Prime Numbers and Proof

Year 7

#MathsEveryoneCan

2019-20

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Algebraic Thinking**						**Place Value and Proportion**					
	Sequences		Understand and use algebraic notation		Equality and equivalence		Place value and ordering integers and decimals			Fraction, decimal and percentage equivalence		
Spring	**Applications of Number**						**Directed Number**			**Fractional Thinking**		
	Solving problems with addition & subtraction		Solving problems with multiplication and division			Fractions & percentages of amounts	Operations and equations with directed number			Addition and subtraction of fractions		
Summer	**Lines and Angles**						**Reasoning with Number**					
	Constructing, measuring and using geometric notation			Developing geometric reasoning			Developing number sense		Sets and probability		Prime numbers and proof	

Summer 2: Reasoning with Number

Weeks 7 to 8: Developing Number Sense

Students will review and extend their mental strategies with a focus on using a known fact to find other facts. Strategies for simplifying complex calculations will also be explored. The skills gained in working with number facts will be extended to known algebraic facts.

National curriculum content covered:

- consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals, fractions, powers and roots
- select and use appropriate calculation strategies to solve increasingly complex problems
- begin to reason deductively in number and algebra

Interleaving/Extension of previous work

- Generating and describing sequences
- Substitution into expressions
- Order of operations

Weeks 9 to 10: Sets and Probability

FDP equivalence will be revisited in the study of probability, where students will also learn about sets, set notation and systematic listing strategies.

National curriculum content covered:

- record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness, fairness, equally and unequally likely outcomes, using appropriate language and the 0-1 probability scale
- understand that the probabilities of all possible outcomes sum to 1
- enumerate sets and unions/intersections of sets systematically, using tables, grids and Venn diagrams
- generate theoretical sample spaces for single and combined events with equally likely and mutually exclusive outcomes and use these to calculate theoretical probabilities
- appreciate the infinite nature of the sets of integers, real and rational numbers

Interleaving/Extension of previous work

- FDP equivalence
- Forming and solving equations
- Adding and subtracting fractions

Weeks 11 to 12: Prime Numbers and Proof

Factors and multiples will be revisited to introduce the concept of prime numbers, and the Higher strand will include using Venn diagrams from the previous block to solve more complex HCF and LCM problems. Odd, even, prime, square and triangular numbers will be used as the basis of forming and testing conjectures. The use of counterexamples will also be addressed.

National curriculum content covered:

- use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation property
- use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5
- make and test conjectures about patterns and relationships; look for proofs or counterexamples
- begin to reason deductively in number and algebra

Interleaving/Extension of previous work

- Generating and describing sequences
- Factors and multiples

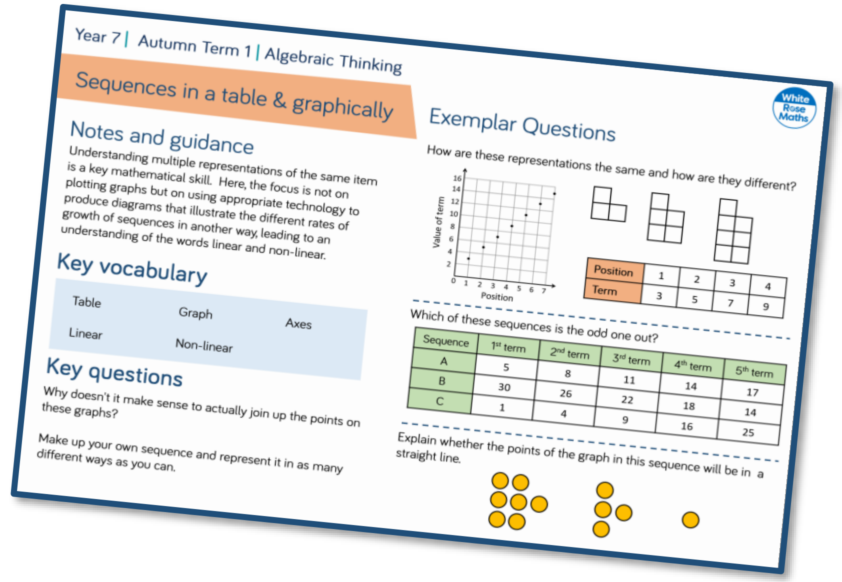
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

