotenuse and the sides opposite and adjacent to the 30° angle.

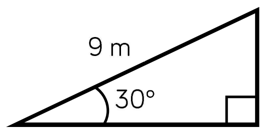


Work out the ratio of the length of sides opposite the given angle and the hypotenuse in each triangle. Investigate other triangles similar to these.



In which triangles is the angle labelled $a = 60°$? How do you know?





Use your knowledge of ratios and Pythagoras' theorem to work out the unknown sides in the triangle .

Ratio and Proportion

Small Steps

- Solve problems with direct proportion (R)
- Direct proportion and conversion graphs (R)
- Solve problems with inverse proportion
- **Graphs of inverse relationships** (H)
- Solve ratio problems given the whole or a part (R)
- Solve 'best buy' problems
- **Solve problems ratio and algebra** (H)

(H) denotes Higher Tier GCSE content

(R) denotes 'review step' – content should have been covered earlier in KS3

Direct proportion

R

Notes and guidance

Students will be familiar with direct proportion and the use of double number lines from their learning earlier in KS3. Here students can review their learning and compare methods for finding solutions, such as the unitary method and using factors of multiples. Students should appreciate that as well as relationships like, “If you multiply one variable by x you multiply the other by x,” that there is a constant multiplier between each pair of values the variables take.

Key vocabulary

Relationship

Ratio

Multiplier

Constant

Scale factor

Key questions

If we know how much 2 items cost how can I work out how much 6 items cost? What about 1 item?

How many different methods can you find to work out the cost of 14 packets if you know the cost of 4 packets?

What’s the significance of 0 items when looking at direct proportion relationships?

Exemplar Questions

One packet of stickers costs £1.20

Complete the table of values to show the cost of buying packets of stickers.

Packets	0	1	2	6	15
Cost (£)		£1.20			

4 bags of sweets cost £3.20

Complete the double number line.

What else can you find out or add to double number line?

A machine fills 3,000 jars in 4 hours.

Show that the relationship between the number of jars filled, y, and the time taken in hours taken, t, is $y = 750t$

- How would the relationship change if the machine filled 6,000 jars in 4 hours?
- How would the relationship change if the machine filled 1,500 jars in 4 hours?

Compare the times taken to fill 120,000 jars in each case.

Proportion and conversion graphs R

Notes and guidance

Again reviewing earlier learning, students explore when graphs show direct proportion relationships and when they do not. Students will need to understand why the graph is linear, why it begins at the origin and explain why other graphs are non-examples of direct proportion. The meaning of the gradient of the graph in terms of the context of the problem should be explored, linking to the constant of proportionality.

Key vocabulary

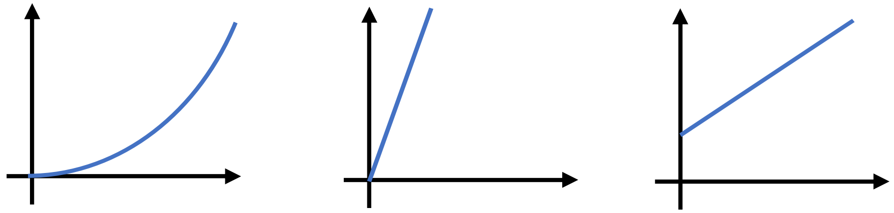
Graph	Relationship	Variable
Linear	Non-linear	Gradient

Key questions

Does a graph showing direct proportion always have to start at the origin?
Do all linear graphs show direct proportion relationships?
How can you use a graph to convert between two currencies? Is this a direct proportion relationship?

Exemplar Questions

Explain which of the graphs show direct proportion and which do not.



Which graph(s) could show conversion between two currencies?

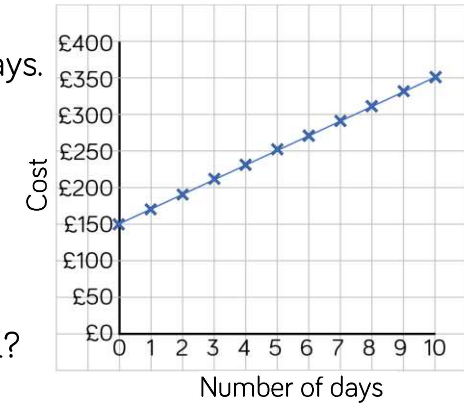
A pack of ten rulers costs £1.50
Complete the table of values to show the cost of buying packs of rulers.

Number of packs	0	1	2	5	10
Cost (£)					

Draw a graph to represent this information. What do you notice?
Is the cost of rulers directly proportional to the number of packs bought?

The graph shows the cost of renting a cement mixer for a given number of days.

What is the y-intercept of the graph?
What is the gradient of the graph?
What does this tell you about the relationship between the cost and number of days?
Is this relationship directly proportional?



Inverse proportion

Notes and guidance

Here students explore inverse proportion relationships discovering that as one variable is multiplied by a scale factor, the other needs to be divided by the same scale factor. Another way of looking at this is to observe that the product of the two variables is constant, i.e. looking at $xy = k$ rather than $y = \frac{k}{x}$. Students should be aware when working with questions like the first exemplar that an assumption is made that all workers perform at the same rate.

Key vocabulary

Inverse	Variables	Constant
Product	Proportional	Relationship

Key questions

What is the difference between direct and inverse proportion?
If two variables are inversely proportional, what happens to the value of one variable if the other is multiplied by a?
Give an example of an inverse proportion relationship.

Exemplar Questions

It takes 3 workers 9 hours to build a wall.
Would it take more or less time if there were more workers? Why?
Would it take more or less time if there were fewer workers? Why?
Comment on the relationship between the number of workers and the time taken to build the wall.
What assumption are you making about the rate at which different workers work? Is this realistic?

Decide whether each of these relationships are directly proportional, inversely proportional or neither.

Number of days it takes to paint a house and the number of workers.	The exchange rate between US dollars and British pounds.	Height of a student and the size of their feet.

T is inversely proportional to g. When $T = 10$ then $g = 2$
Using this information to complete the table of values for the relationship between T and g.

T	1		10	100	200
g		10	2		

Form an equation that links T and g.

Inverse relationships graphs H

Notes and guidance

Here students compare and contrast graphs of inverse and direct proportional relationships. Graphing software can be used to illustrate the key features, including the fact that the lines do not meet the axes. An intuitive example such as speed and time is useful; using the first exemplar question students should appreciate both relationships $t = \frac{120}{s}$ and $st = 120$. Students should be aware that some relationships are neither directly nor inversely proportional.

Key vocabulary

Inverse	Variables	Constant
Non-linear	Proportional	Relationship

Key questions

Is the graph of an inverse proportion relationship linear or non-linear? Why?

Does a graph showing inverse proportion start at the origin?

What's the same and what's different about direct proportion graphs and inverse proportion graphs?

Exemplar Questions

Here is some information about the speed and time taken for a boat to travel a distance of 120 miles.

Speed	60mph	30mph	10mph
Time	2 hours	4 hours	12 hours

What do you notice?

