Forming and solving equations

Year 9

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Reasoning with Algebra**						**Constructing in 2 and 3 Dimensions**					
	Straight line graphs		Forming and solving equations		Testing conjectures		Three-dimensional shapes			Constructions and congruency		
Spring	**Reasoning with Number**						**Reasoning with Geometry**					
	Numbers		Using percentages		Maths and money		Deduction		Rotation and translation		Pythagoras' Theorem	
Summer	**Reasoning with Proportion**						**Representations and Revision**					
	Enlargement and similarity		Solving ratio & proportion problems		Rates		Probability		Algebraic representation	Revision		

Autumn 1: Reasoning with Algebra

Weeks 1 and 2: Straight line graphs

This block builds on Year 8 content where students plotted simple straight line graphs. They now study $y = mx + c$ as the general form of the equation of a straight line, interpreting m and c in abstract and real-life contexts, and reducing to this form in simple cases. This will be explored further in the next block when students rearrange formulae. Higher strand students will also consider inverse relationships and perpendicular lines.

National Curriculum content covered includes:

- develop algebraic and graphical fluency, including understanding linear and simple quadratic functions
- recognise, sketch and produce graphs of linear and quadratic functions of one variable with appropriate scaling, using equations in x and y and the Cartesian plane
- interpret mathematical relationships both algebraically and graphically
- reduce a given linear equation in two variables to the standard form $y = mx + c$; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically and algebraically
- use linear and quadratic graphs to estimate values of y for given values of x and vice versa and to find approximate solutions of simultaneous linear equations
- solve problems involving direct and inverse proportion, including graphical and algebraic representations

Weeks 3 and 4: Equations and inequalities

Students revisit and extend their knowledge of forming and solving linear equations and inequalities, including those related to different parts of the mathematics curriculum. They also explore rearranging formulae seeing how this links to solving equations and reinforcing their understanding of the difference between equations, formulae, identities and expressions. This is a good opportunity to practise non-calculator skills if appropriate.

National Curriculum content covered includes:

- move freely between different numerical, algebraic, graphical and diagrammatic representations [for example…equations and graphs]
- use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement)
- understand and use standard mathematical formulae; rearrange formulae to change the subject
- model situations or procedures by translating them into algebraic expressions or formulae and by using graphs

Weeks 5 and 6: Testing conjectures

Reasoning is encouraged throughout the White Rose Maths scheme of learning, and this block allows time for direct teaching of this. The opportunity is taken to revisit primes, factors and multiples which provides a wealth of opportunity to make and test simple conjectures. As well as testing given conjectures, students should be encouraged to create and test their own. An example given in the block is through looking at relationships in a 100 square; another great source of patterns is Pascal's triangle. Students also develop their algebraic skills through developing chains of reasoning and learning how to expand a pair of binomials, which Higher strand students met in Y8

National Curriculum content covered includes:

- make and test conjectures about patterns and relationships; look for proofs or counterexamples
- begin to reason deductively in geometry, number and algebra
- use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation
- simplify and manipulate algebraic expressions to maintain equivalence by expanding products of two or more binomials

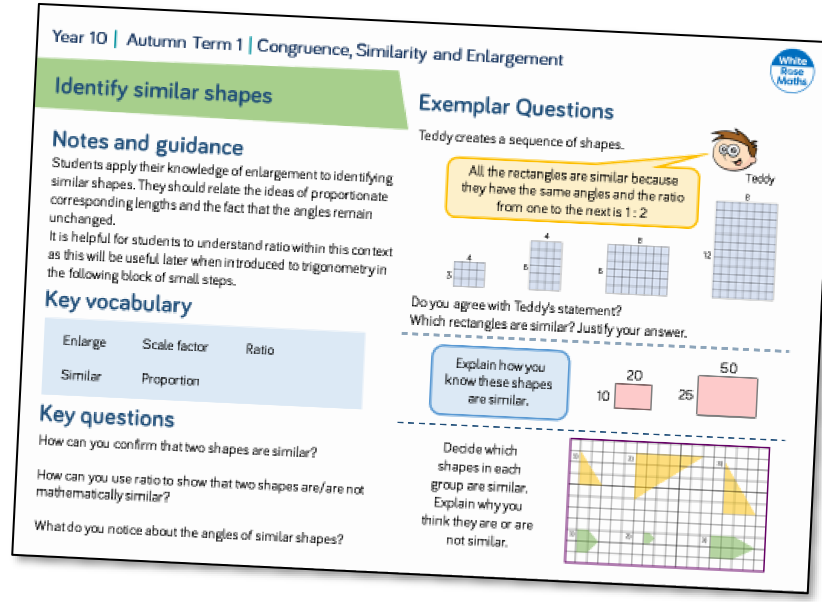
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.