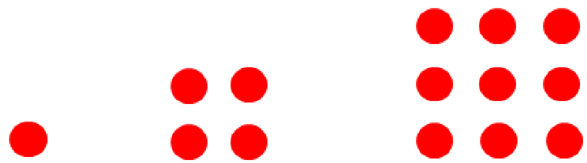
- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



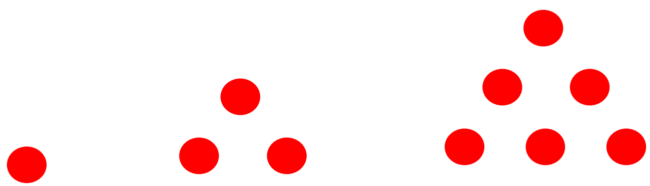
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

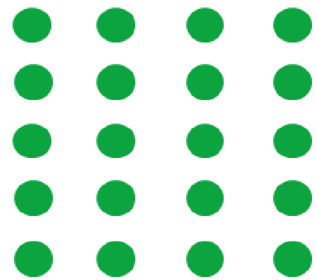
Key Representations



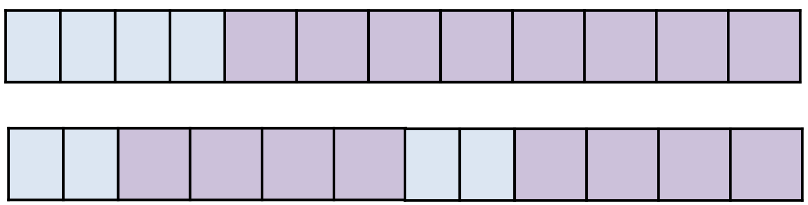
Square numbers

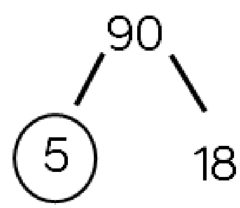


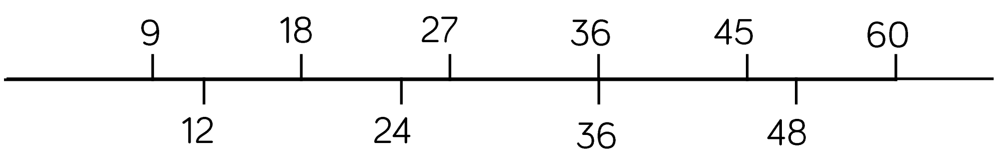
Triangular numbers



Arrays







Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Here are a few ideas for how you might use representations of number to develop reasoning skills.

Many of the small steps are linked to area of a rectangle to support understanding.

For example:
18 × 5 can be calculated in many different ways. It could be partitioned into 10 × 5 and 8 × 5 or 18 could be halved and 5 could be doubled to change the calculation to 9 × 10

Prime Numbers and Proof

Small Steps

- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors of a set of numbers including the HCF
- Find common multiples of a set of numbers including the LCM
- Write a number as a product of its prime factors
- **Use a Venn diagram to calculate the HCF and LCM** H
- Make and test conjectures
- Use counterexamples to disprove a conjecture

H denotes higher strand and not necessarily content for Higher Tier GCSE

Find and use multiples

Notes and guidance

Building on their understanding from KS2, it is important to emphasise that multiples are found by multiplying any number by a positive integer. Skip counting on a number line can support understanding of multiples, as can arrays and Cuisenaire rods. Students often confuse multiples and factors, so this step and the next, need to be planned carefully to overcome this.

Key vocabulary

Multiples	Integer	Positive
Zero		

Key questions

How many multiples of 11 are there?

Can you have multiples of $\frac{1}{3}$?

Does zero have any multiples? Explain your answer.

Exemplar Questions

State whether the statements are true or false. Explain why each time.

4.5 is a multiple of 3

1002 is a multiple of 3

0 is a multiple of 3

Freya's age is a multiple of 7. Next year her age will be a multiple of 5. How old could Freya be?

Freya then says that two years ago her age was a multiple of 4
How old must Freya be?

If x is a positive integer, state whether the following expressions are always, sometimes or never a multiple of 4
Explain your choice of answer each time.