Add some more pairs of values to the table and use the information to plot a graph of speed against time.

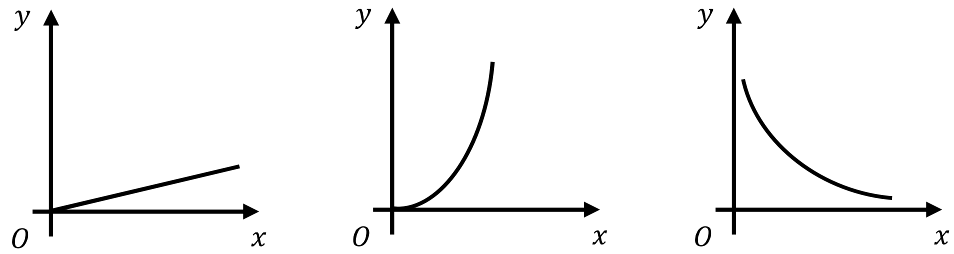
Is the graph linear or non-linear? Why?

What is the relationship between the speed and time taken of the boat?

Decide whether the following statements are examples of inverse or direct proportion.

- The relationship between the number of builders building a wall and time taken to complete the wall.
- Converting between € (euros) and £ (pounds).
- The relationship between number of taps used to fill a tank and the time taken for the tank to fill.

Which of the following graphs show direct proportion, inverse proportion or neither?



Solve ratio problems

R

Notes and guidance

In this step students will revisit and apply their knowledge and understanding of ratio problems. The use of bar models should be encouraged so that students can visualise what information has been given and what they need to work out. Examples should cover a variety of situations where students are given/need to find the total, a single part or a difference. Students should also explore ratios where the comparisons are between more than two items.

Key vocabulary

Divide	Share	Equal parts
Equivalent	Factor	More than/less than

Key questions

How many items are we comparing?
How can we use a bar model to represent the problem?
Which parts are equal and which are not?
Which parts can we label and what can we use them to calculate?

Exemplar Questions

Use the bar model to answer the questions.

Amir and Tommy share £240 in the ratio 3 : 1 How much do they each receive?	Amir and Tommy share some money in the ratio 3 : 1 Tommy gets £240 more than Amir. How much did Amir receive?	Amir and Tommy share some money in the ratio 3 : 1 Amir gets £240 How much money did they share?

What's the same and what's different about the questions and the approaches to solving them?

The angles in a triangle are in the ratio 2 : 5 : 2
Find the size of the largest angle, and show the triangle is not right-angled.
What type of triangle is it? How do you know?

Rosie, Mo and Teddy share some money.
Rosie gets 20% of the money.
Mo and Teddy share the rest of the money in the ratio 4 : 5
Teddy gets £40

- How much does Mo receive?
- What is the ratio of Rosie's share to Mo's share?

'Best buy' problems

Notes and guidance

Students have already considered unit pricing in the 'Maths and Money' block in the Spring term. Here they revise this idea and compare with different alternative methods, as illustrated by the first exemplar question. This presents a great opportunity for class discussion around which methods are most efficient for which problem. Students can then go on to look at more complex problems that include special offers, percentage reductions etc.

Key vocabulary

Unit cost

Factor

Multiple

Direct proportion

Key questions

Is the number of items directly proportional to the cost?

What is the difference between working out the cost per item and the number of items you can buy for a pound?

How does this affect whether best value is the least or greatest value?

Exemplar Questions

Two shops both sell the same type of cartons of juice.

Shop A	Shop B
3 cartons of juice for just £2.10	£1.30 each Buy one get one free.

Annie is working out which shop gives best value for money.
Here is her method

3 cartons = £2.10 6 cartons = £4.10	2 cartons = £1.30 6 cartons = £3.90

What would you conclude? How many other different methods can you find to solve the problem?

In supermarket A 10 eggs costs £3.50, and in supermarket B 12 eggs cost £4.00

Ron attempts to work out which supermarket is better value.
Here is his working out.

Supermarket A	Supermarket B
£3.50 = 10 eggs £1 = 2.86 eggs	£4.00 = 12 eggs £1 = 3 eggs

He says supermarket A is better value as 2.86 is less than 3

- Do you agree with Ron? Why or why not?

Mo solves the problem by finding the cost of 120 eggs in both shops.

- Why do you think Mo used the number 120? What other numbers could he have used?

Ratio problems with algebra H

Notes and guidance

In this higher step students will apply their algebraic understanding and their ratio knowledge to solve complex algebra and ratio problems. Students should be confident with fractions associated with ratios, in particular realising that if $a : b = c : d$ then $\frac{a}{b} = \frac{c}{d}$. This can be checked using numerical examples. Time should be spent considering how to form equations from given problems and what steps are needed to solve these.

Key vocabulary

Equation	Fraction	Divide
Equivalent	Equal parts	

Key questions

If you know the ratio of two quantities a and b, what fractions can you write? How might a bar model help?

What fractions can you write using the given information? How can you rearrange these to form equations?

Exemplar Questions

Given that x is three times the size of y, which of the statements on the cards are true and which are false?

$x : y = 3 : 1$	$y : x = 3 : 1$	$\frac{x}{y} = \frac{1}{3}$	$\frac{y}{x} = \frac{1}{3}$

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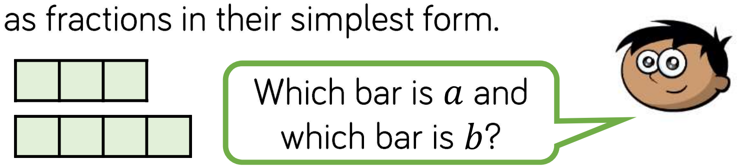
a is three-quarters of b.

- Write $a : b$ as a ratio in its simplest form.
- Write $\frac{a}{b}$ and $\frac{b}{a}$ as fractions in their simplest form.



- Given that $k : 9 = 16 : k$, show that $k = 12$
- w is 60% of x and y is two-thirds of w
  Find the ratio $w : x : y$

Jack and Tommy share some counters in the ratio 5 : 3
Tommy gives 30 counters to Jack.
Now the ratio of Jack's to Tommy's counters is 3 : 13
Which of the following equations would you use to find out how many counters they had initially?

$3(5x + 30) = 5(3x - 30)$	$3(5x - 30) = 13(3x + 30)$
$3(5x + 30) = 13(3x - 30)$	$13(5x + 30) = 3(3x - 30)$

How many counters does Tommy have now?

Rates

Small Steps

- Solve speed, distance and time problems without a calculator
- Solve speed, distance and time problems with a calculator
- Use distance-time graphs
- Solve problems with density, mass and volume
- Solve flow problems and their graphs
- Rates of change and their units
- **Convert compound units** H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

S, D, and T without a calculator

Notes and guidance

Sometimes students don't connect e.g. 60 mph with '60 miles in 1 hour', so this is worth emphasising as the meaning of 'per' underlies the whole study of rates. Double number lines are very useful when working without a calculator, although bar models can also be used if preferred. Students often make errors by treating time as a decimal quantity e.g. thinking 1 hour 25 minutes is 1.25 It is useful to address this here, and in the next step when using calculators.