It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge, Scale factor, Ratio, Similar, Proportion

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

Do you agree with Teddy's statement?
Which rectangles are similar? Justify your answer.

Explain how you know these shapes are similar.

Decide which shapes in each group are similar. Explain why you think they are or are not similar.

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered earlier in Key Stage 3 are labelled R .

White Rose Maths

Forming and solving equations

Small Steps

- Solve one- and two-step equations and inequalities R
- Solve one- and two-step equations and inequalities with brackets R
- Inequalities with negative numbers
- Solve equations with unknowns on both sides
- Solve inequalities with unknowns on both sides
- Solving equations and inequalities in context
- Substituting into formulae and equations
- Rearranging formulae (one-step)

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered earlier in KS3

Forming and Solving Equations

Small Steps

- Rearrange formulae (two-step)
- **Rearrange complex formulae including brackets and squares** H

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered earlier in KS3

1/2-step equations and inequalities R

Notes and guidance

Students will be familiar with equations and inequalities from previous learning. This step will provide an opportunity for students to revisit key ideas before looking at more complex examples. Students could have access to calculators throughout this step if appropriate and examples should include decimals to avoid "spotting" answers. Look out for the common misconceptions of changing an inequality sign for an equals sign.

Key vocabulary

Equation	Inequality	Greater/ less than	
Solution	Unknown	Inverse	Solve

Key questions

What is the difference between an equation and an inequality?

How many solutions does an inequality have?

How many solutions does an equation have?

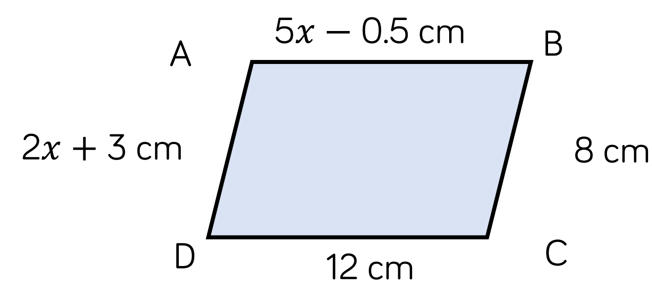
Exemplar Questions

Amir is solving some equations.
What mistakes has he made?

$$36 = 10x + 2$$
$$3.6 = x + 2$$
$$1.6 = x$$

$$\frac{x-3}{2} = 10$$
$$\frac{x}{2} = 13$$
$$x = 26$$

Find the value of x so that ABCD is a parallelogram.



Which of the following have the solution set $x < 7.5$?

$15 > 2x$	$x > 15 - x$
$5x - 17 < 23.5$	$23 > 3x - \frac{1}{2}$
$3.75 > \frac{x}{2}$	$\frac{x}{4} < 1.875$

Equations/inequalities with brackets R

Notes and guidance

Students should now be secure in solving one- and two-step equations. In this step questions that do not have integer solutions should be encouraged. Students need to be clear that they can leave solutions in fractional form. Calculators could be used to support this. Students should be exposed to varying methods to solve the questions and these should be discussed in depth asking 'what is the same? what is different?'

Key vocabulary

Equation	Inequality	Greater/ less than	
Solution	Unknown	Inverse	Expand

Key questions

Do you have to expand the brackets first to be able to solve the equation?

Can we check the solution is correct? How?

Can we have a solution that is not a whole number? Give me an example.

Exemplar Questions

Match the cards that have the same solution or set of solutions.