$4x$	$4x + 2$	$4x + 4$	$x + 4$
$x \div 4$	$x - 4$	$8x$	$4x - 4$

Pens come in packs of 8. Pencils come in packs of 6
Mrs Potts buys packs of both and has 120 pens and pencils in total.
How many packs of each did she buy?
Ravi thinks there are 4 different combinations of packs of pencils and pens that Mrs Potts could have bought. Find the four ways.

Factors of numbers & expressions

Notes and guidance

Students will also have met factors at KS2. They should understand that a factor divides exactly into a number with no remainder. Sometimes students do not realise that a number is a factor of itself. Representing numbers as different arrays can help to find the factors of a number.
It is important to distinguish between factors and multiples e.g. 10 is a multiple of 5, but a factor of 20

Key vocabulary

Factor	Divisible	Remainder	Term
Factorise	Divisor	Multiple	

Key questions

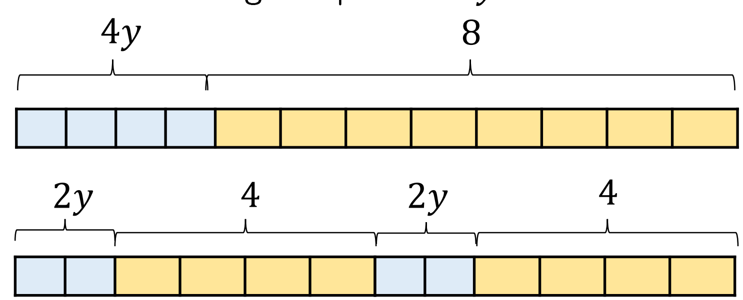
Explain the difference between a factor of a number and a multiple of a number.
Can a number be both a factor and a multiple?
Can zero be a factor of a number?
Can negative numbers be factors of positive numbers?

Exemplar Questions

Use arrays of 24 counters to find all the factors of 24

How many ways can you find to show that 3 is not a factor of 10?
What diagrams can you use? What sentences can you write?

Sally uses a bar modelling to represent $4y + 8$



Sally notices that 2 lots of $2y + 4$ is the same as $4y + 8$
She concludes that both 2 and $2y + 4$ are factors of $4y + 8$
Draw another bar model, or an array, to represent different factors of $4y + 8$

In each list, what's the same? What's different?

6	$6x$	$6xy$	$6x^2y$
$2x + 2$	$4x + 4$	$2x - 2$	$8x - 4$

Find the factors of each expression e.g.
$6x = 3 \times 2x$ so 3 and $2x$ are both factors of $6x$
$6x = 3x \times 2$ so ….

Prime numbers

Notes and guidance

Ensure students know that prime numbers are integers greater than 0 that have exactly two factors. Emphasise that the first prime number is 2, as 1 only has one factor.

There is an opportunity to interleave previous topics such as Venn diagrams into this small step e.g. sets that show prime and odd numbers.

Key vocabulary

Prime number	Factor	Odd
Even	Digit	

Key questions

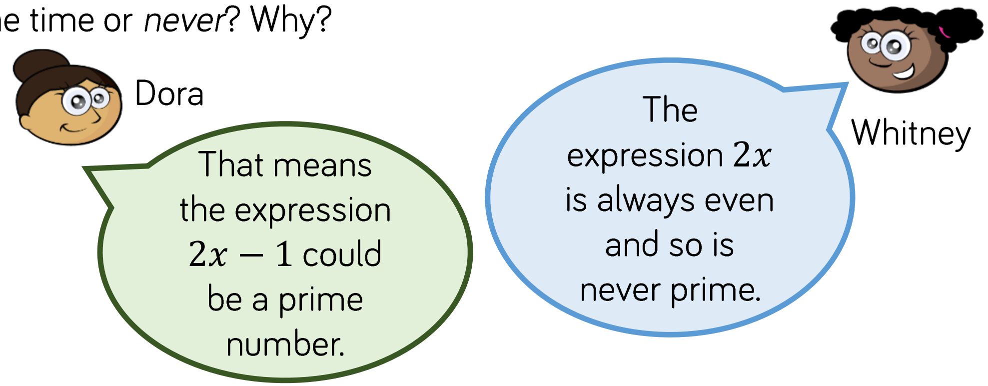
When you add together two prime numbers, do they always give an even number? Explain your answer.

Which large numbers can you tell are not prime just by looking at their digits?

Exemplar Questions

Find the factors of all the integers from 1 to 20
How many of the first 20 integers are prime?
How many have an odd number of factors?
Investigate further.