Key vocabulary

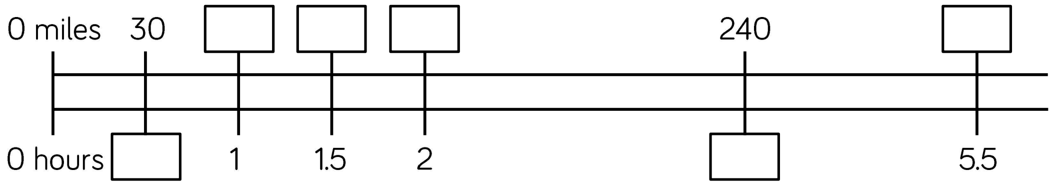
Speed	Distance	Time
Per	Hours	Minutes

Key questions

What can you mark onto your double number line?
What else can you find out?
How many minutes is 0.25 hour?
What fraction of an hour is 15 minutes?
If the speed is constant, is distance travelled directly proportional to time?

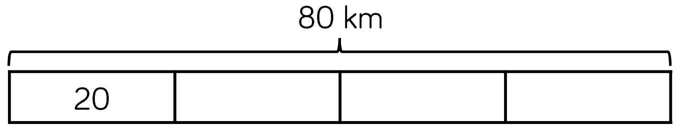
Exemplar Questions

A train travels at a constant speed of 60 mph.
Complete the missing information on the double number line.



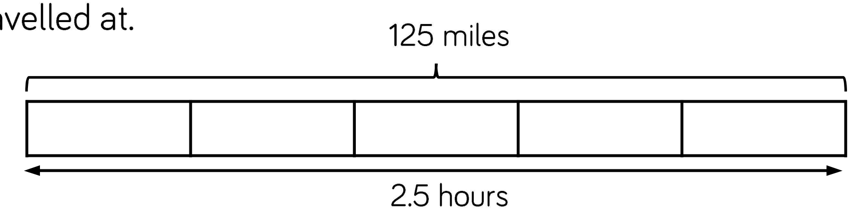
What else can you work out? Add the information onto the number line.

A car travels at a constant speed of 80 km/h.
Dexter draws this bar model to represent the information.



Dexter says, "The car travels 20 km in 0.15 hour."
Explain the mistake that Dexter has made.
How long will it take the car to travel 20 km? 120 km?
How long will it take the car to travel 100 km?

A tram travels at a constant speed for 2.5 hours.
In this time, the tram travels 125 miles.
Use the bar model to help you work out the speed that the tram travelled at.



S, D, and T with a calculator

Notes and guidance

Before solving these problems, students must be confident in converting between hours and minutes. They also need to be able to rearrange an equation with structure $a = \frac{b}{x}$ to find the unknown. Students are more likely to substitute correctly into the formula for speed if they attend to the units given in the question. Remembering the definition of speed and rearranging is often more effective than 'formula triangles'.

Key vocabulary

Convert	Rounding	Speed/distance/time
Accuracy	Average	

Key questions

Is it sensible to round when we convert minutes into hours?
Why shouldn't we round if we are then going to substitute this answer into the S/D/T formula?
How do we know where to substitute the values?
How can we rearrange the formula to find the unknown?

Exemplar Questions

Dora says, "To convert minutes into hours, divide by 60".
Mo says, "To convert hours into minutes, multiply by 60".
Are both Dora and Mo correct? Convert the following into hours.

- 8 minutes
- 1 hour and 8 minutes
- 2 hours and 48 minutes

Convert the following into hours and minutes.

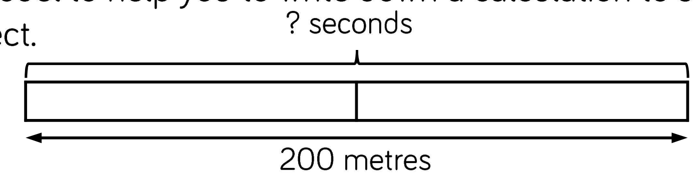
- 0.15 hour
- 1.45 hours
- 1.6 hours

Solve the equations.

$4 = \frac{x}{2}$ $\quad$ $4.1 = \frac{x}{2.3}$ $\quad$ $4 = \frac{8}{x}$ $\quad$ $4.1 = \frac{8.7}{x}$

- Work out the average speed of an aeroplane which travels 5000 miles in 8 hours and 42 minutes.
- Work out the distance a golden eagle flies, when travelling at a constant speed of 125 km/h for 8 hours and 42 minutes.
- Work out the time it takes for a wasp to travel 1 metre if it travels at a constant speed of 200 metres per minute.

Nijah runs 200 metres.
She runs the first 100 metres in 20 seconds.
She runs the next 100 metres in 30 seconds.
Brett says, "Nijah's average speed over 200 m is 25 metres per second".
Use the bar model to help you to write down a calculation to show that Brett is incorrect.



Use distance-time graphs

Notes and guidance

Students start by identifying what different line segments on a distance/time graph represent. They might also consider how they know which line segments represent the same speeds, making the link with gradient. Sometimes students think a line going in the opposite direction, even with the same gradient, indicates a different speed. Calculating speeds can unpick this misconception. Students should have the opportunity to both read from and draw distance/time graphs.

Key vocabulary

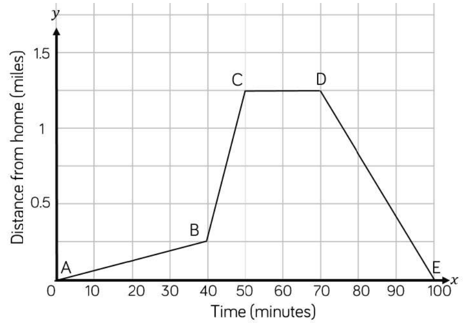
Gradient	Speed	Distance
Axes	Origin	Time

Key questions

What does the gradient of a straight line segment in a distance-time graph tell us?
How can we identify speeds that are slower, quicker, equal?
How can we calculate overall average speed?
How do we know where to start or end a line segment when drawing a distance-time graph?

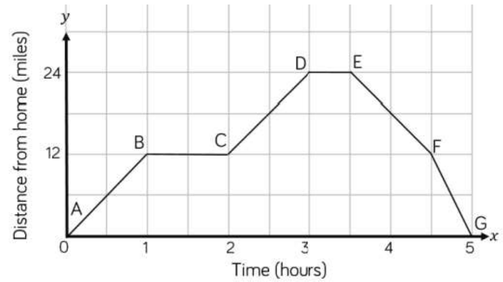
Exemplar Questions

Tom sets off from his house. Sometimes he walks, sometimes he runs and at one point he stops for a break.
Match the line segment to the activity.



AB	Running
BC	Stop for a break
CD	Walking at a quick speed
DE	Walking very slowly

Dani says, "Tom travels for 1.25 miles".
Is Dani correct? Explain your answer.