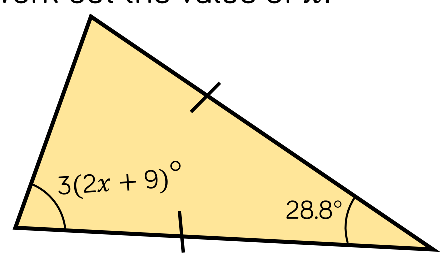
$\frac{3}{2} = 3(x+5)$	$2(x-5) < 1.4$
$2.85 < \frac{x}{2}$	$3x + 9 = 22.5$
$0.7 > x - 5$	$2.8 < 8(\frac{1}{2}x - 2.5)$
$22.5 = 3(3+x)$	$30 + 6x = 3$

Compare the two methods to solve $5(x-2) \geq 17.7$
What is the same? What is different?

$$5(x-2) \geq 17.7$$
$$x - 2 \geq 3.54$$
$$x \geq 5.54$$

$$5(x-2) \geq 17.7$$
$$5x - 10 \geq 17.7$$
$$5x \geq 27.7$$
$$x \geq 5.5.4$$

Mo is working out value of x. Comment on the mistake he has made, and work out the value of x.



$$3(2x+9) = 28.8$$
$$6x + 27 = 28.8$$
$$6x = 1.8$$
$$x = 0.3$$

Inequalities with negative numbers

Notes and guidance

In this step students will explore and therefore understand the need to reverse the inequality when multiplying and dividing by a negative number. Students could compare and contrast with solving equations and check their solution sets by testing values either side of the boundary values found. Number lines are useful to support this. It is useful to include examples with non-integer solutions. Calculators should be used to support fluency with the "change sign" key.

Key vocabulary

Inequality	Satisfy	Reverse
Solve	Greater/less than (or equal)	

Key questions

What is the same and what is different about solving an inequality where the variable has a negative coefficient?

Explain why the direction of inequality sign has changed.

What is the first step you are going to take?

Exemplar Questions

Fill in the blanks.

- $20 \geq 6 - 5x$
  - $+5x$ | $+5x$
  - $5x + 20 \geq 6$
  - -20 | ____
  - $5x \geq -14$
  - ____ | ____
  - $x \geq$ ____
- $-3x < 16$
  - ____ | ____
  - $0 < 16 + 3x$
  - ____ | ____
  - ____ $< 3x$
  - ____ | ____
  - ____ $< x$

Here is an inequality. $-10 < -8$

Is the inequality still true if:

- 2 is added to both sides?
- Both sides are multiplied by 2?
- 2 is subtracted from both sides?
- Both sides are multiplied by -2?
- Both sides are divided by -2?

Which inequality is the same as $x > 5$?

$-x > -5$ $\quad$ $-x < -5$

Solve the inequalities.

- $-8y + 6 > 10$
- $8y - 6 < -10$
- $8 \geq 7 - 10y$
- $-10 - 8y \leq 20$
- $-\frac{1}{3}y > 2$
- $7 > 5 - \frac{2}{5}y$

Unknowns on both sides - equations

Notes and guidance

Students should now be fully confident using the 'balance' method to solve equations and inequalities and we now focus on solving equations where we have unknowns on both sides. Bar models should be used alongside, rather than instead of, the abstract calculation. Students should be exposed to examples with the larger coefficient on the right as well as on the left. Again, non-integer solutions and checking by substitution should be encouraged.

Key vocabulary

Equation	Balance	Coefficient
Solve	Unknown	Check

Key questions

Why do we do the same operation to both sides of an equation?
When solving a four-term equation should we deal with the variables or constants first? Why?
When solving an equation do we always start by subtracting something? Why or why not?

Exemplar Questions

Which of the equations **does** the bar model represent?

x	x	x	x	18.7
x	x	x	23.1	

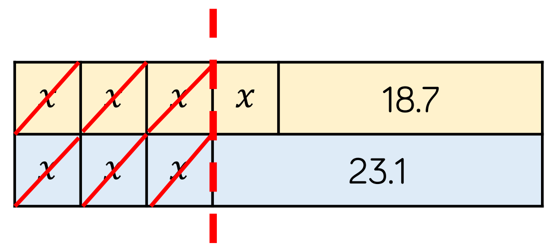
$23.1 + 3x = 18.7 + 4x$

$3x + 23.1 = 4x + 18.7$

$18.7 + 4x = 3x + 23.1$

$18.7 + 3x = 4x + 23.1$

Use the bar model and fill in the blanks to solve



$4x + 18.7 = 3x + 23.1$

$-3x$ ☐

$x + 18.7 = 23.1$

☐ ☐

$x =$ ☐

Match the equations that have the same solutions.