Explore these statements by substituting in values of x into the expressions. Are Dora and Whitney correct *all* of the time, *some* of the time or *never*? Why?



Raffle tickets with the numbers 1 to 100 on are placed in a bag.
To play the game, you randomly select a raffle ticket.
You choose the winning criteria. Which would you choose?
Justify your answer.

Win a prize for a multiple of 8	Win a prize for a prime number.

Square and triangular numbers

Notes and guidance

This small step provides opportunities for students to spot patterns and follow a line of enquiry.
Concrete resources and pictorial representations are very useful for this. Students should be encouraged to notice that the sum of two consecutive triangle numbers is a square number, and this can be easily shown with a diagram.

Key vocabulary

Triangular Number	Relationship	Investigate
Square Number	Expression	

Key questions

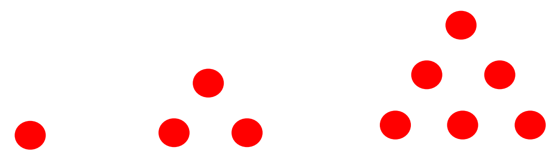
Can a number be both square and triangular?
Why do square numbers always have an odd number of factors?
What's the difference between n^2 and $n + n$?
Can triangular and square numbers be odd or even?

Exemplar Questions

Make the following square numbers using counters.

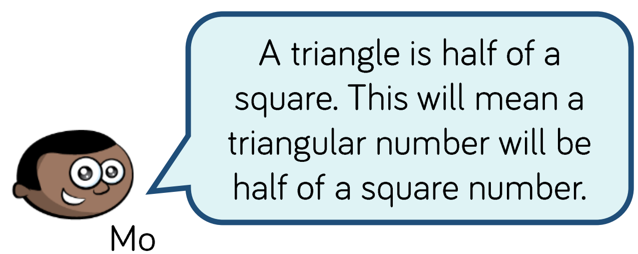
What do you notice about the way square numbers increase.
How many counters will you need for the 10th pattern? 20th pattern?

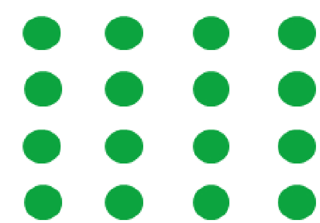
Make the following triangular numbers using counters.



What do you notice about the way triangular numbers increase.
How many counters will you need for the 10th pattern? 20th pattern?

Which triangle numbers can you see within this square number?
Investigate with other square numbers.
What relationships can you find?





White Rose Maths

Common Factors and HCF

Notes and guidance

Secure table knowledge is beneficial for this step, although it can still be accessed by use of supporting manipulatives. At this stage, we are understanding the idea of HCF rather than exploring algorithms for this. We can extend to looking at common factors of algebraic expressions if appropriate. It is important to include finding the HCF of more than two numbers.

Key vocabulary

Factor
Common Factor
Factorising
Factorise
Highest Common Factor

Key questions

What number is a common factor of all numbers?
How do we know when we have found the highest common factor?
What do you notice about the HCF of two numbers when one is a multiple of the other?

Exemplar Questions

Draw as many different rectangles as possible, with integer lengths and widths, with an area of:

$12cm^2$ $18cm^2$

Write down the highest common factor of 12 and 18
Use this fact to help work out the highest common factors of these pairs of numbers.

120 and 180 | 6 and 9 | 24 and 36 | 18 and 27

Bob has two pieces of ribbon, one 75 cm long and one 45 cm long. He wants to cut them up into smaller pieces that are all of the same length, with no ribbon left over. What is the greatest length of ribbon that he can make from the two pieces of ribbon?

Keira works out the HCF of some pairs of numbers.

Pair	HCF
12 and 30	6
30 and 60	30
12 and 60	12

Keira says the HCF of 12, 30 and 60 is 30 because it is the highest HCF of the pairs. Do you agree? Why or why not?

Common Multiples and LCM

Notes and guidance

Students will benefit from the modelling of a systematic method of finding the LCM. It is helpful to make the link to finding the lowest common denominator when adding fractions. It is useful to look at the LCM of more than two numbers and if appropriate, algebraic expressions. Emphasis should be placed on language and student explanation to prevent confusion between HCF and LCM.