The graph shows the distance Aisha travels on her bike over time.
Calculate Aisha's speed from A to B, C to D and E to F.
What do you notice?

Calculate Aisha's average speed whilst she is moving.
Aisha's brother, Ron, sets off from their home on his bike 30 minutes after Aisha. He cycles at a constant speed of 9 mph for 2 hours before returning home at a constant speed of 12 mph. Represent Ron's journey on the same graph.
Who returns home first?

Density, mass and volume

Notes and guidance

One way to introduce the concept of density is to compare the density of e.g. silver to that of e.g. oxygen, and to discuss why these are different. This also helps students to identify the correct units for each of density, mass and volume, explicating linking 'per' with the familiar mph/kmh measures. Students then practise substituting into the formula. There's also an opportunity to interleave volume of different solids here. Students should check units are all consistent.

Key vocabulary

Density	Mass	Volume	Per
Units	Substitute	Rearrange	

Key questions

What is the difference between mass and volume?

Why is the density of a metal likely to be different from the density of a gas?

Why do we measure density in g/cm^3 (or kg/m^3)?

How can we work out the volume of a prism?

Why do we have to check the given units?

Exemplar Questions

A ruby has a density of 4 g per cm^3 (4 g/cm^3)

Complete the table.

Density (g/cm^3)	Mass (g)	Volume (cm^3)
4	4	1
4		2
4		3

Tick the correct formula.

Density = Mass × Volume	$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$	$\text{Density} = \frac{\text{Volume}}{\text{Mass}}$

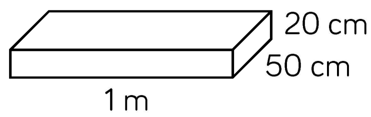
The density of tin is 7260 kg/m^3

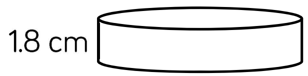
- Calculate the mass of a piece of tin with a volume of 0.2 m^3
- Calculate the volume of a piece of tin with a mass of 8 kg.

This cuboid has a mass of 68 000 g.

Work out the density of this block of plywood.

Give your answer in kg/m^3





The diagram shows a circular disc of ice, radius 2 cm. This disc has a mass of 20.74 g.

An object will float if its density is less than the density of water.

The density of water is 0.997 g/cm^3

Will the circular disc float on water? Justify your answer.

Liquid A has a density of 0.92 g/cm^3

Liquid B has a density of 1.22 g/cm^3

152 g of liquid A is mixed with 98 g of liquid B to create a new liquid.

What do you know? What can you find out?

Flow problems and their graphs

Notes and guidance

Students might start by comparing different shaped containers and considering what's the same and what's different about the rate at which they will fill with a liquid. They need to identify containers that will fill at a constant rate over time (represented by a straight line graph) compared to those that will fill at a varying rate over time (represented by a curve). Students then consider solving flow problems, thinking about units as in the previous steps.

Key vocabulary

Constant rate	Straight line	Curve
Flow rate	Prism	Volume

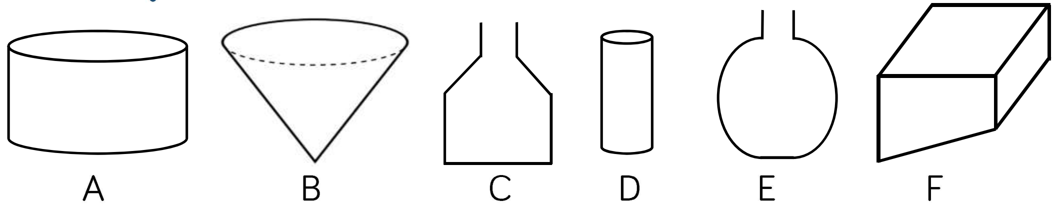
Key questions

Compare two cylinders of the same height, but different diameters. Which will fill more quickly? What will this look like on a graph? Will the lines be straight or curved?

Describe the rate at which the container will fill. Why will it change?

What volume of water is delivered every ____ seconds?

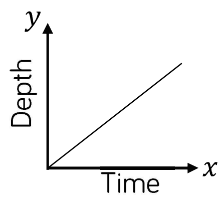
Exemplar Questions

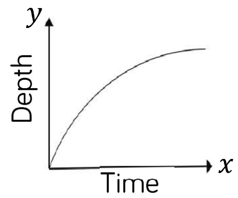


Imagine filling each container with water. Discuss

- Which of the containers will fill at a constant rate?
- Which will fill more slowly/quickly to begin with?

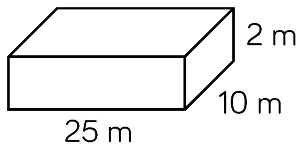
Which container(s) being filled, could these graphs represent?





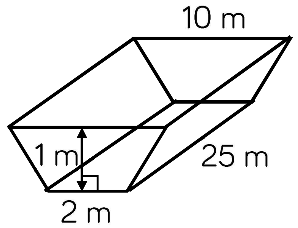
Sketch a graph of the depth of water against time for each of the other containers.

This cuboid container is full of water. The water is drained at a constant rate from the container through a plug in the bottom.



The level of the water in the container goes down by 10 cm every 45 seconds. How many minutes does it take for the container to empty?

If the container is rotated so that it's height is now 25 m, will it take a different amount of time to empty? Explain your answer.



How long would it take for this container to empty, if the level of water decreases by the same rate?

Rates of change and their units

Notes and guidance

This step gives students time to explore the units involved in rates of change questions. It helps to embed their understanding of a rate of change. Interpreting the gradient of a graph in a given context is important in supporting students to connect the rate of change to gradient. They should be encouraged to consider each rate as ___ per ___ making sense of the changes to the variables through models such as the double number line or graphs.

Key vocabulary

Units	Conversions	Per
Gradient	Rate of change	Units

Key questions

How can I convert from ___ to ___ (e.g. m to km)?

What units are used in the question? What units do you need for the answer?

How can you work out the gradient? What does this tell us in terms of the given context?

Exemplar Questions

Use the cards to fill in the sentences:

British Pounds (£)	Kilometres (Km)	Volume (cm^3)	
Hour (h)	Euros (€)	Litres (l)	Mass (g)

- Speed measures _______ per _______
- Exchange rate measures _______ per _______
- Density measures _______ per _______

A machine fills 200 bottles in 8 minutes.
At what rate is the machine filling bottles?
How many bottles can the machine fill in 2 and a half hours?
How long will it take for the machine to fill 1500 bottles?
After a service, the machine operates 10% more efficiently.
How does this change your answers?

Scott types at an average rate of 40 words per minute.
Why do think this rate is given as an average?
How many words can Scott type in three-quarters of an hour?
How long will it take Scott to type a 10 000 word essay?
How realistic do you think your answers are?

Emily is packing cups into boxes. In the first hour she packs one box every two minutes. In the second hour she packs 25 boxes and in the last hour she packs two boxes every 5 minutes.
Compare her rates of packing and find her average rate over the three hours.