$1 + y = 3y + 5$

$7y + 4 = 5 + 5y$

$7y - 7 = 3 + 2y$

$1.5 - y = 2y$

$3y - 5 = -13 - y$

$20 - \frac{y}{2} = 4y + 11$

Unknowns on both sides - inequalities

Notes and guidance

In this step students will extend their learning and understanding of the balancing method for solving equations and inequalities with unknowns on both sides.
Students should only move onto this step once they are fully secure with solving equations and inequalities with unknowns.
Throughout this step students should be encouraged to fully check their solutions through substitution.

Key vocabulary

Equation	Inequality	Substitute
Solve	Unknown	Check

Key questions

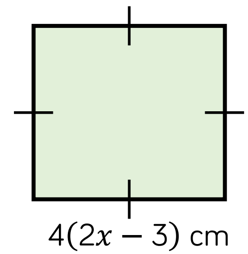
What would be the first step you would take to solve…?

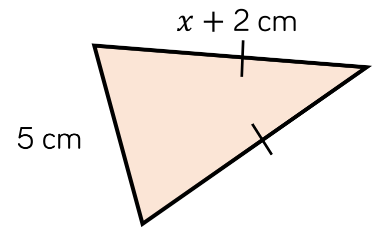
"An equation only has one solution." Is this True or False? Give an example.

How can we check that the solution to an inequality is correct?

Exemplar Questions

The perimeter of the square is greater than the perimeter of the isosceles triangle.
Form and solve the inequality to find the possible values of x.





Rosie is solving $16 - 4x \leq 5.5 + x$.

Find the mistakes in her solution.

Solve the inequality.

$$16 - 4x \leq 5.5 + x$$
$$10.5 - 4x \leq x$$
$$10.5 \leq -3x$$
$$-3.5 \geq x$$

Solve the inequalities. What is the same? What is different?

- $4x + 9 \leq 9x$
- $9 + 4x \geq 9x - 4$
- $-4x + 9 > -9x + 4$
- $-4x - 9 < -9x$
- $4x - 9 \geq 9x + 4$
- $-4x + 9 > 9x$

Equations and inequalities in context

Notes and guidance

Here students look at forming and solving equations in mathematical contexts. This gives them the opportunity to revisit e.g. angles rules, types of triangles and quadrilaterals, probability, the mean and range, and a host of other areas. Teachers can choose the topics their classes need to revise the most and choose/create equation or inequality-based questions accordingly.

Key vocabulary

Form	Solve	Equation
Inequality	Check	

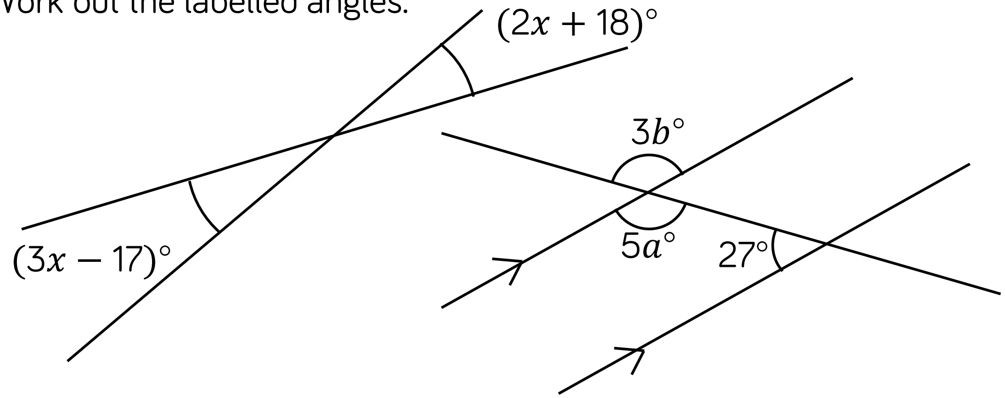
Key questions

Is your answer realistic given the context of the question?
How can you check your answer?