Key vocabulary

Common Multiple	Product
Lowest Common Multiple	Multiple

Key questions

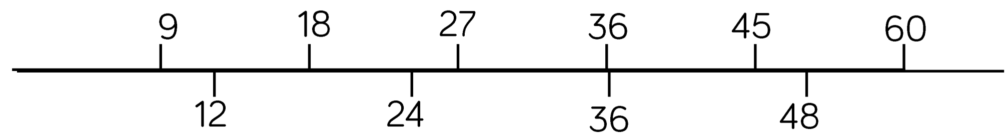
Why will the product of two numbers be a common multiple of the numbers?

When is the LCM of a set of numbers not the same as their product?

Can the HCF and LCM of a pair of numbers be the same?

Exemplar Questions

Marcel uses number lines to find the LCM of 9 and 12



Explain how Marcel uses this to find the LCM of 9 and 12

Find the LCM of the following numbers.

6 and 8 | 6 and 15 | 6, 8 and 15

Marcel notices that the LCM of 6, 8 and 15 is 5 times as large as the LCM of 6 and 8. Explain why this is true.

Explain how the LCM can be used to compare the size of the following pairs of fractions:

$\frac{3}{5}$ and $\frac{7}{10}$ | $\frac{7}{9}$ and $\frac{5}{6}$ | $\frac{7}{12}$ and $\frac{11}{20}$

What other ways could you use to compare the size of the fractions?

At a bus stop, Bus A arrives every 4 minutes and bus B arrives every 6 minutes. Bus A and B both arrive at 10am.
At what time to Bus A and Bus B arrive together next?
Bus C arrives every 10 minutes.
How many times per hour do buses A, B and C arrive at the same time?

Product of prime factors

Notes and guidance

A key concept to cover here is that all non-prime positive integers can be written as a product of prime factors, and that this product is unique. Index form is not required at this stage, but could be explored if appropriate. Students may need help with the language associated with this step, including the command word 'express'. The 'factor tree' method should be distinguished from the familiar additive part-whole model.

Key vocabulary

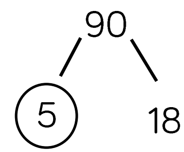
Factor	Prime Factor	Factorise
Product	Express	

Key questions

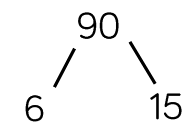
Is there more than one way to factorise 12?
Is there more than one way to express 12 as a product of prime factors? Does the order of the factors matter?
What happens when you find the prime factorisation of a prime number?

Exemplar Questions

Tom knows that $90 = 5 \times 18$
He starts his prime factor tree:



Amir knows that $90 = 6 \times 15$
He starts his prime factor tree:



Complete both trees. What do you notice about your final answers?

List the factors of 80

Express 80 as a product of its prime factors.

Factorise 80

What's the same and what's different about each question?
Which of the questions can you answer in more than one way?
Why is it only possible to express the product of 80 in terms of its prime factors in one way only?

Liam is investigating the prime factorisation of some numbers.
He lists his results so far in a table. Copy and complete the table.

Number	Prediction	How do you know?
12	$2 \times 2 \times 3$	$12 = 4 \times 3$ and $4 = 2 \times 2$ so $12 = 2 \times 2 \times 3$
24	$2 \times 2 \times 3 \times 2$	24 is double 12, so $24 = (2 \times 2 \times 3) \times 2$
72		
36		
144		

Use a prime factor trees to check your answers.

Venn Diagrams to find HCF/LCM

Notes and guidance

Identifying the intersection on a Venn diagram as common elements in both sets reinforces the idea of common factors. This then supports understanding of the calculation for the highest common factor. Finding any common multiples of two numbers using a Venn Diagram, and finally working out a method to calculate the lowest common multiple, supports understanding of the structure underlying this method.

Key vocabulary

Highest Common Factor	Union/Intersection
Lowest Common Multiple	Prime Factors

Key questions

$60 = 2 \times 2 \times 3 \times 5$ $168 = 2 \times 2 \times 2 \times 3 \times 7$

Why don't we write 2, 2, 2, 2, 3, and 3 in the intersect on the Venn diagram?

Why is $2 \times 2 \times 3$ the HCF of the two numbers?