Convert compound units

H

Notes and guidance

Considering which units need converting before starting the question, helps students to plan a solution. They should be encouraged to take a step-by-step approach e.g. moving from metres per second to metres per minute, metres per hour and then km per hour. This is a good opportunity to interleave questions on speed, density and flow rates. Using 'goal free' questions also provides a 'way in' for students in this small step.

Key vocabulary

Imperial	Metric	Convert
Units	Double number line	

Key questions

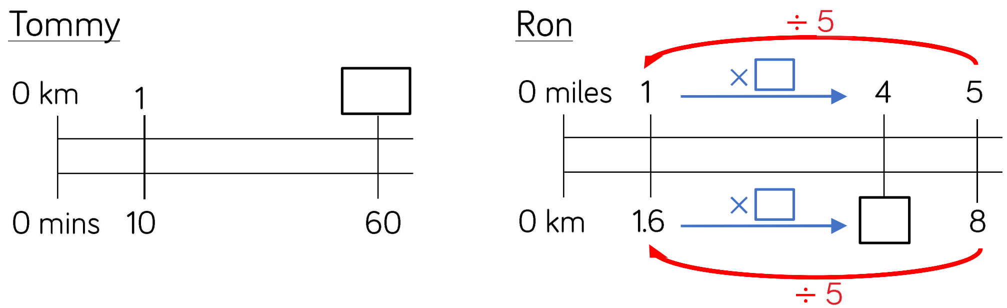
If you know a speed in kilometres per hour, what steps would you take to convert it to metres per second?

What's the same and what's different about the units g/cm^3 and kg/m^3? What do these units both measure?

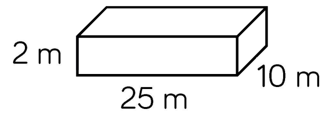
Explain why $1\ m^3$ is not equal to $100\ cm^3$

Exemplar Questions

Tommy walks 1 km every 10 minutes.
Ron's average walking speed in 4 mph.
Ron knows that 5 miles = 8 km.
They are trying to work out who's faster at walking.



Discuss with a partner how the double number lines have been drawn.
Complete the missing numbers on each double number line.
Who is walking at a greater speed? Justify your answer.



This cuboid swimming pool is filled with water at a rate of 5 litres per minute.

What do you know? What can you find out?
How many m^3 are there in 1 litre?
How many minutes will the pool take to fill?
How many hours will the pool take to fill?

Describe how to convert the following pairs of units

- Metres per second to kilometres per hour.
- Miles per hour to feet per second.

What's the same and what's different?

Summer 2 : Representations and Revision

Weeks 1 and 2: Probability

In this block students build on their learning in Year 7 and 8 to calculate the probabilities of single and combined events. A key focus is the introduction of the idea of independent events and the use of the multiplication rule for these. Students also look at a variety of diagrams that support probability such as sample space diagrams,Venn diagrams and two-way tables. Tree diagrams, considering both with and without replacement, are included as Higher steps.
National Curriculum content covered includes:

- record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness, fairness, equally and unequally likely outcomes, using appropriate language and the 0-1 probability scale
- understand that the probabilities of all possible outcomes sum to 1
- enumerate sets and unions/intersections of sets systematically, using tables, grids and Venn diagrams
- generate theoretical sample spaces for single and combined events with equally likely, mutually exclusive outcomes and use these to calculate theoretical probabilities

Week 3: Algebraic representation

Students extend their knowledge of graphs to look at interpretation and creation of different types of graphs. The first non-linear graph explored is the quadratic graph, where students are encouraged to look at the symmetry of the curve and read off x/y values. They also explore reciprocal end exponential graphs. Although students need to be able to plot curves and practising this is important, they can also use graphing software to explore the general forms of the curves as this will save a lot of time and be more accurate. Students knowledge of straight line graphs is extended by looking at inequalities graphical, and these are also represented as number lines. In addition, solution of simultaneous equations by graphical methods is also included as a Higher step.

National Curriculum content covered includes:

- recognise, sketch and produce graphs of quadratic functions of one variable with appropriate scaling, using equations in x and y and the Cartesian plane
- use quadratic graphs to estimate values of y for given values of x and vice versa
- find approximate solutions to contextual problems from given graphs of a variety of functions, including piece-wise linear, exponential and reciprocal graphs
- use linear graphs to estimate values of y for given values of x and vice versa and to find approximate solutions of simultaneous linear equations
- understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors

Weeks 4 to 6: Revision

The last three weeks of the summer term are unassigned in order to allow you time to review any areas of the KS3 curriculum that you feel your students would benefit from as they prepare to transition to KS4, or to deepen their knowledge of an area if appropriate.
You may wish to include:

- Handling Data – there is no explicit data coverage in Year 9, so you could revise the learning of Year 7 and 8, possibly through projects, and include the Y8 Higher steps around mean averages from a frequency table
- Sequences – there is no new sequence content in Year 9. If your class did not cover the Higher step for finding the rule for the nth term of a linear sequence, you could do this here.
- Error intervals – also only covered as a Higher step in Y8
- Trigonometry – you could develop the brief introduction to trigonometry in Summer Block 1 to study this in more detail, but please note this is covered in depth in the first block of our Year 10 scheme of learning

National Curriculum content covered depends on your choices.

Probability

Small Steps

- Single event probability (R)
- Relative frequency
- Expected outcomes
- Independent events
- **Use tree diagrams** (H)
- **Use tree diagrams to solve 'without replacement' problems** (H)
- Use diagrams to work out probabilities

(H) denotes higher strand and not necessarily content for Higher Tier GCSE
(R) denotes 'review step' – content should have been covered earlier in KS3

Single event probability

R

Notes and guidance

You may also wish to revise fraction equivalence and arithmetic before starting this block. Students may need reminding that probability can be stated as a fraction, decimal or percentage but not in words or as a ratio. You could also discuss when it is and when it isn't appropriate to assume that events are equally likely, and the meaning of key words such as bias and fairness.

Key vocabulary

Event	Outcome	Equally likely
Probability	Biased/unbiased	Fair

Key questions

Are the outcomes of the event equally likely or not? How do you know?

When can you/can't you add together the probabilities of events?

If you know the probability that an outcome happens, how can you easily find the probability it doesn't happen?

Exemplar Questions

The ratio of red counters to yellow counters in a bag is 2 : 3

Explain why the probability of selecting a red counter from the bag is not $\frac{2}{3}$

Give the correct probability of selecting a red counter as a fraction, a decimal and a percentage.

Which of these numbers could not be the probabilities of events?

0.0004	1.5%	0	$\frac{2}{9}$	$\frac{9}{2}$
125%	1.5	−0.4		

Explain why not.

Brett buys a ticket for a raffle. He says, "The probability I win a prize is one half, because I either win or I don't."

Explain why Brett could be wrong.

The table shows the probabilities of obtaining some scores when a six-sided die is thrown.

Score	1	2	3	4	5	6
Probability	0.2	0.1	0.05	0.35		

Explain how you can tell from the table that the die is biased.