What facts do we know that will hep us to form an equation/inequality in this question?

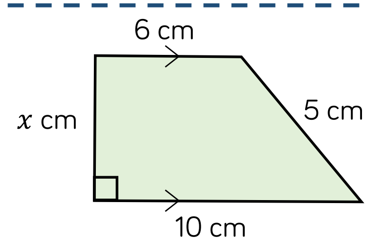
Exemplar Questions

Work out the labelled angles.



Annie has some coins.
Dora has three more coins than Annie.
Mo has three times as many coins as Dora.
Altogether they have more than 70 coins.
What is the smallest number of coins Dora could have?

The area of the trapezium is 32 cm^2.
Work out the perimeter of the trapezium.



The mean of the numbers on the cards is 9
Find the range.

x	$x - 2$	6	$2x - 1$	12

Formulae and equations

Notes and guidance

Here students explore the difference between formulae and equations and substitute numbers into formulae to produce equations to solve. The concept of the subject of the formula can be introduced here, as this step leads into the next few steps which focus on rearrangement. Teachers may wish to interleave familiar formulae that students should know, rounding and limits of accuracy, use of calculators, and/or using numbers expressed in standard form.

Key vocabulary

Formula	Equation	Solve
Variable	Substitute	Subject

Key questions

What is the difference between a formula and an equation?
How do you know which letter represents what quantity in a formula?
Can you substitute into a formula/equation?
Can you solve a formula/equation?

Exemplar Questions

A plumber charges a £35 call out fee, plus £15 per hour worked. Which is correct formula for the total cost, £C, to hire the plumber for t hours?

$C = (35 + 15)t$	$C = 35 + 15t$	$C = 35t + 15$

Form and solve equations to find:

- The cost of a job that takes the plumber 9 hours.
- The length of a job that costs £125

The perimeter of a rectangle is given by the formula $P = 2(l + w)$.
What do each of P, l and w represent?
Find the perimeter of a rectangle of length 11 cm and width 9.3 cm.
Find the width of a rectangle of length 9.2 cm and perimeter 27 cm.

The formula for Ohm's law, which links current, voltage and resistance is $V = IR$.

- Work out V when $I = 14$ and $R = 18$
- Work out I when $V = 14$ and $R = 18$
- Work out R when $I = 14$ and $V = 18$

Pressure (p) is found by dividing force (F) by area (A).

- Write down the formulae connecting p, F and A
- Work out p when $F = 20$ and $A = 0.4$
- Work out F when $p = 1\,000$ and $A = 0.1$
- Work out A when $F = 140$ and $p = 7$

Rearrange formulae (one-step)

Notes and guidance

Here students explore the link between solving one-step equations and rearranging one-step formulae. They could begin with simple “think of a number puzzles” and move from particular solutions to general ones, and then repeat the process with symbols. Bar models are useful tools from which to see both the original and rearranged formulae. Substitution is a useful strategy to check the new formula.

Key vocabulary

Formula	Subject	Rearrange
Make the subject of		Inverse operation

Key questions

Which variable is the subject of the formula? How do you know?

What is the inverse of ____?

Does it make a difference if a formula reads $x =$… or … $= x$? Would it be different if it was an equation?

Exemplar Questions

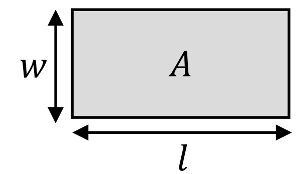
Which of these formulae have A as the subject?

$p = \frac{F}{A}$ | $A = bh$ | $A = \frac{1}{2}bh$ | $V = Ax$

The area of a rectangle is given by the formula $A = lw$.

How can you find the area of a rectangle of width 10 cm and height 3.7 cm?

How do you find the width of a rectangle of area 864 cm^2 and length 32 cm?



Complete the formula for the width of a rectangle given its length and its area.

$A = \frac{\square}{\square}$

Match the formulae on the left with rearrangements on the right. Substitute values of x, y and z to check your answers.