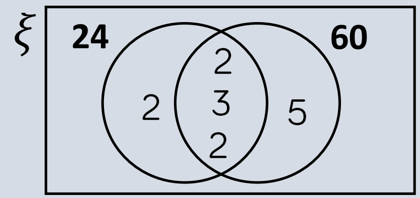
Why is $2 \times 2 \times 2 \times 3 \times 5 \times 7$ the smallest of the common multiples?

How can we find a larger common multiple?

Exemplar Questions

The Venn Diagram shows the prime factors of 24 and 60

- Why does 2 appear twice in the intersection?
- Why does 5 not appear in the circle representing 24?
- How does the intersection help us find the all the common factors, and so the HCF of 24 and 60?
- How can we use the diagram to find the LCM of 24 and 60?



$24 = 2 \times 2 \times 2 \times 3$

$60 = 2 \times 2 \times 3 \times 5$

Express 105 and 120 as products of their prime factors.
Use a Venn diagram to find the HCF and LCM of 105 and 120.
Use your answers to work out:

The HCF of 105 and 60	The HCF of 1050 and 120
The LCM of 105 and 240	The LCM of 105 and 12

Mo expresses two numbers as a product of their prime factors: $30 = 2 \times 3 \times 5$ $36 = 2 \times 2 \times 3 \times 3$

Mo said: "I can use these prime factors to calculate the lowest common multiple of 30 and 36: $2 \times 3 \times 5 \times 2 \times 2 \times 3 \times 3 = 1080$"

Mo

Explain Mo's mistake and find lowest common multiple of 30 & 36

Make and test conjectures

Notes and guidance

Conjectures arise when students notice a pattern that hold for many cases. Students will already have made many conjectures e.g. predicting next terms in sequences, square and triangular numbers etc. Provide opportunities for students to explore the concept of a conjecture by using examples where several conjectures emerge and can be tested.

Key vocabulary

Conjecture	Explain	Relationship	True
False	Proof	Demonstration	

Key questions

How many examples do you need to prove that a conjecture is always true?
Convince me that your conjecture is always true. Give me a mathematical reason.
What's meant by proof?
Why is proof different from demonstration?

Exemplar Questions

Sort these conjectures into: always true, sometimes true, never true.

- Birds can fly.
- Odd + Odd = Even
- 1, 2, 4….. The numbers in the sequence are doubling each time.
- To find the area of a shape, you multiply length by the width.
- In an equilateral triangle, each angle is 60°
- $a \times b = b \times a$

Here is a 5-term Fibonacci sequence.

1, 1, 2, 3, 5

Add together the first and last terms in the sequence.
What do you notice about the relationship between this, and the middle term of the sequence?
Repeat with several other 5-term Fibonacci sequences.
Is this result always true? Can you use counters or cubes to prove it?

Sarah finds the HCF of 12 and 18 is 6
She also finds the LCM of 12 and 18 is 36
She notices that $6 \times 36 = 12 \times 18 = 216$
She conjectures that that the product of the HCF and LCM of two numbers is always the same as the product of the numbers themselves. Investigate Sarah's conjecture.

Counterexamples

Notes and guidance

Students should understand the word counterexample as an example that shows a conjecture is false. It is often easier to disprove a conjecture than prove a conjecture, as only one counterexample is needed. It is useful to reinforce the importance of not making assumptions from a limited number of cases. The already familiar 'always sometimes or never true' activities help here.

Key vocabulary

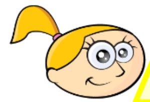
Conjecture	Always	Systematic	Never
Sometimes	Assumption	Counterexample	

Key questions

How many counterexamples do we need to disprove a conjecture?

Is it important to be systematic when looking for a counterexample? Why? What strategies could you use?

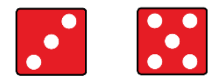
Exemplar Questions



Eva

$4^2 = 16$
So, it must be true that if I square a number, the result is always greater than the number I start with.

Write down a counterexample to show that this conjecture is not always true.



Saffi rolls two dice.
She subtracts the scores on the two dice and makes the conjecture:

The difference between the scores on two dice is always even.

Do you agree, or can you find a counterexample? She conjectures again. Is this conjecture true? If not, find a counterexample.

If the total of the scores on the two dice is even, then the difference between the scores on two dice is also even.

Ali works out the perimeter and area of this square.
Perimeter = 24 cm
Area = 36 cm^2
He thinks "The perimeter of a square can never be equal to its area".
Do you agree? Justify your answer.

6 cm

6 cm