Work out the missing values in the table if

- the probability of scoring 5 is equal to the probability of scoring 6
- the probability of scoring 5 is twice the probability of scoring 6
- the probability of scoring 5 is three times the probability of scoring 6

Relative frequency

Notes and guidance

This step is best approached by performing experiments and seeing how the relative frequency of particular outcomes change. Although there will inevitably be variability, generally experimental results will tend to a limit, and the speed with which this happens can be part of a useful discussion. Plotting a graph of results to see how initial variation settles down to a long-term value is also useful. This can also be illustrated by computer-generated results.

Key vocabulary

Experiment	Outcome	Biased/unbiased
Trial	Frequency	Relative Frequency

Key questions

How do you work out a relative frequency? Why is it different from a probability?
Why is experimental probability different from theoretical probability?
What happens to experimental and theoretical probability when a very large number of trials have been completed?

Exemplar Questions

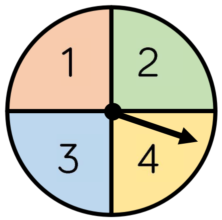
Rosie flips a coin and records the number of times it lands on heads after different numbers of flips.

Number of flips	Number of heads	Relative frequency
10	7	0.7
20	12	0.6
40	21	
100	57	

Explain how the relative frequencies are found.
Work out the missing values in the table.
Which of the relative frequencies gives the best estimate of the probability that Rosie's coin will land on heads? Why?

Roll a six-sided die 60 times and record the number of times it lands on 6 after every 10 throws.
Plot a graph of the relative frequency of rolling a 6 against the number of trials.
Compare your graph with a partner's. What's the same and what's different?
How would you expect the graph to change over larger numbers of trials?

Esther has a four-sided spinner.
The sides of the spinner are labelled 1, 2, 3 and 4
She spins the spinner 150 times and it lands on the number 4 a total of 56 times.
Do you think the spinner is fair? Justify your answer.



Expected outcomes

Notes and guidance

Building from the last step, students need to be aware that the expected number of times an outcome will occur is a long-term average rather than a prediction. The degree of variability can be illustrated by the experiments the students have carried out. Students should be able to find the expected number of times a particular outcome will occur given the probability in any format, not just as a fraction.

Key vocabulary

Event	Outcome	Expected
Probability	Frequency	Trial

Key questions

How do work out the expected frequency of a particular outcome? Is the ‘expected value’ the exact number of times you would expect an event to occur?
Would you be surprised if you (e.g.) flipped a coin 10 times and got heads 6 times? What about 60 times out of 100? 600 times out of 1000?

Exemplar Questions

A fair coin is flipped 500 times.
Explain why the expected frequency of heads is 250
In practice, do you think the coin will land on the heads exactly 250 times? Why or why not?

There are 400 seeds in a packet.
The probability a seed will grow into a flower is 85%
All the seeds are planted.
Work out an estimate for the number of seeds that grow into flowers.

A box contains 5 balls numbered 1, 2, 3, 4 and 5
One ball is chosen at random from the box.
Its number is recorded and the ball replaced in the box.
This process is carried out 240 times altogether.
How many times would you expect

- an even-numbered ball to have been chosen?
- a prime-numbered ball to have been chosen?
- an odd-prime-numbered ball to have been chosen?

A biased spinner labelled 1, 2, 3 and 4 is used in an experiment.
The probabilities of 1, 2 and 4 occurring are shown in the table.

Score	1	2	3	4
Probability	0.29	0.2		0.37

The experiment is carried out 300 times.
Show that a 3 is expected to occur fewer than 50 times.

Independent events

Notes and guidance

Students need to be able to find the probability of independent events, working with both fractions and decimals, in order to support later work on tree diagrams. Questions like the first exemplar help to illustrate where the multiplication rule comes from as e.g. “one quarter of a third”. Students need to be careful not to confuse independent, when outcomes do not affect each other, with mutually exclusive, where they cannot occur together.

Key vocabulary

Event	Outcome	Independent
Probability	Product	Affect

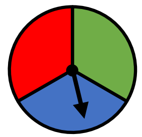
Key questions

Does (e.g.) the score you get on the first roll of the die affect the score you get on the second roll? Why or why not?
What does it mean for two events to be independent?
Is it realistic to assume that these two events are independent?

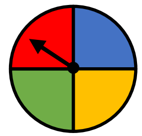
Exemplar Questions

Here are two fair spinners.
Both spinners are spun once.

A



B



- What is the probability spinner A lands on Red?
- What is the probability spinner B lands on Red?

Complete the sample space of the outcomes for both spinners.

Spinner A \ Spinner B	**R**	**B**	**G**	**Y**
R	RR	RB		
G			GG	
B				

- What is the probability that both spinners land on red?
- What is the connection between your answer and the probabilities that each spinner lands on red? Why?

Two fair coins are thrown.

- What is the probability they both show heads?
- How can you illustrate that your answer is correct?

Three fair coins are thrown.

- What is the probability they all show tails?

A code is formed by picking a letter of the alphabet follow by a digit from 0 to 9

- Show that the probability the code is F7 is $\frac{1}{260}$
- Find the probability the code is a vowel followed by a prime number.

Tree diagrams

H

Notes and guidance

Students use tree diagrams to list the possible outcomes from a series of events and support finding their probabilities. They need to be confident with using the fact that the sum of all possible outcomes is 1, both for each pair of 'branches' and for the total of all the combined outcomes. Students can then quickly find e.g. the probability of at least one head by subtracting the probability of no heads from 1. They should also realise they do not always need tree diagrams.

Key vocabulary

Event	Outcome	Independent
Probability	Product	At least one

Key questions

How many branches will this tree diagram need? How do you know?

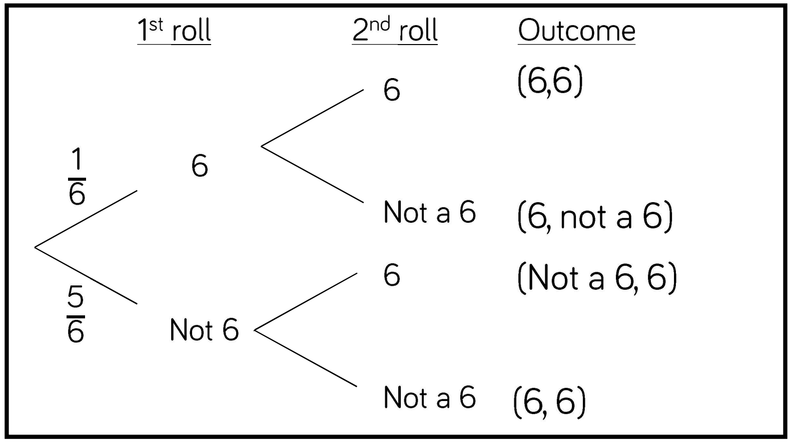
Why do we multiply probabilities along the branches of a tree diagram but add the probabilities of the outcomes?