$x = y + z$	$y = x - z$
$x = yz$	$y = xz$
$x = \frac{y}{z}$	$y = \frac{z}{x}$
$x = y - z$	$y = z + x$

Rearrange the formulae to make z the subject.

Rearrange formulae (two-step)

Notes and guidance

This builds from the last step, and students need to be confident with the more basic formulae before proceeding to these. Again “I think of a number, double it and add 7” type puzzles are a good introduction to the idea. This is a very good opportunity to build on the last block of work on straight line graphs, rearranging into the form $y = mx + c$ and revising the meaning of/how to find the gradient and y-intercept.

Key vocabulary

Formula	Subject	Rearrange
Make the subject of		Inverse operation

Key questions

What is the first step you need to take to rearrange the formula?

If you are multiplying or dividing, why is it important to do this to every term?

How do you choose good values to check your answers by substitution?

Exemplar Questions

Compare solving the equation $3x - 5 = 46$ to making x the subject of the formula in $p = 3x - 5$. What is the same and what is different?

$3x - 5 = 46$

$+5$ $\quad$ $+5$

$3x = 51$

$\div 3$ $\quad$ $\div 3$

$x = 17$

$p = 3x - 5$

$+5$ $\quad$ $+5$

$p + 5 = 3x$

$\div 3$ $\quad$ $\div 3$

$\frac{p+5}{3} = x$

Rearrange the equations of straight lines to the form $y = mx + c$.
State the gradient and y-intercept of each line.

- $x = 4y + 3$
- $2y + 8x = 10$
- $5(y - 8) = x$
- $3 = 6y - 12x$
- $3x - 2y = 0$
- $14(5 - 2x) = 7y$

Make the letter in bold the subject of each formula.
What is the same and what is different?

- $v = \boldsymbol{u} + t$
- $x = \boldsymbol{u} + t$
- $x = \boldsymbol{y} + t$
- $v = 2\boldsymbol{u} + t$
- $x = 2\boldsymbol{u} + t$
- $x = 2\boldsymbol{y} + t$
- $v = t + 2\boldsymbol{u}$
- $x = t + 2\boldsymbol{u}$
- $x = t + 2\boldsymbol{y}$
- $v = \frac{\boldsymbol{u}}{2} + t$
- $x = \frac{\boldsymbol{u}}{2} + t$
- $x = \frac{\boldsymbol{y}}{2} + t$

Rearrange complex formulae H

Notes and guidance

This final step looks at slightly more complex rearrangement that involve more steps. In particular, students explore formulae that include squaring or square rooting and that have terms in brackets. This is a Higher strand step and should only be covered when students are fully confident with the previous two steps. Note that rearrangement where the subject occurs more than once is not covered here, as this is left until KS4

Key vocabulary

Formula	Subject	Rearrange
Make the subject of	Inverse	Square/Root

Key questions

What is the first step you need to take to rearrange the formula?

What is the inverse of squaring/cubing/square rooting/cube rooting?

Do you need to multiply out the brackets or not in this case?

Exemplar Questions

Dexter is rearranging the formula for the area of a circle to make r the subject.

What mistake has he made?

$$A = \pi r^2$$

Divide by π

$$\frac{A}{\pi} = r^2$$

Square root

$$\frac{\sqrt{A}}{\pi} = r$$

Dora and Teddy are rearranging $P = 2(l + w)$ to make w the subject.
Who is correct? How can you verify your answer?

$$P = 2(l + w)$$
$$P = 2l + 2w$$
$$P - 2l = 2w$$
$$\frac{P - 2l}{2} = w$$

$$P = 2(l + w)$$
$$\frac{P}{2} = l + w$$
$$\frac{P}{2} - l = w$$

Make u the subject of each formula.

- $v = u + at$
- $v^2 = u^2 + 2as$
- $s = ut + \frac{1}{2}at^2$

Starting with the same formulae, make a the subject of each.

Make b the subject of each formula.

- $T = \frac{1}{2}ab^2$
- $T = \frac{1}{2}a\sqrt{b + c}$
- $T = \frac{1}{2}a\sqrt{b}$
- $T = \frac{1}{2}a\sqrt{b^2 + c}$