How could I calculate the probability of four heads in a row without drawing a tree diagram?

Exemplar Questions

A fair die is rolled twice.

Here is a tree diagram to show how many sixes are obtained.



Is the probability of getting a 6 on the second roll the same as or different from the probability of getting a 6 on the first roll? Why?

Copy and complete the tree diagram and find the probabilities of each of the four outcomes.

What is the sum of these four probabilities? Why?

A coin is biased so that the probability of getting a head is 0.6

The coin is thrown twice.

Alex works out the probability that the coin lands on heads exactly once.

P(H once) = P(1^{st} throw H) $\times$ P(2^{nd} thrown T) = $0.6 \times 0.4 = 0.24$

Draw a tree diagram to show the possible outcomes and show that Alex is wrong.

Tree diagrams without replacement H

Notes and guidance

It is worth spending time looking at how the probabilities change after particular outcomes before moving on to using these in complex tree diagrams. The first exemplar question illustrates this and can be adapted easily for more practice. Students could also draw sketches at each branching point of the tree diagram to show e.g. how many coins of each type/counters of each colour there are after each trail.

Key vocabulary

Event	Outcome	Independent
Probability	Replacement	At least one

Key questions

After (e.g.) selecting a red sweet, how many sweets are there now? How many of these are red? What is the probability that the second sweet is red?
Why do the probabilities change between trials? How do they change? Can you still multiply probabilities even though the events are not independent?

Exemplar Questions

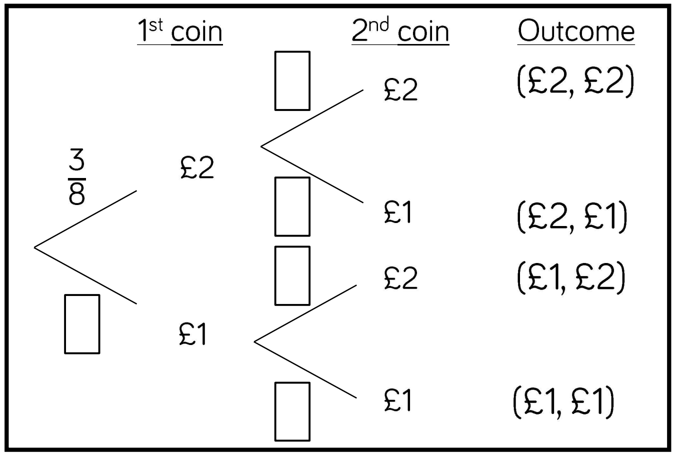
A bag contains 4 red sweets and 5 green sweets.
A sweet is selected at random from the bag.

- What is the probability that the sweet is red?

If the sweet is red, what can you find out about the colours of the sweets that remain in the bag?

- What is the probability that a second sweet selected from the bag is also red?
- How would your answers change if the first sweet had been green?

Tommy has three £2 coins and five £1 coins in his pocket.
He takes two coins without looking.
Copy and complete the tree diagram to show the possible outcomes.



Use your tree diagram to work out the probability Tommy has taken exactly £3 out of his pocket.

Using diagrams for probabilities

Notes and guidance

The step provides an opportunity to review other diagrams used to calculate probabilities so students do not think they have to always use a tree diagram. They may need a reminder as to how to construct a sample space/array for questions like the second exemplar. This is also a good opportunity to revise constructing and interpreting two-way tables and Venn diagrams. If appropriate you can also revise and extend set notation e.g. looking at P(A ∪ B)'

Key vocabulary

Venn diagram	Intersection	Union
Sample Space	Two-way table	

Key questions

How do you set up a two-way table? What values do you know and what can you find?

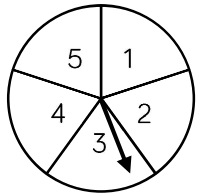
Describe the meaning of each section of the Venn diagram in words.

What is a sample space? How can you list one efficiently?

Exemplar Questions

There are 30 students in a class.
18 are boys and 14 play chess.
6 girls play chess.
Represent this information in a two-way table and find the probability that a boy chosen at random from the class plays chess.

Two identical spinners labelled 1 to 5 are spun once.
The scores on the spinners are added together.
Find the probability that the total score is



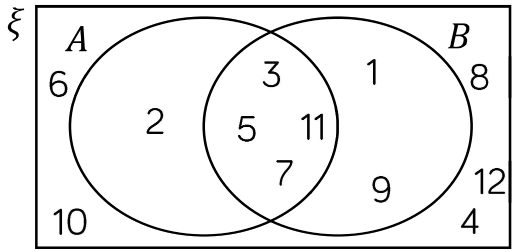
- greater than 7
- a factor of 10
- prime

The Venn diagram shows

ξ = {integers from 1 to 12}

A = {prime numbers}

B = {odd numbers}



A number from the universal set is selected at random.

- What is the probability the number is a member of A ∪ B?
- What is the probability the number is prime?
- Given that the number is prime, what is the probability it is odd?
- Given that the number is odd what is the probability it is prime?
- Explain why there is only one even number in A ∪ B.

Algebraic representation

Small Steps

- Draw and interpret quadratic graphs
- Interpret other graphs, including reciprocal and piece-wise
- **Investigate graphs of simultaneous equations** H
- Represent inequalities

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Quadratic graphs

Notes and guidance

Students need to be confident with substituting numbers, especially negative numbers, into quadratic expressions in order to produce tables of values for their own graphs. They will need support and practise to draw accurate smooth curves. In particular, they should be aware that the turning point is not always at an integer value of x. They should also explore the parabolas formed by quadratic expressions using dynamic software.

Key vocabulary

Quadratic	Parabola	Curve
Vertex	Turning Point	Symmetry

Key questions

How can you tell from an equation whether the graph will be a straight line or a parabola?
Are all quadratic graphs symmetrical?
Why is there only one value of y for a given x but sometimes two values of x for a given y?

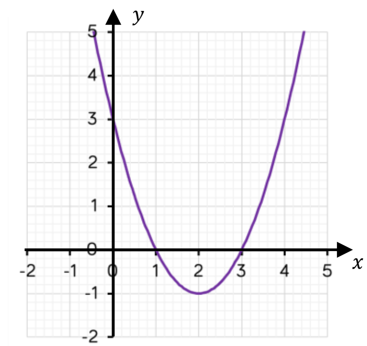
Exemplar Questions

Evaluate each expression for $x = 4$ and $x = -4$

x^2 | $x^2 + 2$ | $x^2 - 2$ | $x^2 + 2x$ | $x^2 - 2x$ | $x^2 + 3x - 1$ | $x^2 - 3x + 1$

What's the same? What's different?

Here is the graph of $y = x^2 - 4x + 3$



Use the graph to find

- the value of y when $x = 0.5$
- the values of x when $y = 4$

Are your answers exact or estimates? Why?

Complete the table of values for $y = x^2 + 2x - 1$, and plot the graph.

x	-3	-2	-1	0	1	2	3
y	2						14

Complete the table of values for $y = x^2 + x - 3$

x	-3	-2	-1	0	1	2	3
y	3						9

Explain how you can tell from the table of values that you need to calculate y for one more value of x before you can plot the graph.

Interpret other graphs

Notes and guidance

Students need to be able to read from any type of graph. As well as the examples included here, you could look at cubic graphs and multiples of the reciprocal and exponential graphs. Encourage students to use a ruler to draw to lines to help them reading off in both directions. Students have met piece-wise graphs before (e.g. distance-time graphs) but the real-life applications may need explaining. Whilst the focus is on interpretation, students could also draw graphs.

Key vocabulary

Reciprocal	Exponential	Curve
Piece-wise	Discontinuous	

Key questions

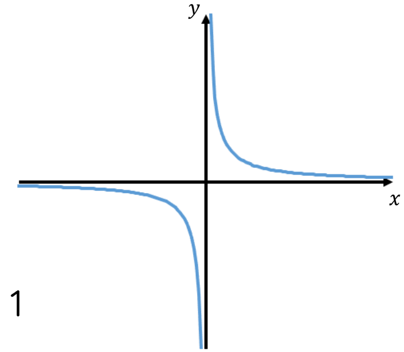
Why is there no y value for $x = 0$ on the graph of $y = \frac{1}{x}$?

What's the same and what's different about the graphs of $y = x^2$ and $y = 2^x$?

Exemplar Questions

Here is a sketch of the graph of $y = \frac{1}{x}$

Why is this graph different from the graphs you have studied before?



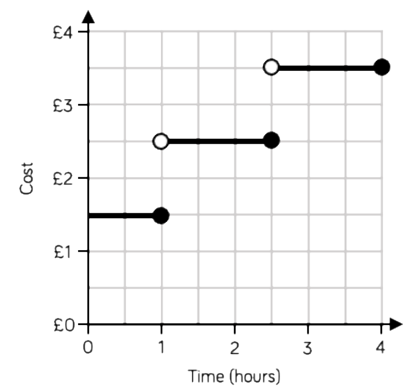
Use an accurate copy of the graph to find

- the values of y when $x = 2$, 0.5 and -1
- the values of x when $y = 2$, 0.5 and -1

What do you notice?

The graph shows the costs of parking in a car park for different lengths of time.

- Why does the graph 'break' at 1 hour and 2 hours?
- Why do you think a graph like this is called a piece-wise graph?



Use graphing software to plot the graphs of $y = 2^x$ and $y = 3^x$

What's the same and what's different about the two graphs?

Use the graphs to solve the equations on the cards.

$2^x = 20$	$3^x = 20$	$2^x = 0.5$	$3^x = 1$

Predict the shapes of $y = 4^x$ and $y = 0.5^x$, and check your answers.

Simultaneous equations

H

Notes and guidance

Students should be aware that a single equation in two unknowns has an infinite number of possible solutions, but a pair of simultaneous equations generally have a single solution pair. You could consider the special cases of parallel lines/equivalent lines if appropriate. Students should also be aware that solutions need not be integers and the graphical method is limited. It is useful to consider where straight lines meet the axes to make graph drawing more efficient.

Key vocabulary

Simultaneous	Solution	Intersection
Satisfy		

Key questions

What values of x and y do you use to find where straight line graphs meet the coordinate axes?
What do you know about the values of x and y at the point of intersection of the two graphs?
How can you check the solution to a pair of simultaneous equations?

Exemplar Questions

Here are two equations. $x + y = 4$ $3x + y = 6$

- Find some values of x and y that satisfy each equation. How many pairs of values can you find?
- Find some values of x and y that satisfy both equations. How many pairs of values can you find?
- Use graphing software to plot the graph of $x + y = 4$ and $3x + y = 6$ How do the graphs support your findings?

Where do the straight lines $x + y = 3$ and $x - 2y = 6$ meet the axes?
Use your answers to draw the lines and solve the simultaneous equations $x + y = 3$ and $x - 2y = 6$

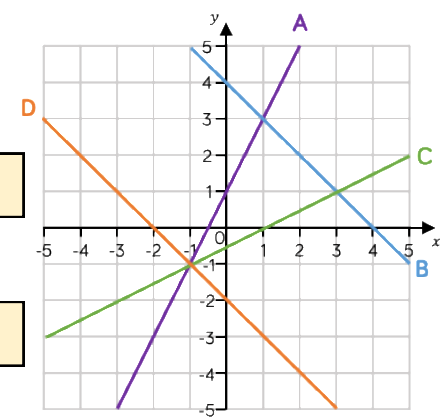
Which equation belongs to which line?

$y = 2x + 1$

$x + y = 4$

$2y = x - 1$

$x + y + 2 = 0$



How many pairs of simultaneous equations can you solve using the graphs?

Representing inequalities

Notes and guidance

Building on their knowledge of straight line graphs, students shade regions to represent given inequalities in one variable, and if appropriate two variables. They should use dotted lines to show when borderline values are not included in the solution set. They can compare this to empty circles for the corresponding number line representation. Students could revise forming and solving inequalities and represent the solutions in a variety of ways.

Key vocabulary

Inequality	Solution set	Satisfy
Test point	Included/excluded	

Key questions

On a graph, what's the difference in meaning between a dotted line and a solid line that border a region?

On a number line representing an inequality, what does it mean if the circle is filled in/empty?

Can the variable only take integer values? How does this affect your solution to the inequality?

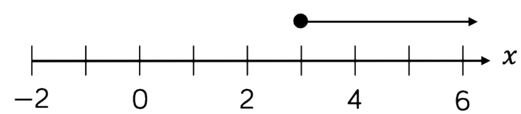
Exemplar Questions

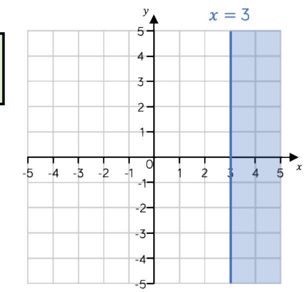
Explain how each representation shows the same inequality.

the smallest possible value of x is 3

$x \leq 3$

$3 \geq x$



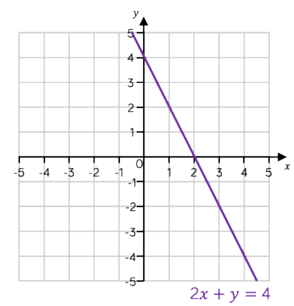


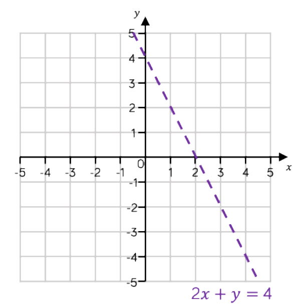
Kim thinks the inequalities $x \geq 2$ and $x > 3$ are identical. Explain why Kim is wrong by

- giving a numerical example
- representing the inequalities on number lines.

Which grid would you use to show the inequality $2x + y > 4$?

Which side of the line would you shade?





Represent the inequality shown

- in words
- using algebra
- using graphs